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Case II: Formulation where the Piezo-Electrolytic Process is Regulated by a Feedback Control Law

The system model is modified by adding a feedback control law $U(t)$ to give Equation (1).

$$\dot{Q}_a(t) = m(V(t) + U(t)) + \Delta I(t) \tag{1}$$

The derivative of the surface $\dot{s}(t)$ and the control law $U(t)$ are expressed as (2) and (3) respectively.

$$\dot{s}(t) = \lambda m(V(t) + U(t)) + \lambda \Delta I(t) - \lambda \dot{Q}_d(t) \tag{2}$$

$$U(t) = -K_d s(t) - \hat{V} + \hat{\phi}\dot{Q}_d(t) - k^* sat\Big(\frac{s}{\epsilon}\Big) \tag{3}$$

$V(t)$ and ϕ are unknown during the control design but are estimated as $\hat{V}$ and $\hat{\phi}$ respectively. Hence, $\tilde{V} = \hat{V} - V$ and $\tilde{\phi} = \hat{\phi} - \phi$. The Lyapunov candidate W used to obtain the adaptive laws while guaranteeing system stability is given as 4 and the derivative of W along the system trajectory is expressed in (5).

$$W(t) = \frac{1}{2}\Big(\frac{1}{m}s_\epsilon^2 + \frac{1}{\gamma}\tilde{\phi}^2 + \frac{1}{\Gamma}\tilde{V}^2\Big) \tag{4}$$

$$\dot{W}(t) = \frac{1}{m}s_\epsilon \dot{s} + \frac{1}{\gamma}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{\Gamma}\tilde{V}\dot{\hat{V}} \tag{5}$$

Equation (2) and (3) are substituted into (5) to give (6).

$$\dot{W}(t) = -\lambda K_d s_\epsilon s + \lambda s_\epsilon\Big(V(t) - \hat{V} + \hat{\phi}\dot{Q}_d(t) - k^* sat\Big(\frac{s}{\epsilon}\Big)\Big) + \lambda s_\epsilon\Big(\frac{\Delta I(t)}{m} - \phi \dot{Q}_d(t)\Big)$$
$$+\frac{1}{\gamma}\tilde{\phi}\dot{\hat{\phi}} + \frac{1}{\Gamma}\tilde{V}\dot{\hat{V}} \tag{6}$$

From (6), the adaptive laws become (7) and (8).

$$\dot{\hat{\phi}} = -\gamma\lambda\dot{Q}_d(t)s_\epsilon \tag{7}$$

$$\dot{\hat{V}} = \lambda\Gamma s_\epsilon \tag{8}$$

Substituting the adaptive laws (7) and (8) into (6) gives the expression in (9).

$$\dot{W}(t) = -\lambda K_d s_\epsilon s - \lambda k^* s_\epsilon sat\Big(\frac{s}{\epsilon}\Big) + \frac{\Delta I(t)}{m}\lambda s_\epsilon \tag{9}$$

$$\dot{W}(t) = -\lambda K_d s_\epsilon\Big(s_\epsilon + \epsilon sat\Big(\frac{s}{\epsilon}\Big)\Big) - \lambda k^* s_\epsilon sat\Big(\frac{s}{\epsilon}\Big) + \frac{\Delta I(t)}{m}\lambda s_\epsilon \tag{10}$$

When $|s| \le \epsilon$, $|s_\epsilon| = 0$ and (10) is zero (11).

$$\dot{W}(t) = 0 \qquad\qquad \forall |s| \le \epsilon \tag{11}$$

When $|s| > \epsilon$, $|s_\epsilon| = s_\epsilon sat(s/\epsilon)$. By also taking into account $k^* \ge \rho/m_{min}$, (10) is expressed to give (14).

$$\dot{W}(t) = -\lambda K_d s_\epsilon^2 - (K_d\epsilon + k^*)\lambda|s_\epsilon| + \frac{\Delta I(t)}{m}\lambda s_\epsilon \tag{12}$$

$$\dot{W}(t) \leq -\lambda K_d s_\epsilon^2 - K_d \epsilon \lambda |s_\epsilon| - \Big(k^* - \frac{\Delta I(t)}{m}\Big)\lambda |s_\epsilon| \tag{13}$$

$$\dot{W}(t) \leq -\lambda K_d s_\epsilon^2 \qquad \forall |s| > \epsilon \tag{14}$$

Again, the above formulations in (11) and (14) indicate that s_ϵ, $\tilde{\phi}$ and $\tilde{V}$ are globally bounded. This also means that $s(t)$ is bounded and the control design guarantees that the system trajectory will converge to the sliding mode.

Rectified Sinusoidal Input

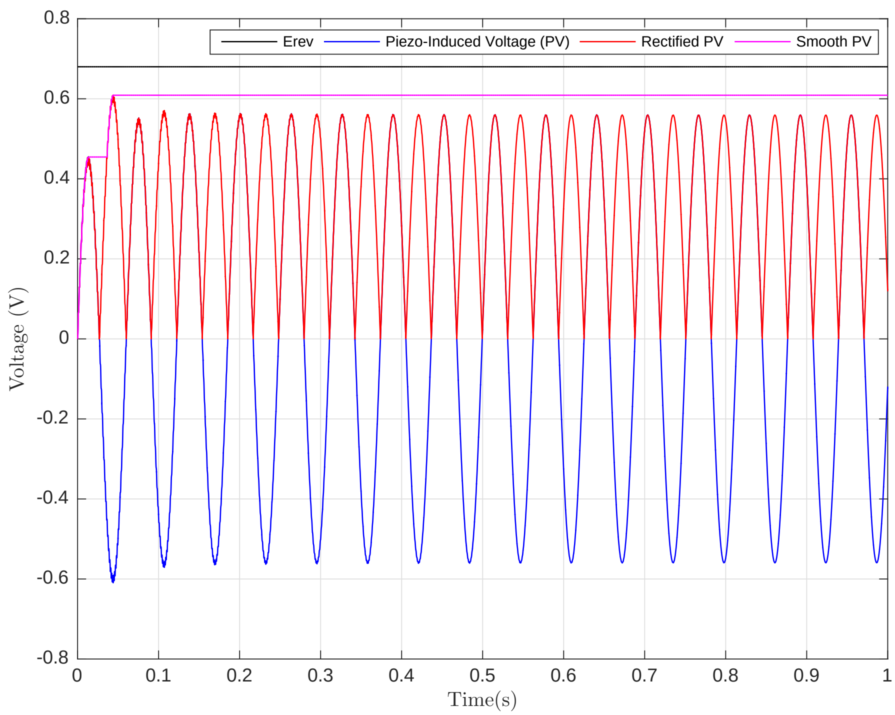

Figure 1. When a sinusoidal load with an amplitude of $300N$ is applied to the self-healing material, the piezoelectric direct effect generates a sinusoidal voltage of amplitude $559mV$. The resulting sinusoidal voltage is rectified and smoothed to create the dc voltage required for electrolysis.