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Travelling and standing waves in magnetoconvection†

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The problem of Boussinesq magnetoconvection with periodic boundary conditions is studied using standard perturbation techniques. It is found that either travelling waves or standing waves can be stable at the onset of oscillatory convection, depending on the parameters of the problem. When travelling waves occur, a steady shearing flow is present that is quadratic in the amplitude of the convective flow. The weakly nonlinear predictions are confirmed by comparison with numerical solutions of the full partial differential equations at Rayleigh numbers 10% above critical. Modulated waves (through which stability is transferred between travelling and standing waves) are found near the boundary between the regions in parameter space where travelling waves and standing waves are preferred.

1. Introduction

This work is concerned with convection that is constrained by a vertical magnetic field, motivated by a desire to understand the form of convection in sunspots, where the inhibiting effect of the vertical magnetic field on convection is responsible for the dark appearance of the spot. Magnetoconvection is oscillatory at onset if the magnetic diffusivity is small and the magnetic field strength is large. Much previous work on magnetoconvection has concentrated on the case with sidewalls (Z_2, or reflecting, boundary conditions), which allows only standing wave (SW) oscillations (Weiss 1981*a*, *b*; Knobloch & Proctor 1981; Knobloch *et al.* 1981; Proctor & Weiss 1982). However, periodic ($O(2)$) boundary conditions are more appropriate than fixed sidewalls when modelling convection in an infinite layer. With periodic boundary conditions, travelling waves (TW) are also possible (Ruelle 1973). The question of whether standing or travelling waves are preferred at onset has not been fully addressed; the aim of the present work is to provide a full investigation of the stability of standing and travelling waves near the onset of convection in a vertical magnetic field, over a wide range of parameter values. In addition to the analytical approach of finding the coefficients in the amplitude equations, we give the results of numerical solutions of the partial differential equations (PDES). This is useful as a check on the analytical results and also illustrates the transitions to modulated waves (MW) that can occur as the amplitude of convection increases.

Weakly nonlinear theory must be used to resolve which of standing or travelling waves will occur at the onset of convection. If the frequency of the oscillation is small, it is known that either standing waves or travelling waves may be preferred

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depending on the values of the parameters (Dangelmayr & Knobloch 1986), but the general case has not been fully investigated. The relevant coefficients were calculated by E. Knobloch (unpublished results) and also by Nagata (1986), but there is an error in the printed form of the coefficients and Nagata did not explore the consequences of his results. Proctor (1986) found that travelling waves are always preferred in the limit of large magnetic fields; however, his scaling is not appropriate for the problem considered here, and indeed we obtain a different result. Magnetoconvection with periodic boundary conditions in a compressible fluid has been studied numerically by Hurlburt *et al.* (1989). This work found that standing waves are stable when the magnetic field is weak, but travelling waves become stable for stronger fields.

The stability of standing and travelling waves has been analysed in other double-diffusive systems. For convection of a binary fluid mixture in a porous medium, travelling waves are preferred at onset (Knobloch 1986*a*). This is also the case for thermosolutal convection (Deane *et al.* 1987), although here the analysis needs to be carried to higher order as one of the cubic terms in the amplitude equation is zero. When convection is constrained by rotation about a vertical axis, either travelling or standing waves may be stable, depending on the Prandtl number and the rate of rotation (Knobloch & Silber 1990). The case of a horizontal field has been analysed by Knobloch (1986*b*), who found that if the frequency of the oscillation is small, travelling waves are preferred.

We describe the model under consideration in §2 and derive the amplitude equations in §3. In §4, we interpret the coefficients computed in §3 and compare our results with numerical solutions of the PDEs and with other work.

2. Equations

The model under consideration is a Boussinesq fluid of density ρ with thermal diffusivity κ, kinematic viscosity ν, magnetic diffusivity η and expansion coefficient α. The fluid is confined to a layer of depth d across which there is a temperature difference ΔT; the layer is permeated by a vertical magnetic field of strength B_0. The system is assumed to be periodic in the horizontal direction (with spatial period $2\pi d/k$) and the upper and lower boundaries are assumed stress-free, with fixed temperature and vertical magnetic field. The flow is assumed to be two-dimensional, with x and z the horizontal and vertical coordinates.

The dimensionless equations for the stream-function ψ, the temperature perturbation θ and the flux function perturbation A are (Knobloch *et al.* 1981)

$$\frac{\partial \nabla^2\psi}{\partial t}+\mathrm{J}(\psi,\nabla^2\psi) = \sigma\nabla^4\psi+\sigma R\frac{\partial\theta}{\partial x}+\sigma\zeta Q\left(\mathrm{J}(A,\nabla^2 A)+\frac{\partial\nabla^2 A}{\partial z}\right), \tag{1}$$

$$\frac{\partial\theta}{\partial t}+\mathrm{J}(\psi,\theta) = \nabla^2\theta+\frac{\partial\psi}{\partial x}, \tag{2}$$

$$\frac{\partial A}{\partial t}+\mathrm{J}(\psi,A) = \zeta\nabla^2 A+\frac{\partial\psi}{\partial z}, \tag{3}$$

where $\sigma = \nu/\kappa$ and $\zeta = \eta/\kappa$ are Prandtl numbers and

$$\mathrm{J}(f,g) = (\partial f/\partial x)(\partial g/\partial z)-(\partial g/\partial x)(\partial f/\partial z).$$

The Rayleigh and Chandrasekhar numbers R and Q are

$$R = g\alpha\,\Delta T d^3/\nu\kappa, \quad Q = B_0^2 d^2/\mu_0 \rho\eta\nu, \tag{4}$$

which measure the destabilizing effect of the temperature gradient and the stabilizing effect of the magnetic field respectively. Lengths are scaled by the depth of the layer, and time is scaled by the thermal diffusion time d^2/κ. It is well known (Chandrasekhar 1961) that the static state undergoes a Hopf bifurcation at a critical Rayleigh number $R^{(o)}$ (defined below) provided that $\zeta < 1$ and Q is sufficiently large. With periodic boundary conditions in x, the Hopf bifurcation may lead to either a travelling or a standing wave.

3. Weakly nonlinear analysis

To study the weakly nonlinear behaviour of the system near the onset of oscillatory convection, a small parameter ϵ is introduced, defined by $R = R^{(o)} + \epsilon^2 R_2$; the variables ψ, θ and A are expanded in powers of ϵ as $\psi = \epsilon\psi_1 + \epsilon^2\psi_2 + \dots$, and so on (see for example, Knobloch *et al.* 1981). These expansions are substituted into the PDEs (1)–(3), which are arranged in powers of ϵ.

At first order in ϵ the equations are linear and have general solutions of the form:

$$\psi_1 = \operatorname{Re}\,(\psi_{1\mathrm{L}}\,\mathrm{e}^{\mathrm{i}(\omega t + kx)} + \psi_{1\mathrm{R}}\,\mathrm{e}^{\mathrm{i}(\omega t - kx)})\sin \pi z, \tag{5}$$

where $\psi_{1\mathrm{L}}$ and $\psi_{1\mathrm{R}}$ are the amplitudes of left-going and right-going travelling waves respectively; these amplitudes vary on a slow timescale $\tau = \epsilon^2 t$. A standing wave is represented by $\psi_{1\mathrm{L}} = \psi_{1\mathrm{R}}$. At the Hopf bifurcation, the frequency of the oscillation is ω. Similar expressions can be written for θ_1 and A_1, with the exception that the z dependence of A_1 is $\cos \pi z$. Substituting these expressions into the governing equations (1)–(3) gives the homogeneous linear system

$$(-\mathrm{i}\omega a^2 - \sigma a^4)\,\psi_{1\mathrm{L}} = \sigma R^{(o)}\mathrm{i}k\theta_{1\mathrm{L}} + \sigma\zeta Q\pi a^2 A_{1\mathrm{L}}, \tag{6}$$

$$(\mathrm{i}\omega + a^2)\,\theta_{1\mathrm{L}} = \mathrm{i}k\psi_{1\mathrm{L}}, \tag{7}$$

$$(\mathrm{i}\omega + \zeta a^2)\,A_{1\mathrm{L}} = \pi\psi_{1\mathrm{L}}, \tag{8}$$

where $a^2 = \pi^2 + k^2$. This linear system has a non-zero solution only if its determinant is zero. Equating the real and imaginary parts of the determinant to zero gives two equations from which $R^{(o)}$ and ω can be found in terms of Q, k, σ and ζ:

$$R^{(o)} = \frac{a^6(\sigma+\zeta)\,(1+\zeta)}{\sigma k^2} + \frac{\pi^2\zeta(\sigma+\zeta)\,a^2}{(1+\sigma)\,k^2}Q, \tag{9}$$

$$\omega^2 = \frac{\pi^2\sigma\zeta Q(1-\zeta)}{1+\sigma} - \zeta^2 a^4. \tag{10}$$

The equations for the functions representing a right-going wave are identical to (6)–(8) but with a sign change in k.

At second order, the nonlinear interactions of the first-order terms generate the following forms for the second-order functions:

$$\psi_2 = \psi_{02}\sin 2\pi z, \tag{11}$$

$$\theta_2 = (\theta_{02} + \operatorname{Re}\,(\theta_{022}\,\mathrm{e}^{2\mathrm{i}\omega t}))\sin 2\pi z, \tag{12}$$

$$A_2 = \operatorname{Re}\,(A_{02}\cos 2\pi z + A_{20\mathrm{L}}\,\mathrm{e}^{2\mathrm{i}(\omega t + kx)} + A_{20\mathrm{R}}\,\mathrm{e}^{2\mathrm{i}(\omega t - kx)} + A_{20}\,\mathrm{e}^{2\mathrm{i}kx}). \tag{13}$$

The seven second-order coefficients can be expressed as combinations of products of the first-order coefficients. The second-order stream-function represents a steady shear flow, and is proportional to $|\psi_{1L}|^2 - |\psi_{1R}|^2$; hence the shear flow is present for a travelling wave but not for a standing wave. This shearing flow is responsible for the triangular appearance of travelling waves apparent in numerical simulations (Hurlburt *et al.* 1989), and is directed in the same sense that the wave is travelling at the centre of the layer, but in the opposite sense at the top and bottom of the layer. The ψ_{02} term is not present at this order in convection in a binary fluid mixture (Knobloch 1986*a*) or thermosolutal convection (Deane *et al.* 1987). However, it does appear in convection in a rotating layer (Knobloch & Silber 1990), for which the equations are very similar to those considered here (essentially the only difference is the presence of a linear Coriolis force instead of the nonlinear Lorentz force).

At third order in ϵ, the nonlinear interactions generate terms that are cubic in the first-order amplitudes. The slow-time derivatives of these amplitudes and terms proportional to the bifurcation parameter R_2 also appear at this order. Only those terms with the same spatial structure as the first-order functions need be retained. Exploiting the degeneracy of (6)–(8), an evolution equation for ψ_{1L} can be obtained; the equation for ψ_{1R} can be deduced by symmetry. The amplitude equations have the form

$$\mathrm{d}\psi_{1L}/\mathrm{d}\tau = \beta R_2 \psi_{1L} + \gamma|\psi_{1L}|^2\psi_{1L} + \delta|\psi_{1R}|^2\psi_{1L}, \tag{14}$$

$$\mathrm{d}\psi_{1R}/\mathrm{d}\tau = \beta R_2 \psi_{1R} + \gamma|\psi_{1R}|^2\psi_{1R} + \delta|\psi_{1L}|^2\psi_{1R}. \tag{15}$$

The coefficients β, γ and δ are given by

$$\beta = \sigma k^2/\alpha(a^2+\mathrm{i}\omega), \tag{16}$$

where

$$\alpha = \frac{2\mathrm{i}\omega a^2(a^2(1+\sigma+\zeta)+\mathrm{i}\omega)}{(a^2+\mathrm{i}\omega)\,(a^2\zeta+\mathrm{i}\omega)}, \tag{17}$$

and

$$\alpha\gamma = \left\{k^2 - 3\pi^2 + \frac{(\sigma+\zeta)\,a^2(a^2-\mathrm{i}\omega)}{(1-\zeta)\,(a^2+\mathrm{i}\omega)} + \frac{(1+\sigma)\,a^2(a^2\zeta-\mathrm{i}\omega) - 8\pi^4\sigma(1-\zeta)}{(1-\zeta)\,(a^2\zeta+\mathrm{i}\omega)}\right\}$$
$$\times\frac{\mathrm{i}\omega\pi^2k^2Q}{(Q+4\pi^2)\,(a^4\zeta^2+\omega)^2} - \frac{2\pi^2k^2(1+\sigma)\,((k^2-\pi^2)\,a^2\zeta+2\mathrm{i}\omega k^2)}{(1-\zeta)\,(2k^2\zeta+\mathrm{i}\omega)\,(a^2\zeta+\mathrm{i}\omega)} - \frac{(\sigma+\zeta)\,a^4k^2}{2(1-\zeta)\,(a^2+\mathrm{i}\omega)}, \tag{18}$$

$$\alpha(\gamma+\delta) = -\frac{2\pi^2(1+\sigma)\,(3(k^2-\pi^2)\,a^2\zeta k^2 + \mathrm{i}\omega(3k^4-\pi^4))}{(1-\zeta)\,(a^2\zeta+\mathrm{i}\omega)\,(2k^2\zeta+\mathrm{i}\omega)} - \frac{a^2(\sigma+\zeta)\,(3\pi^2k^2a^2+\mathrm{i}\omega k^4)}{(1-\zeta)\,(a^2+\mathrm{i}\omega)\,(2\pi^2+\mathrm{i}\omega)}. \tag{19}$$

This system (14), (15) has been analysed in much previous work (for example, Knobloch 1986*a*). The real part of β is always positive, confirming that the linear system is unstable for $R_2 > 0$. A travelling wave is represented by a state in which $\psi_{1R} = 0$ and $|\psi_{1L}|^2 = -\mathrm{Re}\,(\beta)\,R_2/\mathrm{Re}\,(\gamma)$. This is stable if $\mathrm{Re}\,(\gamma+\delta) < 2\,\mathrm{Re}\,(\gamma) < 0$. A standing wave is given by $|\psi_{1L}|^2 = |\psi_{1R}|^2 = -\mathrm{Re}\,(\beta)\,R_2/(\mathrm{Re}\,(\gamma+\delta))$ and is stable if $2\,\mathrm{Re}\,(\gamma) < \mathrm{Re}\,(\gamma+\delta) < 0$.

To guard against the possibility of algebraic errors, the coefficients were computed by two different methods: by hand and by using the symbolic algebra package Reduce (Hearn 1991). The special case of a standing wave only has been considered by Knobloch *et al.* (1981); our results agree with theirs in this case.

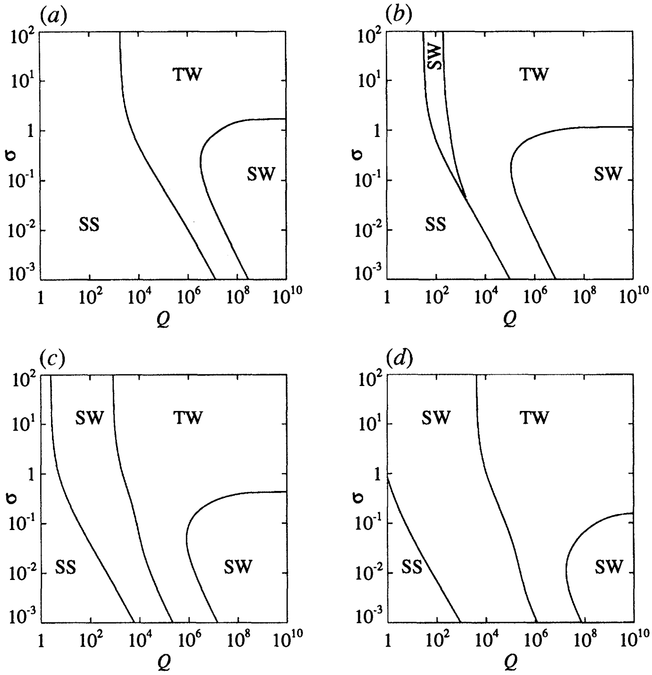

Figure 1. Regions of stability of steady convection (SS), standing waves (SW) and travelling waves (TW) at onset, at the optimum wave-number. (*a*) $\zeta = 0.90$; (*b*) $\zeta = 0.50$; (*c*) $\zeta = 0.10$; (*d*) $\zeta = 0.02$.

4. Discussion

The coefficients in (14), (15) are complicated functions of the parameters Q, k, σ and ζ, so it is not possible to give a simple criterion for the stability of travelling and standing waves. We reduce the number of parameters involved from four to three by considering the most unstable wave-number: the value of k that minimizes $R^{(o)}$, given by solving

$$a^6 - \tfrac{3}{2}\pi^2 a^4 = \frac{\pi^4 \sigma \zeta Q}{2(1+\sigma)(1+\zeta)} \tag{20}$$

for k (recall that $a^2 = \pi^2 + k^2$). To compare our results with previous work, we also consider fixed k below.

The coefficients γ and δ were computed numerically to determine whether standing or travelling waves are preferred at the onset of oscillatory convection. For four values of ζ, a wide range of σ and Q was considered (see figure 1). For each set of parameter values, the linear problem was solved to determine whether steady (SS) or oscillatory convection becomes unstable first as R is increased. If it was found that the oscillatory mode becomes unstable first, the coefficients γ and δ were determined at the value of k that minimized $R^{(o)}$, to investigate the stability of travelling or standing waves. Figure 1*a–d* shows the results as a function of σ and Q at $\zeta = 0.90$, 0.50, 0.10 and 0.02 respectively. For all the parameter values investigated, both oscillatory branches were found to be supercritical, ensuring the stability of either travelling or standing waves. If k is not equal to its optimum value, subcritical behaviour is possible: see below.

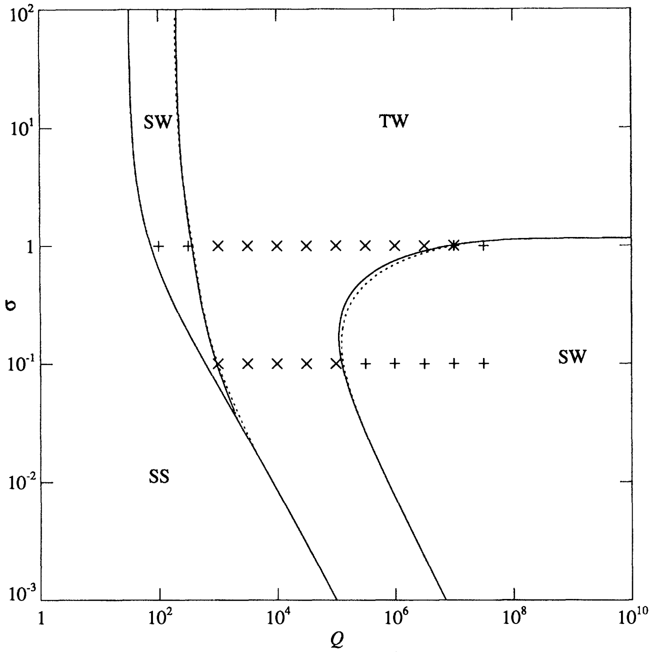

Figure 2. Regions of stability of steady convection (SS), standing waves (SW) and travelling waves (TW) at onset, with $\zeta = 0.50$ and k equal to its optimum value. The regions in which each type of convection is preferred are separated by solid lines. The results of direct numerical integrations of the PDEs are indicated: $+$ indicates standing waves, $\times$ indicates travelling waves and $*$ indicates modulated waves. Dotted lines indicate parameter values at which the PDEs have a Hopf bifurcation from travelling waves to modulated waves, at $R = 1.1R^{(o)}$.

For $\zeta = 0.90$, travelling waves are seen first as Q is increased with σ fixed. For smaller ζ, standing waves may be seen first, depending on the value of σ. If standing waves are seen first, they are replaced by travelling waves as Q is increased. As Q is increased further, the solution may revert to standing waves again if σ is sufficiently small.

In certain limits, the coefficients can be simplified so that a simple stability criterion can be found. When Q is large, and k is scaled as $k \sim Q^{\frac{1}{6}}$ so as to minimize $R^{(o)}$, the equations become degenerate in the sense that $\mathrm{Re}\,(\gamma) = \mathrm{Re}\,(\delta)$ if only the largest terms are retained. Taking the next term in the asymptotic series, we find that standing waves are stable if $\zeta > \sigma^2/(2+\sigma)$; otherwise travelling waves are stable. This disagrees with the results of Proctor (1986), who found that travelling waves were always preferred in the limit of large magnetic field. This discrepancy is due to the different scalings adopted: Proctor scaled the amplitude of convection as $Q^{-\frac{1}{6}}$, while we have taken the limit of small amplitude at a fixed magnetic field, then taken the limit of large magnetic field. This suggests that if standing waves are seen at onset, they will lose stability to travelling waves soon after the initial bifurcation. When ζ is small and σ and Q are of order unity, we find that standing waves are always preferred at onset.

Ideally, one would like a physical explanation of the preferred behaviour of the system. However, as the interaction between standing and travelling waves is essentially nonlinear, and the system is capable of switching back and forth between the two types of oscillation, it seems unlikely that a simple physical mechanism can be found.

The full PDEs (1)–(3) were integrated using a fully spectral method based on that of Veronis (1966); the results are shown in figure 2. At selected values of σ and Q (with fixed $\zeta = 0.50$), the Rayleigh number was set at 10% above its critical value $R^{(o)}$ and the ultimate behaviour of the PDEs determined. These integrations confirm the predictions made by the weakly nonlinear calculation, and reveal (in the nonlinear domain) the presence of intervals of modulated waves (MW) near the boundary between standing waves and travelling waves. This is to be expected from the work of Knobloch (1986*c*), who extended the amplitude equations (14), (15) to seventh order.

A further check on the correctness of the weakly nonlinear predictions was made. As R increases, travelling waves may lose stability to modulated waves via a Hopf bifurcation. The locus of points in the (σ, Q)-plane at which this occurs when $R = 1.1R^{(o)}$ is shown in figure 2 (dotted lines). The Hopf bifurcation was found by requiring that the PDEs linearized about the travelling waves have a pair of pure imaginary eigenvalues. The dotted lines lie very close to the solid lines of the weakly nonlinear calculation, but not exactly on them as the PDE simulation was done with $R \neq R^{(o)}$.

The above calculations were all done at the optimum wave-number; however, numerical simulations are usually carried out at fixed wave-number. In figure 3, we show which of standing or travelling waves are preferred as a function of k and Q, for four representative cases. Enlargements of figure 3*a*, *b* are shown in figure 4. The optimum value of the wave-number k as a function of Q is indicated by a dashed line. This line is discontinuous at the boundary between steady and oscillatory behaviour, indicated by a thick line. In the region of steady convection (to the left of the thick line), the bifurcation is subcritical below the dotted line and supercritical above. In the region of oscillatory convection (to the right of the thick line), the bifurcation to travelling waves is subcritical below the dash-dotted line, and that to standing waves is subcritical in two regions: first, when k is small and Q is large (below the dotted line in figure 3), and second, just below the thick line with Q large. This second region is too narrow to be seen in figure 3, but it is visible in figure 4*b*. Recall that in the regions where either bifurcation is subcritical, our analysis does not determine the preferred behaviour at onset.

The thick lines in figure 3 and figure 4 are lines of Takens–Bogdanov bifurcation points with $O(2)$ symmetry where the Hopf bifurcation to standing and travelling waves coincides with the pitchfork bifurcation to steady convection (see Dangelmayr & Knobloch 1987). Near the Takens–Bogdanov bifurcation, the frequency at the Hopf bifurcation is small. Dangelmayr & Knobloch (1986) have investigated the occurrence of standing and travelling waves near the Takens–Bogdanov bifurcation and found (as in, for example, figure 4*a*, *b*) that standing or travelling waves may be preferred and may be sub- or supercritical, depending on the precise parameter values. We recover their expressions for the stability of standing and travelling waves at the Takens–Bogdanov bifurcation by taking the limit of small frequency in (18), (19).

For each set of σ and ζ considered in figure 3, there is at least one point where the boundary between standing and travelling waves intersects the (thick) line of Takens–Bogdanov bifurcations. The analysis of this degenerate Takens–Bogdanov bifurcation point is not attempted here.

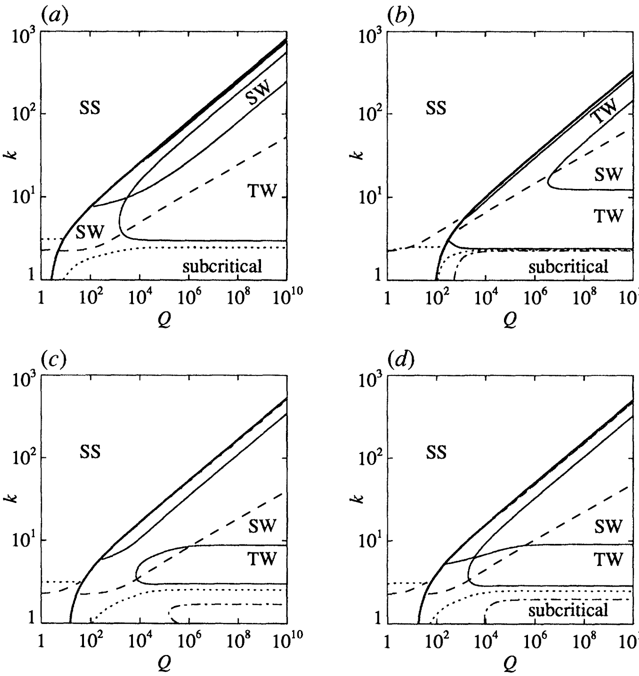

Figure 3. Regions of stability of steady convection (SS), standing waves (SW) and travelling waves (TW) at onset. In the regions labelled 'subcritical', the analysis does not determine which of standing or travelling waves are preferred. (*a*) $\sigma = 1.0$, $\zeta = 0.1$; (*b*) $\sigma = 1.0$, $\zeta = 0.8$; (*c*) $\sigma = 0.1$, $\zeta = 0.1$; (*d*) $\sigma = 0.2$, $\zeta = 0.2$. The meaning of the lines is described in the text. (*a*) and (*b*) are shown in more detail in figure 4.

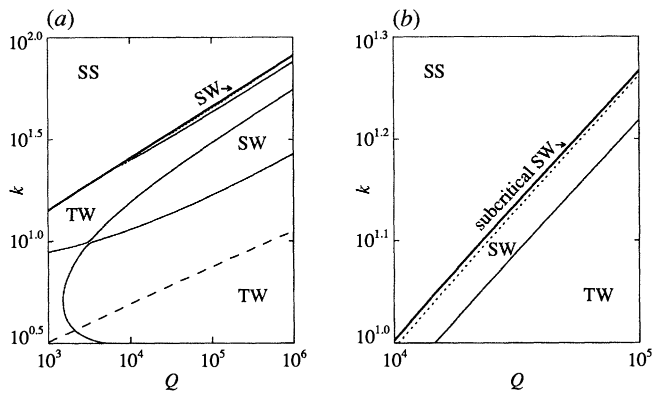

Figure 4. Regions of stability of steady convection (SS), standing waves (SW) and travelling waves (TW) at onset (detail of figure 3). (*a*) $\sigma = 1.0$, $\zeta = 0.10$; (*b*) $\sigma = 1.0$, $\zeta = 0.80$ (much enlarged). The bifurcation to standing waves is subcritical between the dotted line and the thick line in (*b*).

Calculations done in a box with sidewalls may be unstable to travelling waves if periodic boundary conditions are used instead. We have examined a range of published investigations of the PDEs (1)–(3); as might be expected, for some calculations done in a box, standing waves are preferred at onset, while for others, travelling waves are preferred. Weiss (1981*a*, *b*) solved the PDEs in a box with $\sigma =$

1.0, $\zeta = 0.1$ at $k = \pi$ and $Q = 1000$, and at $k = 2\pi$ and $Q = 7840$; we find that standing waves are preferred in the first case and travelling waves in the second (see figure 3*a*). Knobloch *et al.* (1981) studied the PDEs in a box with $\sigma = 0.2$, $\zeta = 0.2$ at $k = \pi$ and $Q = 98.7$, and at $k = 2\pi$ and $Q = 617$; again, we find that standing waves are preferred in the first case and travelling waves in the second (see figure 3*d*). Rucklidge (1992, 1993) has examined the limit of large wave-number and found chaotic standing waves in the PDEs near the Takens–Bogdanov bifurcation point with $\sigma = 1.0$, $\zeta = 0.8$ and k large. The appropriate limit for this case is $k \sim Q^{\frac{1}{4}}$ with Q large and σ and ζ order unity; we find that travelling waves are unstable to standing waves and that standing wave bifurcate subcritically in this limit (see figure 4*b*). The analysis of Rucklidge *et al.* (1993), who considered the case of the Takens–Bogdanov bifurcation with standing waves only but with $\mathrm{Re}\,(\gamma+\delta)$ positive, suggests that the subcritical standing waves become stable in a saddle-node bifurcation at Rayleigh numbers just below the Takens–Bogdanov bifurcation point, that is, stable standing waves exist in regions of parameter space where the primary bifurcation is a pitchfork bifurcation to steady convection.

Hurlburt *et al.* (1989) have studied the related problem of compressible convection in a vertical magnetic field and found, with $\sigma = \zeta = 0.1$ and $k = \pi$ and 2π, a transition from standing to travelling waves as Q is increased, in qualitative agreement with figure 3*c*. This work is being extended to three-dimensional convection (Matthews 1993; Matthews *et al.* 1993); the results of these simulations show that there are regions of parameter space where the two-dimensional travelling waves discussed here are stable, but standing waves are replaced by alternating rolls, which are two standing waves at 90° to each other and 90° out of phase. For the Boussinesq case in three dimensions, an analysis, similar to that in §3 but much more complicated, is being carried out by T. Clune & E. Knobloch (personal communication).

In summary, we have calculated whether standing waves or travelling waves are preferred at the onset of oscillatory convection in a vertical magnetic field over a wide range of parameter values and confirmed the analysis by comparison with numerical solutions of the governing PDEs. Near the boundary between the regions where standing waves and travelling waves are preferred, modulated waves are found at mildly supercritical Rayleigh numbers.

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