Proceedings Paper:

Oguejiofor, OS and Zhang, LX (2016) Global Optimization of Weighted Sum-Rate for Downlink Heterogeneous Cellular Networks. In: Telecommunications (ICT), 2016 23rd International Conference on. 2016 23rd International Conference on Telecommunications (ICT), 16-18 May 2016, Thessaloniki, Greece. IEEE . ISBN 978-1-5090-1990-8

https://doi.org/10.1109/ICT.2016.7500345

Global Optimization of Weighted Sum-Rate for Downlink Heterogeneous Cellular Networks

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Abstract—**It is envisioned that Heterogeneous cellular network is key technology in 5G that can be used to meet the ever increasing demand of data rate. The most critical problem of HetNet is interference. One of our objectives is to design beamformers to mitigate interference and achieve the maximum throughput while satisfying some power and interference constraints. In this paper we are able to determine the global solution of the non-convex NP-hard weighted sum-rate problem using branch and bound method. It involves searching for the best individual rates among many feasible rates achievable in the system that maximizes the weighted sum-rate of the system while fulfilling the power and interference constraints. Results obtained show that our proposed method outperformed other methods such as egoistic beamforming method and the relaxed convex optimization heuristic method which produces sub-optimal solution to the original non-convex problem.**

I. Introduction

Heterogeneous Networks (HetNets) [1], composed of small cells distributed around the coverage area of conventional macrocellular network, are regarded as a cost-efficient solution to improve system coverage and capacity. This improved capacity can only be achieved if good interference management scheme is in place for the system under consideration.

So in this paper, we determine the global solution of the weighted sum-rate maximization problem for a 2-tier HetNet, while using optimized beamforming vectors for spatial separation of user equipment's (UE) signals to manage interference subject to constraints. Maximizing the weighted sum-rate of a system is generally regarded as an NP-hard problem because there are no known efficient algorithms that can solve it in polynomial time. Usually most authors shy away from this problem because of its non-convex nature; instead, local optimization methods are most widely adopted. In local optimization methods [2], the global optimal solution is usually sacrificed for a local optimal solution which can be achieved in polynomial time. Similarly, others solve the reformulated convex version of the non-convex problem, this usually can be solved efficiently and also produces (roughly) global solution, but the downside to it is that the solution found is not for the exact problem hence is suboptimal too. Many works have been done for maximizing the weighted sum-rate of a system but most of these works are targeted at either single tier coordinated multi-cell system [3] or single tier single-cell system [4] where there are no variations in the power class of the base stations (BSs). However, in our work we consider the impact of the the superior interfering power generated by the macro-base station (MBS) to other co-users in the multi-tier heterogeneous system together with the interference between small cells.

In this work we propose an approach which first solves convex feasibility problem and then performs an exhaustive search within the feasible set of the sum-rate optimization problem in order to find the global optimum of the non-convex optimization problem. This method is regarded as branch and bound (B&B) method [5], [10], which was adapted to solve the weighted sum-rate maximization optimization problem of a 2-tier HetNet. In our approach, the feasible set that satisfies the constraints of the optimization problem are represented in a box interval which is assumed to be compact and a subset of the non-negative orthant $\mathbb{R}_{+}^{K}$, where the optimal solution can be selected from. The B&B algorithm efficiently computes a lower bound and upper bound on the optimal value over this box. The lower bound of this box is initially found using heuristic reformulation of the non-convex optimization problem into a convex one, while the upper bound is found by assuming each UE achieved the best individual rate using a beamforming scheme that maximizes individual UE rates. This algorithm is iterative and will only terminate if the difference between the upper bound and the lower bound is smaller than a threshold. If not, the initial box is split into two using bisection method where their respective upper and lower bound are determined again, in each iteration the convex feasibility of the point gotten through line search is checked to make sure it satisfies the constraints of the feasible set, otherwise it is discarded. The iterative process continues until a global optimal value is achieved.

The compromise in achieving the global solution is efficiency. However, in this work we limit our consideration to small number of variables and total number of users considered to ensure efficiency. The rest of the paper is organized as follows. Section 2 describes the system model and problem formulation and it shows how the non-convex problem can be relaxed into convex heuristic problem which can be easily solved by efficient algorithms. It also shows how to formulate convex feasibility problems. Section 3 describes the B&B methods while in section 4 we show using simulation results how our proposed method outperforms other existing methods. We conclude our work in section 5.

Notations: $(\cdot)^H$ is the transpose-conjugate operation, $(\cdot)^T$ is the transpose operation, $\|\cdot\|$ is the norm of a vector, $|\cdot|$ is the magnitude of a complex variable, $\mathbb{E}\{\cdot\}$ is the expectation over a random variable. We use upper-case boldface letters for matrices and lower-case boldface for vectors.

II. System Model

We consider the downlink of a 2-tier HetNet as depicted in Fig. 1, which consist of K_t pico cells and a single macrocell making it a total of K cells in the system. Each BS has N antennas and communicate with a single active UE assumed to have a single effective antenna per cell, making the total number of cells to be equal to the total number of UEs in the system. The pico cells are underlaid in the coverage area of the macro cell and all cells use the same carrier frequency. The respective BSs are connected through a limited backhaul link, hence each BS will only send data to UE belonging to its cell while the beamformers can be jointly optimized by the all the BSs in the network. We denote the set of BSs in the HetNet by $\Upsilon = \{0, 1, \ldots, K_t\}$ where 0 represent the macro BS. The complex-baseband received signal at UE i is $y_i \in \mathbb{C}$ and given by

$$y_i = \sum_{k=1}^{K} \sqrt{g_{i,k}} (\mathbf{h}^s_{i,k})^H \mathbf{x}_k + n_i, \tag{1}$$

where $\sqrt{g_{i,k}}$ is the large-scale pathloss from the kth BS

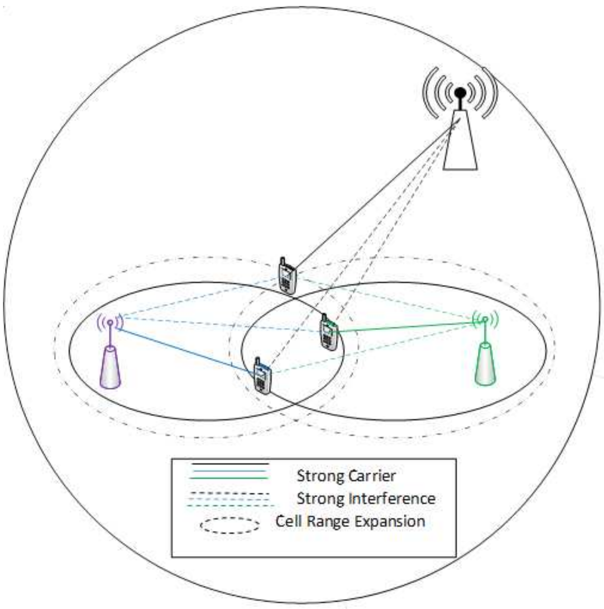

Fig. 1. Downlink 2-tier HetNet model with two Pico cells in the coverage area of MBS

to UE i. Also $\mathbf{h}^s_{i,k} \in \mathbb{C}^{N\times 1}$ is the small scale (fading) channel vector from the kth BS to UE i. $n_i \in \mathbb{C}$ is the additive noise from the surrounding and is modelled as circularly symmetric complex gaussian, distributed as $n_i \sim \mathcal{N}\ (0, \sigma^2)$, where σ^2 is the noise power. Consequently, the signal-to-noise-and-interference ratio (SINR) at UE i is

$$SINR_i = \frac{|\mathbf{h}^H_{i,i}\mathbf{x}_i|^2}{\sigma_i^2 + \sum_{\substack{k\neq i\\ k\in\Upsilon}} |\mathbf{h}^H_{i,k}\mathbf{x}_k|^2}. \tag{2}$$

Where $\mathbf{h}_{i,k} \triangleq \sqrt{g_{i,k}}\mathbf{h}^s_{i,k}$.

A. Coordinated Beamforming: Problem Formulation

Recall that in coordinated beamforming [6], BS i transmit signal to UE i while the beamformers from each BSs are jointly optimized by all BS in the system considered. The transmitted signal by each BS to its served UE is

$$\mathbf{x}_i = \mathbf{w}_i s_i, \tag{3}$$

where $\mathbf{w}_i \in \mathbb{C}^{N\times 1}$ and $s_i \in \mathbb{C}$ are transmit beamforming vector and information symbol for UE i respectively, s_i is normalized to unit power, $\mathbf{E}[|s_i|^2] = 1$.
Hence the achievable data rate for the UE i is

$$r_i(\{\mathbf{w}_i\}) = log_2(1 + SINR_i)\ \forall i = 1, \ldots, K, \tag{4}$$

which can be expressed in a more detailed form as

$$r_i(\{\mathbf{w}_i\}) = log_2\left(1 + \frac{|\mathbf{h}^H_{i,i}\mathbf{w}_i|^2}{\sigma_i^2 + \sum_{k\neq i} |\mathbf{h}^H_{i,k}\mathbf{w}_k|^2}\right), \tag{5}$$

where $\{\mathbf{w}_i\}$ denotes the set of beamforming vectors of the system.
In this paper, the target is to select $\mathbf{w}_i\ \forall i = 1, \ldots, K$, to maximize the weighted sum-rate, which is

$$\begin{aligned} &\underset{\{\mathbf{w}_i\}}{\text{maximize}} \quad \sum_{i=1}^{K} u_i r_i(\{\mathbf{w}_i\}) \\ &\underset{(Const)}{\text{subject to}} \quad \begin{cases} ||\mathbf{w}_i||_2^2 \leq P_m,\ \forall m = 0, m \in \Upsilon, \\ ||\mathbf{w}_i||_2^2 \leq P_s,\ \forall s \neq 0, s \in \Upsilon, \\ \mathbf{w}_m^H \mathbf{G}_{i,m}\mathbf{w}_m \leq \tau_i,\ \forall m = 0. \end{cases} \end{aligned} \tag{6}$$

Where the utility function represents the weighted sum-rate of the system with the nonnegative factor u_i denoting the individual weights assigned to each UE i, determined based on individual channel gain. A larger gain has larger weight and vice versa, also the second, third and fourth row of (6) represent MBS power constraint, low power node (LPN) power constraint and interference power constraint (i.e., interference generated from MBS to UE i), henceforth these constraints will be denoted by $Const$. $\mathbf{G}_{i,m} \triangleq \mathbf{h}_{i,m}\mathbf{h}^H_{i,m}$ is a positive semidefinite (PSD) matrix ($\mathbf{G}_{i,m} \geq \mathbf{0}$), where $\mathbf{h}_{i,m}$ is the channel vector from the MBS to UE i and τ_i is the threshold which controls the allowable level of interference in UE i .
These constraints ($Const$) are convex but the utility function

is not convex thanks to the $SINR_i$ which are non-convex functions of the beamforming vectors $\{\mathbf{w}_i\}$. To make the problem formulation more elaborate it can be rewritten as

$$\begin{aligned} \underset{\{\mathbf{w}_i\}}{\text{minimize}} \quad & -\sum_{i=1}^{K} u_i r_i(\{\mathbf{w}_i\}) \\ \text{subject to} \quad & |\mathbf{h}_{i,i}^H \mathbf{w}_i|^2 \geq \gamma_i(\sigma_i^2 + \sum_{k\neq i} |\mathbf{h}_{i,k}^H \mathbf{w}_k|^2), \\ and \quad & Const\ in\ (6). \end{aligned} \tag{7}$$

In (7) the second row represent the quality of service (QoS) constraint expected of each UE i in the system and is generally known as the $SINR_i$ constraint, where $SINR_i \geq \gamma_i \ \forall i = 1, \dots, K$ and $Const$ denotes all the power and interference constraints as in (6). In this case γ_i denotes the QoS threshold for each UE in the system while the term $\sum_{k\neq i} |\mathbf{h}_{i,k}^H \mathbf{w}_k|^2$ represent the total interference towards the desired UE i. This formulation is still non-convex but in the next subsection, we show how (7), an NP-hard non-convex problem can be made convex.

B. Convex Heuristic Reformulation

To solve the non convex problem, convex heuristics are easily adopted by researchers because of its efficiency. However, it produces suboptimal solution to the non-convex problem. To reformulate (7) into a convex problem, this can be achieved by either fixing the γ_i value at each user or by fixing the interference term. In this paper we prefer limiting the interference to a particular fixed threshold Γ_i which is more practical and not equating it to zero as in the case when zero forcing technique is applied, which is seen as an overreaction. Hence we obtain our convex reformulation as

$$\begin{aligned} \underset{\{\mathbf{w}_i\}}{\text{minimize}} \quad & -\sum_{i=1}^{K} u_i r_i(\{\mathbf{w}_i\}) \\ \text{subject to} \quad & |\mathbf{h}_{i,i}^H \mathbf{w}_i|^2 \geq \gamma_i(\sigma_i^2 + \Gamma_i), \\ & \sum_{k\neq i} \mathbf{w}_k^H (\mathbf{h}_{i,k}\mathbf{h}_{i,k}^H)\mathbf{w}_k \leq \Gamma_i, \\ and \quad & Const\ in\ (6). \end{aligned} \tag{8}$$

Then semidefinite relaxation [7] can be applied to the quadratic terms in (8) after which it can be efficiently solved by a solver known as SeDuMi or SDPT3 implemented in CVX [8] - a Matlab-based modeling system for convex optimization. We will use the result from here to initialize the proposed procedure which will lead to global optimal solution of the non-convex problem. We will also use it as a benchmark to compare with our proposed method.

C. Convex Feasibility Problem

This feasibility problem will help us in our proposed method to always check if a selected solution from a box interval is feasible or not. If not, it can be discarded because it cannot be the optimal solution. Convex feasibility problem is to find any feasible solutions without regard to the utility function. In our case we seek the set of beamformers $\{\mathbf{w}_i\}$ that satisfy the convex constraints. In this case γ_i value is believed to be known a priori but can be computed as $\gamma_i \triangleq 2^{r_i} - 1$ obtainable from (4), hence our convex feasibility problem formulation can be formulated as

$$\begin{aligned} \text{find} \quad & \{\mathbf{w}_i\}\ \forall i = 1, \dots, K \\ \text{subject to} \quad & |\mathbf{h}_{i,i}^H \mathbf{w}_i|^2 \geq \gamma_i(\sigma_i^2 + \sum_{k\neq i} |\mathbf{h}_{i,k}^H \mathbf{w}_k|^2), \\ and \quad & Const\ in\ (6). \end{aligned} \tag{9}$$

In order to be easily solved, the feasibility problem can be formulated as a power control problem such as minimizing some transmitted power in the system subject to QoS constraint, power and interference constraints. If we assume a fraction of the total power in the system to be a non-negative value and denoted as ϑ, also the upperbound of the MBS power constraint and the LPN power constraints of the $Const$ in (6) be replaced with ϑP_m and ϑP_s respectively then the power minimization problem can be formulated as

$$\begin{aligned} \underset{\{\mathbf{w}_{k_i}\},\vartheta}{\text{minimize}} \quad & \vartheta \\ \text{subject to} \quad & |\mathbf{h}_{i,i}^H \mathbf{w}_i|^2 \geq \gamma_i(\sigma_i^2 + \sum_{k\neq i} |\mathbf{h}_{i,k}^H \mathbf{w}_k|^2), \\ and \quad & Const\ in\ (6). \end{aligned} \tag{10}$$

This can be easily solved. Note, the optimization problem in (10) is convex if the SINR constraint is rewritten as a second order cone (SOC) constraint [9]. After we find a feasible solution, we can use other steps in the (B&B) algorithm to obtain the global solution.

III. Branch and Bound Method

Branch and Bound (B&B) method is the method through which we can get the global optimal solution of an NP-hard non-convex weighted sum-rate maximization problem for a 2-tier HetNet. It is an iterative method that requires at least two procedures that can efficiently calculate a lower and an upper bound on the optimal value of the non-convex problem over a given set or region. In our case, the set or region considered is a subset of a box interval. This set is the feasible set that satisfies our problem formulation, also the utility function in our optimization problem is lipschitz continous and increasing over this box interval. We denote the initial box as $\beta = [\mathbf{a}\ \mathbf{b}] \subseteq \mathbb{R}_+^K$, this box is assumed to be compact and normal [11] and houses all kind of rates from the worst to the best rates. $\mathbf{a}$ denotes the worst rate vector achievable by UEs in the system thus $\mathbf{a} = \mathbf{0} \in \mathbb{R}_+^K$ while $\mathbf{b} \in \mathbb{R}_+^K$ is the best rate vector achievable by UEs in the system using egoistic beamforming scheme such that $\mathbf{a} < \mathbf{b}$, also $[\mathbf{a}\ \mathbf{b}]$ is defined to be the set of all rates achievable in the system such that $\mathbf{a} \leq \mathbf{r} \leq \mathbf{b}$. Egoistic beamforming is a beamforming scheme where beamformers are designed to maximize the array gain of a single UE in a system. Note this beamforming scheme will always be suboptimal if there are other sources of interference, hence

$$\mathbf{b} = [b_1 \dots b_K]^T = log_2 \left(1 + \frac{|\mathbf{h}_{i,i}^H \mathbf{w}_i|^2}{\sigma_i^2} \right) \forall i = 1, \dots, K. \tag{11}$$

This best rate vector is not always feasible when co-users interference are considered in the system while designing the beamformers.

Our feasible set from the original problem formulation for the r_i that optimizes the sum-rate can be denoted as

$$\chi = \left\{ (r_1, \dots, r_K) | (\mathbf{w}_1, \dots, \mathbf{w}_K) \in \mathbb{C}^{N \times 1}, Const\ as\ in(6) \right\}. \tag{12}$$

Where χ denotes the set of all feasible solution $(r_1, \dots, r_K)$ for which $(\mathbf{w}_1, \dots, \mathbf{w}_K)$ is feasible and satisfy the $Const$ in (6). Therefore, our optimization problem for maximizing the sum-rate of the system in this section is equivalent to searching for a feasible solution in the box that has the minimum L-2 norm to $\mathbf{b}$, and this is formulated as

$$\begin{aligned} \underset{\{\mathbf{r}\}}{\text{maximize}} \quad & f(\mathbf{r}) \\ \text{subject to} \quad & \mathbf{r} \in \chi. \end{aligned} \tag{13}$$

Where our utility function is denoted as

$$f(\mathbf{r}) = \sum_{i=1}^{K} u_i r_i, \tag{14}$$

where $\mathbf{r} = [r_1 \dots r_K]^T$ is the rate vector achievable by UEs in the system. The lower bound on the optimal value of the non-convex problem can be found from its convex relaxation, and in this paper (8) gives a feasible solution on the optimal solution of the non-convex problem. Let $\mathbf{r}$ denotes this feasible solution of the box β, hence the lower bound on the optimal value of this box is denoted $f_{min}^{\beta}(\mathbf{r})$. Similarly, since $\mathbf{b}$ represent the best rate vector in the system, though might not be feasible, $f_{max}^{\beta}(\mathbf{b})$ denotes the upper bound on the optimal value of this box hence $f_{min} \leq f_{opt} \leq f_{max}$. Where f_{opt} denotes the optimal value of the sum-rate of the system, f_{min} and f_{max} denote lower bound and upper bound on the optimal value of the weighted sum-rate of the system respectively, also $\mathbf{0} \leq \mathbf{r}_{opt} \leq \mathbf{b}$ where $\mathbf{r}_{opt}$ denotes the optimal solution of the system while $\mathbf{0}$ and $\mathbf{b}$ denote the worst feasible solution and the best feasible solution achievable in the system .

A. Branching

This is the process of spliting the initial box β into more than one partitions. Branching will only be necessary if $f_{max} - f_{min} > \epsilon$. Where ϵ is the accuracy of the sum-rate in the B&B method. The splitting of box β is done along the longest edge using line bisection principles in Euclidean space, after which the upper and lower bound on the optimal value are calculated for each. After splitting, $\beta = \beta_1 \cup \beta_2$ where β_1 denotes box 1 and β_2 denotes box 2. Assuming $\beta_1 = [\mathbf{a}_1\ \mathbf{b}_1]$ and $\beta_2 = [\mathbf{a}_2\ \mathbf{b}_2]$ where $\mathbf{a}_1$ and $\mathbf{a}_2$ denote the lower conners of boxes 1 and 2 respectively, also $\mathbf{b}_1$ and $\mathbf{b}_2$ denote the upper conners of boxes 1 and 2 respectively. Note that $\mathbf{b}_2 = \mathbf{b}$ and $\mathbf{a}_1 = \mathbf{a}$ of the initial box β . The feasible solution of the new boxes can be chosen by comparing the feasible solution of the initial box to the lower conner of box 2, if greater than or equal to it, will give rise to a new feasible solution for boxes 1 and 2, which can be computed as

$$\begin{aligned} \mathbf{r}^{\beta_1} &= \begin{cases} \mathbf{r} - [\mathbf{r} - \mathbf{b}_1], & \mathbf{r} \geq \mathbf{a}_2, \\ \mathbf{r}, & \text{otherwise}, \end{cases} \\ \mathbf{r}^{\beta_2} &= \mathbf{r}, \end{aligned} \tag{15}$$

summarily, the new feasible solution for boxes 1 and 2 becomes $\mathbf{r}$ and $\mathbf{r}$ if for box 1, $\mathbf{r} \leq \mathbf{a}_2$ in the first row of (15). While the upper bounds on the optimal value for both boxes can also be chosen as

$$\begin{aligned} f_{max}^{\beta_1} &= min(f(\mathbf{b}_2),\ f(\mathbf{b}_1)), \\ f_{max}^{\beta_2} &= f(\mathbf{b}_2) \end{aligned} \tag{16}$$

respectively. Where the $\min(\cdot)$ operator selects the smallest value of its argument.

Futhermore, we shall proceed by removing parts of the boxes which cannot contain the optimal solution, knowing that $f_{min}^{\beta}(\mathbf{r}) \leq f_{opt} \leq f_{max}^{\beta}(\mathbf{b}2)$, note that $\mathbf{b}_2 = \mathbf{b}$. This can be done by checking for any part that is less than $f_{min}^{\beta}(\mathbf{r})$ or greater than $f_{max}^{\beta}(\mathbf{b}2)$ which cannot contain the optimal solution.

Generally we assume w.l.o.g that $f_{max}^{\beta_t}$ $\forall t = 1, 2$ is non increasing while $f_{min}^{\beta_t}$ $\forall t = 1, 2$ is non decreasing. After prunning of the boxes, the lower conners of the new boxes can be computed as

$$\begin{aligned} \tilde{\mathbf{a}}_1 &= (1 - \nu^{\beta_1}) \mathrm{b}_1^k + \nu^{\beta_1} \mathrm{a}_1^k, \\ \tilde{\mathbf{a}}_2 &= (1 - \nu^{\beta_2}) \mathrm{b}_2^k + \nu^{\beta_2} \mathrm{a}_2^k, \end{aligned} \tag{17}$$

where $\tilde{\mathbf{a}}_1$ and $\tilde{\mathbf{a}}_2$ denote the lower conners of the new boxes after prunning, b_1^k and a_1^k also denote the kth element of the upper and lower conners of the new boxes respectively.

The parameter ν^{β_t} can be gotten through line search and can take values between zero and one, it is computed as

$$\nu^{\beta_t} = \frac{f(\mathrm{b}_t)^k - f_{min}^{\beta}(\mathbf{r})}{f(\mathrm{b}_t^k - \mathrm{a}_t^k)}, \ \forall t = 1, 2, \ k = 1, \dots, K. \tag{18}$$

Similarly, the upper conners of the new boxes can be computed as

$$\begin{aligned} \tilde{\mathbf{b}}_1 &= (1 - \mu^{\beta_1}) \tilde{\mathbf{a}}_1 + \mu^{\beta_1} \mathrm{b}_1^k, \\ \tilde{\mathbf{b}}_2 &= (1 - \mu^{\beta_2}) \tilde{\mathbf{a}}_2 + \mu^{\beta_2} \mathrm{b}_2^k, \end{aligned} \tag{19}$$

while parameter μ^{β_t} can also be computed as

$$\mu^{\beta_t} = \frac{f_{max}^{\beta_t} - f(\tilde{\mathbf{a}}_t)}{f(\mathrm{b}_t^k - \mathrm{a}_t^k)}, \ \forall t = 1, 2, \ k = 1, \dots, K. \tag{20}$$

The prunned new boxes are denoted as $\tilde{\beta}_t = [\tilde{\mathbf{a}}_t \; \tilde{\mathbf{b}}_t] \; \forall t = 1, 2$. One of these boxes contain the optimal value, and the most likely one is box 2. This is because maximizing the weighted sum-rate is equivalent to searching for the feasible point with the minimum L-2 norm to the best infeasible individual rate achievable in the system. We check if this box is feasible by solving (10) using $\tilde{\mathbf{a}}_2$ to get the QoS constraint. This leads us to bounding procedure in the next subsection.

B. Bounding

If the box is feasible, bounding procedure involves searching for the best lower and upper bound on the optimal value in each iteration using line search technique. This line search corresponds to looking for the best feasible point with minimum Euclidean length to the best infeasible individual rates in the box. This is achieved by starting with an initial feasible point $\tilde{\mathbf{a}}_2$ which is then added to the product of the step size (positive scalar) and the search direction. Where the search direction is denoted as $s_d = \frac{(\tilde{\mathbf{b}}_2 - \tilde{\mathbf{a}}_2)}{||(\tilde{\mathbf{b}}_2 - \tilde{\mathbf{a}}_2)||_1}$, also the step size is denoted as $\alpha \in [0, \; ||(\tilde{\mathbf{b}}_2 - \tilde{\mathbf{a}}_2)||_1]$ whose set is searched for the best value using line bisection search method; also every value selected must satisfy the feasibility condition, the bounding procedure can be computed for any box using

$$\mathbf{n} = \tilde{\mathbf{a}}_2 + \alpha s_d \tag{21}$$

where $\mathbf{n}$ is a feasible point better than $\mathbf{a}_2$. In each iteration we check to know if the present optimal value of the feasible point is greater than the previous ones, if so we finally update the value to be the best optimal value based on the feasible point. Finally we set $f_{min} = max\big(f^{\beta}_{min}(\mathbf{r}), f^{\tilde{\beta}_t}_{min}(\mathbf{n})\big)$, and $f_{max} = max\big(f^{\beta}_{max}(\mathbf{b}), f^{\tilde{\beta}_t}_{max}(\tilde{\mathbf{b}})\big) \; \forall t = 1, 2$.
We summarized the B&B method using Algorithm 1.

Algorithm 1 Branch and Bound Method

Require: B&B accuracy tolerance $\epsilon > 0$
Require: compute best infeasible individual rate $\mathbf{b}$ using (11);
Require: compute feasible solution $\mathbf{r}$ of initial box using (8);
Require: initial box $\beta = [\mathbf{a} \; \mathbf{b}]$;
Ensure: $f_{min} = f^{\beta}_{min}(\mathbf{r})$ and $f_{max} = f^{\beta}_{min}(\mathbf{b})$;
while $f_{max} - f_{min} > \epsilon$ **do**
  split the initial box along the longest edge;
  prune the new boxes using (18),(20);
  check feasibility of the outermost box using (10);
  If feasible,
  apply bounding procedures using (21);
  obtain best feasible point $\mathbf{n}$ and upper bound $f^{\tilde{\beta}_t}_{max}$;
  set $f_{min} = max\big(f^{\beta}_{min}(\mathbf{r}), f^{\tilde{\beta}_t}_{min}(\mathbf{n})\big)$;
  set $f_{max} = max\big(f^{\beta}_{max}(\mathbf{b}), f^{\tilde{\beta}_t}_{max}(\tilde{\mathbf{b}})\big)$;
end while
Ensure: final optimal bound $[f_{min}, f_{max}]$;
Ensure: final optimal solution $\mathbf{r}_{opt} = max(\mathbf{r}, \mathbf{n})$.

IV. Simulation

Our considered HetNet system model is depicted in Fig. 1. The Simulation parameters are as follows: the transmit powers of the macro and pico BSs are respectively 46dBm and 30dBm, while the receiver noise power is -75dBm. The large-scale path loss model of the macro and pico cells are respectively $PL(dB) = 128.1 + 37.6log(\frac{d_0}{10^3})$ and $PL(dB) = 140.7 + 36.7log(\frac{d_0}{10^3})$ where d_0 is the distance of a user to the BS. The channel vectors are generated as uncorrelated rayleigh fading while the large-scale pathloss given by

$$\sqrt{g_{i,k}} = \frac{\beta}{d^n_{i,k}}, \tag{22}$$

where β is a constant which accounts for system losses and can be determine through the large scale path loss models for both macro and pico cells respectively. n is the path-loss exponent, typically $n > 2$, while $d_{i,k}$ is the distance between kth BS and the ith UE. The default system setting for the simulation are as follows; $N = 3, K = 3$. 10000 monte carlo runs are used for the channel realizations, while the maximum number of iteration and evaluation function for the B&B algorithm are 3000 and 4000 respectively. The B&B accuracy tolerance $\epsilon = 0.001$. This settings will be used except otherwise indicated. Fig. 2 shows the average sum-rate achievable as a function of SNR. It compares the average sum-rate achieved in the system using our proposed method, the heuristic convex method and the egoistic beamforming method. Our proposed method outperforms both methods in both low and high SNR, the lowest performing method is achieved by egoistic beamforming which shows single cell processing without beamforming coordination.

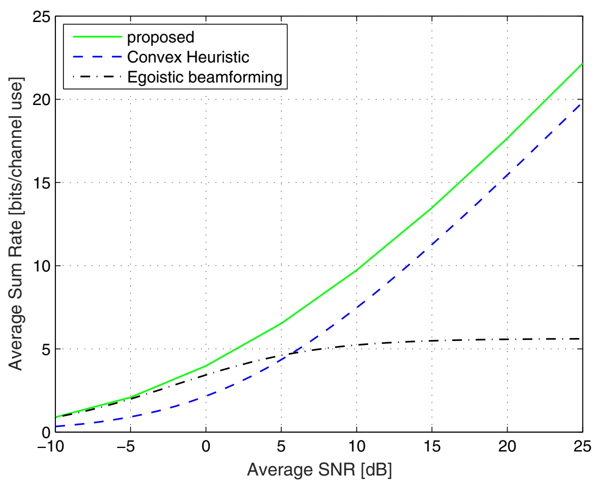

Fig. 2. Average sum-rate achievable at different SNR for $N = 3$

In Fig. 3 the cumulative distribution function (CDF) of user average rate achieved for the system by different methods are illustrated clearly. The proposed B&B scheme outperforms the heuristic convex and the egoistic schemes.

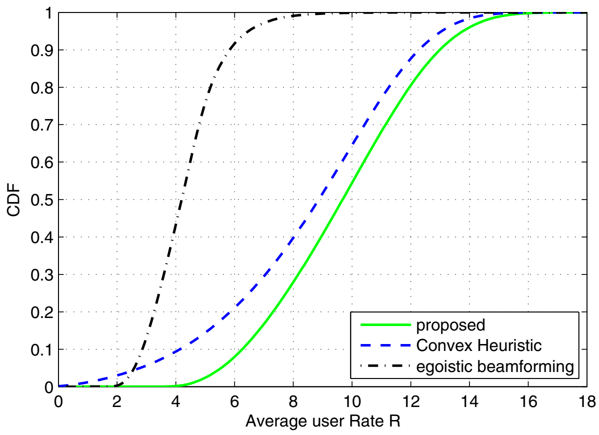

Fig. 3. The CDF of the users rate achieved by different methods

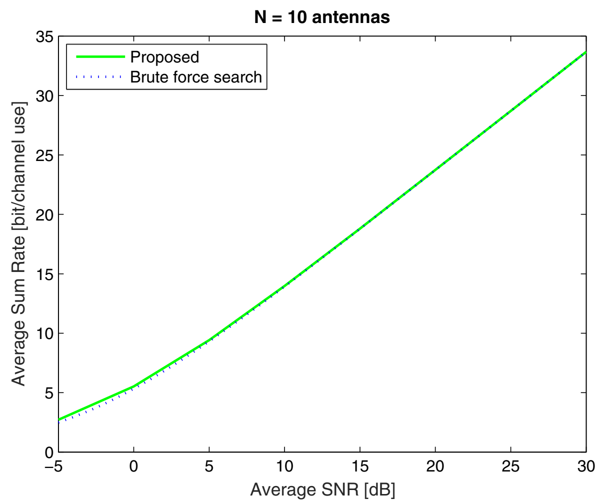

Fig. 4. Average sum-rate achievable at different SNR for $N = 10$

Fig. 4 compares our proposed method with the brute force search method which also gives global optimum solution and is usually a baseline for global convergence of non-convex optimization problems. The result shows that our proposed method only slightly outperforms the brute force search method at low SNR between -5dB and 5dB. Nevertheless, the brute force search method is not recommended because of computational complexity involved in each iteration where the utility function is evaluated for each feasible solution in the search space. However, our proposed method involves an intelligent search procedure that searches only parts of the feasible set that contain the optimal solution thereby reducing the computational complexity of our proposed algorithm. Having said that, the brute force search method is not recommended for implementation in a system setting with more than 6 UEs.

V. Conclusion

In this paper, we have shown that B&B method can outperform popular methods using relaxed convex-optimization for finding the optimal solution to non-convex NP-hard weighted sum-rate problem in HetNet. B&B method involves searching of a box interval to get the best feasible solution that maximizes the weighted sum-rate of the system; but this search is not like the brute force search that brings a lot of computational complexity. It is more of an intelligent search because only part of the box that contain the optimal solution is searched, hence reducing computational complexity. The search can be proved to be global because the utility function which is maximized is Lipschitz continuous and increasing over the box interval. A function $f : [\mathbf{a}\ \mathbf{b}] \to \mathbb{R}$ is said to be lipschitz continuous with lipschitz constant L_f, if $|f(\mathbf{r}) - f(\hat{\mathbf{r}})| \leq L_f||\mathbf{r} - \hat{\mathbf{r}}||_1$, $\forall\ \mathbf{r}, \hat{\mathbf{r}} \in [\mathbf{a}\ \mathbf{b}]$ and $\mathbf{r} \geq \hat{\mathbf{r}}$. The global optimal value is guaranteed because f_{min} is nondecreasing in the box while f_{max} is non increasing in the box.

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