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Platoon based Cooperative Driving Model with Consideration of Realistic Inter-vehicle Communication

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Abstract
Recent developments of information and communication technologies (ICT) have enabled vehicles to timely communicate with each other through wireless technologies, which will form future (intelligent) traffic systems (ITS) consisting of so-called connected vehicles. Cooperative driving with the connected vehicles is regarded as a promising driving pattern to significantly improve transportation efficiency and traffic safety. Nevertheless, unreliable vehicular communications also introduce packet loss and transmission delay when vehicular kinetic information or control commands are disseminated among vehicles, which brings more challenges in the system modelling and optimization. Currently, no data has been yet available for the calibration and validation of a model for ITS, and most research has been only conducted for a theoretical point of view. Along this line, this paper focuses on the (theoretical) development of a more general (microscopic) traffic model which enables the cooperative driving behaviour via a so-called inter-vehicle communication (IVC). To this end, we design a consensus-based controller for the cooperative driving system (CDS) considering (intelligent) traffic flow that consists of many platoons moving together. More specifically, the IEEE 802.11p, the *de-facto* vehicular networking standard required to support ITS applications, is selected as the IVC protocols of the CDS, in order to investigate how the vehicular communications affect the features of intelligent traffic flow. This study essentially explores the relationship between IVC and cooperative driving, which can be exploited as the reference for the CDS optimization and design.

keyword
cooperative driving,inter-vehicle communication,consensus control, switching network, vehicle platoons, connected vehicles, car-following model

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1. Introduction

The emerging vehicular networking technology enables vehicles to timely communicate with each other and exchange important information. These connected vehicles with some common interests can cooperatively drive on the road, e.g., platoon-based driving pattern, which may significantly improve the traffic safety and efficiency (Farah and Koutsopoulos, 2014, Kesting et al., 2010a, 2008, Milans et al., 2014, Monteil et al., 2013, Ngoduy, 2013a, Sau et al., 2014, van Arem et al., 2006).

Basically, in such a Cooperative Driving System (CDS), a vehicle obtains neighboring information via inter-vehicle communication (IVC), and then adopts a suitable control law to achieve certain objective, such as maintaining a constant inter-vehicle spacing within the same platoon. To this end, four major components in CDS are supposed to be considered: (1) the vehicle dynamics which inherently characterize vehicle's behaviour stemming from manufacture, e.g., actuator lag; (2) the information to be exchanged among vehicles, e.g., the position and velocity of a vehicle; (3) the communication topology describing the connectivity structure of vehicular networks, such as predecessor-follower, leader-follower, bidirectional, etc.; (4) the control law such as sliding-mode control, consensus control, etc. to be implemented on each vehicle in order to define the car-following rule in the connected traffic flow.

The issues of CDS have been extensively studied in recent years (Jia et al., 2015, Liu and Khattak, 2016, Luo et al., 2016). For example, some typical control schemes for CDS include the *cooperative adaptive cruise control* (CACC) design (Naus et al., 2010) which adopts the constant time-headway policy with the predecessor-follower information, and the sliding-mode control (Fernandes, 2012) with the leader-follower information. Wan et al. (2016) have investigated how connected vehicles can obtain and utilize upcoming traffic signal information to manage their speed in advance in order to reduce fuel consumption and improve ride comfort by reducing idling at red lights. In terms of information content, Xu et al. (2014) quantified the impact of communication information structures and contents on the platoon safety. The results showed that event data (e.g., drivers braking events) may contain more effective information for platoon management than some traditional information such as distance and vehicle speed. Another important issue is the heterogeneity of vehicle dynamics in CDS, such as the effect of intelligent vehicles on the multi-class traffic flow stability (Ngoduy, 2012, 2013a,b), the mixed operation of the different vehicle classes (e.g. trucks and cars) on the stability of traffic flow (Ngoduy, 2015), the impact of heterogeneous parasitic time delays and lags on ACC-equipped vehicle longitudinal dynamics (Ling and Gao, 2011), etc. Specifically, due to the natural limitations and uncertainties in practical vehicular networking, such as transmission range, packet loss, and probabilistic transmission delay, substantial work has been concerning how to design the CDS under such communication constraints and uncertainties (Ghasemi et al., 2013, Hao and Barooah, 2012, Jin and Orosz, 2014, Kesting et al., 2010b, Middleton and Braslavsky, 2012, Monteil et al., 2014, Oncu et al., 2011, Ploeg et al., 2013, Wang et al., 2014, 2013). This paper focuses on bridging the gap between traffic flow modeling and communication approaches in order to build up better cooperative systems via a realistic inter-communication design.

In view of communication topology, due to high traffic mobilities, the unreliable vehicular networking with packet loss and transmission error cannot guarantee the fixed topology (e.g., predecessor-follower and leader-follower) among vehicles within the CDS. Therefore, it is imperative to explore a more generic communication structure and control algorithm suitable for cooperative vehicle driving with vehicular networking. To this end, we propose to adopt the *consensus control* approach to build up a model for connected traffic flow. In general, the *con-*

sensus control approach is considered a distributed control law which can efficiently facilitate the convergence of collective behavior among multiple agents and can well adapt to the characteristics of the *time-varying communication topology* of the IVC in the CDS. The related work was initially reported by Fax and Murray (2004), in which dynamical systems as the paradigm are used to model the information exchange within a platoon, and cooperative driving vehicles are formulated as a typical *consensus control* problem. Thereafter, considerable studies were conducted on the issues of cooperative driving and formulated these issues into different consensus problems under various communication assumptions (Bernardo et al., 2015, Ren, 2007, Santini et al., 2015, Wang et al., 2012). In Wang et al. (2012), the cooperative driving vehicles are required to converge the weighted headway spacing to a constant. Moreover, the authors proposed a two-stage stochastic approximation algorithm with post-iterate averaging to mitigate the observation noises. Numerical simulations showed the effectiveness of the vehicle-to-vehicle (V2V) communication in vehicles deployment compared to the sensor-based communication. Later on, Bernardo et al. (2015) considered vehicle platooning in the presence of the time-varying heterogeneous communication delays. They adopted the leader-follower control topology, and calculated the upper bound delay by Lyapunov-Razumikhin theorem which guarantees the stability of the platooning system. Besides, some other studies generalized similar issues as cooperative driving and provided theoretical frameworks for the analysis of the consensus problem in multi-agent networked systems, with an emphasis on the role of the directed information flow, changing network topology due to the impaired communication, as well as the design technologies (Olfati-Saber et al., 2007, Zhang et al., 2012).

Despite the advantage of consensus control design for distributed multi-agent coordination, there still exist some issues unclear about the practical implementation of cooperative driving, especially regarding the communication topology:

i) the realistic inter-vehicle communication (IVC) has not been fully considered in the cooperative driving model. Most of the previous work only assumed a general IVC condition, regardless of the communication protocols being applied in the CDS. Actually, IVC protocols play a critical role in the CDS and different protocols show system performance at different levels (Fernandes, 2012). Therefore, it is important to clarify and theoretically analyze what the critical metrics (packet delay/loss or other criteria) of the IVC are important to meet the requirement of the consensus-based control for the CDS, and how the communication protocols affect the system performance.
ii) most consensus-related work only considered general multi-agent systems in which the relative position of the agents and the direction of the information delivery are barely specified. Actually, due to some practical requirements for the vehicle platooning, such as all vehicle driving in the same direction and collision avoidance, the impact of the communication topology on the platoon-based CDS needs to be further explored.
iii) the platoon-based CDS has not been comprehensively studied before, where large-scale vehicles are grouped into a series of platoons driving along the road (i.e. connected traffic flow is modelled as many platoons moving together). In this case, not only the vehicles within the same platoon are required to drive cooperatively (intra platoon), but also the cooperation among platoons (inter platoons) should be taken into account.

To this end, in this paper, we attempt to build up a novel platoon-based cooperative driving model with consideration of the realistic IVC. Specifically, the IEEE 802.11p, the *de-facto* vehicular networking standard, is selected as the IVC protocols, which concerns how to design suitable consensus control algorithms and how the IVC protocols affect the system performance.

By definition, the IEEE 802.11p is an approved amendment to the IEEE 802.11 standard to add wireless access in a vehicular communication system. It defines enhancements to 802.11 (the basis of products marketed as Wi-Fi) required to support ITS applications. This IEEE 802.11p supports data exchange between high-speed vehicles and between the vehicles and the roadside infrastructure. Our main contributions in this paper are threefold. 1) We proposed a platoon-based CDS with consideration of the realistic IVC. 2) The consensus-based control algorithm is implemented in the CDS, where the impact of the IVC, e.g., heterogeneous inter-vehicular communication delay and packet loss, on the system performance is theoretically studied. 3) The model is verified by numerical simulations which couple the traffic dynamics and the vehicular communication. More specifically, the system performance is fully evaluated under various traffic scenarios.

The rest of this paper is organized as follows. We describe the system model and formulate the control problem in Section 2. In Section 3, we first propose the consensus algorithm, then theoretically analyse the system performance with and without packet loss, respectively. Some numerical simulations are conducted in Section 4 to support our theoretical results and evaluate the system performance. We conclude the paper in Section 6.

2. System modelling

In this section, we first model the traffic dynamics and inter-vehicle communication, respectively, then demonstrate the specifications and assumptions on the proposed platoon-based CDS. Finally, we formulate the vehicle platooning into a consensus problem.

2.1. Generic car-following model

The dynamics of individual vehicles can be described by microscopic (car-following) traffic flow models, which illustrate the acceleration of vehicle j in relation to its leading vehicle $(j-1)$. Traditionally, the acceleration of a vehicle is mainly determined by its velocity, the inter-vehicle spacing, and the relative velocity with respect to the leader(s). With the help of the IVC, a vehicle may obtain more information from neighboring vehicles, which can facilitate the optimal velocity and improve traffic safety and efficiency. In this context, the acceleration of a vehicle j can be represented with a more general form:

$$\frac{dv_j(t)}{dt} \doteq \dot{v}_j(t) = f\left(v_j(t), \underset{i\in\mathcal{N}_1(t)}{\Gamma_1(\Delta x_{i,j}(t))}, \underset{i\in\mathcal{N}_2(t)}{\Gamma_2(\Delta v_{i,j}(t))}, ... \right) \tag{1}$$

where $v_j(t)$, $\Delta x_{i,j}(t)$, and $\Delta v_{i,j}(t)$ are the velocity of the considered vehicle j, its space gap, and relative velocity with respect to its neighbouring vehicle i, respectively. $\mathcal{N}_1(t), \mathcal{N}_2(t), ...$ denote the time-varying communication topologies of these available reference information, while $\Gamma_1, \Gamma_2, ...$ describe the corresponding control algorithms. This model can be further extended according to the availability of other type of information, e.g., acceleration of the neighbouring vehicles.

It shall be noted that potentially the general car-following model Eq. (1) can be applied to the complicated mixed traffic flow which consists of both connected vehicles and conventional (human-driven) vehicles. This is because the conventional vehicle essentially can be regarded as the connected vehicle with limited communication capability. In case of such heterogeneous traffic flow, we thus specify two differential functional forms, one for each vehicle class. That is, $f(.) = f_{hu}(.)$ for the conventional vehicles and $f(.) = f_{ca}(.)$ for the connected vehicles. To this end, we can adopt the model proposed in this paper for the specification of f_{ca} and any current car-following model such as the Intelligent Driver Model of Treiber et al. (2006) or the multi-anticipative model of Ngoduy and Wilson (2014) for the specification of f_{hu}. In the ensuing paper, we only focus on the fully connected traffic flow and define the corresponding functional form $f(.)$ using *consensus control* algorithms for the regulation of traffic flow dynamics. As an initial step of the study, we only consider the velocity, space gaps and relative velocities the reference information of the system. In the next section, we will introduce some preliminaries of the consensus, then formulate the platoon-based CDS into the consensus control problem.

2.2. Inter-vehicle communication

To support the cooperative driving, each vehicle within the same group is supposed to periodically disseminate its current kinematic status (including position, velocity, acceleration, etc.) to the neighbours, namely *beacon message dissemination*. In this paper, each vehicle is assumed to be equipped with GPS and on-board sensors to measure its absolute position, speed and acceleration, and adopts the IEEE Wireless Access in Vehicular Environment (WAVE) suite, the *de-facto* vehicular networking standards, as the IVC protocols.

According to the standard of IEEE 1609.4, the channel access time is divided into synchronized intervals (SI) and each SI contains a guard interval and an alternating fixed-length interval, including the control channel interval (CCHI) and the service channel interval (SCHI). All beacons are broadcasted during the CCHI via contention-based carrier sense multiple access with collision avoidance (CSMA/CA) mechanism.

For vehicular communications, we consider that the communication topology among vehicles can be represented as a directed graph (digraph) $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = 1, 2, ..., n$ is the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is an adjacency matrix with nonnegative elements which represents the communication link between node i and j. In this paper, we assume $a_{ij} = 1$ in the presence of a communication link from node j to node i, otherwise $a_{ij} = 0$. In additions, we assume no self-loops in the digraph, i.e., $a_{ii} = 0$ for all $i = 1, ..., n$. The degree matrix $D = diag\{d_1, ..., d_n\}$ is diagonal matrix, whose diagonal elements are given by $d_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix of the weighted digraph is defined as $L = D - A$. To study the leader-following problem, we also define a diagonal matrix $B = \beta \cdot diag\{b_1, ..., b_n\} \in \mathbb{R}^{n \times n}$ to be a leader adjacency matrix associated with the system consisting of n nodes and one leader (labeled with 0),where β is the control weight, $b_i = 1$ in presence of a communication link from node i to leader 0, otherwise $b_i = 0$. We also define $\mathcal{I} = \{i | b_i > 0, i \in \mathcal{V}\}$ the index set of the vertexes whose neighbours include the leader. In case of switching topology, all adjacency matrices are labeled with the subscript of switching signal σ. All possible topology set is defined as $\Lambda = \{\mathcal{G}_0, \mathcal{G}_1, ..., \mathcal{G}_K\}$, where K denotes the total number of all possible communication graphs.

2.3. System specifications and assumptions

place Fig. 1 about here

In this paper, we consider the platoon-based CDS, i.e., a series of platoons are supposed to drive cooperatively with the help of IVC, as illustrated in Fig. 1. To this end, in contrast to conventional (human driven) traffic flow which is considered to consist of many individuals (or particles) moving together, connected traffic flow is modelled as many platoons moving together. It shall be noted that there are various practical uncertainties in such a complicated system, which can be classified by the predefined four components of the system, such as the heterogeneous actuator lag, sensing delays, measurement errors, etc. However, since in this paper we mainly focus on the effects of imperfect vehicular communication in CDS, we make some reasonable assumptions on the system modelling to enable the theoretical analysis. Based on the illustration of the vehicle dynamics and IVC, the specifications and assumptions for the platoon-based CDS are summarized as follows.

1. The vehicles are subject to the uniform platoon-based distribution with smaller intra-platoon spacing s and larger inter-platoon spacing S.
2. All vehicles are assumed identical, each platoon being composed of N member vehicles plus a leader vehicle labeled with index 0.
3. The state of vehicle (position and speed) is assumed to be precisely and timely measured by on-board sensors. [1]
4. Each vehicle has the same fixed transmission range R and the platoon leader can receive the information from the preceding platoon leader.
5. The beacon frequency is set to $1/\tau$ (typically 10Hz), and the consensus control is implemented at each end of the CCHI.
6. The position and velocity function of vehicle are time-continuous, and all vehicles are assumed to keep constant speed during each CCHI, i.e., $v_j(t-\tau_j) \approx v_j(t)$, where τ_j is the information communication delay within each CCHI.
7. The leader's acceleration is assumed with an upper bound $\bar{\alpha}$: $\|\dot{v}_{0,k}\| = \|\alpha(t)\| \leq \bar{\alpha}$.

2.4. Consensus problem formulation

In this paper, the cooperative driving strategies for a series of platoons are defined as follows:

i) Within a platoon, let each member follow the leader asymptotically and maintain the constant smaller inter-vehicle spacing s.
ii) The leader shall follow the average behavior of the preceding platoon and maintain the constant larger inter-platoon spacing S.

Normally, the continuous-time dynamics of vehicle i in platoon k can be represented as follows

$$\dot{x}_{i,k}(t) = v_{i,k}(t) \tag{2}$$

$$\dot{v}_{i,k}(t) = u_{i,k}(t) \tag{3}$$

where $x_{i,k} \in R$ and $v_{i,k} \geq 0$ are the position and velocity of vehicle i in platoon k. $u_{i,k} \in R$ is the control input to follow the generic car-following function $f(.)$ in Eq. (1) which can use the neighboring information and be adjusted to achieve the control goal. For the (platoon) members, the control goal is to follow the leader's behavior within the same platoon, while for the (platoon) leader, the control goal is to follow the average behavior of the preceding platoon.

The consensus of the platoon-based CDS is deemed to be achieved if the state of system Eq. (2)-Eq. (3) satisfies:

a) For each member $i \in 1, ..., N$ in platoon k,

$$x_{i,k}(t) \rightarrow x_{0,k}(t) - i \cdot s, \quad v_{i,k}(t) \rightarrow v_{0,k}(t) \tag{4}$$

[1] the factors such as sensing delays and measure inaccuracies are ignored in the system, which could be addressed by the advanced high-precision sensors.

b) For the leader in platoon k labeled with index 0,

$$x_{0,k}(t) \to \frac{1}{N+1}\sum_{i=0}^{N}(x_{i,k-1}(t) - i \cdot s - S), \quad v_{0,k}(t) \to \frac{1}{N+1}\sum_{i=0}^{N} v_{i,k-1}(t) \tag{5}$$

Accordingly, we can define both platoon stability and traffic flow stability from the consensus perspective as below:

Definition 1 (Platoon stability)**.** *Given the system Eq. (2)-Eq. (3), if the state of any member i within the same platoon k satisfies*

$$\lim_{t->\infty} |x_{i,k}(t) - (x_{0,k}(t) - i \cdot s)| \le C_0, \lim_{t->\infty} |v_{i,k}(t) - v_{0,k}(t)| \le C_0, \tag{6}$$

where C_0 is the constant bounded value, then the platoon k is said to reach the stability.

Definition 2 (Traffic flow stability)**.** *Given the system Eq. (2)-Eq. (3), if the state of leader for the platoon k satisfies*

$$\lim_{t->\infty} |x_{0,k}(t) - \frac{1}{N+1}\sum_{i=0}^{N}(x_{i,k-1}(t) - i \cdot s - S)| \le C_0, \lim_{t->\infty} |v_{0,k}(t) - \frac{1}{N+1}\sum_{i=0}^{N} v_{i,k-1}(t)| \le C_0, \tag{7}$$

where C_0 is the constant bounded value, then the traffic flow consisting of k platoons is said to reach the stability.

Remark 1. *The definition of the platoon stability in this paper is different from that of the string stability, which is guaranteed if the spacing error is not amplified to the upstream of the platoon.*

3. System analysis

3.1. Consensus control algorithms

As stated in Section 2, the control goal is to follow the leader's state for the (platoon) members, and follow the average state of the preceding platoon for the (platoon) leaders. Accordingly, we need to design consensus control algorithms for both members and leaders, respectively. Due to the packet loss caused by the beacon delivery, the leader's information can not always be obtained by the members within the same platoon. In case of the packet loss of the leader's information, we adopt the last available state of the leader to estimate the its current state. Thus the consensus algorithms for the platoon based CDS are proposed as follows.

i) For platoon member $i = 1, ..., N$, the consensus algorithm is

$$u_{i,k}(t) = \sum_{j=1}^{N} a_{ij}\{\gamma_1[x_{j,k}(t-\tau_j) - x_{i,k}(t) + b_i v_0(t-\tau_0(t))\tau_j + (1-b_i)v_{0,k}(t-\hat{\tau}_0)\tau_j - (i-j)\cdot s] \tag{8a}$$

$$+ \gamma_2[v_{j,k}(t-\tau_j) - v_{i,k}(t)]\} \tag{8b}$$

$$+ \beta b_i\{\gamma_1[x_{0,k}(t-\tau_0(t)) - x_{i,k}(t) + v_{0,k}(t-\tau_0)\tau_0(t) - i\cdot s] \tag{8c}$$

$$+ \gamma_2[v_{0,k}(t-\tau_0(t)) - v_{i,k}(t)]\} \tag{8d}$$

$$+ \beta(1-b_i)\{\gamma_1[x_{0,k}(t-\hat{\tau}_0(t)) + v_{0,k}(t-\hat{\tau}_0)(\hat{\tau}_0 - \tau_0) - x_{i,k}(t) - i\cdot s] \tag{8e}$$

$$+ \gamma_2[v_{0,k}(t-\hat{\tau}_0) - v_{i,k}(t)]\} \tag{8f}$$

where a_{ij} is the (i, j)th entry of the adjacency matrix, and β, γ_1 and γ_2 are the positive control parameters. $x_{0,k}(t - \hat{\tau}_0)$ and $v_{0,k}(t - \hat{\tau}_0)$ are the last available state of the leader of platoon k. τ_j is the time-varying communication delays from vehicle j to other members within the same platoon (Here we neglect the effect of the space gap and assume that all neighbouring vehicles can simultaneously receive the beacon from vehicle j). b_i indicates whether the current leader's information is globally reachable to the members within the same platoon.

The detailed physical meanings of Eq. (8) are represented as follows.

- (8a) represents the estimated position error between the gap of member i and j at time t with respect to the desired gap $(i - j) \cdot s$. Due to the time-delay τ_j of $x_{j,k}$, the term $b_i v_0(t - \tau_0(t))\tau_j + (1 - b_i)v_{0,k}(t - \hat{\tau}_0)\tau_j$ is added as the gap supplement, where $v_{0,k}$ adopts the latest available value depending on the time of successfully received information from leader 0.
- (8b) denotes the velocity error between member i and j.
- (8c) denotes, in case of successfully received information from leader 0, the the estimated position error between the gap of member i and leader 0 at time t with respect to the desired gap $i \cdot s$.
- (8d) denotes, in case of successfully received information from leader 0, the velocity error between member i and leader 0.
- (8e) and (8f) represent similar meanings to (8c) and (8d), respectively, for the leader's information from the last successful beacon dissemination (at $t - \hat{\tau}_0$).

Based on the proposed consensus algorithm, we can see the desired acceleration is determined by the state difference (position and velocity) between the considered vehicle and its neighbours. The first and second lines of Eq. (8) represent the vehicle's position and velocity difference between itself and platoon members, respectively, while the remaining lines denote the vehicle's position and velocity difference between itself and the platoon leader. Obviously, the delay τ_j is bounded with one CCHI, i.e., $\tau_j \leq \tau/2$. Moreover, the maximum different delays within one platoon is $n + 1$. The edges associated with time delay τ_j define a subgraph $\mathcal{G}_j$ with corresponding degree matrix D_j and adjacency matrix A_j. Clearly, $D = \sum_{j=1}^{N} D_j$ and $A = \sum_{j=1}^{N} A_j$. For the proposed beacon transmission scheme,

$$A_j = \begin{pmatrix} 0 & \cdots & a_{1j} & \cdots & 0 \\ 0 & \cdots & a_{2j} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & a_{Nj} & \cdots & 0 \end{pmatrix}$$

ii) For the leader in platoon k, the consensus algorithm is

$$\begin{aligned} u_{0,k}(t) = &\frac{1}{d_{i,k-1}} \sum_{j=0}^{N} p_j(t)[\gamma_1(x_{j,k-1}(t - \tau_j) + v_{j,k-1}(t - \tau_j)\tau_j \\ &- x_{0,k}(t) - (N - j) \cdot s - S) + \gamma_2(v_{j,k-1}(t - \tau_j) - v_{0,k}(t))] \end{aligned} \tag{9}$$

where $d_{j,k-1} = \sum_{j=0}^{N} p_j$ is the partial degree in platoon $k - 1$ of leader in platoon k, i.e., the number of vehicles in preceding platoon $k - 1$ establishing a communication link with the leader in platoon k.

Due to the packet loss caused by the beacon broadcast, connected topologies are dynamic in the platoon-based CDS. In the following, we will discuss the system performance under the given consensus algorithms with fixed and switching topologies, respectively. Moreover, we take into account the impact of the heterogeneous time delay of the beacon delivery.

3.2. Related lemmas and theories

Before discussing the convergence of the system in the next section, we first recall some important lemmas to be used in this paper.

Lemma 1 (proposed by Parks and Hahn (1992)). *Given a complex-coefficient polynomial*

$$p(s) = s^2 + (a + ib)s + c + id \tag{10}$$

where $a, b, c, d \in \mathbb{R}$, $p(s)$ is stable if and only if $a > 0$ and $abd + a^2c - d^2 > 0$.

Let $C([-r, 0], \mathbb{R}^n)$ be a Banach space of continuous functions defined on an interval [-r,0] and take values in $\mathbb{R}^n$ with a norm $||\phi||_c = \max_{\theta\in[-r,0]} ||\phi(\theta)||$. Consider the following time-delay system:

$$\begin{aligned} \dot{x} &= f(t, x_t), t > 0, \\ x(\theta) &= \phi(\theta), \theta \in [-r, 0] \end{aligned} \tag{11}$$

where $x_t(\theta) = x(t + \theta), \forall\theta \in [-r, 0]$, $f : \mathbb{R} \times C([-r, 0], \mathbb{R}^n) \to \mathbb{R}$ is a continuous function and $f(t, 0) = 0, \forall t \in \mathbb{R}$. Then the following lemma holds:

Lemma 2 (Lyapunov-Razumikhin Theorem proposed by Hale and Lunel (1993)). *Let ϕ_1, ϕ_2 and ϕ_3 be continuous, nonnegative, nondecreasing functions with $\phi_1(s) > 0$, $\phi_2(s) > 0$ and $\phi_3(s) > 0$ for $s > 0$ and $\phi_1(0) = \phi_2(0) = 0$. If there is a continuous function $V(t, x)$ such that*

$$\phi_1(||x||) \leq V(t, x) \leq \phi_2(||x||), t \in \mathbb{R}, x \in \mathbb{R}^n. \tag{12}$$

In additions, there exists a continuous nondecreasing function $\phi(s)$ with $\phi(s) > s$, $s > 0$ such that the derivative of V along the solution $x(t)$ of Eq. (11) satisfies

$$\begin{aligned} &\dot{V}(t, x) \leq -\phi_3(||x||) \\ &if\ V(t + \theta, x(t + \theta)) < \phi(V(t, x(t))), \theta \in [-r, 0]; \end{aligned} \tag{13}$$

then the solution $x = 0$ is uniformly asymptotically stable.

Lemma 3 (proposed by Horn and Johnson (1985)). *For any $a, b \in \mathbb{R}^n$ and any positive-definite matrix $\Phi \in \mathbb{R}^{n\times n}$, then*

$$2a^T b \leq a^T \Phi^{-1} a + b^T \Phi b \tag{14}$$

3.3. Intra-platoon consensus analysis with fixed topology

This section investigates the intra-platoon consensus with fixed topology in case of successful beacon delivery from the leader, that is, all members within the same platoon can obtain information from the leader at each CCHI. Successful beacon delivery from the leader means $b_i \equiv 1$. Accordingly, Eq. (8) is rewritten as:

$$\begin{aligned} u_{i,k}(t) = \sum_{j=1}^{N} & a_{ij}\{\gamma_1[x_{j,k}(t-\tau_j) - x_{i,k}(t) + v_0(t-\tau_0(t))\tau_j - (i-j)\cdot s] \\ & + \gamma_2[v_{j,k}(t-\tau_j) - v_{i,k}(t)]\} \\ & + \beta\{\gamma_1[x_{0,k}(t-\tau_0(t)) - x_{i,k}(t) + v_0(t-\tau_0)\tau_0(t) - i\cdot s] \\ & + \gamma_2[v_{0,k}(t-\tau_0(t)) - v_{i,k}(t)]\} \end{aligned} \tag{15}$$

Note that Eq. (15) refers to a constant spacing policy, which has been well studied in literature, for example in Swaroop et al. (1994). Nevertheless, our contribution in the platoon stability analysis of this policy (for fixed topology) is more generic which considers the heterogeneous communication delay, the realistic 802.11p protocols on CDS as well as more feasible platooning-based driving strategy (i.e. when traffic is modelled as many platoons moving together).

Let us define the position and speed errors with respect to the leader as $\bar{x}_{i,k} \triangleq x_{i,k} + i\cdot s - x_{0,k}$ and $\bar{v}_{i,k} \triangleq v_{i,k} - v_{0,k}$. Using the assumption $v_{j,k}(t-\tau_j) \approx v_{j,k}(t)$ and substituting Eq. (15) into Eq. (2)-Eq. (3) lead to:

$$\dot{\bar{x}}_{i,k}(t) = \bar{v}_{i,k}(t) \tag{16}$$

$$\dot{\bar{v}}_{i,k}(t) = \sum_{j=1}^{N} a_{ij}\{\gamma_1[\bar{x}_{j,k}(t-\tau_j) - \bar{x}_{i,k}(t)] + \gamma_2[\bar{v}_{j,k}(t-\tau_j) - \bar{v}_{i,k}(t)]\} - \beta[\gamma_1\bar{x}_{i,k}(t) + \gamma_2\bar{v}_{j,k}(t)] \tag{17}$$

Let $\bar{x}_k \triangleq [\bar{x}_{1,k}, ..., \bar{x}_{n,k}]^T$, $\bar{v}_k \triangleq [\bar{v}_{1,k}, ..., \bar{v}_{n,k}]^T$, $\bar{\chi}_k \triangleq [\bar{x}_k^T \bar{v}_k^T]^T$, Eq. (16)-Eq. (17) can be transformed into:

$$\dot{\bar{\chi}}_k(t) = \mathcal{A}_0\bar{\chi}_k(t) + \sum_{j=1}^{N} \mathcal{A}_j\bar{\chi}_k(t-\tau_j) \tag{18}$$

where

$$\mathcal{A}_0 = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -\gamma_1(D+\beta I) & -\gamma_2(D+\beta I) \end{bmatrix} \text{ and } \mathcal{A}_j = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} \\ \gamma_1 A_j & \gamma_2 A_j \end{bmatrix}$$

Using the Leibniz-Newton formula leads to:

$$\begin{aligned} \bar{\chi}_k(t-\tau_j) &= \bar{\chi}_k(t) - \int_{-\tau_j}^{0} \dot{\bar{\chi}}_k(t+s)ds \\ &= \bar{\chi}_k(t) - \sum_{i=0}^{N} \mathcal{A}_i \int_{-\tau_j}^{0} \bar{\chi}_k(t+s-\tau_i)ds \end{aligned} \tag{19}$$

where $\tau_0 \equiv 0$. To substitute Eq. (19) into Eq. (18), we can obtain:

$$\dot{\bar{\chi}}_k(t) = F\bar{\chi}_k(t) - \sum_{j=1}^{N}\sum_{i=0}^{N} \mathcal{A}_j\mathcal{A}_i \int_{-\tau_j}^{0} \bar{\chi}_k(t+s-\tau_i)ds \tag{20}$$

where

$$F = \sum_{i=0}^{N} \mathcal{A}_i = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -\gamma_1 H & -\gamma_2 H \end{bmatrix}, H = L + \beta I \tag{21}$$

We propose the following **Lemma** 4 and **Theorem** 1 for the intra-platoon consensus analysis. Details of the proof are given in A and B of this paper.

Lemma 4. *Let the matrix F and H be defined in Eq. (21). F is Hurwitz stable if and only if H is positive stable and*

$$\frac{\gamma_2}{\sqrt{\gamma_1}} > \max_{\theta_i \in \sigma(H)} \frac{|Im(\theta_i)|}{\sqrt{|Re(\theta_i)|} \cdot |\theta_i|} \tag{22}$$

where $\sigma(H)$ is the set of all eigenvalues of H.

In the proposed beacon dissemination scheme, a successful beacon dissemination from the leader indicates there exists a spanning tree with the root of the leader in a platoon communication topology. In this case, matrix H is positive stable according to Hu and Hong (2007). Then we have the following theorem.

Theorem 1. *If the leader can always successfully broadcast its beacon to the others within the same platoon, i.e., there exists a spanning tree from the leader for each topology graph in Λ, and the control parameters γ_1 and γ_2 satisfy*

$$\frac{\gamma_2}{\sqrt{\gamma_1}} > \max_{\theta_i \in \sigma(H)} \frac{|Im(\theta_i)|}{\sqrt{|Re(\theta_i)|} \cdot |\theta_i|}$$

where H is defined in Eq. (21) and $\sigma(H)$ is the set of all eigenvalues of H, then there exists a constant $\tau_0 > 0$ such that when $0 \leq \tau_j \leq \tau_0$ (j=1,...,N) the members within the same platoon can achieve the consensus as defined in Eq. (4).

3.4. Beacon Performance Analysis

Due to the lack of RTS/CTS acknowledgement scheme in the beacon dissemination, when more than two messages are transmitted simultaneously they will collide with each other. Such issue consequently leads to the packet loss problem which has been regarded as a major challenge in the vehicle platooning system (Jia et al., 2015). To further explore the system performance in case of the packet loss, we first identify the beacon performance under the specification of the IEEE 802.11P and WAVE 1609.4 with a similar methodology proposed in Campolo and Vinel (2011).

In this paper, the application layer of VANET is assumed to be aware of channel CCH/SCH and each beacon is uniformly disseminated during the available CCHI time. This uniform transmission policy in general outperforms the synchronized one which disseminates all beacons at the beginning of CCHI. In addition, we assume all vehicles are within the transmission range with good signal-to-noise ratio (i.e., no transmission errors and hidden terminals) and the packet loss is considered the only factor of the IVC which can cause the switching communication topology of the CDS. This assumption is typical in communication literature to evaluate the MAC performance.

The radio channel can be in one of three possible states: i) idle with no transmission, ii) success with one transmission, and iii) collision with multiple simultaneous transmission. Let us denote ϱ as the time duration of a generic slot being idle, which equals the fixed unit slot time given in the IEEE 802.11 standard. T_s is the time duration for a successful transmission and is calculated by $T_s = \frac{T_h + L/R_d + T_{AIFS}}{\varrho}$, where T_h is the transmission time periods of the frame header, L is the packet length, R_d is the data rate, T_{AIFS} is an extension time of the backoff procedure in the Distributed Coordination Function (DCF), which is calculated by $T_{AIFS} = AIFSN \times \varrho + T_{SIFS}$. Here the value of $AIFSN$ is allocated by the access category in the IEEE 802.11p. T_c is the time duration for data collision and is calculated by $T_c = \frac{T_h + L/R_d + T_{EIFS}}{\varrho}$, where T_{EIFS} is the costed time whenever the physical layer indicates an unsuccessful transmission event. The useful duration T is calculated as $T = \frac{T_{CCH} - T_g - L/R_d - T_h}{\varrho} - W$, where W is the contention window size.

According to the uniform transmission scheme, a beacon will first wait for a random delay $T_d = rand(0, T)$, then uniformly choose a backoff slot time $T_w = rand(0, W-1)$. Thus the beacon may start a transmission at time slot $T_b = T_d + T_w$. The corresponding probability distribution can be calculated by $P_{T_b}(t_b) = p\,(T_b = t_b) = \sum_{t_d=0}^{t_b} p_{T_d}(t_d) p_{T_w}(t_b - t_d)$. The beacon dissemination for M vehicles within one CCHI can be modelled by a Bernoulli process. Thus the probability $p\,(l, M, (T + W - 1), k)$ with $(0 \leq l \leq (T + W - 1)$ and $1 \leq k \leq M)$ that M vehicles in the system select backoffs time slot from the time duration $T + W - 1$, $(l - 1)$ empty slots passed before the first transmission attempts, and κ vehicles transmit in slot l, is computed by the following equation:

$$\begin{aligned} p(l, M, (T + W - 1), \kappa) = &\left(1 - \sum_{t_b=0}^{l-1} p(T_b = t_b)\right)^M \\ &\cdot \begin{pmatrix} M \\ \kappa \end{pmatrix} \left(\frac{p(T_b = 0)}{1 - \sum_{t_b=0}^{l-1} p(T_b = t_b)}\right)^{\kappa} \left(1 - \frac{p(T_b = l)}{1 - \sum_{t_b=0}^{l-1} p(T_b = t_b)}\right)^{M-\kappa} \end{aligned} \tag{23}$$

where $\dfrac{p(T_b = l)}{1 - \sum_{t_b=0}^{l-1} p(T_b = t_b)}$ denotes the probability of choosing any slot out of the remaining $(T + W - 1)$ slots.

Then let $X(T + W - 1, M)$ be the mean number of successful beacon dissemination during each CCHI, where $(T + W - 1)$ is the total possible transmission time slots for M vehicles. $X(T + W - 1, M)$ can be calculated as:

$$\begin{aligned} X(T + W - 1, M) = &\sum_{l=0}^{T+W-1} \{p(l, M, (T + W - 1), 1)[1 + X(T + W - l - T_s, M - 1)] \\ &+ \sum_{\kappa=2}^{M} p(l, M, (T + W - 1), \kappa) X(T + W - l - T_c, M - \kappa)\} \end{aligned} \tag{24}$$

The first two terms of Eq. (24) indicate the probability that only one out of M vehicle successfully transmits the beacon in the l^{th} slot, and $X(T + W - l - T_s, M - 1)$ is the mean number of successful beacon dissemination in the remaining $T + W - l - T_s$ slots with $M - 1$ vehicles. The last term indicates that more than two vehicles transmit the beacons in the same l^{th} slot leading to data collision. It is worth noticing that this calculation assumes that the backoff counter is decreased by one during the backoff process, i.e, backoff freezing is neglected.

To combine Eq. (23) with Eq. (24), the probability of a successful packet delivery p_{suc} is computed as:

$$p_{suc} = \frac{X(T + W - 1, M)}{M} \tag{25}$$

In view of a given vehicle j out of M vehicles, the probability of successful packet delivery of j is also p_{suc}. Moreover, the process of the packet delivery of vehicle j can be regarded as a Bernoulli process with probability p_{suc}. Consequently, the probability of intervals between two successful packet deliveries of vehicle j is:

$$p_{int}(r) = (1 - p_{suc})^{r-1} p_{suc}, \quad \text{for } r = 1, 2, ... \tag{26}$$

The expected number of intervals between two successful beacon delivery is calculated by $1/p_{suc}$. If the confidence level $P_0 < 1$ is given, then the maximum number of intervals π can be estimated by

$$\sum_{r=1}^{\pi} p_{int}(r) \geq P_0 \tag{27}$$

Obviously, $\pi = 1$ when $p_{suc} = 1$.

3.5. Intra-platoon consensus analysis with switching topology

In case of switching topology, the leader's state information can be lost at some CCHIs due to the data collision. To deal with such issue, the proposed consensus algorithm estimates the current leader's state by its last available value, thus it can be envisaged that the leader's state information has been successfully broadcasted at these CCHIs (i.e., globally reachable). However, this method also introduces the estimation errors which may impair the system performance. In the following, we analyse the system performance under the switching topology and the proposed consensus algorithm.

In case of switching topology we have $b_i \equiv 0$ and we can obtain the closed-loop dynamics of the platoon members:

$$\dot{\bar{x}}_{i,k}(t) = \bar{v}_{i,k}(t) \tag{28}$$

$$\begin{aligned}\dot{\bar{v}}_{i,k}(t) = &\sum_{j=1}^{N} a_{ij}\{\gamma_1[\bar{x}_{j,k}(t-\tau_j) - \bar{x}_{i,k}(t)] + \gamma_2[\bar{v}_{j,k}(t-\tau_j) - \bar{v}_{i,k}(t)]\} - \beta[\gamma_1\bar{x}_{i,k}(t) + \gamma_2\bar{v}_{j,k}(t)] \\ &+ \sum_{j=1}^{N} a_{ij}\tau_j\gamma_1[v_{0,k}(t-\hat{\tau}_0) - v_{0,k}(t-\tau_0)] \\ &+ \beta\{\gamma_1[x_{0,k}(t-\hat{\tau}_0) - x_{0,k}(t-\tau_0) + v_{0,k}(t-\hat{\tau}_0)(\hat{\tau}_0-\tau_0)] + \gamma_2[v_{0,k}(t-\hat{\tau}_0) - v_{0,k}(t-\tau_0)]\}\end{aligned} \tag{29}$$

In Eq. (29), we can see that the system consists of two parts: the fixed topology system as described in Section 3.3 and the leader's state error system. Obviously, for the time-continuous velocity of the leader, $x_{0,k}(t-\hat{\tau}_0) - x_{0,k}(t-\tau_0) = \int_{t-\tau_0}^{t-\hat{\tau}_0} v_{0,k}(\tau)d\tau = -v_{0,k}(t-\bar{\tau})(\hat{\tau}_0 - \tau_0)$, where $\bar{\tau} \in [\tau_0, \hat{\tau}_0]$. Due to $v_j(t-\tau_j) \approx v_j(t)$, $v_{0,k}(t-\hat{\tau}_0) - v_0(t-\tau_0)$ is equivalent to $v_0(t-r\tau) - v_0(t)$, where $r\tau$ denotes the intervals between the time of the leader's last successful beacon delivery and the current time, and the value of r can be evaluated by Eq. (27). Thus we can obtain the closed-loop dynamics of members as follows:

$$\dot{\bar{\chi}}_k(t) = \mathcal{A}_{0,\sigma}\bar{\chi}_k(t) + \sum_{j=1}^{N} \mathcal{A}_{j,\sigma}\bar{\chi}_k(t-\tau_j) + \Delta \tag{30}$$

where

$$\mathcal{A}_{0,\sigma} = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -\gamma_1(D_\sigma + \beta I^2) & -\gamma_2(D_\sigma + \beta I) \end{bmatrix}, \ \mathcal{A}_{j,\sigma} = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} \\ \gamma_1 A_{j,\sigma} & \gamma_2 A_{j,\sigma} \end{bmatrix},$$

and

$$\Delta = \begin{bmatrix} 0_{N\times 1} \\ \delta_{N\times 1}(t) \end{bmatrix} = \begin{bmatrix} 0_{N\times 1} \\ \gamma_1 A_\sigma T_{N\times 1} + \beta\gamma_2 \mathbf{1}_{N\times 1} \end{bmatrix} \cdot (v_{0,k}(t-\hat{\tau}_0) - v_0(t-\tau_0))$$
$$+ \begin{bmatrix} 0_{N\times 1} \\ \beta\gamma_1 \mathbf{1}_{N\times 1} \end{bmatrix} \cdot (v_{0,k}(t-\hat{\tau}_0) - v_0(t-\bar{\tau}_0))(\hat{\tau}_0 - \tau_0), \ T_{N\times 1} = [\tau_1, ..., \tau_N]^T$$

To combine with the analysis of the packet loss of the beacon delivery, we can further estimate the bounded value of $\delta_i(t)$:

$$\begin{aligned} |\delta_i(t)| &\le (\gamma_1 N p_{suc} T + \beta\gamma_2)(\pi - 1)\tau\alpha(t - \tau_0^{'}) + \beta\gamma_1(\pi - 1)\alpha(t - \tau_0^{''})(\pi - 1)\tau \\ &\le (N p_{suc} + \beta)(\gamma_1 \tau/2 + \gamma_2)(\pi - 1)\tau\bar{\alpha} \end{aligned} \tag{31}$$

where $T \approx 2/\tau$, $\tau_0^{'} \in [\tau_0, \hat{\tau}_0]$, and $\tau_0^{''} \in [\bar{\tau}_0, \hat{\tau}_0]$. Using the Leibniz-Newton formula leads to:

$$\begin{aligned} \bar{\chi}_k(t - \tau_j) &= \bar{\chi}_k(t) - \int_{-\tau_j}^{0} \dot{\bar{\chi}}_k(t+s)ds \\ &= \bar{\chi}_k(t) - \sum_{i=0}^{N} \mathcal{A}_{i,\sigma} \int_{-\tau_j}^{0} \bar{\chi}_k(t+s-\tau_i)ds - \int_{-\tau_j}^{0} \Delta(t+s)ds \end{aligned} \tag{32}$$

where $\tau_0 \equiv 0$. Substituting Eq. (32) into Eq. (30) results in:

$$\dot{\bar{\chi}}_k(t) = F_\sigma \bar{\chi}_k(t) - \sum_{j=1}^{N}\sum_{i=0}^{N} \mathcal{A}_{j,\sigma}\mathcal{A}_{i,\sigma} \int_{-\tau_j}^{0} \bar{\chi}_k(t+s-\tau_i)ds - \sum_{j=1}^{N} \mathcal{A}_{j,\sigma} \int_{-\tau_j}^{0} \Delta(t+s)ds + \Delta \tag{33}$$

where

$$F_\sigma = \sum_{i=0}^{N} \mathcal{A}_{i,\sigma} = \begin{bmatrix} 0_{N\times N} & I_{N\times N} \\ -\gamma_1 H_\sigma & -\gamma_2 H_\sigma \end{bmatrix}, \ H_\sigma = L_\sigma + \beta I \tag{34}$$

We propose the following theorem for our analysis in case of the packet loss of the beacon delivery. The detailed proof of **Theorem** 2 is given in C.

Theorem 2. *Let the matrix H_σ be defined in Eq. (34). In case of the packet loss with the leader's beacon delivery, if the control parameters γ_1 and γ_2 satisfy*

$$\begin{bmatrix} \gamma_1^2(H_\sigma + H_\sigma^T) - \gamma_2 I & \gamma_1\gamma_2(H_\sigma + H_\sigma^T) - (\gamma_1 + \gamma_2)I \\ \gamma_1\gamma_2(H_\sigma + H_\sigma^T) - (\gamma_1 + \gamma_2)I & \gamma_2^2(H_\sigma + H_\sigma^T) - (2\gamma_1 + \gamma_2)I \end{bmatrix} > 0 \tag{35}$$

then there exist a constant $\tau_0 > 0$, such that when $0 \le \tau_j \le \tau_0$ (j=1,...,N), the members within the same platoon can achieve the following consensus:

$$\lim_{t\to\infty} \|\bar{\chi}_k\| \le C_0 \tag{36}$$

for some constant C_0 depending on $\bar{\alpha}$, beacon delivery ratio p_{suc}, platoon size N. Morover, if $\bar{\alpha} = 0$ or $p_{suc} = 1$, then $\lim_{t\to\infty} \bar{\chi}_k = 0$

Theorem 2 reveals that in case of the packet loss of the beacon delivery, the state (position and velocity) errors between the platoon members and the leader can converge within a certain bound. Moreover, the bound value is determined by the system parameters, such as platoon size, beacon delivery ratio, and acceleration perturbation magnitude. In general, given other parameters, the more beacon delivery ratio is, the smaller state errors are. This analytical result is consistent with **Theorem** 1. In addition, the analytical results highlight the importance of the globally reachable leader's state information to the system performance, and provide a general design guideline for the IVC protocols.

3.6. Inter-platoon stability analysis

Let us define the position and speed errors with respect to the preceding platoon as $\bar{x}_{0,k} \triangleq x_{0,k} - \dfrac{1}{N+1}\sum_{i=0}^{N}(x_{i,k-1}(t) - (N-i)\cdot s - S)$ and $\bar{v}_{0,k} \triangleq v_{0,k} - \dfrac{1}{N+1}\sum_{i=0}^{N} v_{i,k-1}(t)$. To use the assumption $v_{j,k}(t-\tau_j) \approx v_{j,k}(t)$ and substitute Eq. (9) into Eq. (2)-Eq. (3), we can obtain the closed-loop dynamics of the leader:

$$\dot{\bar{x}}_{0,k}(t) = \bar{v}_{0,k}(t) \tag{37}$$

$$\begin{aligned}\dot{\bar{v}}_{0,k}(t) = & -\gamma_1\bar{x}_{0,k}(t) - \gamma_2\bar{v}_{0,k}(t)\\ & + [\frac{1}{d_{i,k-1}}\sum_{j=0}^{N} p_j(t)(x_{j,k-1}(t) - (N-j)\cdot s - S) - \frac{1}{N+1}\sum_{j=0}^{N}(x_{j,k-1}(t) - (N-j)\cdot s - S)]\\ & + [\frac{1}{d_{i,k-1}}\sum_{j=0}^{N} p_j(t)v_{j,k-1}(t) - \frac{1}{N+1}\sum_{j=0}^{N} v_{j,k-1}(t)]\end{aligned} \tag{38}$$

Clearly, if the intra-platoon consensus holds, the latter two terms of the above equation can be omitted. Accordingly, Eq. (38) can be transformed to:

$$\dot{\bar{v}}_{0,k}(t) + \gamma_2\bar{v}_{0,k}(t) + \gamma_1\bar{x}_{0,k}(t) = 0 \tag{39}$$

The stability of the above system holds if $\gamma_2 > 0$. Moreover, if $\gamma_2^2 > 4\gamma_1$, the system has negative real eigenvalues, which means the position error $\bar{x}_{0,k}$ will monotonically converge to zero. However, it shall be noted that velocity error $\bar{v}_{0,k}(t)$ cannot be guaranteed based on the proposed consensus algorithm. To achieve the the equilibrium state defined in Eq. (5), some additional information, e.g., acceleration, may be needed for the consensus algorithms. This will be left in our future research.

4. Numerical studies

In this section, we conduct some numerical experiments to support our theoretical results in the previous sections and to evaluate the performance of the proposed cooperative driving strategies.

4.1. Simulation settings and scenarios

We use the PLEXE (Segata et al., 2014) simulator in this paper, an open source IVC simulation framework which consists of the network simulator OMNeT++/MiXiM and the road traffic simulator SUMO. OMNET++/MiXiM is used to simulate V2V communication based on the 802.11p standard, while SUMO can simulate the vehicle dynamics with the proposed consensus algorithms. Both components are coupled with each other through a standard traffic control interface (TraCI) by exchanging the Transmission Control Protocol (TCP) messages, while OMNeT++/MiXiM is acting as the TraCI client and SUMO is acting as the TraCI server. The simulation parameters for VANET are based on the IEEE 802.11p standard, as listed in Table 1. The traffic related parameters used in our experiments are summarized in Table 2. *It shall be noted that to model a more realistic vehicle dynamics, the actuator lag (i.e., the delay between the acceleration command and its actual realization in the vehicle due to inertial and mechanical limits) is considered and implemented in PLEXE.*

place Table 1 about here

place Table 2 about here

The measured traffic flow is composed of three consecutive platoons with an identical platoon size. Three typical traffic scenarios are considered for the system evaluation: (1) an initial phase during which all following vehicles launch from predefined positions to finally cooperatively driving at the same constant speed 25 m/s regulated by the leader, (2) a single large perturbation wherein the leader first decelerates from 25 m/s to 5 m/s, then maintains this speed for a period of time, and finally accelerates to the original speed 25 m/s, and (3) the continuous small perturbations wherein the leader experiences a sinusoidal disturbance in speed, defined as follows:

$$\delta(t) = Asin(0.2\pi t), A = 5m/s \tag{40}$$

4.2. Verification of the beacon dissemination

This section verifies the performance analysis presented in Sections 3.4 on the beacon dissemination. The related networking parameters of the IEEE802.11p are given in Table 1. Fig. 2 illustrates the beacon reception ratio with respect to the number of vehicles. We can observe that analytical results closely match the simulation results. The uniform transmission policy performs very well for fewer vehicles in a transmission range. However, with a larger number of vehicles, the beacon reception ratio also decreases dramatically. This is because all vehicles have to contend with the limited time slots within the CCH interval.

place Fig. 2 about here

4.3. Impact of communication topology

This section investigates the impact of IVC topology on the system performance. We assume that all vehicles can receive beacons without packet loss, that is, only the heterogenous beacon delays are considered in the IVC. We apply two typical IVC topologies to the proposed consensus algorithm: (a) the *general communication topology* including backward and forward neighboring information, and (b) the *forward communication topology* where only the front vehicles' information is taken into account (another popular *leader-preceding topology* can be regarded as a special case of the forward topology). To facilitate the comparison, we fix the values of control parameters for those two different IVC topologies adopted in (Santini et al., 2015), as listed in Table 2. We will discuss the issue of control parameter values in Section 5.

First, we evaluate the intra-platoon performance in an initial-phase traffic scenario defined as follows. The platoon is composed of several vehicles initially separated by an equal length of

30m distance between two consecutive ones, with the same start-up speed 25m/s. The control objective is to attain a stable traffic flow state with the constant 15m inter-vehicle spacing and speed 90km/h (or 25m/s).

place Fig. 3 about here

place Fig. 4 about here

We first adopt the general communication topology for the platoon with 8 vehicles, and select vehicle member 1, 4, and 7 as our system state outputs, and compare them with the leader's state. The simulation results are shown in Fig. 3. We can see that after about 10s transient stage, all members' states (including position and speed) finally converged to the leader's. However, the velocity of vehicle 1 experienced some disturbance (first decreased then increased) during its initial phase. This is because vehicle 1 locates in the front part of the platoon, its acceleration is mostly determined by the information from following vehicles, which makes vehicle 1 decelerate to minimize the relative position error with respect to following vehicles. This phenomenon may bring a negative effect in the initial-phase traffic scenario, even lead to a collision. For example, in case of 16 vehicles in one platoon, shown in Fig. 4, we can see that the gap between vehicle 3 and vehicle 2 ($x_{3,k} - x_{2,k}$) as well as the one between vehicle 2 and vehicle 1 ($x_{2,k} - x_{1,k}$) gradually shrank at the initial stage, and finally led to a collision. This is due to the negative impact of massive backward information on the vehicle's acceleration decision.

place Fig. 5 about here

To minimize this disadvantage, one possible way is to limit the number of the referred neighboring information. Another method is to request the referred neighboring information only from the front vehicles, i.e., adopting the forward communication topology in CDS. In Fig. 5, we can see that under the forward communication topology with the large platoon size of 16, all vehicles could start to accelerate to minimize the position error with respect to their front ones, and finally converge to the leader's state (similar results can be obtained by applying the leader-preceding topology). This is because the acceleration is only decided by the information from the front vehicles, which can meet the requirement of platoon driving in the same direction (i.e., negative speed avoidance).

place Fig. 6 about here

place Fig. 7 about here

Next, we evaluate the system performance under two typical perturbation scenarios: the single large perturbation (to mimic traffic emergency like collision avoidance as in Fig. 8(a)) and the continuous small perturbations of the leader speed (to mimic common traffic disturbance caused by abnormal driving behavior as in Fig. 9(a)). We adopt the forward topology and choose vehicle member 1, 4, and 7 as our reference outputs. The simulation results are given in Fig. 6 and Fig. 7, respectively.

We can observe that in both scenarios, a little disturbance occurred on the state error between members and the leader during the period of traffic perturbations. This is due to the assumption of $v(t - \tau_j) \approx v(t)$ in the proposed consensus algorithms, which introduces the track lag in case of the leader's perturbation. However, the magnitude of the track lag can be bounded by the maximum acceleration and the time delay in both traffic scenarios. This conclusion could be utilized as a criterion for the driving safety design, that is, to guarantee the collision avoidance between vehicles in the same platoon, the desired inter-vehicle spacing shall be larger than the maximum position errors, which can practically be estimated by some system parameters such as the maximum acceleration of the leader, packet reception ratio, platoon size, etc., according to the proof of Theorem 2. In addition, we can see that the consensus is achieved (i.e., no state errors) among members, which is because all members receive the same information from the leader at each beacon dissemination. Likewise, we can obtain similar results from the CDS that adopts a general topology. To minimize the state errors between the leader and members, one possible method is to predict more precisely the leader's current state by introducing additional acceleration information, e.g., let $v_0(t) = v_0(t - \tau_0) + a_0(t - \tau_0) \cdot \tau_j$. This issue will be left in our future work.

Finally, we verify the inter-platoon performance under the proposed consensus algorithms.

place Fig. 8 about here

place Fig. 9 about here

Fig. 8(a) and Fig. 9(a) describe the velocity profiles of three consecutive platoon's leaders in two perturbation traffic scenarios. Obviously, when the first leader $0, 0$ experienced disturbances, the following two leaders started to oscillate simultaneously. Fig. 8(b) and Fig. 9(b) show that the inter-platoon spacing errors also amplified along the upstream direction, which indicates the instability of the system controlled by the proposed consensus algorithms. The simulation results have verified our theoretical analysis in Section 3.6. Essentially, the proposed inter-platoon control algorithm can be regarded as the preceding-information based adaptive cruise control, which cannot guarantee the constant inter-platoon spacing with only reference information of the velocity and position, as has been proven in the literature. To further improve the system performance, additional information e.g., acceleration information, or changing communication topology is necessary.

4.4. Impact of uncertainties

In this section, we evaluate the proposed platoon-based CDS with some typical uncertainties. Specifically, we focus on the impact of the communication uncertainty, i.e., packet loss of the beacon dissemination, as well as the possibly introduced measurement errors on the system performance. We assume the platoon is composed of 1 leader and 7 members with the forward communication topology.

First, we evaluate the system performance under different beacon reception ratios.

place Fig. 10 about here

Fig. 10 describes the state errors of platoon member 4, the centering vehicle of the platoon, with respect to the leader 0 under different beacon reception ratios such as 90%, 80%, and 70% in both perturbation scenarios. With more packet loss in beaconing, the magnitude of state error (both velocity and position) increases accordingly, which is consistent with our conclusion in Theorem 2. The reason is that the current leader's information cannot be obtained by the members in existence of the packet loss, which introduced estimation error of the leader's state in our proposed consensus algorithms. The more packet loss is, the larger estimation error of the leader's state is, which leads to an increased state error between the members and the leader. Consequently, the globally achievable leader's information is critical to stabilize the platoon-based CDS. This conclusion can be utilized as a principle of the efficient IVC protocols design, especially the beacon dissemination in vehicular networks.

Second, we consider the imperfect measurements of positions and velocities of each vehicle, and evaluate their impact on the system performance. We assume a standard derivation of zero mean Gaussian noise for both position and speed state measurement, denoted as $\mathcal{N}(0, \rho_x)$ and $\mathcal{N}(0, \rho_v)$, respectively, then explore the state errors between the leader and member 7.

place Fig. 11 about here

In Fig. 11, we can observe that the state errors between the leader and the member are enlarged by the measurement errors in both constant speed and continuous small perturbations traffic scenarios. With the larger measured errors, the magnitude of state errors increases accordingly. However, the state errors can be bounded by the maximum measurement errors in both traffic scenarios, which is critical for vehicle platooning to avoid traffic collision. To mitigate such negative impacts, in principle, the adaptive weight (e.g. the smaller weight assigned to the reference information with large measurement errors) in the improved control algorithm should be further investigated, or some data processing, e.g. Kalman filter, should be applied to the raw traffic reference information.

5. Discussions

There are a few points which still cannot be addressed yet in this paper:

a) According to Theorem 2, the proposed consensus control algorithms guarantee a nearly constant inter-vehicle spacing within the platoon, however, it cannot essentially mitigate the traffic perturbations because the control objective is to let all following vehicle follow the leader's state. To address this issue, an alternative constant time-headway policy can be adopted in the platoon control algorithm.

b) As stated previously, to facilitate the performance comparison of different communication topologies on platooning system, we adopt the same control parameters for different IVC topologies. These are the default values which are consistent with those in Santini et al. (2015). Nonetheless, it shall be noted that control parameters design is another critical issue to the system performance. In the future, we will focus on investigating how the control parameters, essentially the weight factor of kinetic information, affect the system performance. Based on the analysis, we will further investigate how to select the optimal (control parameter) values to meet the system requirement.

c) The system analysis is performed using **second-order vehicle dynamics** described by Eq. (2)-Eq. (3). In the presence of system uncertainties and physical limitations, including *actuator lags* and *sensing delays*, the vehicle dynamics will be modelled by a **third-order system** as below:

$$\dot{x}_{i,k}(t) = v_{i,k}(t) \tag{41}$$

$$\dot{v}_{i,k}(t) = a_{i,k}(t) \tag{42}$$

$$\dot{a}_{i,k}(t) = -\frac{1}{\omega_{i,k}}[a_{i,k}(t) - u_{i,k}(t - \varphi_{i,k})] \tag{43}$$

where $\omega_{i,k}$ is the actuator lag, and $\varphi_{i,k}$ is the sensing delay. For such a third-order system, the envisioned control algorithm with the help of acceleration information may further improve the robust performance of CDS (Jin and Orosz, 2014). However, it is very cumbersome, even not possible to derive the analytical stability of our proposed control algorithms for the third-order system. Nevertheless, we have numerically investigated the effect of such factors (i.e. actuator delay) in the simulation. On the other hand, more reference information could introduce measurement errors which may impair the platoon performance. This issue should be left in our future research.

6. Conclusion

Recently, cooperative driving with the help of vehicular communication has attracted more concerns in both transportation society and control society. Due to the advantages in modelling such dynamical networking-based system, consensus control has been applied as a promising method to deal with these issues. However, there still exist some issues unclear about the practical implementation of the cooperative driving model, especially regarding the uncertainties of the communication topology in the CDS such as the packet loss and the transmission delay.

To contribute to the state-of-the-art in modelling the dynamics of connected vehicles, in this paper, we have tried to build up a novel platoon-based cooperative driving model with consideration of the realistic IVC. To this end, we proposed consensus-based control algorithms on the multi-platoons cooperative driving, wherein the IEEE 802.11p- the *de-facto* vehicular networking standard, is selected as the practical IVC protocols, and the effects of heterogeneous IVC delays and the packet loss in the beacon dissemination have been taken into account. We then theoretically analyzed the system performance under these uncertainties in the IVC. Some numerical simulations have been conducted to verify our analysis in various traffic scenarios. Both theoretical analysis and simulation results showed that the globally achievable leader's information plays a critical role in stabilizing the platoon-based CDS. Compared with the general topology, the forward topology is more suitable to meet the requirement of vehicle platooning. In addition, the proposed consensus control algorithm shows very high resilience to some typical uncertainties, such as the packet loss and measurement errors.

Acknowledgements

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Appendix

A. Proof of Lemma 4

Proof. Let λ be the eigenvalue of F, then

$$\begin{aligned} det(\lambda I_{2N} - F) &= \begin{vmatrix} \lambda I_{N\times N} & -I_{N\times N} \\ \gamma_1 H & \lambda I_{N\times N} + \gamma_2 H \end{vmatrix} \\ &= det(\lambda^2 I_{N\times N} + \gamma_2 H \lambda I_{N\times N} + \gamma_1 H) \\ &= \prod_{i=1}^{N} (\lambda^2 + \gamma_2 \theta_i \lambda + \gamma_1 \theta_i) \end{aligned}$$

where $\theta_i \in \sigma(H)$. Thus the Hurwitz stability of matrix F is equivalent to that of polynomial: $R(\lambda) = \lambda^2 + \gamma_2 \theta_i \lambda + \gamma_1 \theta_i$, for all $\theta_i \in \sigma(H)$. Based on Lemma 1, we have:

(1) $Re(\theta_i) > 0$, which holds by the positive stable matrix H.

(2) $\gamma_2^2 \gamma_1 Re(\theta_i)(Im(\theta_i))^2 + \gamma_2^2 \gamma_1 (Re(\theta_i))^3 - \gamma_1^2 (Im(\theta_i))^2 > 0$, which can be satisfied by the condition Eq. (22).

Thus the Lemma 4 holds. □

B. Proof of Theorem 1

Proof. Based on Lemma 4, F is Hurwitz stable. Therefore, there exists a positive definite matrix $P \in \mathbb{R}^{2N\times 2N}$ such that

$$PF + F^T P = -I_{2N\times 2N} \tag{44}$$

Consider Lyapunov-Razumikhin candidate function $V(\bar{\chi}_k) = \bar{\chi}_k^T P \bar{\chi}_k$ satisfying $z_{min}(P)\|\bar{\chi}_k\|^2 \leq V(\bar{\chi}_k) \leq z_{max}(P)\|\bar{\chi}_k\|^2$. Combining Eq. (20), we have

$$\begin{aligned}\dot{V}(\bar{\chi}_k) &= \dot{\bar{\chi}}_k^T P\bar{\chi}_k + \bar{\chi}_k^T P\dot{\bar{\chi}}_k \\ &= \bar{\chi}_k^T(PF + F^T P)\bar{\chi}_k - 2\sum_{j=1}^{N}\sum_{i=0}^{N}\bar{\chi}_k^T P\mathcal{A}_j\mathcal{A}_i\int_{-\tau_j}^{0}\bar{\chi}_k(t+s-\tau_j)ds\end{aligned} \tag{45}$$

Based on Lemma 3, let $a = -\mathcal{A}_i\mathcal{A}_j P\bar{\chi}_k$, $b = \bar{\chi}_k(t+s-\tau_j)$ and $\Phi = P$, integrating both side of Eq. (14), then

$$\begin{aligned}\dot{V}(\bar{\chi}_k) \leq \bar{\chi}_k^T(PF + F^T P)\bar{\chi}_k &+ \sum_{j=1}^{N}\sum_{i=0}^{N}\tau_j\bar{\chi}_k^T P\mathcal{A}_j^T\mathcal{A}_i^T P^{-1}\mathcal{A}_i\mathcal{A}_j P\bar{\chi}_k \\ &+ \sum_{j=1}^{N}\sum_{i=0}^{N}\int_{-\tau_j}^{0}\bar{\chi}_k^T(t+s-\tau_j)P\bar{\chi}_k(t+s-\tau_j)ds\end{aligned} \tag{46}$$

Choose $\phi_s = \zeta s$ where constant $\zeta > 1$. According to Lemma 2, in case:

$$V(\bar{\chi}_k(t+s-\tau_j)) = \bar{\chi}_k^T(t+s-\tau_j)P\bar{\chi}_k(t+s-\tau_j) \leq \zeta V(\bar{\chi}_k), \tau_j \leq \tau/2$$

we have

$$\begin{aligned}\dot{V}(\bar{\chi}_k) &\leq -\bar{\chi}_k^T\bar{\chi}_k + \tau_j\sum_{j=1}^{N}\sum_{i=0}^{N}\bar{\chi}_k^T(P\mathcal{A}_j^T\mathcal{A}_i^T P^{-1}\mathcal{A}_i\mathcal{A}_j P + \zeta P)\bar{\chi}_k \\ &\leq -\bar{\chi}_k^T\bar{\chi}_k + \frac{\tau}{2}\sum_{j=1}^{N}\sum_{i=0}^{N}\bar{\chi}_k^T(P\mathcal{A}_j^T\mathcal{A}_i^T P^{-1}\mathcal{A}_i\mathcal{A}_j P + \zeta P)\bar{\chi}_k\end{aligned} \tag{47}$$

Therefore, if the value of τ satisfies:

$$\tau < \tau_0 = \frac{2}{\|\sum_{j=1}^{N}\sum_{i=0}^{N}(P\mathcal{A}_j^T\mathcal{A}_i^T P^{-1}\mathcal{A}_i\mathcal{A}_j P + \zeta P)\|} \tag{48}$$

then $\dot{V}(\bar{\chi}_k) \leq \xi\bar{\chi}_k^T\bar{\chi}_k$ for $\xi > 0$. □

C. Proof of Theorem 2

Proof. Consider Lyapunov-Razumikhin candidate function $V(\bar{\chi}_k) = \bar{\chi}_k^T\Phi\bar{\chi}_k$, where

$$\Phi = \begin{bmatrix}\gamma_2 I_{N\times N} & \gamma_1 I_{N\times N} \\ \gamma_1 I_{N\times N} & \gamma_2 I_{N\times N}\end{bmatrix}, \gamma_2 > \gamma_1 > 0$$

is positive definite. then

$$\begin{aligned}
\dot{V}(\bar{\chi}_k) &= \bar{\chi}_k^T(\Phi F + F^T\Phi)\bar{\chi}_k - 2\sum_{j=1}^{N}\sum_{i=0}^{N}\bar{\chi}_k^T\Phi\mathcal{A}_{j,\sigma}\mathcal{A}_{i,\sigma}\int_{-\tau_j}^{0}\bar{\chi}_k(t+s-\tau_j)ds \\
&- 2\sum_{j=1}^{N}\bar{\chi}_k^T\Phi\mathcal{A}_{j,\sigma}\int_{-\tau_j}^{0}\Delta(t+s)ds + 2\bar{\chi}_k^T\Phi\Delta \\
&\le \bar{\chi}_k^T(\Phi F + F^T\Phi)\bar{\chi}_k + \tau_j\sum_{j=1}^{N}\sum_{i=0}^{N}\bar{\chi}_k^T(\Phi\mathcal{A}_{j,\sigma}^T\mathcal{A}_{i,\sigma}^T\Phi^{-1}\mathcal{A}_{i,\sigma}\mathcal{A}_{j,\sigma}\Phi)\bar{\chi}_k \\
&+ \sum_{j=1}^{N}\sum_{i=0}^{N}\int_{-\tau_j}^{0}\bar{\chi}_k^T(t+s-\tau_j)\Phi\bar{\chi}_k(t+s-\tau_j)ds + \tau_j\sum_{j=1}^{N}\bar{\chi}_k^T\Phi\mathcal{A}_{j,\sigma}\Phi^{-1}\mathcal{A}_{j,\sigma}^T\Phi^T\bar{\chi}_k \\
&+ \sum_{j=1}^{N}\int_{-\tau_j}^{0}\Delta^T(t+s)ds\Phi\Delta(t+s)ds + \bar{\chi}_k^T\Phi\bar{\chi}_k + \Delta^T\Phi\Delta \\
&\le \bar{\chi}_k^T(\Phi F + F^T\Phi)\bar{\chi}_k + \tau_j\sum_{j=1}^{N}\sum_{i=0}^{N}\bar{\chi}_k^T(\Phi\mathcal{A}_{j,\sigma}^T\mathcal{A}_{i,\sigma}^T\Phi^{-1}\mathcal{A}_{i,\sigma}\mathcal{A}_{j,\sigma}\Phi)\bar{\chi}_k \\
&+ \sum_{j=1}^{N}\sum_{i=0}^{N}\int_{-\tau_j}^{0}\bar{\chi}_k^T(t+s-\tau_j)\Phi\bar{\chi}_k(t+s-\tau_j)ds + \tau_j\sum_{j=1}^{N}\bar{\chi}_k^T\Phi\mathcal{A}_{j,\sigma}\Phi^{-1}\mathcal{A}_{j,\sigma}^T\Phi^T\bar{\chi}_k \\
&+ \sum_{j=1}^{N}\int_{-\tau_j}^{0}\Delta^T(t+s)ds\Phi\Delta(t+s)ds + \bar{\chi}_k^T\Phi\bar{\chi}_k + \Delta^T\Phi\Delta
\end{aligned} \tag{49}$$

Similar to the analysis in the proof of Theorem 1, let us choose $\phi_s = \zeta s$ where constant $\zeta > 1$, in case $V(\bar{\chi}_k(t+s-\tau_j)) \le \zeta V(\bar{\chi}_k), \tau_j \le \tau/2$, we have:

$$\begin{aligned}
\dot{V}(\bar{\chi}_k) \le -\bar{\chi}_k^T\Big\{Q_\sigma - \frac{\tau}{2}\Big[\sum_{j=1}^{N}\sum_{i=0}^{N}(\Phi\mathcal{A}_{j,\sigma}^T\mathcal{A}_{i,\sigma}^T\Phi^{-1}\mathcal{A}_{i,\sigma}\mathcal{A}_{j,\sigma}\Phi + \zeta\Phi) \\
+ \sum_{j=1}^{N}\Phi\mathcal{A}_{j,\sigma}\Phi^{-1}\mathcal{A}_{j,\sigma}^T\Phi^T\Big]\Big\}\bar{\chi}_k + |\bar{\delta}|^2(\gamma_2^2 - \gamma_1^2)(1 + \sum_{j=1}^{N}\tau_j)
\end{aligned} \tag{50}$$

where

$$|\bar{\delta}| \equiv (\frac{1}{2}\gamma_1 N p_{suc}\tau + \beta\gamma_2 + \beta(\pi-1)\tau)(\pi-1)\tau\bar{\alpha}$$

$$Q_\sigma = -(\Phi F_\sigma + F_\sigma^T\Phi + \Phi) = \begin{bmatrix} \gamma_1^2(H_\sigma + H_\sigma^T) - \gamma_2 I & \gamma_1\gamma_2(H_\sigma + H_\sigma^T) - (\gamma_1+\gamma_2)I \\ \gamma_1\gamma_2(H_\sigma + H_\sigma^T) - (\gamma_1+\gamma_2)I & \gamma_2^2(H_\sigma + H_\sigma^T) - (2\gamma_1+\gamma_2)I \end{bmatrix}$$

According, Q_σ is positive definite if the control parameter γ_1 and γ_2 satisfies Eq. (35).

Therefore, if

$$\tau < \tau_0 = \frac{\min\lambda_{min}(Q_\sigma)}{\|\sum_{j=1}^{N}\sum_{i=0}^{N}(\Phi\mathcal{A}_{j,\sigma}^T\mathcal{A}_{i,\sigma}^T\Phi^{-1}\mathcal{A}_{i,\sigma}\mathcal{A}_{j,\sigma}\Phi + \zeta\Phi) + \sum_{j=1}^{N}\Phi\mathcal{A}_{j,\sigma}\Phi^{-1}\mathcal{A}_{j,\sigma}^T\Phi^T\|} \tag{51}$$

then $\dot{V}(\bar{\chi}_k) \le -\eta\bar{\chi}_k^T\bar{\chi}_k + C_0$ for some $\eta > 0$, where $C_0 \equiv |\bar{\delta}|^2(\gamma_2 + \gamma_1)(1 + N\tau/2)$. □

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Table 1: 802.11p Parameter Setting

Parameter	**Value**	**Parameter**	**Value**
Channel data rate	6Mbps	Slot time	13 μs
SIFS	32 μs	AIFS	71 μs
Preamble length	32 μs	Plcp duration	8 μs
Propagation delay	2 μs	CWmin	7
Beacon frequency	0.1 s	Beacon priority	2
Beacon size	200 bytes	Transmission range R	500m
CCH interval	46ms	Sync interval	4ms

Table 2: Traffic Related Parameters

Parameter	**Value**	**Parameter**	**Value**
Vehicle length	5 m	Intra-platoon spacing	15 m
Maximum acceleration	3 m/s^2	Inter-platoon spacing	35 m
Maximum deceleration	6 m/s^2	Average speed	25 m/s
Maximum velocity	41 m/s	Platoon size	8 and 16
Control gains:	β=10	γ_1=1,γ_2=2	
Actuator lag	0.25s	Measured error $\rho_x = \rho_v$	0.5, 1

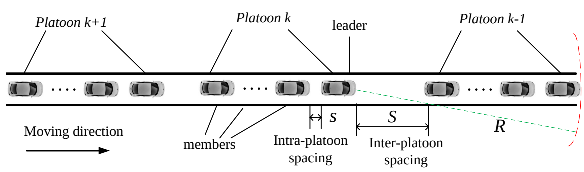


Figure 1: Platoon-based cooperative driving pattern

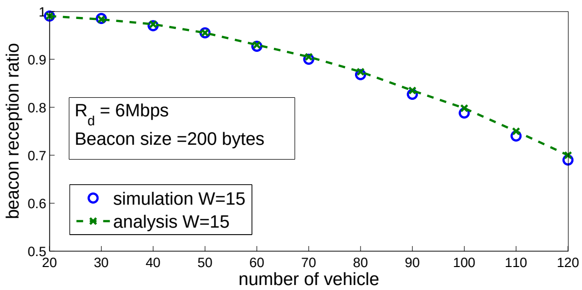
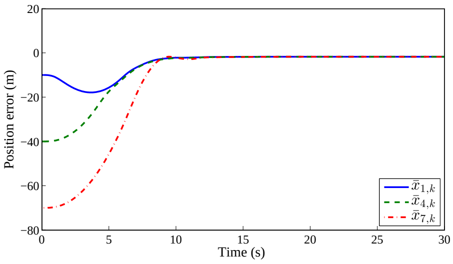
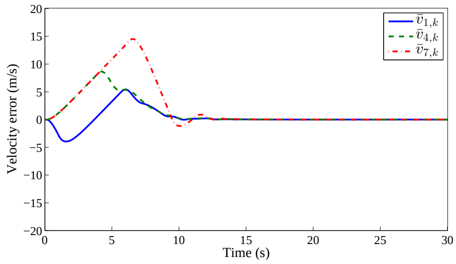


Figure 2: Probability of successful beacon dissemination with respect to number of vehicles

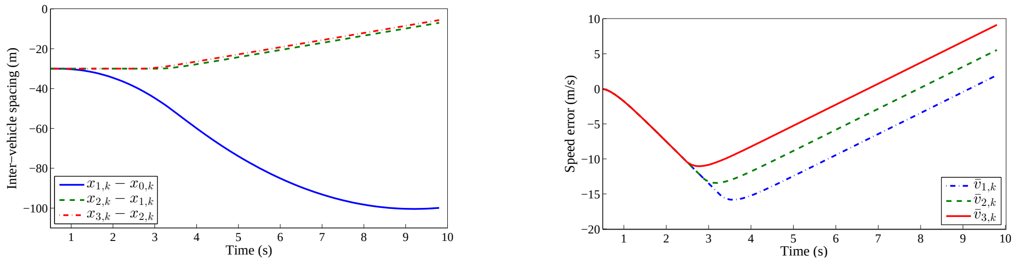


(a) Position error vs. time



(b) Speed error vs. time

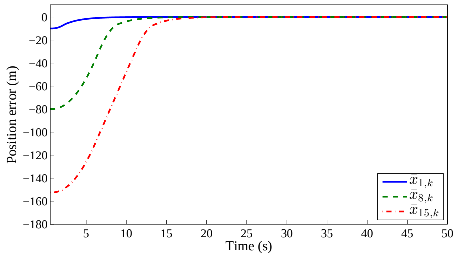
Figure 3: Intra-platoon performance in initial-phase scenario with general topology and platoon size of 8



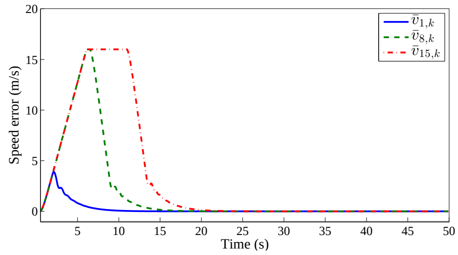
(a) Inter-vehicle spacing vs. time

(b) Speed error vs. time

Figure 4: Intra-platoon performance in initial-phase scenario with general topology and platoon size of 16

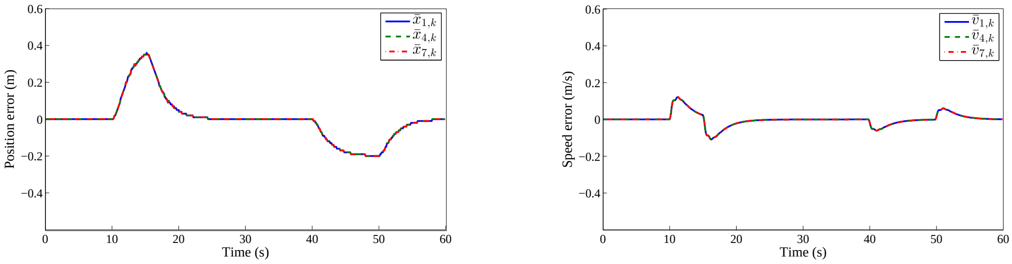


(a) Position error vs. time



(b) Speed error vs. time

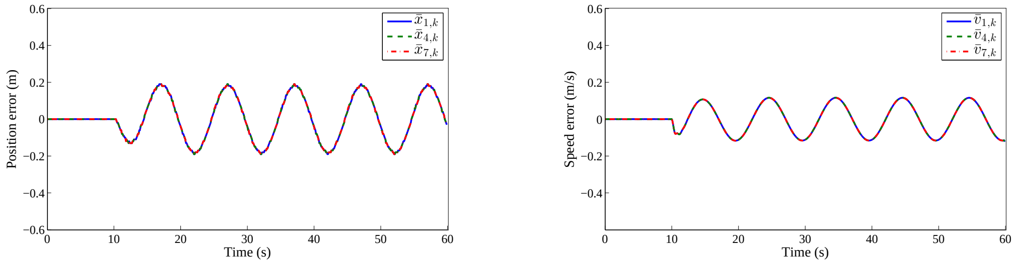
Figure 5: Intra-platoon performance in initial-phase scenario with forward topology and platoon size of 16



(a) Position error vs. time

(b) Speed error vs. time

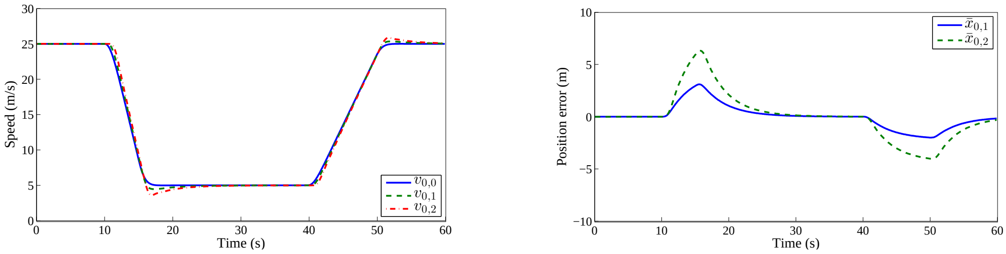
Figure 6: Intra-platoon performance in single large perturbation traffic scenario with forward topology



(a) Position error vs. time

(b) Speed error vs. time

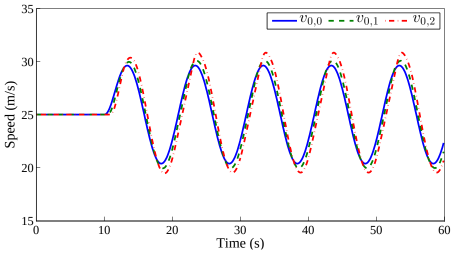
Figure 7: Intra-platoon performance in continuous small perturbation traffic scenario with forward topology



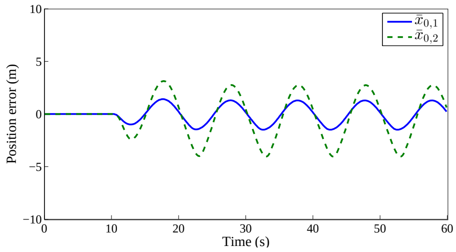
(a) Speed of the platoon leaders

(b) Position error of the platoon leaders

Figure 8: Inter-platoon performance in single large perturbation scenario with forward topology

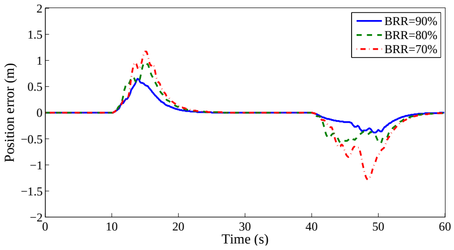


(a) Speed of the platoon leaders

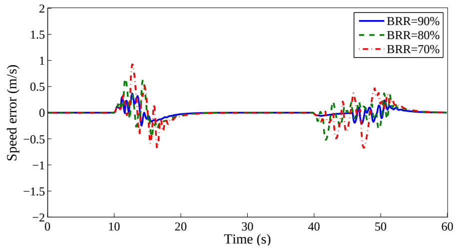


(b) Position error of the platoon leaders

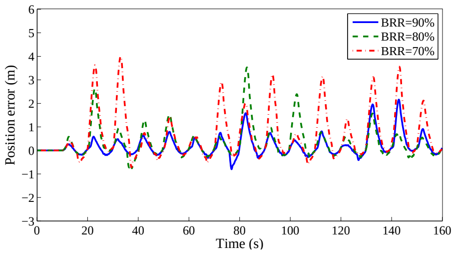
Figure 9: Inter-platoon performance in continuous small perturbations scenario with forward topology



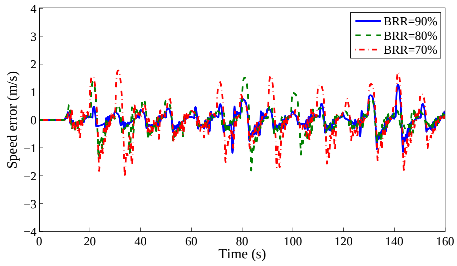
(a) Position error in single large perturbation



(b) Speed error in single large perturbation

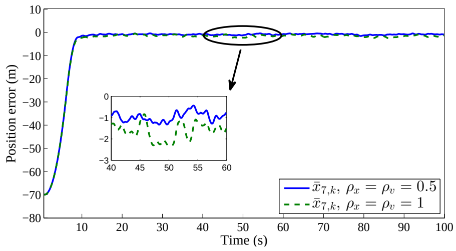


(c) Position error in continuous small perturbations

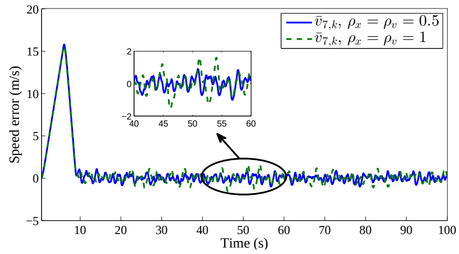


(d) Speed error in continuous small perturbations

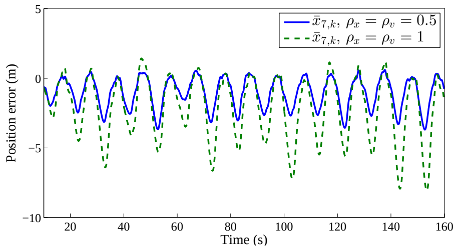
Figure 10: Intra-platoon performance in case of packet loss with forward topology



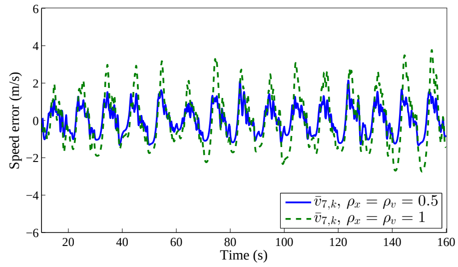
(a) Position error in constant speed traffic condition



(b) Speed error in constant speed traffic condition



(c) Position error in continuous small perturbations



(d) Speed error in continuous small perturbations

Figure 11: Impact of measurement errors on the system performance