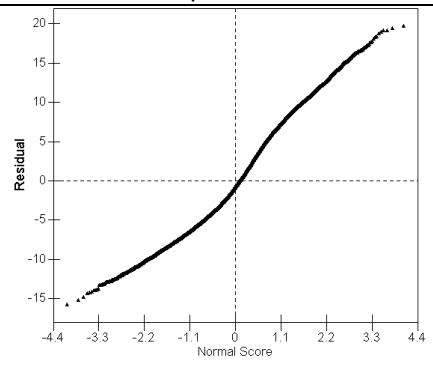
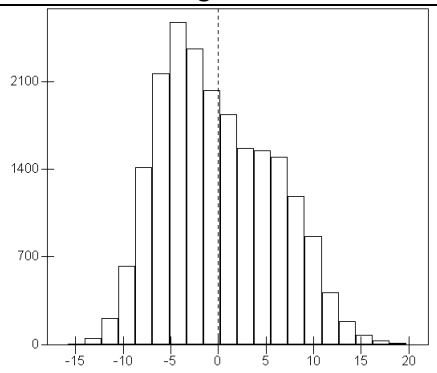
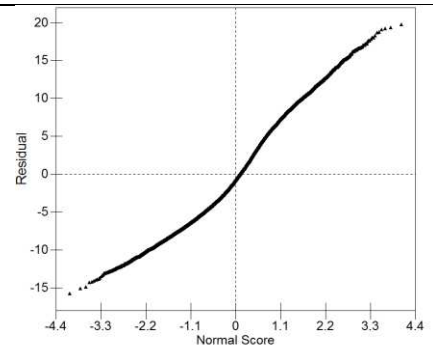
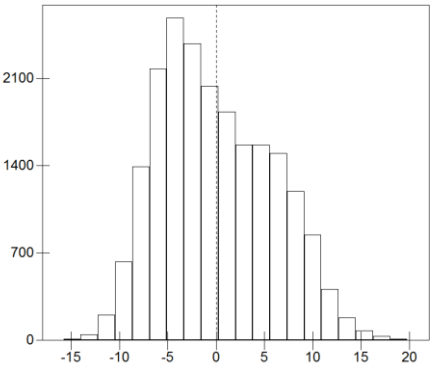
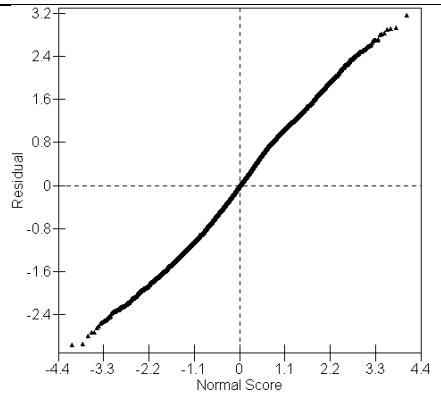
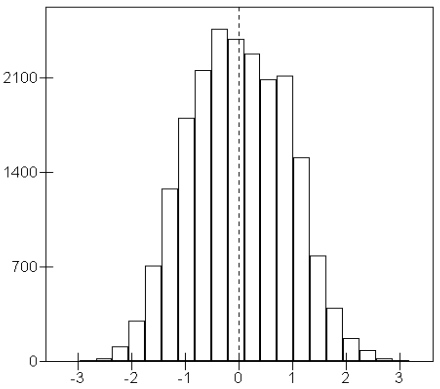
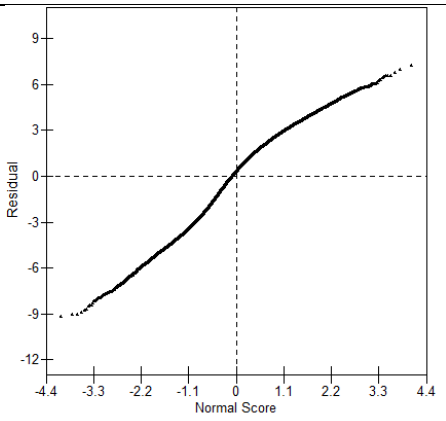
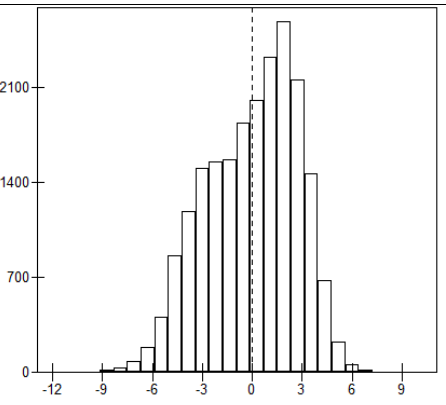
Online appendix to *Formula for Success: Multilevel modelling of Formula One driver and constructor performance, 1950-2014*.

This appendix provides further figures and tables that could not be included in the printed version of the paper, but which may nonetheless be of interest to some readers. The contents of this document are as followed:

- Table A1: shows the sensitivity of the models to different outcome variable transformations, as mentioned in **section 3.3.2**.
- Table A2: shows the insignificance of the race-level and driver-year level random effects (as mentioned in **section 3.1.**
- Table A3: shows basic null models in the form of equation 2 in the paper, including the model that produced **Figure 1** and the variances mentioned in **section 4.2**.
- Table A4: shows complex models in the form of equation 3 in the paper, including the model that produced **Figure 2.**
- Table A5: shows complex models that produced differential random effects for different weather conditions, including the model that produces **Figure 3.**
- Table A6: shows complex models in the form of equation 7 in the paper, including the model that produced **Figure 4.**
- Table A7: shows the separately coded version of the models shown in table A5.
- Table A8: shows the separately coded version of the models shown in table A6.
- Table A9: shows a list of the top 50 drivers, extending **Figure 1.**
- Table A10: shows predicted and actual champions in each season, as mentioned in **section 4.3**.
- Table A11: shows predicted and actual champions for the 2015 season, as mentioned in **section 4.3.**
- Figure A1: a visual representation of the team, team-year and driver variances from the model in table A3.
- Figure A2: shows all the drivers in a single graph
- Figure A3: shows the top 20 team-level residuals
- Figure A4: shows the top 20 team-year-level residuals
- Figure A5: shows the top 20 drivers under different weather conditions, as mentioned in **section 4.1.**
- Figure A6: shows the top 20 drivers on different track types, as mentioned in **section 4.1**.

Table A1: Level 1 residual plots, and predicted top 10 drivers, according to a variety of different dependent variables and model specifications.

Y variable	L1 Normal Q-Q plot	L1 Residuals Histogram	Top 10 drivers
Finishing Position			1. Fangio 2. Prost 3. Alonso 4. Clark 5. Senna 6. Stewart 7. Piquet 8. E Fittipaldi 9. Schumacher 10. Vettel
Finishing position, with additional driver-year level in the model			1. Fangio 2. Prost 3. Alonso 4. Clark 5. Senna 6. Stewart 7. Piquet 8. E Fittipaldi 9. Schumacher 10. Vettel
Square root of Finishing Position			1. Fangio 2. Clark 3. Prost 4. Alonso 5. Stewart 6. Senna 7. Schumacher 8. Vettel 9. E Fittipaldi 10. Piquet
Log of Points scored			1. Fangio 2. Prost 3. Alonso 4. Clark 5. Senna 6. Stewart 7. Piquet 8. E Fittipaldi 9. Vettel 10. Schumacher

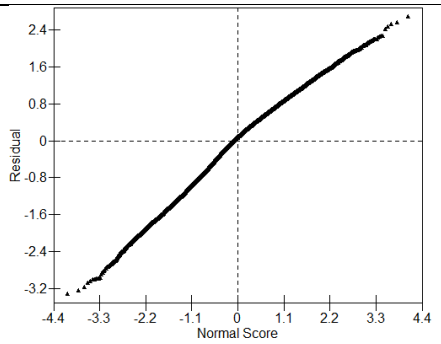
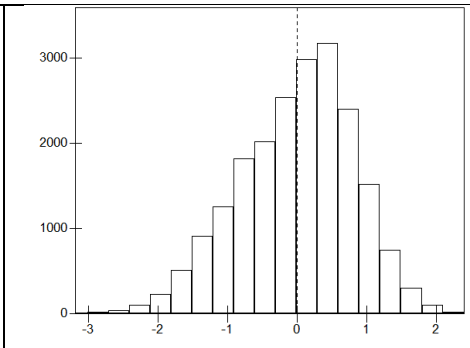
Rankit of Points scored			1. Fangio 2. Clark 3. Prost 4. Alonso 5. Stewart 6. Senna 7. Schumacher 8. Vettel 9. E Fittipaldi 10. Piquet

Table A2: Model results for 5 and 6 level model including a driver-year level and a race level (for a model with 500,000 iterations).

	6-level model				5-level model			
	Estimate	95% CIs		ESS	Estimate	95% CIs		ESS
Fixed Part								
Constant	0.434	0.403	0.464	19157	0.434	0.404	0.464	19492
Ndrivers -gm	-0.042	-0.046	-0.038	150153	-0.042	-0.046	-0.038	153110
NewComp -gm	0.977	0.343	1.614	166553	0.978	0.346	1.611	168164
Random Part								
Driver-Year Variance	0.003	0.001	0.006	984	0.003	0.001	0.006	1007
Race Variance	0.000	0.000	0.000	7314				
Team Variance	0.023	0.016	0.032	32087	0.023	0.016	0.031	32399
Team-Year Variance	0.021	0.017	0.025	29449	0.021	0.017	0.025	28677
Driver Variance	0.013	0.010	0.017	32126	0.013	0.010	0.017	31736
Level 1 Variance	0.288	0.282	0.294	349694	0.288	0.282	0.294	350524
DIC:	33786.549				33762.679			

Table A3: Models showing variance partitioning, controlling for competitiveness and the number of drivers.

	(a) 1979-2014			(b) 1950-2014 - Schumacher as 1 driver **(produces Figure 1)**			(c) 1950-2014 - Schumacher as 2 drivers		
	Estimate	95% CIs		Estimate	95% CIs		Estimate	95% CIs	
Fixed Part									
Constant	0.065	-0.011	0.139	0.434	0.403	0.464	0.433	0.403	0.463
Ndrivers -gm	-0.049	-0.060	-0.039	0.986	0.353	1.615	0.982	0.350	1.615
Comp -gm	1.343	0.047	2.641	-0.042	-0.046	-0.038	-0.042	-0.046	-0.038
Random Part									
Team Variance	0.066	0.040	0.102	0.022	0.015	0.031	0.023	0.016	0.032
Team-Year Variance	0.042	0.034	0.051	0.022	0.018	0.026	0.021	0.018	0.025
Driver Variance	0.017	0.012	0.025	0.013	0.010	0.017	0.013	0.010	0.017
Level 1 Variance	0.468	0.457	0.480	0.289	0.284	0.295	0.289	0.283	0.295
DIC:	29638.77			33776.32			33764.87		

Table A4: Models with variance as a function of Year, 1979-2014

	(a) Year Fixed Effect Only			(b) Year effect random at Team level			(c) Year effect random at Team-Year level			(d) Year effect random at Driver level			(e) Year effect random at all higher levels **(produces figure 2)**		
	Estimate	95% CIs		Estimate	95% CIs		Estimate	95% Cis		Estimate	95% CIs		Estimate	95% CIs	
Fixed Part															
Cons	0.065	-0.012	0.140	0.063	-0.017	0.138	0.066	-0.011	0.141	0.064	-0.012	0.137	0.065	-0.013	0.141
Ndrivers -gm	-0.052	-0.063	-0.040	-0.052	-0.063	-0.041	-0.052	-0.063	-0.040	-0.052	-0.064	-0.041	-0.053	-0.064	-0.042
Comp -gm	1.564	0.203	2.927	1.657	0.311	3.009	1.569	0.204	2.934	1.482	0.124	2.839	1.634	0.292	2.976
Year -gm	-0.002	-0.006	0.002	-0.002	-0.008	0.004	-0.001	-0.005	0.003	-0.002	-0.006	0.002	-0.002	-0.008	0.005
Random Part															
Team Level															
Cons	0.067	0.041	0.104	0.058	0.033	0.094	0.068	0.041	0.105	0.062	0.037	0.098	0.059	0.034	0.095
Covariance				0.001	-0.000	0.002							0.001	-0.000	0.002
Year				0.000	0.000	0.000							0.000	0.000	0.000
Team-Year Level															
Cons	0.042	0.034	0.051	0.037	0.030	0.045	0.041	0.033	0.050	0.032	0.021	0.043	0.028	0.018	0.038
Covariance										0.000	- 0.000	0.001	0.000	-0.000	0.001
Year										0.000	0.000	0.000	0.000	0.000	0.000
Driver Level															
Cons	0.017	0.012	0.025	0.016	0.011	0.023	0.013	0.007	0.020	0.017	0.012	0.024	0.012	0.007	0.019
Covariance							0.000	-0.001	0.000				- 0.000	-0.001	- 0.000
Year							0.000	0.000	0.000				0.000	0.000	0.000
Level 1 Var	0.468	0.457	0.480	0.468	0.457	0.480	0.468	0.457	0.479	0.468	0.457	0.480	0.468	0.457	0.479
DIC:	29639.30			29634.08			29632.00			29638.52			29625.41		

Table A5: Models with variance as a function of weather (dry/wet conditions), 1979-2014

	(a) Weather Fixed effects only			(b) Weather random at Team Level			(c) Weather random at Team-Year Level			(d) Weather random at Driver Level			(e) Weather random at all higher levels **(produces figure 3)**		
	Estimate	95% CIs		Estimate	95% CIs		Estimate	95% CIs		Estimate	95% CIs		Estimate	95% CIs	
Fixed Part															
Cons	0.067	-0.010	0.140	0.060	-0.019	0.136	0.066	-0.010	0.141	0.065	-0.011	0.139	0.060	-0.019	0.136
Ndrivers -gm	-0.049	-0.060	-0.039	-0.049	-0.060	-0.039	-0.050	-0.060	-0.039	-0.049	-0.060	-0.039	-0.050	-0.060	-0.039
Comp -gm	1.345	0.051	2.639	1.350	0.050	2.654	1.353	0.064	2.648	1.347	0.053	2.647	1.357	0.062	2.654
Wet	0.002	-0.032	0.035	0.040	-0.010	0.092	0.003	-0.033	0.038	0.013	-0.025	0.051	0.042	-0.010	0.096
Random Part															
Team Level															
Cons	0.066	0.040	0.102	0.071	0.044	0.109	0.065	0.040	0.101	0.065	0.040	0.102	0.070	0.044	0.108
Covariance				-0.015	-0.030	-0.003							-0.013	-0.028	0.000
Wet				0.010	0.004	0.022							0.009	0.003	0.020
Team-Year Level															
Cons	0.042	0.034	0.051	0.042	0.034	0.051	0.044	0.035	0.054	0.042	0.034	0.051	0.044	0.035	0.053
Covariance							-0.008	-0.018	0.000				-0.007	-0.016	0.001
Wet							0.011	0.003	0.026				0.010	0.003	0.023
Driver Level															
Cons	0.017	0.012	0.025	0.018	0.012	0.025	0.017	0.012	0.025	0.019	0.013	0.026	0.019	0.013	0.026
Covariance										-0.005	-0.010	0.000	-0.003	-0.009	0.002
Wet										0.005	0.001	0.012	0.004	0.001	0.010
Level 1 Var	0.468	0.457	0.480	0.467	0.456	0.479	0.467	0.456	0.478	0.468	0.457	0.479	0.466	0.455	0.477
DIC:	29640.87			29625.10			29635.45			29637.32			29623.84		

Table A6: Models with variance as a function of track type (permanent/temporary/street), 1979-2014

	(a) Track type Fixed Effects only			(b) Track type random at Team level			(c) Track type random at Team-Year level			(d) Track type random at Driver level			(e) Street and temp random at all higher levels **(produces figure 4)**		
	Estimate	95% CIs		Estimate	95% CIs		Estimate	95% CIs		Estimate	95% CIs		Estimate	95% CIs	
Fixed Part															
Cons	0.066	-0.010	0.140	0.059	-0.021	0.136	0.067	-0.009	0.141	0.064	-0.013	0.139	0.062	-0.018	0.139
Ndrivers -gm	-0.049	-0.060	-0.039	-0.049	-0.059	-0.039	-0.050	-0.060	-0.039	-0.049	-0.060	-0.039	-0.049	-0.060	-0.039
Comp -gm	1.342	0.041	2.638	1.354	0.059	2.652	1.342	0.048	2.639	1.317	0.010	2.613	1.339	0.041	2.634
Temp	0.002	-0.043	0.047	0.026	-0.034	0.088	0.002	-0.046	0.051	0.014	-0.038	0.066	0.027	-0.038	0.092
Street	0.000	-0.033	0.033	0.022	-0.018	0.064	0.001	-0.033	0.036	0.012	-0.031	0.055	0.016	-0.029	0.062
Random Part															
Team Level															
Cons	0.066	0.040	0.102	0.074	0.047	0.114	0.064	0.039	0.099	0.066	0.041	0.103	0.072	0.045	0.111
temp/cons				-0.011	-0.028	0.004							-0.009	-0.025	0.005
Temp				0.010	0.003	0.023							0.008	0.002	0.019
street/cons				-0.009	-0.020	-0.002							-0.005	-0.014	0.001
street/temp				0.002	-0.002	0.007							0.001	-0.001	0.005
Street				0.003	0.001	0.008							0.001	0.000	0.004
Team-Year Level															
Cons	0.042	0.034	0.051	0.042	0.034	0.051	0.047	0.038	0.057	0.042	0.034	0.051	0.045	0.037	0.055
temp/cons							-0.005	-0.018	0.007				-0.002	-0.015	0.010
Temp							0.036	0.013	0.068				0.029	0.010	0.058
street/cons							-0.015	-0.024	-0.007				-0.011	-0.020	-0.004
street/temp							0.010	-0.001	0.024				0.005	-0.003	0.016
Street							0.011	0.004	0.023				0.006	0.002	0.015
Driver Level															
Cons	0.017	0.012	0.025	0.017	0.012	0.024	0.018	0.012	0.025	0.019	0.013	0.026	0.018	0.012	0.025
temp/cons										-0.005	-0.012	0.002	-0.002	-0.010	0.005
Temp										0.014	0.005	0.030	0.011	0.004	0.025
street/cons										-0.004	-0.011	0.002	-0.002	-0.009	0.004
street/temp										0.011	0.002	0.022	0.009	0.001	0.019
Street										0.019	0.009	0.034	0.017	0.007	0.030
Level 1 Var	0.468	0.457	0.480	0.468	0.457	0.479	0.465	0.454	0.476	0.465	0.454	0.477	0.463	0.452	0.474
DIC:	29642.78			29636.556			29619.56			29618.313			29605.69		

Table A7: Separately coded model with variance as a function of weather, 1950-2014

	Weather with separate coding		
	Estimate	95% CIs	
Fixed Part			
Cons	0.458	0.42	0.494
Ndrivers -gm	0.997	0.364	1.63
Comp -gm	-0.042	-0.046	-0.038
Wet	-0.029	-0.06	0.002
Random Part			
Team Level			
Dry	0.025	0.017	0.034
Covariance	0.02	0.013	0.029
Wet	0.018	0.011	0.028
Team-Year Level			
Dry	0.022	0.019	0.027
Covariance	0.019	0.014	0.024
Wet	0.02	0.013	0.029
Driver Level			
Dry	0.014	0.011	0.018
Covariance	0.012	0.008	0.016
Wet	0.012	0.007	0.018
Level 1 Variance	0.288	0.282	0.294
DIC:	33746.002		

Table A8: Separately coded model with variance as a function of track type, 1950-2014

	Track type with separate coding		
	Estimate	95% CIs	
Fixed Part			
Cons	0.431	0.399	0.462
Ndrivers -gm	0.969	0.339	1.602
Comp -gm	-0.042	-0.046	-0.038
Temporary	0.018	-0.035	0.071
Street	0.011	-0.02	0.043
Random Part			
Team level			
Permanent	0.026	0.018	0.036
Perm/Temp Cov	0.022	0.014	0.033
Temporary	0.025	0.014	0.041
Perm/Street Cov	0.021	0.014	0.03
Street/Temp Cov	0.019	0.012	0.029
Street	0.02	0.013	0.029
Team-Year level			
Permanent	0.024	0.019	0.028
Perm/Temp Cov	0.021	0.014	0.028
Temporary	0.037	0.021	0.056
Perm/Street Cov	0.017	0.013	0.021
Street/Temp Cov	0.019	0.012	0.028
Street	0.016	0.011	0.023
Driver level			
Permanent	0.014	0.01	0.018
Perm/Temp Cov	0.011	0.006	0.017
Temporary	0.016	0.008	0.029
Perm/Street Cov	0.013	0.01	0.018
Street/Temp Cov	0.013	0.007	0.02
Street	0.018	0.012	0.025
Level 1 Variance	0.287	0.281	0.293
DIC:	33754.69		

Table A9: Top 50 drivers based on the driver level residuals from model A1d (Michael Schumacher treated as two drivers, pre 2006 and post 2010)

Rank	Driver	Residual	Rank	Driver	Residual
1	Juan Manuel Fangio	0.333	26	Robert Kubica	0.129
2	Alain Prost	0.300	27	Carlos Reutemann	0.128
3	Michael Schumacher (pre-2006)	0.286	28	Tom Pryce	0.128
4	Jim Clark	0.276	29	Stirling Moss	0.123
5	Ayrton Senna	0.265	30	Martin Brundle	0.121
6	Fernando Alonso	0.263	31	Rubens Barrichello	0.119
7	Nelson Piquet	0.238	32	Daniel Ricciardo	0.119
8	Jackie Stewart	0.232	33	Alan Jones	0.119
9	Emerson Fittipaldi	0.217	34	Kimi Raikkonen	0.118
10	Sebastian Vettel	0.213	35	Patrick Depailler	0.118
11	Christian Fittipaldi	0.198	36	Carlos Pace	0.117
12	Lewis Hamilton	0.175	37	Richie Ginther	0.116
13	Graham Hill	0.169	38	Denny Hulme	0.115
14	Dan Gurney	0.166	39	Thierry Boutsen	0.113
15	Jody Scheckter	0.165	40	Mike Hawthorn	0.111
16	Jenson Button	0.160	41	Jean-Pierre Beltoise	0.106
17	Marc Surer	0.158	42	Heinz-Harald Frentzen	0.105
18	Damon Hill	0.157	43	Prince Bira	0.102
19	Louis Rosier	0.143	44	Keke Rosberg	0.100
20	Elio de Angelis	0.141	45	Clay Regazzoni	0.098
21	Ronnie Peterson	0.140	46	Luigi Fagioli	0.097
22	Nino Farina	0.130	47	Jack Brabham	0.093
23	Nick Heidfeld	0.130	48	Jacques Villeneuve	0.093
24	Pedro Rodríguez	0.129	49	Nico Rosberg	0.092
25	John Watson	0.129	50	Phil Hill	0.090

Table A10: Comparison between predictions of the champion (from the model as in equation 3) and the actual champion, for years 1979-2014. Schumacher is treated as two drivers.

Year	Model's predicted champion	Actual Position of model's champion	Actual Champion	Model's position of actual champion	Notes
1979	J Scheckter	1st	J Scheckter	1st	
1980	C Reutemann	3rd	A Jones	2nd	
1981	C Reutemann	2nd	N Piquet	3rd	
1982	Keke Rosberg	1st	K Rosberg	1st	
1983	A Prost	2nd	N Piquet	7th	Piquet beaten in the model by 2nd-4th place drivers (Prost, Arnoux and Tambay, who were within 20 points of him), Jonathan Palmer (who only raced one race and outperformed his Williams team-mate in that race), Jacques Laffite (who benefits in the model compared to the championship because he missed two races, and Keke Rosberg, who won the championship the previous year.
1984	A Prost	1st	A Prost	1st	
1985	A Prost	1st	A Prost	1st	
1986	N Piquet	3rd	A Prost	2nd	Only 3 points between 1st and 3rd in the championship
1987	N Piquet	1st	N Piquet	1st	
1988	A Prost	2nd	A Senna	2nd	Only 3 points between 1st and 2nd in the championship; Very close in model predictions between 1st and second as well
1989	A Prost	1st	A Prost	1st	
1990	A Senna	1st	A Senna	1st	
1991	A Senna	1st	A Senna	1st	
1992	R Patrese	2nd	N Mansell	3rd	Very small differences in the model predictions of 1st and 3rd. Mansell gained lots of 1st places, so won by a long way in points (but there is a less clear gap in finishing position, reducing the advantage when points are transformed).
1993	A Prost	1st	A Prost	1st	
1994	A Senna	Not classified	M Schumacher	3rd	Senna only raced 3 races, finishing none (the third race was, tragically, his last). However because he didn't race in many races, it doesn't count against him or his team-year too much. Thus, his high driver residual was weighted heavily in his favour.
1995	M Schumacher	1st	M Schumacher	1st	
1996	D Hill	1st	D Hill	1st	
1997	M Schumacher	2nd/DSQ	J Villeneuve	4th	Schumacher was 2nd (3 points behind Villeneuve) but was disqualified from the final standings for dangerous driving.
1998	M Schumacher	2nd	M Hakkinen	2nd	
1999	M Schumacher	5th	M Hakkinen	3rd	Schumacher only completed seven races when he broke his leg, at which point he was second in the championship
2000	M Schumacher	1st	M Schumacher	1st	
2001	M Schumacher	1st	M Schumacher	1st	
2002	M Schumacher	1st	M Schumacher	1st	
2003	M Schumacher	1st	M Schumacher	1st	
2004	M Schumacher	1st	M Schumacher	1st	

2005	F Alonso	1st	F Alonso	1st	
2006	M Schumacher	2nd	F Alonso	2nd	Model produces a close result between Alonso and Schumacher. In the championship there was only a 13 point difference.
2007	F Alonso	3rd	K Raikkonen	2nd	Only 1 point between 1st and 3rd on the championship
2008	L Hamilton	1st	L Hamilton	1st	
2009	J Button	1st	J Button	1st	
2010	F Alonso	2nd	S Vettel	2nd	Only 4 points between 1st and 2nd in the championship
2011	S Vettel	1st	S Vettel	1st	
2012	F Alonso	2nd	S Vettel	2nd	Only 3 points between 1st and 2nd in the championship
2013	S Vettel	1st	S Vettel	1st	
2014	L Hamilton	1st	L Hamilton	1st	

Table A11: Out of sample predictions for the 2015 F1 season. Team year residuals are assumed not to change from 2014, and all trends are extrapolated. Based on a model including random slopes on year, using data from 1979. Michael Schumacher is treated as two drivers.

2015 Actual Results	Driver	2015 team	Predicted Ranking
1	Lewis Hamilton	Mercedes	5
2	Nico Rosberg	Mercedes	6
3	Sebastian Vettel	Ferrari	2
4	Kimi Räikkönen	Ferrari	4
5	Valtteri Bottas	Williams	12
6	Felipe Massa	Williams	13
7	Daniil Kvyat	Red Bull	9
8	Daniel Ricciardo	Red Bull	7
9	Sergio Pérez	Force India	16
10	Nico Hülkenberg	Force India	14
11	Romain Grosjean	Lotus	11
14	Pastor Maldonado	Lotus	10
16	Jenson Button	McClaren	3
17	Fernando Alonso	McClaren	1
18	Marcus Ericsson	Sauber	15
21	Will Stevens	Marussia	17
—	Kevin Magnussen	McClaren	8

Figure A1: Plot of the top 20 driver-level residuals, representing the top 20 drivers of all time (1950-2014) according to our model. Number of drivers and race competitiveness are controlled. Michael Schumacher is treated as two drivers (pre and post retirement), with only his pre-retirement performances represented in the graph. Based on model (c) in table A3. 95% credible intervals are shown.

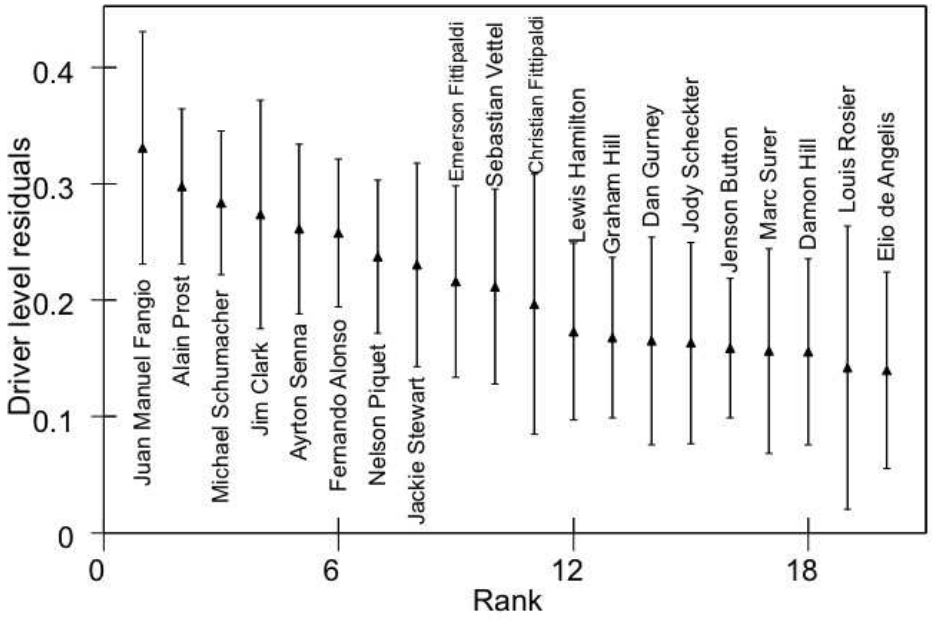


Figure A2: Driver level residuals for all 695 drivers, based on predictions from model (b) in table A3. It can be seen that the better drivers (with large negative residuals) generally have narrower confidence intervals as a result of competing in more races (many of those in the lower ranks competed in as few as one race). This graph is an extended version of figure 2.

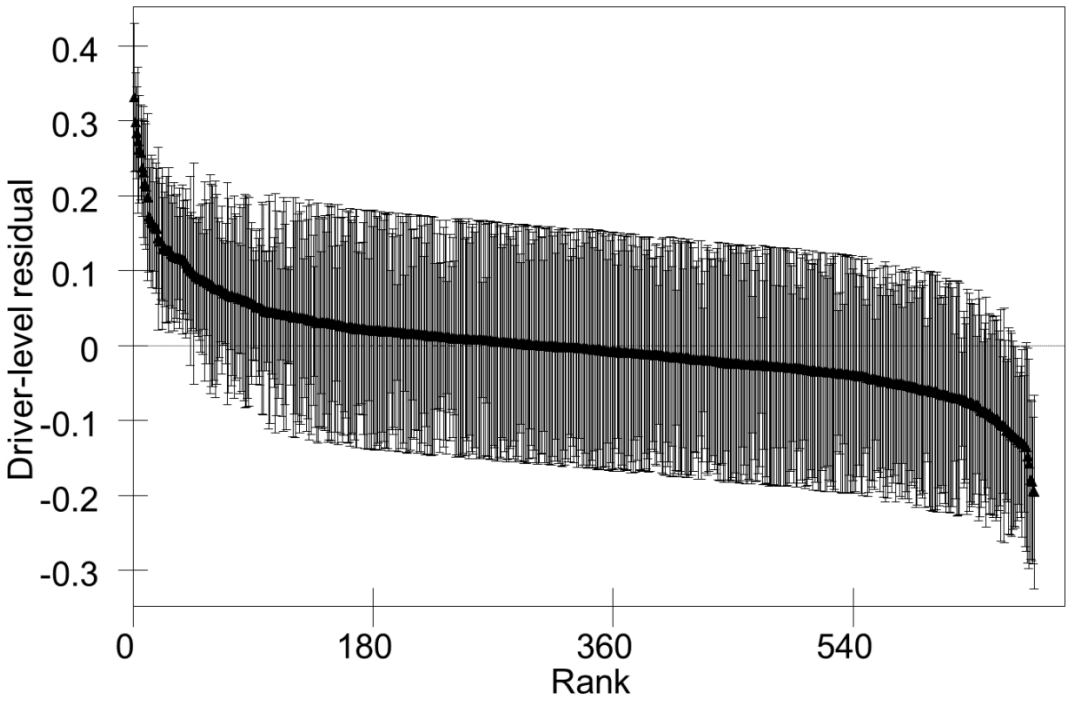


Figure A3: Top 20 team-level residuals, based on model (b) in table A3

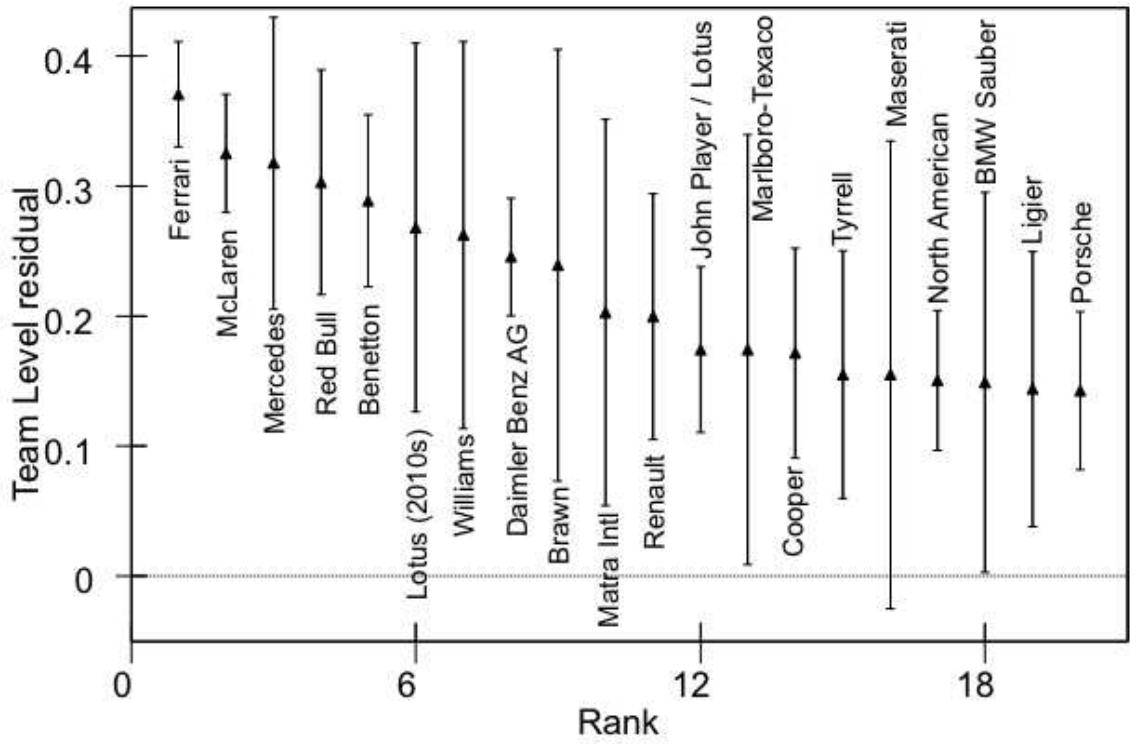


Figure A4: Top 20 team-year level residuals, based on model (b) in table A3. Red Bull in 2011 performed best relative to the same team's average performance.

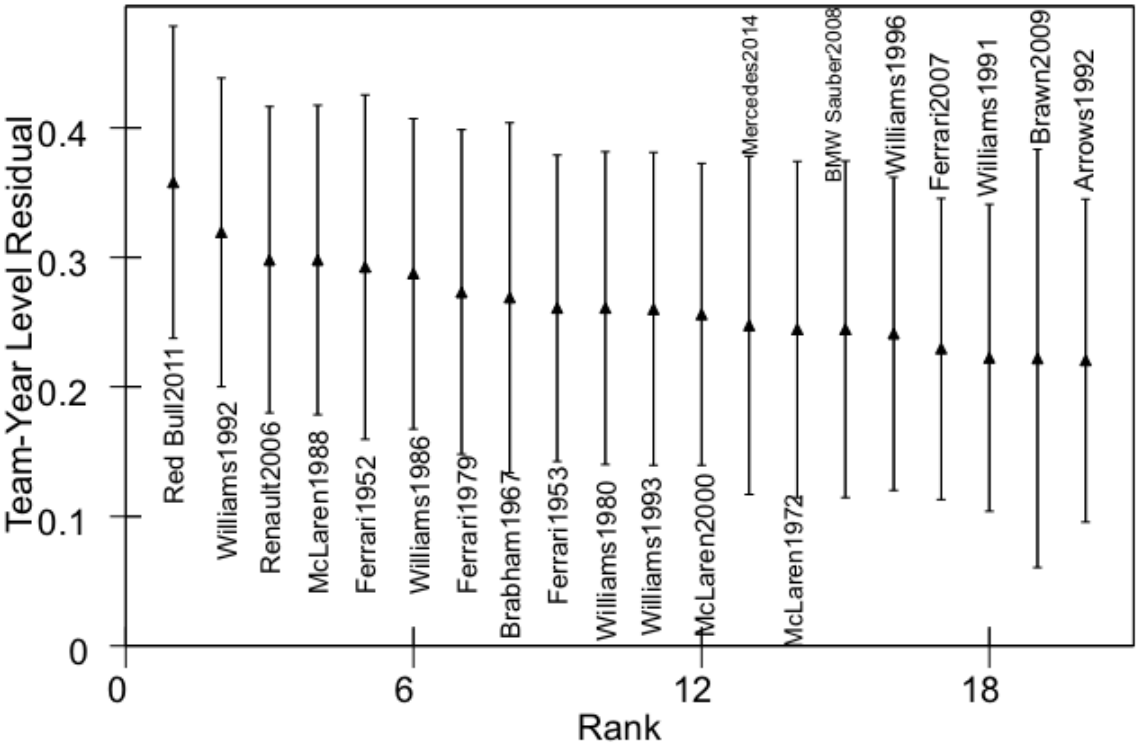
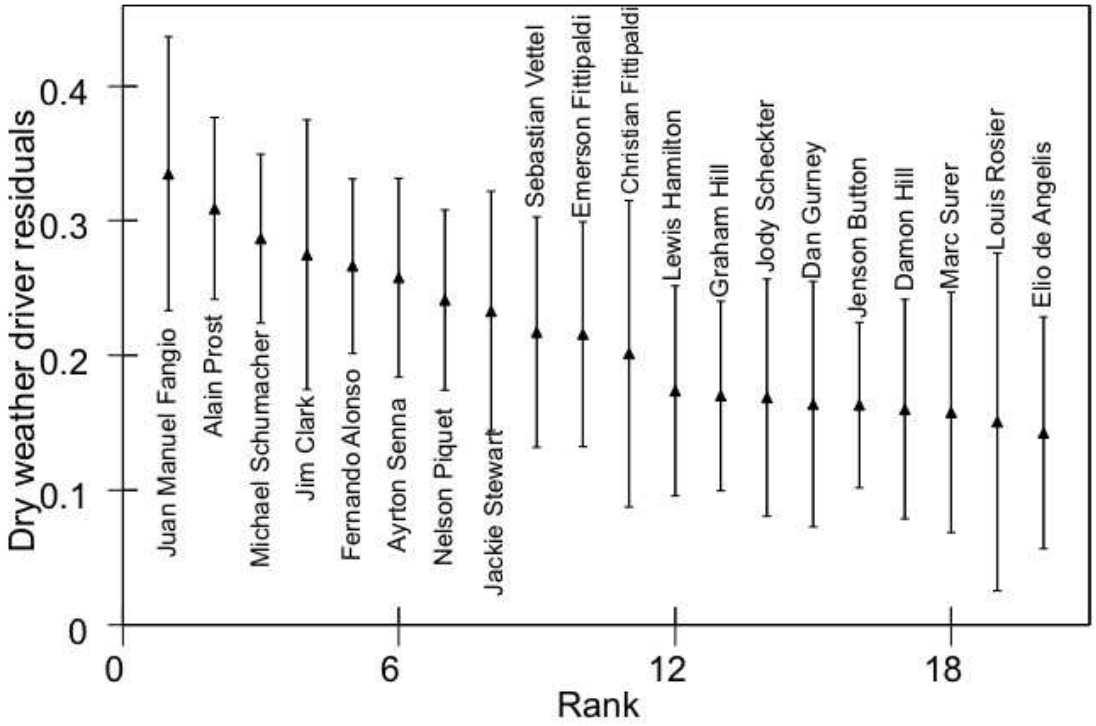
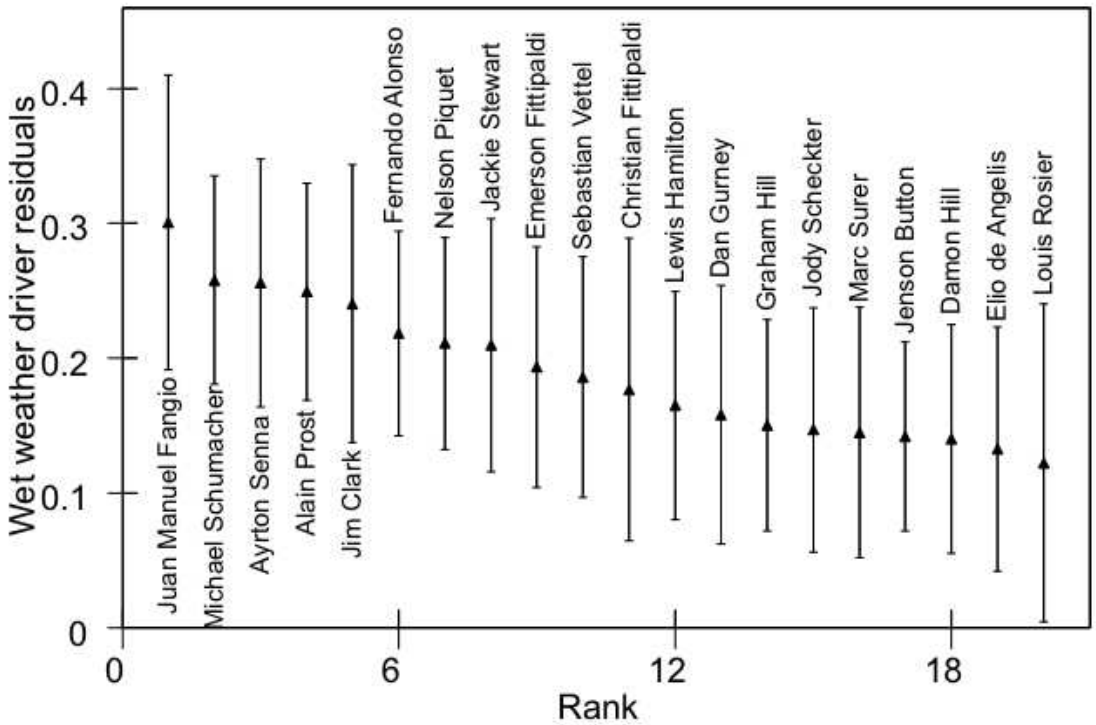
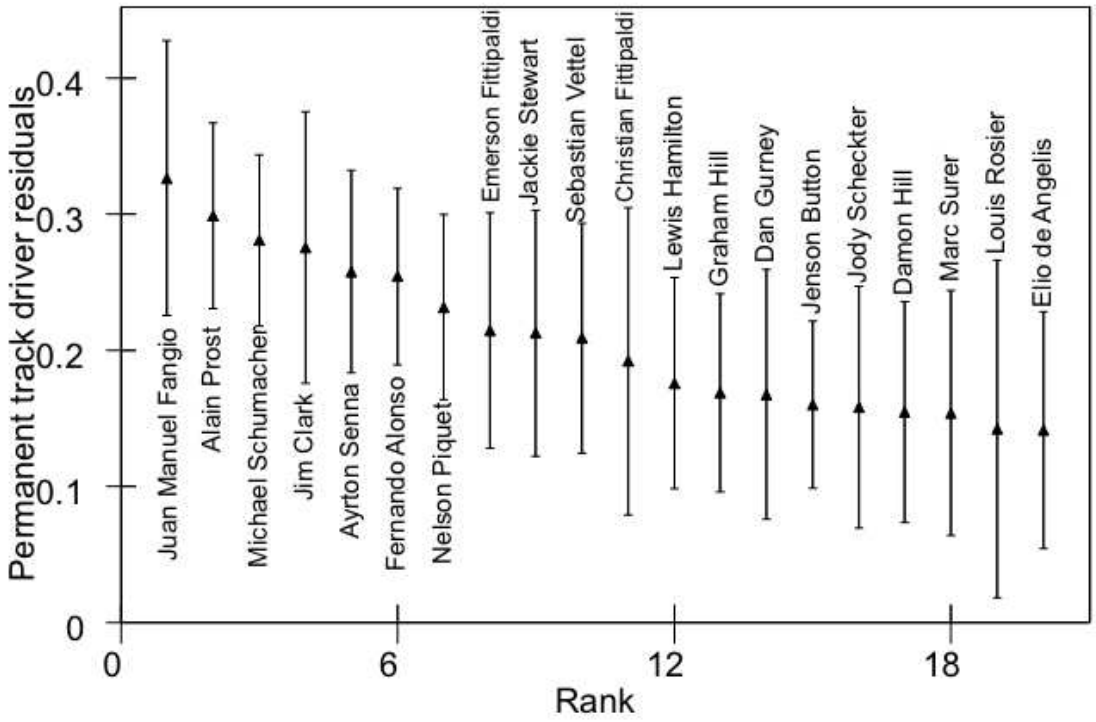


Figure A5: Plot of the top 20 driver-level residuals, representing the top 20 drivers of all time (1950-2014) according to our model in (a) wet and (b) dry conditions. Number of drivers and race competitiveness are controlled. Michael Schumacher is treated as two drivers (pre and post retirement), with only his pre-retirement performances represented in the graph. 95% credible intervals are shown. Based on the model in table A7.

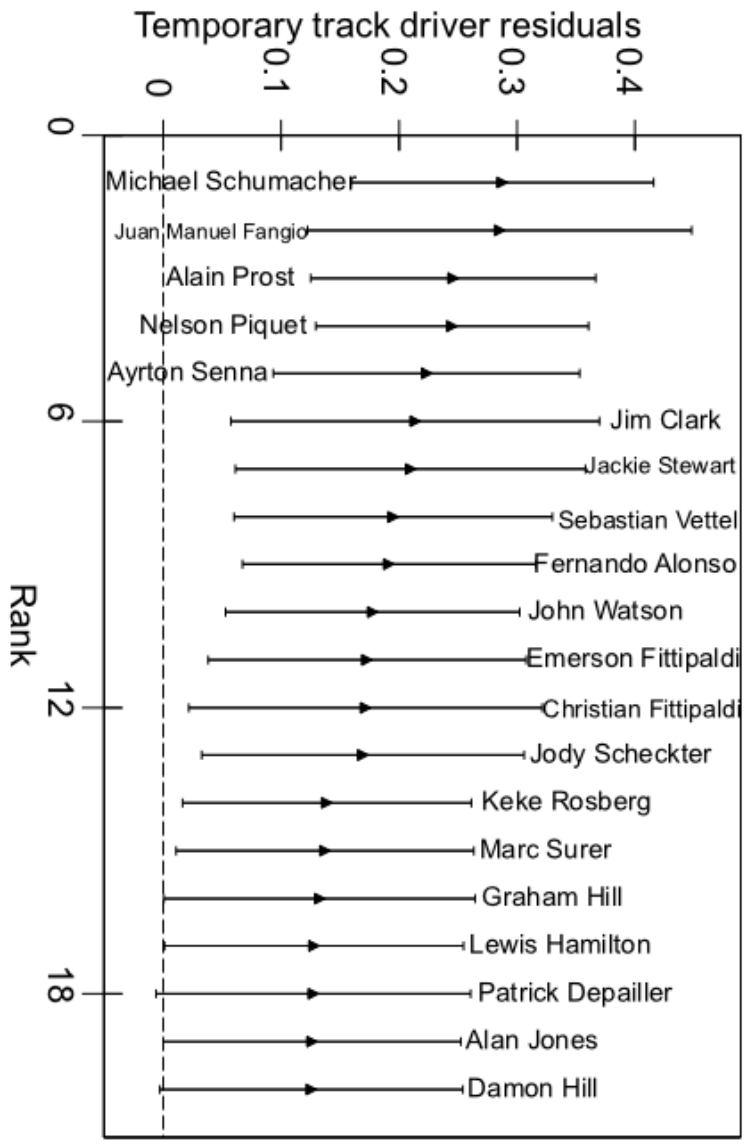


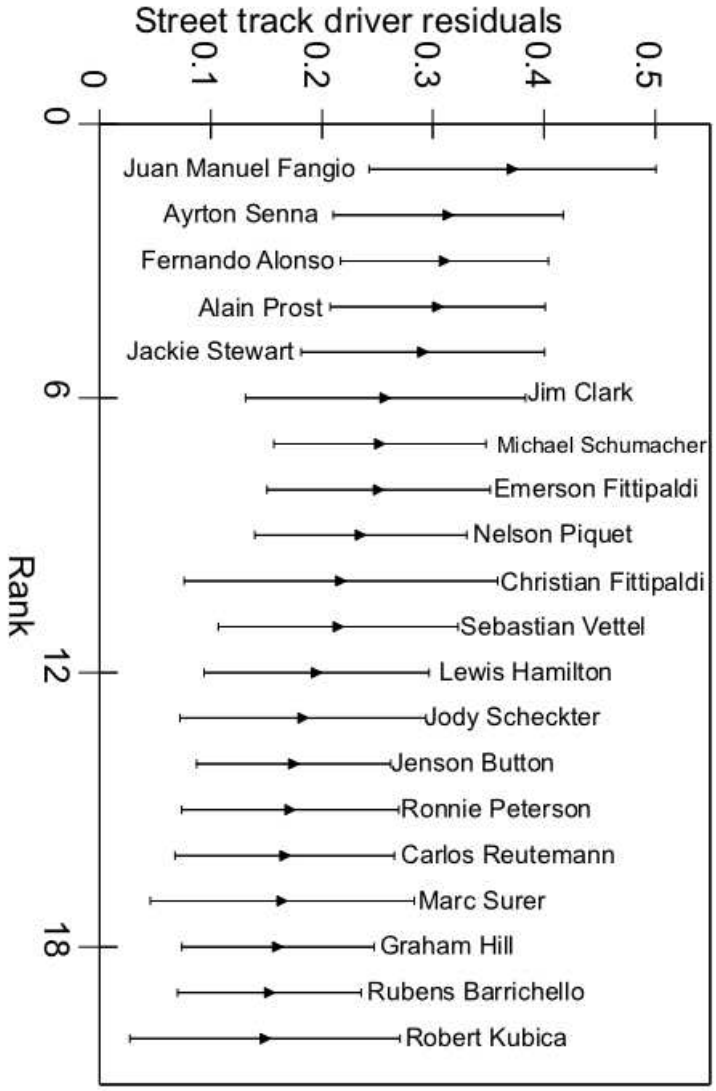


[1]Figure A6: Plot of the top 20 driver-level residuals, representing the top 20 drivers of all time (1950-2014) according to our model, on (a) permanent, (b) temporary and (c) street circuits. Number of drivers and race competitiveness are controlled. Michael Schumacher is treated as two drivers (pre and post retirement), with only his pre-retirement performances represented in the graph. 95% credible intervals are shown. Based on the model in table A8.



[1] These graphs (figures A6 and A7) were produced by models that used 'separate coding' in the random part of the model, to allow effects and their uncertainty to be most easily computed. These models produce exactly equivalent results to the 'contrast coding' expressed in equation 7; see Bullen et al. (1997). Schumacher is treated as two drivers in both figures. The coefficients for these models are given in the online appendix (tables A7 and A8).


Temporary track driver residuals
0
0.1
0.2
0.3
0.4
Rank
0
6
12
18
Michael Schumacher
Juan Manuel Fangio
Alain Prost
Nelson Piquet
Ayrton Senna
Jim Clark
Jackie Stewart
Sebastian Vettel
Fernando Alonso
John Watson
Emerson Fittipaldi
Christian Fittipaldi
Jody Scheckter
Keke Rosberg
Marc Surer
Graham Hill
Lewis Hamilton
Patrick Depailler
Alan Jones
Damon Hill


Street track driver residuals
0
0.1
0.2
0.3
0.4
0.5
Rank
0
6
12
18
Juan Manuel Fangio
Ayrton Senna
Fernando Alonso
Alain Prost
Jackie Stewart
Jim Clark
Michael Schumacher
Emerson Fittipaldi
Nelson Piquet
Christian Fittipaldi
Sebastian Vettel
Lewis Hamilton
Jody Scheckter
Jenson Button
Ronnie Peterson
Carlos Reutemann
Marc Surer
Graham Hill
Rubens Barrichello
Robert Kubica