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Dilatonic Black Holes in Higher Curvature String Gravity

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Abstract

We give analytical arguments and demonstrate numerically the existence of black hole solutions of the $4D$ Effective Superstring Action in the presence of Gauss-Bonnet quadratic curvature terms. The solutions possess non-trivial dilaton hair. The hair, however, is of "secondary type", in the sense that the dilaton charge is expressed in terms of the black hole mass. Our solutions are not covered by the assumptions of existing proofs of the "no-hair" theorem. We also find some alternative solutions with singular metric behaviour, but finite energy. The absence of naked singularities in this system is pointed out.

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1 Introduction

It has become evident in recent years that the properties of black holes are modified when the theory of matter fields has sufficient structure. In the presence of the low energy degrees of freedom characteristic of String Theory [1], i.e. dilatons, axions and Abelian or Yang-Mills fields, it is possible to have non-trivial static configurations for these fields outside the horizon, i.e. have black holes with hair [2] [3]. It is not clear however whether these cases, in which the "no-hair theorem" [4] does not apply [5], represent stable solutions. Explicit black hole solutions have been found also in string-effective theories involving higher-order curvature corrections to the Einstein gravity. They exhibit secondary hair of the dilaton, axion and modulus fields. The solutions were approximate, in the sense that only a perturbative analysis to $\mathcal{O}(\alpha')$ [6] and $\mathcal{O}(\alpha'^2)$ [7] has been performed. This analysis motivates the search for exact (to all orders in α') solutions within the framework of curvature-squared corrections to Einstein's theory. Although the effect of the higher-order curvature terms is not small for energy scales of order α', from a local field theory point of view it makes sense to look for this kind of solutions, with the hope of drawing some useful conclusions that might be of relevance to the low-energy limit of string theories.

In the present article we shall demonstrate the existence of black hole solutions of the Einstein-dilaton system in the presence of the higher-derivative, curvature squared terms. These solutions will be endowed with a non-trivial dilaton field outside the horizon, thus possessing dilaton hair. The treatment of the quadratic terms will be non-perturbative and the solutions are present for any value of α'/g^2. What we shall argue in this paper is that the presence of these terms provides the necessary 'repulsion' in the effective theory that balances the gravitational attraction, thereby leading to black holes dressed with non-trivial classical dilaton hair. An analogous phenomenon occurs already in the case of Einstein-Yang-Mills systems[3]. There, the presence of the non-abelian gauge field repulsion balances the gravitational attraction leading to black hole solutions with non-trivial gauge and scalar (in Higgs systems) hair.

It is useful to discuss briefly the situation in effective theories obtained from the string. We shall concentrate on the bosonic part of the gravitational multiplet which consists of the dilaton, graviton, and antisymmetric tensor fields. In this work we shall ignore the antisymmetric tensor for simplicity[1]. As is well known in low-energy effective field theory, there are ambiguities in the coefficients of such terms, due to the possibility of *local* field redefinitions which leave the S-matrix amplitudes of the effective field theory invariant, according to the *equivalence* theorem. To $\mathcal{O}(\alpha')$ the

[1] In four dimensions, the antisymmetric tensor field leads to the axion hair, already discussed in ref.[6]. Modulo unexpected surprises, we do not envisage problems associated with its presence as regards the results discussed in this work, and, hence, we ignore it for simplicity.

freedom of such redefinitions is restricted to two generic structures, which cannot be removed by further redefinitions [8]. One is a curvature-squared combination, and the other is a four-derivative dilaton term. Thus, a generic form of the string-inspired $\mathcal{O}(\alpha')$ corrections to Einstein's gravitation have the form

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}(\partial_\mu\phi)^2 + \frac{\alpha'}{8g^2}e^{\phi}(c_1\mathcal{R}^2 + c_2(\partial_\rho\phi)^4) \tag{1}$$

where α' is the Regge slope, g^2 is some gauge coupling constant (in the case of the heterotic string that we concentrate for physical reasons), and $\mathcal{R}^2$ is a generic curvature-dependent quadratic structure, which can always be fixed to correspond to the Gauss-Bonnet (GB) invariant

$$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2 \tag{2}$$

The coefficients c_1, c_2 are fixed by comparison with string scattering amplitude computations, or σ-model β-function analysis. It is known that in the three types of string theories, Bosonic, Closed-Type II Superstring, and Heterotic Strings, the ratio of the c_1 coefficients is 2:0:1 respectively [8]. The case of superstring II effective theory, then, is characterized by the absence of curvature-squared terms. In such theories the fourth-order dilaton terms can still be, and in fact they are, present. In such a case, it is straightforward to see from the modern proof of the no-scalar hair theorem of ref. [4] that such theories, cannot sustain to order $\mathcal{O}(\alpha')$, any non-trivial dilaton hair. On the other hand, the presence of curvature-squared terms can drastically change the situation, as we shall discuss in this article. There is a simple reason to expect that in this case the no-scalar-hair theorem can be bypassed. In the presence of curvature-squared terms, the modified Einstein's equation leads to an effective stress tensor that involves the gravitational field. This implies that the assumption of positive definiteness of the time-component of this tensor, which in the Einstein case is the local energy density of the field, may, and as we shall show it does indeed, break down. The second, but equally important, reason, is that as a result of the higher-curvature terms, there is an induced modification of the relation $T^t_t = T^\theta_\theta$ between the time and angular components of the stress tensor, which was valid in the case of spherically-symmetric Einstein theories of ref. [4].

The structure of the article is the following: In section 2 we give analytic arguments for the existence of scalar (dilaton) hair of the black hole solution, which bypasses the conditions for the no-hair theorem. In section 3 we present an analysis of the black hole solutions. In section 4 we discuss alternative solutions, some of which are interesting due to the finite energy-momentum tensor they possess. Finally, conclusions and outlook are presented in section 5.

2 Existence of Hair in Gravity with a Gauss-Bonnet term: analytic arguments

Following the above discussion we shall ignore, for simplicity, the fourth-derivative dilaton terms in (1), setting from now on $c_2 = 0$. However, we must always bear in mind that such terms are non-zero in realistic effective string cases, once the GB combination is fixed for the gravitational $\mathcal{O}(\alpha')$ parts. Then, the lagrangian for dilaton gravity with a Gauss Bonnet term reads

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}(\partial_\mu\phi)^2 + \frac{\alpha'}{8g^2}e^\phi R^2_{GB} \tag{3}$$

where R^2_{GB} is the Gauss Bonnet (GB) term (2).

As we mentioned in the introduction, although we view (3) as a heterotic-string effective action, for simplicity, in this paper we shall ignore the modulus and axion fields, assuming reality of the dilaton ($S = e^{i\phi}$ in the notation of ref. [6]). We commence our analysis by noting that the dilaton field and Einstein's equations derived from (3), are

$$\frac{1}{\sqrt{-g}}\partial_\mu[\sqrt{-g}\partial^\mu\phi] = -\frac{\alpha'}{4g^2}e^\phi R^2_{GB} \tag{4}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}g_{\mu\nu}(\partial_\rho\phi)^2 - \alpha'\mathcal{K}_{\mu\nu} \tag{5}$$

where

$$\mathcal{K}_{\mu\nu} = (g_{\mu\rho}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\rho})\eta^{\kappa\lambda\alpha\beta}D_\gamma[\tilde{R}^{\rho\gamma}{}_{\alpha\beta}\partial_\kappa f] \tag{6}$$

and

$$\begin{aligned} \eta^{\mu\nu\rho\sigma} &= \epsilon^{\mu\nu\rho\sigma}(-g)^{-\frac{1}{2}} \\ \epsilon^{0ijk} &= -\epsilon_{ijk} \\ \tilde{R}^{\mu\nu}{}_{\kappa\lambda} &= \eta^{\mu\nu\rho\sigma}R_{\rho\sigma\kappa\lambda} \\ f &= \frac{e^\phi}{8g^2} \end{aligned} \tag{7}$$

From the right-hand-side of the modified Einstein's equation (5), one can construct a conserved "energy momentum tensor", $\nabla_\mu T^{\mu\nu} = 0$,

$$T_{\mu\nu} = \frac{1}{2}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}g_{\mu\nu}(\partial_\rho\phi)^2 + \alpha'\mathcal{K}_{\mu\nu} \tag{8}$$

It should be stressed that the time component of $-T_{\mu\nu}$, which in Einstein's gravity would correspond to the local energy density $\mathcal{E}$, may *not be positive* . Indeed, as

we shall see later on, for spherically-symmetric space times, there are regions where this quantity is negative. The reason is that, as a result of the higher derivative GB terms, there are contributions of the gravitational field itself to $T_{\mu\nu}$. From a string theory point of view, this is reflected in the fact that the dilaton is part of the string gravitational multiplet. Thus, this is the *first* important indication on the possibility of evading the no-scalar-hair theorem of ref. [4] in this case. However, this by itself is not sufficient for a rigorous proof of an evasion of the no-hair conjecture. We shall come to this point later on.

At the moment, let us consider a spherically symmetric space-time having the metric

$$ds^2 = -e^{\Gamma}dt^2 + e^{\Lambda}dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2) \tag{9}$$

where Γ, Λ depend on r solely. Before we proceed to study the above system it is useful to note that if we turn off the Gauss-Bonnet term, equation (4) can be integrated to give $\phi' \sim \frac{1}{r^2}e^{(\Lambda-\Gamma)/2}$. A black hole solution should have at the horizon r_h the behaviour $e^{-\Gamma}$, $e^{\Lambda} \to \infty$. Therefore the radial derivative of the dilaton would diverge on the horizon resulting into a divergent energy-momentum tensor

$$T^t_t = -T^r_r = T^{\theta}_{\theta} = -\frac{e^{-\Lambda}}{4}\phi'^2 \to \infty \tag{10}$$

Rejecting this solution we are left with the standard Schwarzschild solution and a trivial (ϕ = *constant*) dilaton, in agreement with the no-hair theorem. This behaviour will be drastically modified by the Gauss-Bonnet term.

The r component of the energy-momentum conservation equations reads:

$$\partial_r[\sqrt{-g}T^r_r] - \frac{1}{2}\sqrt{-g}(\partial_r g_{\alpha\beta})T^{\alpha\beta} = 0 \tag{11}$$

The spherical symmetry of the space-time implies $T^{\theta}_{\theta} = T^{\varphi}_{\varphi}$. Then eq. (11) becomes:

$$(e^{(\Gamma+\Lambda)/2}r^2T^r_r)' = \frac{1}{2}e^{(\Gamma+\Lambda)/2}r^2[\Gamma'T^t_t + \Lambda'T^r_r + \frac{4}{r}T^{\theta}_{\theta}] \tag{12}$$

where the prime denotes differentiation with respect to r. It can be easily seen that the terms containing Λ cancel to give

$$(e^{\Gamma/2}r^2T^r_r)' = \frac{1}{2}e^{\Gamma/2}r^2[\Gamma'T^t_t + \frac{4}{r}T^{\theta}_{\theta}] \tag{13}$$

Integrating over the radial coordinate r from the horizon r_h to generic r yields

$$T^r_r(r) = \frac{e^{-\Gamma/2}}{2r^2}\int_{r_h}^{r} e^{\Gamma/2}r^2[\Gamma'T^t_t + \frac{4}{r}T^{\theta}_{\theta}]dr \tag{14}$$

The boundary terms on the horizon vanish, since scalar invariants such as $T_{\alpha\beta}T^{\alpha\beta}$ are finite there. For the first derivative of T^r_r we have

$$\begin{aligned}(T^r_r)'(r) &= \frac{\Gamma'}{2}T^t_t + \frac{2}{r}T^\theta_\theta - \frac{e^{-\Gamma/2}}{r^2}(e^{\Gamma/2}r^2)'T^r_r \\ &= \frac{\Gamma'}{2}(T^t_t - T^r_r) + \frac{2}{r}(T^\theta_\theta - T^r_r) \end{aligned} \tag{15}$$

Taking into account (8) and (9), one easily obtains

$$\begin{aligned}T^t_t &= -e^{-\Lambda}\frac{\phi'^2}{4} - \frac{\alpha'}{g^2r^2}e^{\phi-\Lambda}(\phi'' + \phi'^2)(1-e^{-\Lambda}) + \frac{\alpha'}{2g^2r^2}e^{\phi-\Lambda}\phi'\Lambda'(1-3e^{-\Lambda}) \\ T^r_r &= e^{-\Lambda}\frac{\phi'^2}{4} - \frac{\alpha'}{2g^2r^2}e^{\phi-\Lambda}\phi'\Gamma'(1-3e^{-\Lambda}) \\ T^\theta_\theta &= -e^{-\Lambda}\frac{\phi'^2}{4} + \frac{\alpha'}{2g^2r}e^{\phi-2\Lambda}[\Gamma''\phi' + \Gamma'(\phi'' + \phi'^2) + \frac{\Gamma'\phi'}{2}(\Gamma' - 3\Lambda')] \end{aligned} \tag{16}$$

In the relations (16) there lies the *second* reason for a possibility of an evasion of the no-hair conjecture. Due to the presence of the higher curvature contributions , the relation $T^t_t = T^\theta_\theta$ assumed in ref. [4], is no longer valid. The alert reader must have noticed, then, the similarity of the rôle played by the Gauss-Bonnet $\mathcal{O}(\alpha')$ terms in the lagrangian (3) with the case of the non-Abelian gauge black holes studied in ref. [5]. There, the presence of the non-abelian gauge field repulsive forces also lead to non-trivial contributions to $T^\theta_\theta \neq T^t_t$, leading to a sort of 'balancing' between this repulsion and the gravitational attraction. We stress once, again, however, that in our case *both* the non-positivity of the "energy-density" T^t_t, and the modified relation $T^t_t \neq T^\theta_\theta$, play equally important rôles in leading to a possibility of having non-trivial classical scalar (dilaton) hair in GB black holes systems. Below we shall demonstrate rigorously this, by showing that there is *no contradiction* between the results following from the conservation equation of the "energy-momentum tensor" $T_{\mu\nu}$ and the field equations, in the presence of non trivial dilaton hair.

One can define functionals $\mathcal{E}$, $\mathcal{J}$ and $\mathcal{G}$ by

$$\begin{aligned}\mathcal{E} &= -T^t_t \\ \mathcal{J} &= T^r_r - T^t_t \\ \mathcal{G} &= T^\theta_\theta - T^t_t \end{aligned} \tag{17}$$

Then, we can rewrite (14), (15) in the form

$$\begin{aligned}T^r_r(r) &= \frac{e^{-\Gamma/2}}{r^2}\int_{r_h}^r [-(e^{\Gamma/2}r^2)'\mathcal{E} + \frac{2}{r}\mathcal{G}]dr \\ (T^r_r)'(r) &= -\frac{e^{-\Gamma/2}}{r^2}(e^{\Gamma/2}r^2)'\mathcal{J} + \frac{2}{r}\mathcal{G} \end{aligned} \tag{18}$$

From the Einstein's equations of this system we have

$$e^{-\Lambda}[\frac{1}{r^2} - \frac{\Lambda'}{r}] - \frac{1}{r^2} = T^t_t = -\mathcal{E} \tag{19}$$

which can be integrated to give

$$e^{-\Lambda} = 1 - \frac{1}{r}\int_{r_h}^{r} \mathcal{E} r^2 dr - \frac{2}{r}\mathcal{M}_0 \tag{20}$$

where $\mathcal{M}_0$ is a constant of integration. In order that $e^{\Lambda} \to \infty$ as $r \to r_h$, $\mathcal{M}_0$ is fixed by

$$\mathcal{M}_0 = \frac{r_h}{2} \tag{21}$$

Far away from the origin the unknown functions $\phi(r)$, $e^{\Lambda(r)}$, and $e^{\Gamma(r)}$ can be expanded in a power series in $1/r$. These expansions, substituted back into the equations, are finally expressed in terms of three parameters only, chosen to be ϕ_∞, the asymptotic value of the dilaton, the ADM mass M, and the dilaton charge D defined as [10]

$$D = -\frac{1}{4\pi}\int d^2\Sigma^\mu \nabla_\mu \phi \tag{22}$$

where the integral is over a two-sphere at spatial infinity. The asymptotic solutions are

$$e^{\Lambda(r)} = 1 + \frac{2M}{r} + \frac{4M^2 - D^2}{r^2} + \mathcal{O}(1/r^3) \tag{23}$$

$$e^{\Gamma(r)} = 1 - \frac{2M}{r} + \mathcal{O}(1/r^3) \tag{24}$$

$$\phi(r) = \phi_\infty + \frac{D}{r} - \frac{2MD}{r^2} + \mathcal{O}(1/r^3) \tag{25}$$

To check the possibility of the evasion of the no-hair conjecture we first consider the asymptotic behaviour of T^r_r as $r \to \infty$. In this limit, $e^{\Gamma/2} \to 1$, and so the leading behaviour of $(T^r_r)'$ is

$$(T^r_r)' \sim \frac{2}{r}[\mathcal{G} - \mathcal{J}] = \frac{2}{r}(T^\theta_\theta - T^r_r) \tag{26}$$

Since Γ' and $\Lambda' \sim \mathcal{O}(\frac{1}{r^2})$ as $r \to \infty$, we have the following asymptotic behaviour

$$\begin{aligned} T^\theta_\theta &\sim -\frac{1}{4}(\phi')^2 + \mathcal{O}(\frac{1}{r^6}) \\ T^r_r &\sim \frac{1}{4}(\phi')^2 + \mathcal{O}(\frac{1}{r^6}) \end{aligned} \tag{27}$$

Hence, the integral defining T^r_r converges and

$$(T^r_r)' \sim -\frac{1}{r}(\phi')^2 < 0 \qquad \text{as r } \to \infty \tag{28}$$

Thus, T^r_r is positive and decreasing as $r \to \infty$.

We now turn to the behaviour of the unknown functions at the event horizon. When $r \sim r_h$, we make the ansatz

$$\begin{aligned} e^{-\Lambda(r)} &= \lambda_1(r-r_h) + \lambda_2(r-r_h)^2 + ... \\ e^{\Gamma(r)} &= \gamma_1(r-r_h) + \gamma_2(r-r_h)^2 + ... \\ \phi(r) &= \phi_h + \phi'_h(r-r_h) + \phi''_h(r-r_h)^2 + ... \end{aligned} \tag{29}$$

with the subscript h denoting the value of the respective quantities at the horizon. The consistency of (29) will be checked explicitly in the next section. As we can see, $\phi(r_h) \sim$ *constant* while Γ' and Λ' diverge as $(r-r_h)^{-1}$ and $-(r-r_h)^{-1}$ respectively. Then, the behaviour of the components of the energy-momentum tensor near the horizon is

$$\begin{aligned} T^r_r &= -\frac{\alpha'}{2g^2r^2}e^{\phi-\Lambda}\phi'\Gamma' + \mathcal{O}(r-r_h) \\ T^t_t &= \frac{\alpha'}{2g^2r^2}e^{\phi-\Lambda}\phi'\Lambda' + \mathcal{O}(r-r_h) \\ T^\theta_\theta &= \frac{\alpha'}{2g^2r}[\Gamma''\phi' + \frac{\Gamma'\phi'}{2}(\Gamma' - 3\Lambda')] + \mathcal{O}(r-r_h) \end{aligned} \tag{30}$$

Taking into account the above expressions the leading behaviour of T^r_r near the horizon is

$$\begin{aligned} T^r_r(r) &\simeq \frac{e^{-\Gamma/2}}{r^2}\int_{r_h}^{r}(e^{\Gamma/2})'\frac{\alpha'}{2g^2}e^{\phi-\Lambda}\phi'\Lambda' dr + \mathcal{O}(r-r_h) \\ &= -\frac{e^{-\Gamma/2}}{r^2}\int_{r_h}^{r}\frac{\alpha'}{4g^2}e^{\Gamma/2}(\Gamma')^2e^{-\Lambda}e^{\phi}\phi' dr + \mathcal{O}(r-r_h) \end{aligned} \tag{31}$$

Therefore one observes that for r sufficiently close to the event horizon, T^r_r has *opposite* sign to ϕ'.

For $(T^r_r)'$ near the horizon, we have

$$\begin{aligned} -\frac{\Gamma'}{2}\mathcal{J} + \frac{2}{r}(\mathcal{G}-\mathcal{J}) &= \frac{\alpha'}{2g^2}\frac{e^{\phi}}{r^2}e^{-\Lambda}\{-\Gamma'(\phi'' + \phi'^2) + \phi'[\frac{\Gamma'}{2}(\Gamma'+\Lambda') + 2e^{-\Lambda}\Gamma'' \\ &\quad -\frac{2}{r}\Lambda']\} - \frac{1}{4}\Gamma'e^{-\Lambda}\phi'^2 + \mathcal{O}(r-r_h) \end{aligned} \tag{32}$$

where $\Gamma' + \Lambda' \sim \mathcal{O}(1)$ for $r \sim r_h$.

In order to simplify the above expression further, we turn to the field equations. Using the static, spherically symmetric ansatz (9) for the metric, the dilaton equation as well as the (tt), (rr) and $(\theta\theta)$ component of the Einstein's equations take the form

$$\phi'' + \phi'(\frac{\Gamma' - \Lambda'}{2} + \frac{2}{r}) = \frac{\alpha' e^{\phi}}{g^2 r^2}\left(\Gamma'\Lambda' e^{-\Lambda} + (1 - e^{-\Lambda})[\Gamma'' + \frac{\Gamma'}{2}(\Gamma' - \Lambda')]\right) \quad (33)$$

$$\Lambda'\left(1 + \frac{\alpha' e^{\phi}}{2g^2 r}\phi'(1 - 3e^{-\Lambda})\right) = \frac{r\phi'^2}{4} + \frac{1 - e^{\Lambda}}{r} + \frac{\alpha' e^{\phi}}{g^2 r}(\phi'' + \phi'^2)(1 - e^{-\Lambda}) \quad (34)$$

$$\Gamma'\left(1 + \frac{\alpha' e^{\phi}}{2g^2 r}\phi'(1 - 3e^{-\Lambda})\right) = \frac{r\phi'^2}{4} + \frac{e^{\Lambda} - 1}{r} \quad (35)$$

$$\Gamma'' + \frac{\Gamma'}{2}(\Gamma' - \Lambda') + \frac{\Gamma' - \Lambda'}{r} = -\frac{\phi'^2}{2} + \frac{\alpha' e^{\phi - \Lambda}}{g^2 r}\left(\phi'\Gamma'' + (\phi'' + \phi'^2)\Gamma' + \frac{\phi'\Gamma'}{2}(\Gamma' - 3\Lambda')\right) \quad (36)$$

At the event horizon $r \sim r_h$ the (tt) and (rr) components reduce to

$$\begin{aligned} e^{-\Lambda}\Lambda' &= -\frac{1}{r} - \frac{\alpha'}{2g^2}\frac{e^{\phi}}{r}e^{-\Lambda}\Lambda'\phi' + \mathcal{O}(r - r_h) \\ e^{-\Lambda}\Gamma' &= \frac{1}{r} - \frac{\alpha'}{2g^2}\frac{e^{\phi}}{r}e^{-\Lambda}\Gamma'\phi' + \mathcal{O}(r - r_h) \end{aligned} \quad (37)$$

Hence, $e^{-\Lambda}\Lambda' = -\frac{1}{\mathcal{F}r} + \mathcal{O}(r - r_h)$, $e^{-\Lambda}\Gamma' = \frac{1}{\mathcal{F}r} + \mathcal{O}(r - r_h)$, with

$$\mathcal{F} = 1 + \frac{\alpha'}{2g^2}\frac{e^{\phi_h}}{r_h}\phi'_h \quad (38)$$

From the $(\theta\theta)$ component of the Einstein's equations we obtain

$$\begin{aligned} e^{-2\Lambda}\Gamma'' &= -\frac{1}{2}e^{-2\Lambda}(\Gamma')^2 + \frac{1}{2}e^{-2\Lambda}\Gamma'\Lambda' + \mathcal{O}(r - r_h) \\ &= -\frac{1}{r_h^2\mathcal{F}^2} + \mathcal{O}(r - r_h) \end{aligned} \quad (39)$$

Finally, adding the (tt) and (rr) components we obtain

$$\Gamma' + \Lambda' = \frac{1}{\mathcal{F}}[\frac{1}{2}r_h(\phi'_h)^2 + \frac{\alpha'}{g^2}\frac{e^{\phi_h}}{r_h}(\phi''_h + (\phi'_h)^2)] + \mathcal{O}(r - r_h) \quad (40)$$

Substituting all the above formulae into (32) yields, near r_h

$$(T_r^r)'(r) \sim -\frac{1}{4}\frac{(\phi'_h)^2}{r_h^2\mathcal{F}} - \frac{\alpha'}{2g^2}\frac{e^{\phi_h}}{r_h^3\mathcal{F}^2}(\phi''_h + (\phi'_h)^2) - \frac{\alpha'}{4g^4}\frac{e^{2\phi_h}}{r_h^5\mathcal{F}^2}(\phi'_h)^2 + \mathcal{O}(r - r_h) \quad (41)$$

Next, we turn to the dilaton equation (33). Combining eqs.(37) and (39) we obtain

$$e^{-\Lambda}\{\Gamma'' + \frac{\Gamma'}{2}(\Gamma' - \Lambda')\} = -\frac{2}{r_h^2 \mathcal{F}^2} + \mathcal{O}(r - r_h) \tag{42}$$

Then, the dilaton equation (33) at $r \sim r_h$ takes the form

$$\frac{\phi_h'}{r_h \mathcal{F}} = -\frac{3}{\mathcal{F}^2}\frac{\alpha'}{g^2}\frac{e^{\phi_h}}{r_h^4} + \mathcal{O}(r - r_h) \tag{43}$$

from which it follows that

$$\phi_h' = -\frac{3}{r_h^3 \mathcal{F}}\frac{\alpha'}{g^2}e^{\phi} \tag{44}$$

Substituting for $\mathcal{F}$ (38), the following equation for ϕ_h' is derived

$$\frac{\alpha'}{2g^2}\frac{e^{\phi_h}}{r_h}(\phi_h')^2 + \phi_h' + \frac{3}{r_h^3}\frac{\alpha'}{g^2}e^{\phi_h} = 0 \tag{45}$$

which has as solutions

$$\phi_h' = \frac{g^2}{\alpha'} r_h e^{-\phi_h}\left(-1 \pm \sqrt{1 - \frac{6(\alpha')^2}{g^4}\frac{e^{2\phi_h}}{r_h^4}}\right) \tag{46}$$

As we will see below the relation (46) guarantees the *finiteness* of ϕ_h'', and hence of the "local density" T_t^t (16). Both these solutions for ϕ_h' are negative, and hence, since $T_r^r(r_h)$ has the opposite sign to ϕ_h', T_r^r will be *positive* sufficiently close to the horizon. Since $T_r^r \geq 0$ also at infinity, we observe that there is *no contradiction* with Einstein's equations, thereby allowing for the existence of black holes with scalar hair. We observe that near the horizon the quantity $\mathcal{E}$ $(-T_t^t)$ (17), which in Einstein's gravitation would be the local energy density of the field ϕ, is *negative*. As we mentioned earlier, this constitutes one of the reasons one should expect an evasion of the no-scalar-hair conjecture in this black hole space time. Crucial also for this result was the presence of additional terms in (16), leading to $T_t^t \neq T_\theta^\theta$. Both of these features, whose absence in the case of Einstein-scalar gravity was crucial for the modern proof of the no-hair theorem, owe their existence in the presence of the higher-order $\mathcal{O}(\alpha')$ corrections in (3).

The physical importance of the restriction (46) lies on the fact that according to this relation, black hole solutions of a given horizon radius can *only exist* if the coupling constant of the Gauss-Bonnet term in (3) is smaller than a critical value, set by the magnitude of the horizon scale. In fact from (46), reality of ϕ_h' is guaranteed if and only if

$$e^{\phi_h} < \frac{g^2}{\sqrt{6}\alpha'} r_h^2 \tag{47}$$

By a conformal rescaling of the dilaton field we can always set $\alpha'/g^2 \to 1$, in which case ϕ_h can be viewed as a constant added to the dilaton field. In this picture, $\beta \equiv \frac{1}{4}e^{\phi_h}$ can then be viewed as the (appropriately normalized with respect to the Einstein term) coupling constant of the GB term in the effective lagrangian (3). For a black hole of unit horizon radius $r_h = 1$, the critical value of β, above which black hole solutions cannot exist, is then $\beta_c = 4/\sqrt{6}$. One is tempted to compare the situation with the case of $SU(2)$ sphaleron solutions in the presence of Gauss Bonnet terms [9]. Numerical analysis of sphaleron solutions in such systems reveals the existence of a critical value for the GB coefficient above which solutions do not exist. In the sphaleron case this number depends on the number of nodes of the Yang-Mills gauge field. In our case, if one fixes the position of the horizon, then it seems that in order to construct black hole solutions with this horizon size the GB coefficient has to satisfy (47). The difference of (47) from the result of ref. [9] lies on the existence of an extra scale, as compared to the sphaleron case, that of the black hole horizon r_h. Thus, the most correct way of interpreting (46) is that of viewing it as providing a necessary condition for the *absence of naked singularities* in space-time. To understand better this latter point, we have to discuss the simpler case of a dilatonic black hole in the presence of an Abelian gauge field in four dimensions [10]. Such black holes admit dilatonic non-constant hair outside their horizon only in the presence of gauge fields conformally coupled to the dilaton

$$S \propto \int d^4x\sqrt{-g}[-R - \frac{1}{2}(\partial_\rho\phi)^2 - \frac{1}{2}e^{\phi}g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu}F_{\lambda\rho}] \tag{48}$$

where $F_{\mu\nu}$ is the field strength of the (Abelian) gauge field. The metric space-time strongly resembles the Schwarzschild solution, with an horizon $r_h = 2M$, with M the mass of the black hole, and a curvature singularity at $r = a = \frac{Q^2}{2M}e^{-\phi_0}$, where ϕ_0 is an arbitrary constant added to the dilaton, reflecting the conformal coupling, and Q is the magnetic charge of the black hole, associated with $F_{\theta\phi} = Qsin\theta$. If $Q \neq 0$, the black hole admits non-constant dilaton configurations outside the horizon

$$e^{\phi} = e^{\phi_0}(1 - \frac{a}{r}) \tag{49}$$

From the above equation it becomes clear that consistency of the dilaton solution requires $r > a$, which is equivalent to the absence of naked singularities, since a curvature singularity arises at the point $r = a$ for this dilatonic black hole. A similar thing we conjecture as happening in our case, where (47) is interpreted as the necessary condition for the absence of naked singularities. We conjecture that in our black hole solution a curvature singularity occurs at $r_0^2 = \frac{\sqrt{6}\alpha'}{g^2}e^{\phi_h}$, and, thus, due to (47), $r_h > r_0$.

Above, we have argued on the possibility of having black holes in the system (3) that admit non-trivial dilaton hair outside their horizon. The key is the bypassing of the no-hair theorem [4], as a result of the curvature-squared terms. In what follows

we shall write down explicit solutions of the equations of motion originating from (3) and provide evidence for the existence of black hole solutions to all orders in α'. Unfortunately a complete analytic treatment of these equations is not feasible, and one has to use numerical methods. This complicates certain things, in particular it does not allow for a clear view of what happens inside the horizon, thereby not giving any information on the curvature singularity structure.

3 Black Hole Solutions

3.1 Analytic Solution near the Horizon

We now turn to the behaviour of the fields near the horizon. For calculational convienence we set $\frac{\alpha'}{g^2} \to 1$, shifting $\phi \to \phi - \log\left(\frac{\alpha'}{g^2}\right)$. We start by observing that the (rr) component can be solved analytically to yield an expression for e^{Λ}

$$e^{\Lambda} = \frac{-\beta + \delta\sqrt{\beta^2 - 4\gamma}}{2} \ , \ \delta = \pm 1 \tag{50}$$

where

$$\begin{aligned} \beta &= \frac{\phi'^2 r^2}{4} - 1 - \Gamma'(r + \frac{e^{\phi}\phi'}{2}) \\ \gamma &= \frac{3}{2}\Gamma'\phi' e^{\phi} \end{aligned} \tag{51}$$

We, then eliminate Λ' using $\frac{d}{dr}(rr)$. Choosing two of the remaining equations (33), (34) and (36) (only two of them are linearly independent) we obtain the system of equations

$$\phi'' = -\frac{d_1}{d} \tag{52}$$

$$\Gamma'' = -\frac{d_2}{d} \tag{53}$$

where

$$\begin{aligned} d = \ & 4e^{2\Lambda+\phi} r\left(-4 + 8e^{\Lambda} - 4e^{2\Lambda} - 4\Gamma' r + 4\Gamma' e^{\Lambda} r + 5\phi'^2 r^2 - \phi'^2 e^{\Lambda} r^2\right) \\ & + 4\phi' e^{\Lambda+2\phi}\left(6 - 12e^{\Lambda} + 6e^{2\Lambda} + 6\Gamma' r - 8\Gamma' e^{\Lambda} r + 2\Gamma' e^{2\Lambda} r - 3\phi'^2 r^2\right. \\ & \left. + \phi'^2 e^{\Lambda} r^2\right) - 12\Gamma'\phi'^2 e^{3\phi}(1 - e^{\Lambda})^2 - 8\phi' e^{3\Lambda} r^4 \end{aligned} \tag{54}$$

$$\begin{aligned} d_1 = \ & 2\Gamma'\phi'^3 e^{3\phi}\left(9\Gamma' - 6\phi' - 6\Gamma' e^{\Lambda} + 12\phi' e^{\Lambda} + \Gamma' e^{2\Lambda} - 6\phi' e^{2\Lambda}\right) \\ & + \phi'^2 e^{\Lambda+2\phi}\left(24\phi' - 8\Gamma' e^{\Lambda} - 48\phi' e^{\Lambda} + 8\Gamma' e^{2\Lambda} + 24\phi' e^{2\Lambda} - 42\Gamma'^2 r\right. \end{aligned}$$

$$
\begin{aligned}
&-30\phi'^2 r + 20\Gamma'^2 e^{\Lambda} r - 32\Gamma'\phi' e^{\Lambda} r + 16\phi'^2 e^{\Lambda} r - 2\Gamma'^2 e^{2\Lambda} r \\
&-2\phi'^2 e^{2\Lambda} r + 3\Gamma'\phi'^2 r^2 - 3\Gamma'\phi'^2 e^{\Lambda} r^2 + 24\Gamma'\phi' r + 8\Gamma'\phi' e^{2\Lambda} r\Big) \\
&+\phi' e^{2\Lambda+\phi}\Big(-24 + 48e^{\Lambda} - 24e^{2\Lambda} - 4\Gamma' r - 16\phi' r + 8\Gamma' e^{\Lambda} r \\
&+32\phi' e^{\Lambda} r - 4\Gamma' e^{2\Lambda} r - 16\phi' e^{2\Lambda} r + 32\Gamma'^2 r^2 - 16\Gamma'\phi' r^2 + 38\phi'^2 r^2 \\
&+16\Gamma'\phi' e^{\Lambda} r^2 - 6\phi'^2 e^{\Lambda} r^2 - 3\Gamma'\phi'^2 r^3 + \Gamma'\phi'^2 e^{\Lambda} r^3 - 8\Gamma'^2 e^{\Lambda} r^2\Big) \\
&+2e^{3\Lambda} r\Big(8 - 16e^{\Lambda} + 8e^{2\Lambda} + 4\Gamma' r - 4\Gamma' e^{\Lambda} r - 4\Gamma'^2 r^2 - 6\phi'^2 r^2 \\
&+\Gamma'\phi'^2 r^3 - 2\phi'^2 e^{\Lambda} r^2\Big) \qquad (55)
\end{aligned}
$$

$$
\begin{aligned}
d_2 = \;& \Gamma'\phi' e^{\Lambda+2\phi} r\Big(18\Gamma'^2 + 6\phi'^2 - 4\Gamma'^2 e^{\Lambda} + 8\phi'^2 e^{\Lambda} + 2\Gamma'^2 e^{2\Lambda} + 2\phi'^2 e^{2\Lambda} \\
&+5\Gamma'\phi'^2 e^{\Lambda} r - 8\phi'^3 e^{\Lambda} r - 9\Gamma'\phi'^2 r\Big) - 2\Gamma'^3\phi'^2 e^{3\phi}\Big(3 + e^{2\Lambda}\Big) \\
&+\phi' e^{3\Lambda} r^2\Big(8 - 8e^{\Lambda} - 4\Gamma' r - 4\Gamma' e^{\Lambda} r - 4\Gamma'^2 r^2 - 2\phi'^2 r^2 + \Gamma'\phi'^2 r^3\Big) \\
&+e^{2\Lambda+\phi}\Big(8\Gamma' - 16\Gamma' e^{\Lambda} + 8\Gamma' e^{2\Lambda} - 4\Gamma'^2 r + 8\phi'^2 r + 8\Gamma'^2 e^{\Lambda} r \\
&-4\Gamma'^2 e^{2\Lambda} r - 8\phi'^2 e^{2\Lambda} r - 12\Gamma'^3 r^2 - 10\Gamma'\phi'^2 r^2 - 8\phi'^3 r^2 + 4\Gamma'^3 e^{\Lambda} r^2 \\
&+2\Gamma'\phi'^2 e^{\Lambda} r^2 + 8\phi'^3 e^{\Lambda} r^2 + 13\Gamma'^2\phi'^2 r^3 + 4\Gamma'\phi'^3 r^3 + 6\phi'^4 r^3 \\
&+4\Gamma'\phi'^3 e^{\Lambda} r^3 - 2\phi'^4 e^{\Lambda} r^3 - 3\Gamma'\phi'^4 r^4 - 3\Gamma'^2\phi'^2 e^{\Lambda} r^3\Big) \qquad (56)
\end{aligned}
$$

Assuming ϕ_h and ϕ'_h to be finite and $\Gamma' \to \infty$ when $r \to r_h$, we expand the rhs of (50) near the horizon

$$
e^{\Lambda} = \frac{1}{2}(e^{\phi}\phi' + 2r)\Gamma' - \frac{8e^{\phi}\phi' - 8r + e^{\phi}\phi'^3 r^2 + 2\phi'^2 r^3}{4(e^{\phi}\phi' + 2r)} + \mathcal{O}\left(\frac{1}{\Gamma'}\right) \qquad (57)
$$

for $(e^{\phi}\phi' + 2r) \neq 0$ and $\delta = 1$ [2]. Substituting (57) in (52), (53) we obtain

$$
\begin{aligned}
\phi'' &= -\frac{1}{2}\frac{(e^{\phi}\phi' + 2r)(6e^{\phi} + e^{\phi}\phi'^2 r^2 + 2\phi' r^3)}{-6e^{2\phi} + e^{\phi}\phi' r^3 + 2r^4}\Gamma' + \mathcal{O}(1) \qquad (58) \\
\Gamma'' &= -\frac{1}{2}\frac{-6e^{2\phi} + e^{2\phi}\phi' r^2 + 4e^{\phi}\phi' r^3 + 4r^4}{-6e^{2\phi} + e^{\phi}\phi' r^3 + 2r^4}\Gamma'^2 + \mathcal{O}(\Gamma') \qquad (59)
\end{aligned}
$$

[2]This has to be understood by the following facts: (i) the choice $\delta = -1$ leads to $e^{\Lambda} = \mathcal{O}(1)$ near the horizon, which is not a black hole solution, and (ii) if $(e^{\phi}\phi' + 2r) \simeq 0$, then one obtains $e^{\Lambda} = \sqrt{3r}(\Gamma')^{1/2} + ...$ and $\phi'' = \sqrt{3r}e^{-\phi}(\Gamma')^{1/2} + ...$ which implies that ϕ''_h is finite at the horizon only if $\phi_h \to \infty$. This is inconsistent with our initial assumption for finite ϕ_h and ϕ'_h.

We now observe that in order to keep ϕ_h'' finite we have to impose the boundary condition $6e^{\phi} + e^{\phi}\phi'^2 r^2 + 2\phi' r^3 = 0$ which relates ϕ_h' with ϕ_h

$$\begin{aligned} \phi_h' &= r_h e^{-\phi_h}\left(-1 + \sigma\sqrt{1 - 6\frac{e^{2\phi_h}}{r_h^4}}\right) \ , \sigma = \pm 1 \\ \phi_h &< \log(\frac{r_h^2}{\sqrt{6}}) \end{aligned} \tag{60}$$

and implies

$$\begin{aligned} \phi'' &= \mathcal{O}(1) \\ \Gamma'' &= -\Gamma'^2 + \mathcal{O}(1) \Rightarrow \Gamma' = \frac{1}{r - r_h} + \mathcal{O}(1) \end{aligned} \tag{61}$$

or

$$\begin{aligned} e^{\Gamma(r)} &= \gamma_1(r - r_h) + \mathcal{O}(r - r_h)^2 \\ e^{-\Lambda(r)} &= \lambda_1(r - r_h) + \mathcal{O}(r - r_h)^2 \end{aligned} \tag{62}$$

where γ_1 is an arbitrary constant, and $\lambda_1 = 2/(e^{\phi_h}\phi_h' + 2r_h)$. Notice that (60) is exactly the equation (46), obtained previously, from the dilaton equation of motion near the horizon. This shows that if (60) is satisfied, then not only the finiteness of ϕ_h'' is guaranteed but also the expected singular behaviour of the metric is assured. The above analysis leads to the conclusion that the asymptotic solution (60)-(62), is the only acceptable black hole solution with finite ϕ_h, ϕ_h' and ϕ_h''.

3.2 Numerical Analysis

We now proceed to the numerical integration. Starting from the solution (60)-(62), at $r = r_h + \epsilon$, $\epsilon \simeq O(10^{-8})$, we integrate the system (52), (53) towards $r \to \infty$ using the fourth order Runge-Kutta method with an automatic step procedure and accuracy 10^{-8}. The integration stops when the flat space-time asymptotic limit (23)-(25) is reached. Since $\Lambda(r)$ is not an independent variable, and ϕ_h' is related to ϕ_h and r_h through eq.(60), it seems that the only independent parameters of the problem are ϕ_h, r_h and γ_1. Note that the equations of motion do not yield any constraint for γ_1. This is due to the fact that the equations of motion (33)-(36) do not involve $\Gamma(r)$ but only $\Gamma'(r)$. Thus, only $\Gamma'(r)$ can be determined by them and in order to obtain $\Gamma(r)$ a final integration has to be performed. This integration involves an integration constant , γ_1, which will be fixed by demanding the asymptotically flat limit (24). That means that the only independent parameters are just ϕ_h and r_h. Note also that only the choice $\sigma = +1$ in (60) leads to solutions which have the desired behaviour (25) for the dilaton field at infinity. Plots involving the dilaton

field $\phi(r)$, for three different allowed values of the solution parameter ϕ_h, are given in Figure 1. The metric functions $e^{\Lambda(r)}$, $e^{\Gamma(r)}$ as well as the three components T^t_t, T^r_r and T^θ_θ of the energy-momentum tensor for $r_h = 1$ are presented in Figures 2 and 3 respectively.

As we said before the asymptotic solution near the horizon is characterized by only two independent parameters, ϕ_h and r_h. However, the independent parameters that characterize the solution near infinity (23)-(25) are three, M, D and ϕ_∞. From this, we can infer that a relation must hold between the above parameters in order to be able to classify our solution as a two parameter family of black hole solutions. After some manipulation, the set of equations (33)-(36) can be rearranged to yield the identity

$$\frac{d}{dr}\left(r^2 e^{(\Gamma-\Lambda)/2}(\Gamma'-\phi') - \frac{\alpha' e^{\phi}}{g^2} e^{(\Gamma-\Lambda)/2}[(1-e^{-\Lambda})(\phi'-\Gamma') + e^{-\Lambda} r\phi'\Gamma']\right) = 0 \quad (63)$$

Integrating this relation over the interval (r_h, r) we obtain the expression

$$2M - D = \sqrt{\gamma_1\lambda_1}(r_h^2 + \frac{\alpha' e^{\phi_h}}{g^2}) \quad (64)$$

This equation is simply a connection between the set of parameters describing the solution near the horizon and the set M and D. The rhs of this relation clearly indicates that the existing dependence of the dilaton charge on the mass does not take the simple form of an equality encountered in EYMD regular solutions of ref.[9]. In order to find the relation between M and D we follow refs.[6] and [7] and take into account the $\mathcal{O}(\alpha'^2)$ expression of the dilaton charge in the limit $r \to \infty$

$$\begin{aligned} \phi(r) &= \phi_\infty + \frac{D}{r} + ... \\ &= \phi_\infty + \left(\frac{e^{\phi_\infty}}{2M}\frac{\alpha'}{g^2} + \frac{73 e^{2\phi_\infty}}{60(2M)^3}\frac{\alpha'^2}{g^4}\right)\frac{1}{r} + ... \end{aligned} \quad (65)$$

This relation can be checked numerically. The result is shown in Figure 4. Any deviations from this relation are due to higher order terms which turn out to be small.

The above relation (65) implies that the dilaton hair of the black hole solution, discussed in this section, is a kind of 'secondary hair', in the terminology of ref. [11]. This hair is generated because the basic fields (gravitons) of the theory associated with the primary hair (mass) act as sources for the non-trivial dilaton configurations outside the horizon of the black hole.

4 Additional Solutions

A second class of solutions can be obtained if we allow $\phi'(r)$ to be infinite at some finite value r_s of the coordinate r. This choice, as we shall argue below, is not incompatible with the finiteness of the energy-momentum tensor. This is due to the fact that the Gauss-Bonnet term does not have a definite signature. These solutions have the same asymptotic characterization in terms of ϕ_∞, M and D as the black hole. Near $r \simeq r_s$ one obtains

$$\begin{aligned} e^{-\Lambda(r)} &= \lambda_1(r - r_s) + ... \\ \Gamma'(r) &= \frac{\gamma_1}{\sqrt{r - r_s}} + ... \\ \phi(r) &= \phi_s + \phi'_s\sqrt{r - r_s} + ... \end{aligned} \tag{66}$$

To lowest order, the equations of motion yield the constraints

$$\frac{\alpha'}{4g^2}e^{\phi_s}\phi'_s\gamma_1 = \frac{1}{\lambda_1} + \frac{\phi'^2_s r_s^2}{16} \tag{67}$$

and

$$a\gamma_1^2 + b\gamma_1 + c = 0 \tag{68}$$

with

$$\begin{aligned} a &= 16e^{2\phi_s}(2e^{\phi_s}\phi'^2_s + 8r_s + \phi'^2_s r_s^2) \\ b &= 4e^{\phi_s}(e^{2\phi_s}\phi'^3_s - 4e^{\phi_s}\phi'^3_s r_s^2 - 16\phi'_s r_s^3 - 2\phi'^3_s r_s^4) \\ c &= -8e^{2\phi_s}\phi'^2_s r_s - e^{2\phi_s}\phi'^4_s r_s^2 + 2e^{\phi_s}\phi'^4_s r_s^4 + 8\phi'^2_s r_s^5 + \phi'^6_s \end{aligned} \tag{69}$$

These apparently singular solutions comprise a three-parameter family. The behaviour of the dilaton field and the metric components is shown in Figures 5 and 6 respectively. As we can see, these solutions can not be classified as black hole solutions since the metric component g_{tt} does not exhibit any singular behaviour : $e^\Gamma \to$ *constant* when $r \to r_s$. Only g_{rr} takes on an infinite value when r_s is approached.

In order to determine whether the spacetime geometry is really singular at r_s, the scalar curvature R as well as the "curvature invariant" $I = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$ were calculated. It turns out that both of the above quantities do not exhibit any singular behaviour at r_s which implies that the pathology of the metric is due to a pathology of the coordinate system and not of the spacetime geometry itself. Moreover, this guarantees the finiteness of the action (it can be easily checked that the Gauss-Bonnet combination is also finite which is consistent with the field redefinition ambiguity arguments given in the introduction). It is a simple exercise to

verify that the components of the energy-momentum tensor are also finite. They are shown in Figure 7. Unfortunately, at present, we are not in a position to discuss the nature of the solutions for $r \leq r_s$, and hence the only safe conclusion to be made from the above analysis concerns the absence of a *naked singularity*.

It is interesting to mention the existence of another class of solutions which are regular in the metric, do not possess any horizon, but the dilaton becomes infinite at $r \simeq 0$. Near the origin, these solutions are

$$\begin{aligned} e^{\Lambda(r)} &= 1 + \lambda_1 e^{-4\gamma_1/r} \\ \Gamma'(r) &= \gamma_1 e^{-\phi_1/r} \\ \phi(r) &= \frac{\phi_1}{r} + ... \end{aligned} \tag{70}$$

This is also a three parameter family of solutions. The far asymptotic behaviour is still given by (23)-(25), and it is again characterized by the parameters ϕ_∞, M and D. These solutions appear to have no curvature singularities ($R(r \simeq 0) \simeq 0$), but the components of the energy-momentum tensor are infinite at $r \simeq 0$.

Note that our black hole solution appears to be a boundary surface in the phase space between solutions (66) and (70). This means that, if ϕ_0, ϕ_0', Γ_0 and Γ_0' are the values of the fields for the black hole solution at $r = r_0 >> 1$, then integration of the system (52)-(53) starting from r_0 with $\phi'(r_0) > \phi_0'$ leads to the solution (66), whilst the case $\phi'(r_0) < \phi_0'$ leads to the solution (70).

5 Conclusions and Outlook

In this paper we have dealt with solutions of the coupled dilaton-graviton system in four dimensions, in the presence of higher-curvature terms in the Gauss-Bonnet combination. We have demonstrated the existence of black hole solutions for this system, characterized by non-trivial scalar (dilaton) hair. This hair is of 'secondary' type, in the sense that it is not accompanied by the presence of any new quantity that characterizes the black hole. Indeed, it was shown above that the dilaton charge is not an independent quantity, but it can be expressed in terms of the mass of the black hole. It should be stressed, however, that irrespectively of the precise type of hair the set of solutions examined in this work bypasses the conditions of the no-hair theorem [4]. Thus, our solutions may be viewed as demonstrating that there is plenty of room in the gravitational structure of Superstring Theory to allow for physically sensible situations that are not covered by the theorem as stated. Although our results were derived in the framework of the $\mathcal{O}(\alpha')$ effective superstring action, they are non-perturbative in nature and they will persist at least in situations of moderate curvatures.

In addition to the black hole solutions, we were able to find two other families of solutions, one of which had the interesting feature of having finite energy density. At present, the physical significance of the solutions is not fully clear to us. We hope to be able to return and study these structures in the near future.

There are many features of the solutions which we did not address in this work, one of which is their stability under either linear time-dependent perturbations of the graviton-dilaton multiplet, or under generic perturbations (beyond linearity). Such an analysis has been performed for the Einstein-Yang-Mills-Higgs system [5], and one could think of extending it to incorporate higher-curvature gravity theories. A stability analysis, when completed, will prove essential in understanding better the physical significance of the black hole solutions found in this work. This is of particular interest due to the connection of the solutions with superstring theory. We hope to return to these issues in a future publication.

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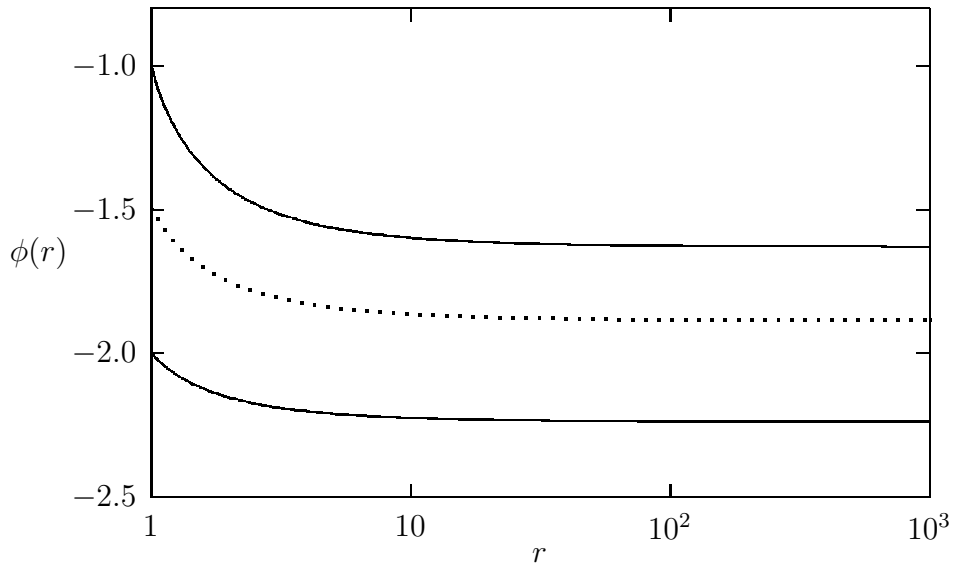

Figure 1: Dilaton field for $r_h = 1$ black hole. Each curve corresponds to a different solution characterized by a different initial value of ϕ_h.

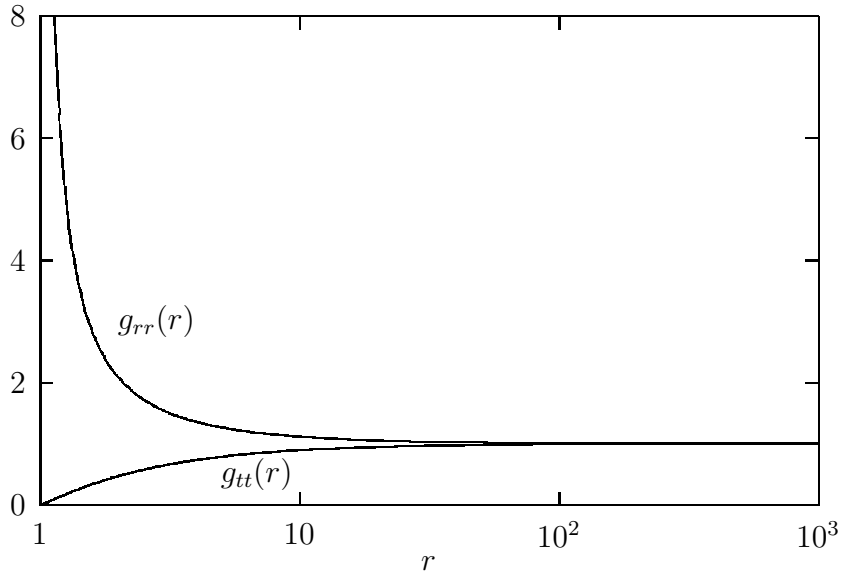

Figure 2: Metric components g_{tt} and g_{rr} for $r_h = 1$ black hole.

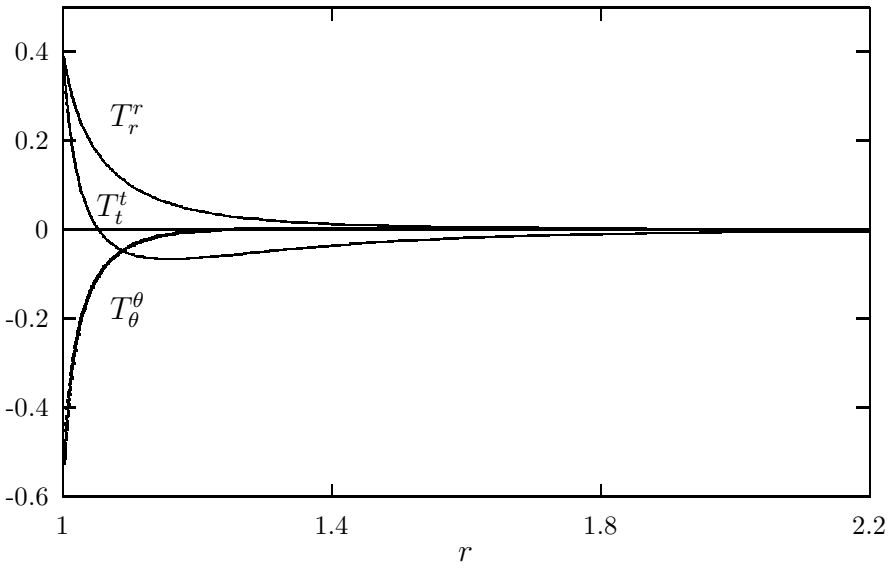

Figure 3: Components of the energy-momemtum tensor for $r_h = 1$ black hole.

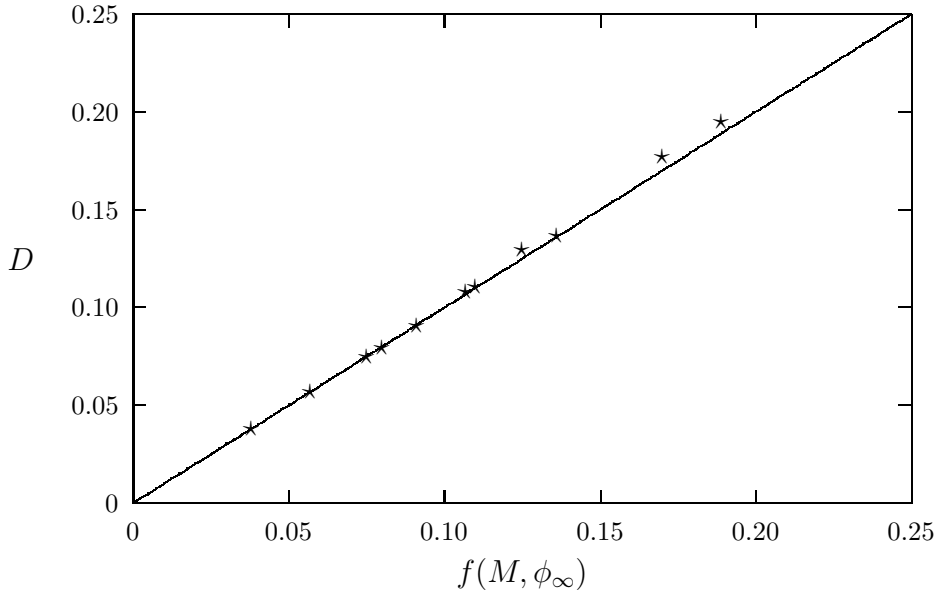

Figure 4: Dependence of the dilaton charge D on M and ϕ_∞ for the $r_h = 1$ black hole. The function $f(M, \phi_\infty)$ stands for the coefficient of $1/r$ in the eq.(65).

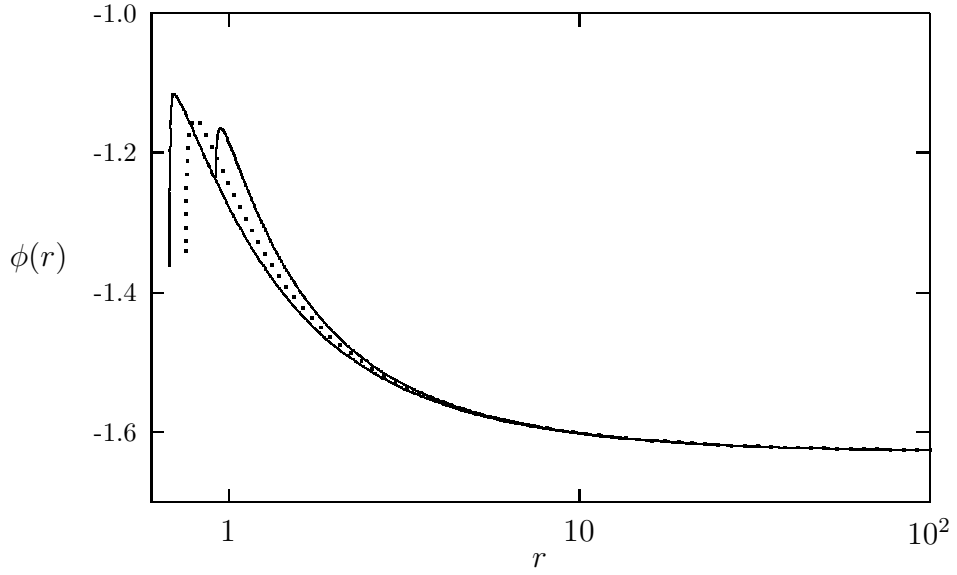

Figure 5: Dilaton field for the singular solution (66). Each curve corresponds to a different solution characterized by a different value of r_s ($r_s = 0.92, 0.75, 0.62$).

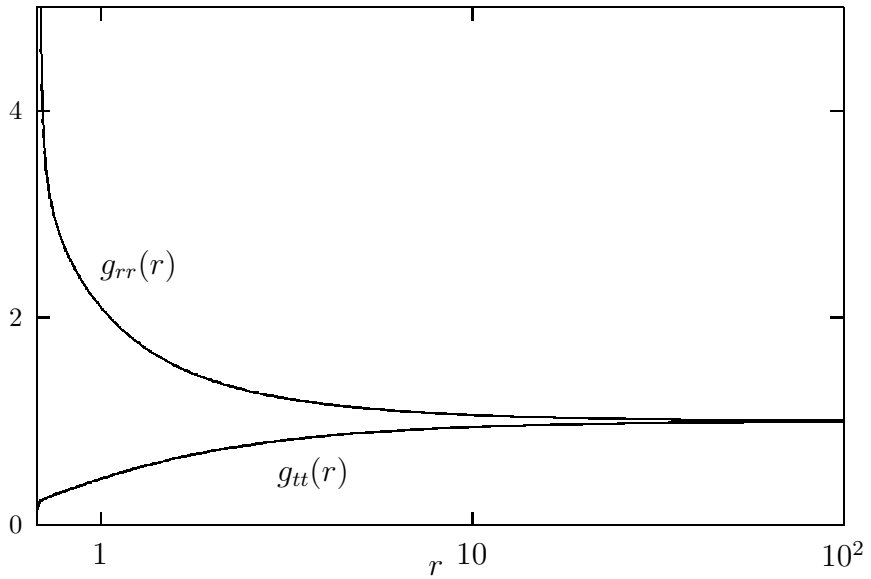

Figure 6: Metric components for the $r_s = 0.68$ singular solution (66).

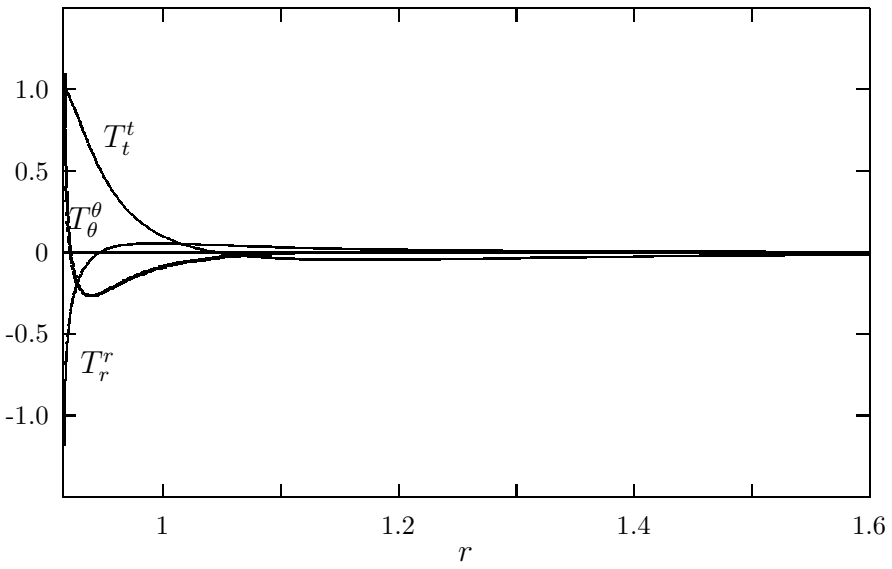

Figure 7: Components of the energy-momentum tensor for the $r_s = 0.92$ singular solution (66).