

# Performance evaluation of fractional-slot tubular permanent magnet machines with low space harmonics

JIABIN WANG

*Department of Electronic and Electrical Engineering  
The University of Sheffield  
Mappin Street, Sheffield S1 3JD  
United Kingdom  
e-mail: j.b.wang@sheffield.ac.uk*

(Received: 18.09.2015 revised: 20.09.2015)

**Abstract:** This paper evaluates the performance of fractional-slot per pole winding configurations for tubular permanent magnet (PM) machines that can effectively eliminate the most undesirable space harmonics in a simple and cost-effective manner. The benefits of the proposed machine topology winding configurations are illustrated through comparison with 9-slot, 10-pole tubular PM machine developed for a free piston energy converter under the same specification and volumetric constraints. It has been shown that the proposed machine topology results in more than 7 times reduction in the eddy current loss in the mover magnets and supporting tube, and hence avoids potential problem of excessive mover temperature and risk of demagnetization.

**Key words:** Linear machines, permanent magnet machines,

## 1. Introduction

Tubular permanent magnet (PM) machines with modular (or fractional-slot concentric) windings are capable of providing high force density, excellent specific force ability, significantly high efficiency and high overloading capability. They have been used in a wide range of applications, such as vehicle active suspensions [1], free-piston energy converter [2], refrigeration applications [3], aerospace applications [4], etc. However, the fraction-slot windings produce a large number of space harmonics in the stator magneto-motive-force (mmf), which travels at different speeds relative to the permanent magnet mover, and hence cause many undesirable effects such as localized core saturation, eddy current losses in permanent magnets and back-iron, acoustic noise and vibrations.

To fully exploit the advantages offered by the fractional-slot per pole winding configurations, it is necessary to mitigate the above detrimental effects. There are a number of techniques reported in literature. The most common approach is to segment the mover magnets both axially and circumferentially in order to reduce the eddy current loss in the mover. Without this, the heat generated by the eddy current loss in the mover magnets may be excessive and may result in irreversible demagnetization.

zation [5]. However, this significantly increases the manufacturing cost and material usage (materials being wasted during segmentation process), and does not address the other undesirable effects. Further, a metallic tube is often required to provide a rigid support for the magnets and mover assembly, and it is also exposed in the time-varying field comprising of sub- and high order space harmonics. Obviously, the support tube cannot be segmented as this would destroy mechanical strength.

This paper describes new winding configurations for tubular permanent magnet machines, which can significantly reduce the eddy current losses due to cancellations of most significant space harmonics. The benefits of the proposed machine topology and winding configurations are illustrated through comparison with 9-slot, 10-pole tubular PM machine developed for a free piston energy converter under the same specification and volumetric constraints.

The rest of the paper is organized as follows. Section 2 outlines the proposed machine topology and winding configurations and analyzes their space harmonic distributions. Section 3 evaluates the performance of the proposed machine design for the same specification required for a free-piston energy converter and under the same volumetric and thermal constraints. Section 4 compares the mover eddy current loss with that of a 9-slot, 10-pole design. The main findings are summarized in conclusion.

## 2. Fractional slot winding configurations with low space harmonics

Without loss of generality, Figure 1 (a) shows the schematic of a conventional 9-slot, 10-active-pole tubular permanent magnet machine and Figure 1 (b) shows the resultant mmf harmonics normalized to the harmonic component with the largest magnitude. As will be seen, the 4<sup>th</sup> order space harmonic has a larger magnitude than the 5<sup>th</sup> working harmonic. It travels in the opposite direction with respect to the mover, and hence induces eddy current loss in the magnets and supporting tube.

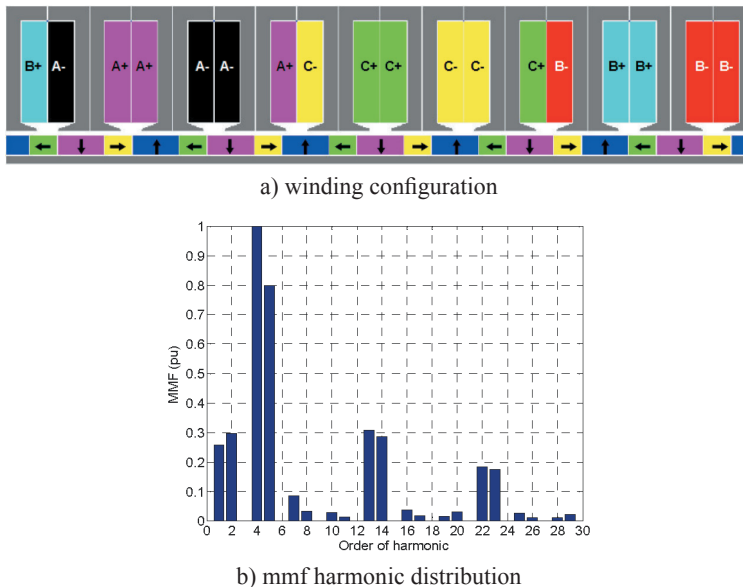
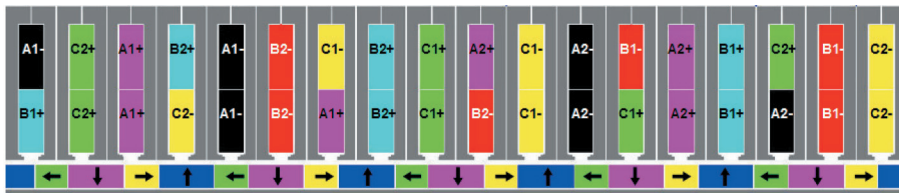


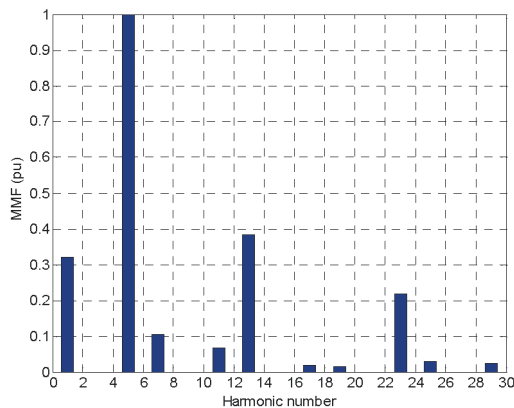
Fig. 1. Winding configuration and mmf harmonics of 9-slot, 10-pole tubular PM machine

To reduce the undesirable harmonics significantly, the number of slots is doubled to 18, and the stator windings are separated into two 3-phase sets, as shown in Figure 2 (a). The first 3-phase windings, denoted as A1, B1 and C1 and the second 3-phase windings as A2, B2 and C2, follow the same pattern as that shown in Figure 1 (a). However, the second 3-phase windings, A2, B2 and C2 are positioned with respect to the first 3-phase windings A1, B1 and C1 by an offset of 5 pole-pitches or 9 slots. The offset in electric degrees is  $900^\circ$  or  $5\pi$ . Thus, if phase windings A1 and A2 are connected in series or parallel with the opposite polarity, and the similar connections are made for B1 and B2, and C1 and C2, respectively, the resultant 5<sup>th</sup> order mmf space harmonic produced by the first 3-phase windings and that by the second 3-phase windings are in phase with respect to each other. The 5<sup>th</sup> order space harmonic has the same number of pole-pairs as that of the active part of the mover, and its interaction with the 5 pole-pair mover magnetic field produces the electromagnetic force. For all even ( $n = 2, 4, 8, \dots$ ) space harmonics, the phase shift between the harmonics produced by the first 3-phase windings and those by the second 3-phase windings is  $(n + 1)\pi$ . Thus these harmonics have the same magnitude but in the opposite direction and hence will be cancelled with each other by the proposed winding configuration. The resultant mmf harmonics are shown in Figure 2 (b).

It follows that the winding configuration shown in Fig. 2 (a) results in the highest possible winding factor for the 5<sup>th</sup> order working space harmonics, and complete elimination of all even space harmonics, while preserving the key advantages of the fractional-slot winding schemes.



a) winding configuration



b) mmf harmonic distribution

Fig. 2. Winding configuration and mmf harmonics of 18-slot, 10-pole tubular PM machine

The above techniques can be generalised for a class of fraction-slot tubular PM machines [6-8]. For the purpose of description, let  $p$  denote the number of pole-pairs and  $q$  being defined as the ratio of the number of slots per phase to the number of poles. For a 3-phase machine,  $q$  is given by:

$$q = N_s / (3 \times 2p) = N_s / (6p),$$

where  $N_s$  denotes the number of slots in the stator. For the conventional distributed winding configuration,  $q$  is an integer. For 3-phase machines with concentric winding configurations in prior arts,  $q$  is a fractional number, where the term ‘‘fractional-slot’’ is derived.

Many feasible slot and pole combinations exist for fractional-slot per phase per pole machines. In a subset of these combinations the slot number  $N_s$  is related to the number of pole-pairs  $p$  by  $N_s = 2p \pm 1$ . For 3-phase machines,  $N_s$  must be divisible by 3 and the phase shift in electrical degrees between phases must equal  $\pm 360k + 120$ , where  $k = 0, 1, 2, \dots$ . The resulting slot and pole combinations which are known in the prior art [9] are listed in Table 1 for  $p \geq 4$  and up to 23.

For the purpose of description, the corresponding  $q$  for each slot and pole combination is also shown in Table 1. It should be noted that any integer multiple of the slot and pole numbers of a given slot and pole combination in Table 1 also results in a feasible combination with the same  $q$ . For example, 18-slot and 16-pole, 27-slot and 24-pole, etc. They have the same  $q = 3/8$ . While the machine designs with these combinations have all the advantages of fractional slot permanent magnet machines, they also result in a large number of undesirable mmf harmonics.

Table 1. Slot-pole combinations of prior art

Number of pole-pairs	Feasible slot number	$q$
4	9	3/8
5	9	3/10
7	15	5/14
8	15	5/16
10	21	7/20
11	21	7/22
13	27	9/26
14	27	9/28
16	33	11/32
17	33	11/34
19	39	13/38
20	39	13/40
22	45	15/44
23	45	15/46
...	...	...

This paper is concerned with the techniques for significantly reducing the undesirable mmf space harmonics associated with the slot and pole combinations shown in Table 1, and any other

combination derived from integer multiple of a given combination listed in Table 1. The proposed technique can be summarized in the following steps:

- 1) Configure a 3-phase winding for a given slot and pole combinations described above according to the prior art.
- 2) Double the number of slots of the design.
- 3) Divide the 3-phase winding into two 3-phase sets denoted by (A1, B1, C1) and (A2, B2, C2). Each 3-phase winding is configured in the same pattern as that in step (1)
- 4) Position the second 3-phase winding (A2, B2, C2) 180 mechanical degrees away with respect to the first 3-phase winding (A1, B1, C1). A1 and A2 are connected either in series or in parallel with the same polarity if the number of pole-pairs is even, or opposite polarity if the number of pole-pairs is odd. Similar connections are made for B1 and B2, and C1 and C2.

It has been shown with the 9-slot, 10-pole ( $p = 5$ ) example that by adopting the above steps, all even harmonics for odd  $p$  are eliminated without affecting the fundamental winding factor. The subsequent example with 9-slot and 8-pole ( $p = 4$ ) will show that all odd harmonics for even  $p$  are also eliminated by adopting the proposed steps for winding configurations.

To reduce the undesirable harmonics significantly with even number of pole-pairs, the number of slots is doubled to 18, and the stator windings are separated into two 3-phase sets, as shown in Figure 3. The first 3-phase windings, denoted as A1, B1 and C1 and the second 3-phase windings as A2, B2 and C2, follow the same pattern as that shown in Fig. 1 (a). In this case, however, the coil span is now two slot-pitches instead of one slot-pitch because the number of slots has been doubled, but it has the same mechanical angle of 40 degrees because one slot-pitch is 20 ( $360/18$ ) mechanical degrees. For an 8-pole mover, the electrical degree of a slot-pitch is  $360 \times 4/18 = 80$  degrees.

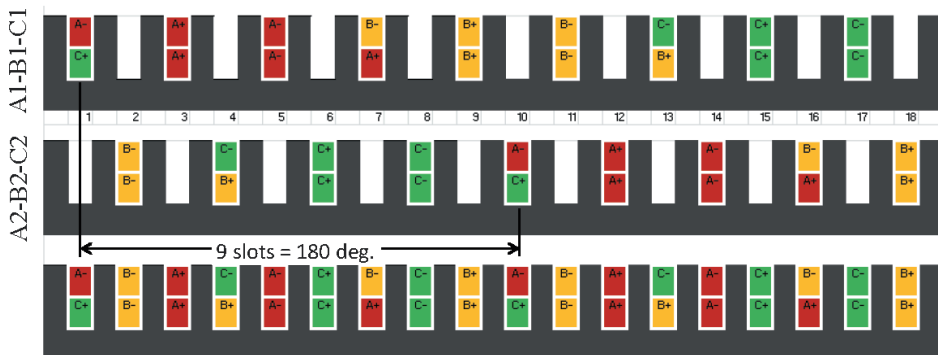


Fig. 3. Schematic of proposed 18-slots, 8-pole winding configuration

The second 3-phase windings A2, B2 and C2 are positioned with respect to the first 3-phase windings A1, B1 and C1 by an offset of 180 mechanical degrees or 9 slot-pitches. The offset in electric degrees is  $720^\circ$  or  $4\pi$ . Thus if phase windings A1 and A2 are connected in series or parallel with the same polarity, and the similar connections are made for B1 and B2, and C1 and C2, respectively, the resultant 4<sup>th</sup> order mmf space harmonics produced by the first 3-phase

windings and the second 3-phase windings are in phase with respect to each other. The 4<sup>th</sup> order space harmonic has the same number of pole-pairs as that of the mover, and its interaction with the 4 pole-pair rotor magnetic field produces the electromagnetic force. For all odd ( $n = 1, 3, 5, \dots$ ) space harmonics, the phase shift between the harmonics produced by the first 3-phase winding and those by the second 3-phase winding is  $n\pi$ . Thus, these harmonics have the same magnitude but in the opposite direction and hence will be cancelled with each other by the proposed winding configuration.

Therefore, the winding configuration shown in Fig. 3 results in the highest possible winding factor for the 4<sup>th</sup> order working space harmonics, and complete elimination of all odd space harmonics, while preserving the key advantages of the fractional-slot per phase per pole winding schemes.

Figure 4 shows the normalized mmf space harmonics distribution produced by the 3-phase windings in Figure 1 or one of the 3-phase windings, viz., A1-B1-C1 or A2-B2-C2, in Figure 3. As will be seen, the mmf contains forward rotating harmonics for  $n = 1, 4, 7, \dots$ , backward rotating harmonics for  $n = 2, 5, 8, \dots$ , and zero triplen harmonics. For the 8-pole machine, however, only the 4<sup>th</sup> mmf harmonic interacts with the magnetic field of the mover to produce continuous force. The other harmonics, in particular the backward-rotating 5<sup>th</sup>, which has relatively large magnitude, and travels at the twice of speed with respect to the mover, may cause undesirable effects, such as localized core saturation, additional iron loss, and eddy current loss in the magnets, etc, as described previously.

Figure 5 shows the normalized mmf space harmonic distribution produced by the two sets of 3-phase windings shown in Figure 3. As will be seen, all odd space harmonics have disappeared. The remaining harmonics have relative low magnitude except for the 2<sup>nd</sup>, 14<sup>th</sup> and 22<sup>th</sup>. However, since the wavelength of the 14<sup>th</sup> and 22<sup>nd</sup> harmonics is relatively short, the resulting magnetic field due to these harmonics will be attenuated rapidly in the radial direction towards to the mover, as can be seen from the analytical expression given in [10]. Therefore the undesirable effects caused by these harmonics will be much less significant. Likewise, the frequency of the 2<sup>nd</sup> harmonic seen by the mover is relatively low, and the resultant undesirable effect is also significantly less.

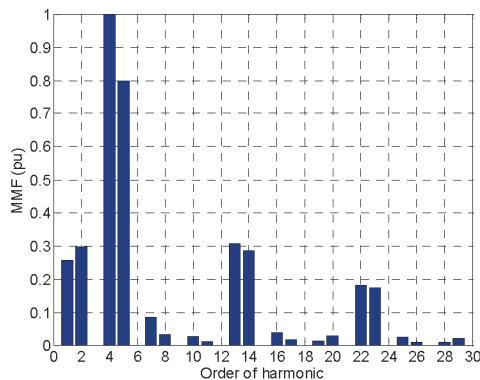


Fig. 4. Normalized mmf distribution by one set of 9-slot, 8-pole, 3-phase windings shown in Fig. 3

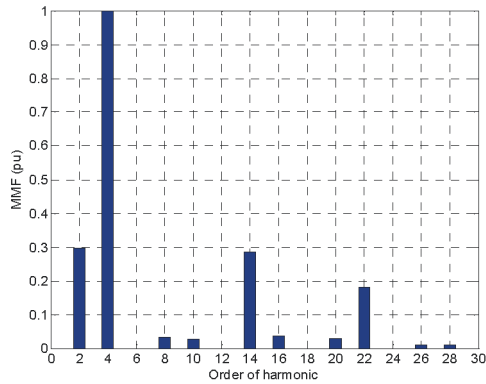


Fig. 5. Normalized mmf distribution of proposed winding configuration for 18-slot, 8-pole machine

By applying the same concept to all slot-pole combinations listed in Table 1, a new set of slot-pole combinations in Table 2 are derived that yield significant reductions in undesirable harmonics.

The same concept is applicable to any slot-pole combination with  $q$  listed in Table 2, for example, 36-slot 16-pole, 54-slot 24-pole, ..., etc, machines whose slot and pole number are integer multiples of 18-slot 8-pole with  $q = 3/4$ . As will be seen, the phase shift between the two 3-phase windings is always equal to 180 mechanical degrees or half of the number of slots, whilst the polarity of the two 3-phase windings is the same for even number of pole-pairs and opposite for odd number of pole-pairs.

Table 2. Proposed slot-pole combinations with low space harmonics

Number of pole-pairs $p$	Slot number $N_s$	$q$	Offset between two 3-phase windings (number of slots)	Polarity between two 3-phase windings
4	18	3/4	$N_s/2 = 9$	same
5	18	3/5	$N_s/2 = 9$	opposite
7	30	5/7	$N_s/2 = 15$	opposite
8	30	5/8	$N_s/2 = 15$	same
10	42	7/10	$N_s/2 = 21$	same
11	42	7/11	$N_s/2 = 21$	opposite
13	54	9/13	$N_s/2 = 27$	opposite
14	54	9/14	$N_s/2 = 27$	same
16	66	11/16	$N_s/2 = 33$	same
17	66	11/17	$N_s/2 = 33$	opposite
19	78	13/19	$N_s/2 = 39$	opposite
20	78	13/20	$N_s/2 = 39$	same
22	90	15/22	$N_s/2 = 45$	same
23	90	15/23	$N_s/2 = 45$	opposite
...	...	...	...	

### 3. Performance evaluation

To illustrate the key benefits of the proposed new winding configurations with low space harmonics, an 18-slot 10-pole tubular PM machine is designed against the same specifications under the same volumetric and thermal constraints for applications in a free-piston energy converter [2]. The key design parameters and specifications are listed in Table 3. The mover magnets employ quasi-Halbach magnetization with the ratio of the width of radially magnetized magnet to the pole-pitch being 0.6.

Table 3. Leading design parameters and specifications

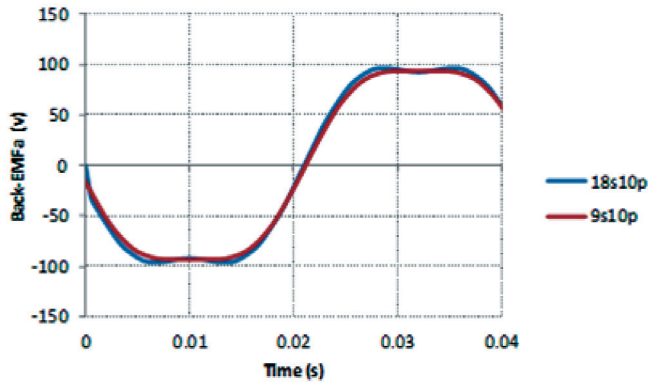
Outer stator radius	0.097 m
Pole-pitch	0.022 m
Outer mover radius	0.064 m
Air-gap length	0.001 m
Magnet thickness	0.006 m
Rate power	44.0 kW
Rated velocity	11 m/s
Rated thrust force	4000 N
Rated current (peak)	285 A

To minimize the mover mass for high reciprocating frequency and hence power density, the magnets are supported by a 2.5mm thick Titanium tube. The performance of the machine may be assessed analytically as described in [10] or by finite element (FE) analysis.

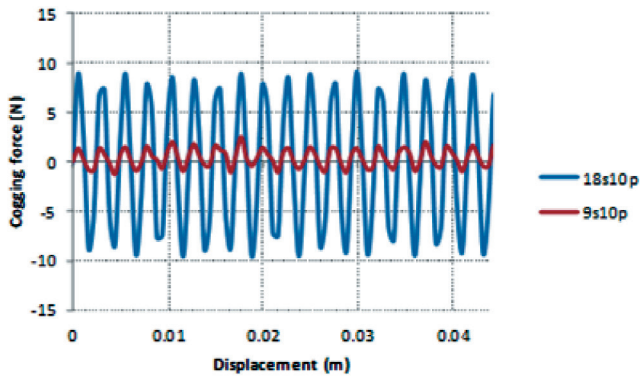
#### 3.1. Back-emf and cogging forces

For simplicity, the performance of the proposed and conventional machine topologies is assessed by 2D axi-symmetric FE models with periodic boundary conditions imposed over the active length of the machine at both axial ends. Therefore, the end effect of the finite stator and mover length is not accounted. This will not affect performance predictions significantly except for thrust force ripple. Figure 6 (a) compares the back-emf of the proposed machine at the mover speed of 11 m/s with that of the 9-slot, 10-pole machine designed with the same specification under the same volumetric and thermal constraints. Similar comparison of cogging force variations with the mover axial displacement is given in Figure 6(b). The magnitudes of the fundamental back-emf and the peak-to-peak cogging forces of the two designs are listed in Table 4. It can be seen that back-emf magnitudes are very close to each other, while the peak-to-peak cogging force of the 18-slot, 10-pole machine is much greater. However, compared with the force ripple due to the end effect and emf harmonics, the cogging forces of the both machines are negligible. The slight higher back-emf of the 18-slot, 10-pole design is due to the fact that the saturation level in its stator back-iron is marginally lower.





a) back-emf at mover speed of 11 m/s



b) cogging force

Fig. 6. Back-emf and cogging force waveforms

Table 4. Comparison of back-emf and cogging force

	18s10p	9s10p
Magnitude of back-emf (V)	96.07	93.80
Cogging Force (p-p) (N)	18.67	3.72

### 3.2. Thrust force

The thrust force variations of the 18-slot, 10-pole and 9-slot, 10-pole machines with mover displacement are compared in Figure 7 when both the machines are excited with the rated current in phase with their back-emfs at the rated velocity. It is seen that the peak-to-peak force ripple of the 18-slot, 10-pole machine is also slightly higher, while the average thrust forces of the two designs are very close, being 3922 N and 3903 N for the 18-slot and 9-slot machines, respectively. The percentage difference in thrust forces between the two designs is lower than that in back-emfs, since the 18-slot machine is slightly more sensitive to magnetic saturation in

teeth. Figure 8 compares thrust force variations with mover velocity when the winding temperatures of the both machines are kept at 135°C.

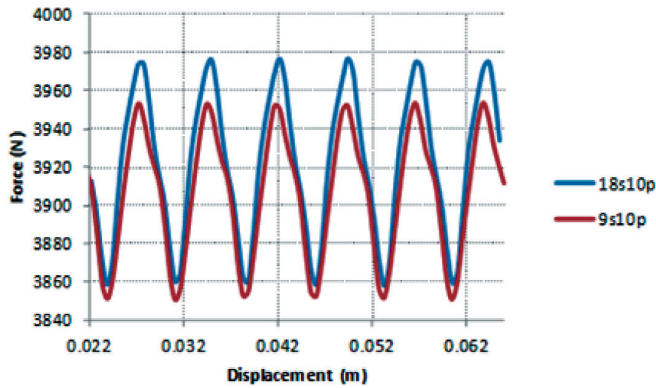


Fig. 7. Thrust force variations with mover displacement

As can be seen, the thrust force of the 9-slot, 10-pole machine drops faster than that of the 18-slot, 10-pole machine as the speed increases. This is mainly due to higher mover losses in the 9-slot, 10-pole machine, which will become clearer in Section 4.

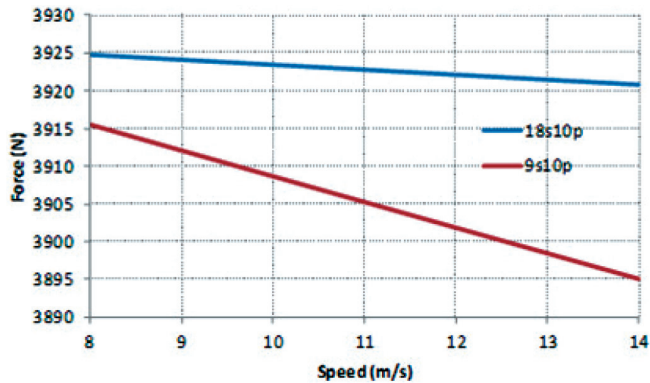
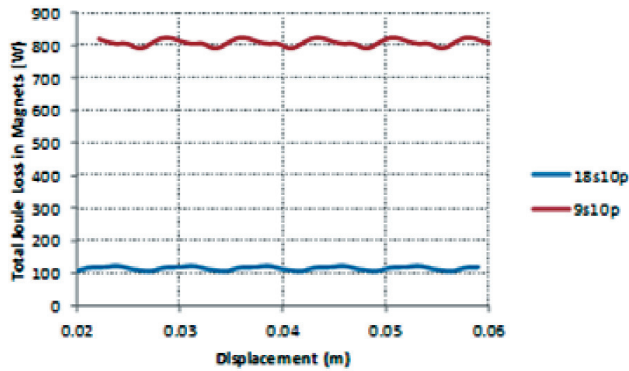


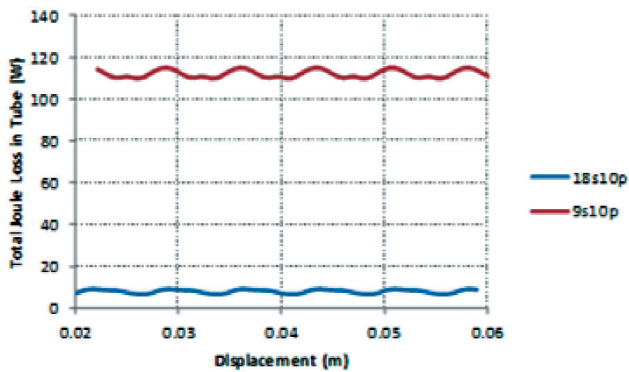
Fig. 8. Variations of thrust forces with speed when winding temperatures fixed to 135 °C

#### 4. Mover eddy current loss

The proposed 18-slot, 10-pole machine eliminates all even order mmf space harmonics of the armature reaction field, and hence the mover eddy current losses associated stator mmf harmonics will be much lower. Fig. 9 compares eddy current loss (Joule loss) variations in the magnets and supporting tube of the 18-slot, 10-pole and 9-slot, 10-pole machines with time when they operate at the maximum speed of 14 m/s and rated current. The magnets are assumed to have a circumferential slit to break circular eddy current flow.



a) eddy current loss in magnets



b) eddy current loss in supporting tube

Fig. 9. Variations of mover eddy current losses in 9-slot, 10-pole and 18-slot, 10-pole machines at 14 m/s and rated current

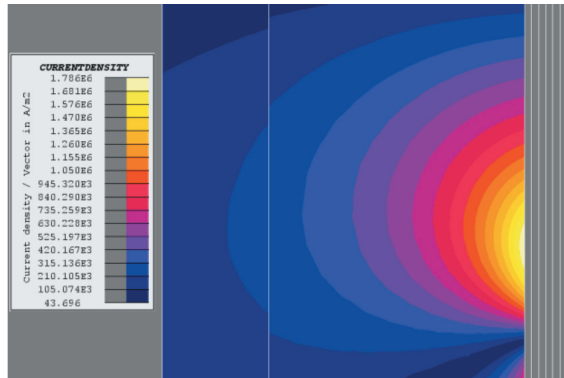
It can be seen that at the maximum speed 14m/s, the losses in magnets and the Titanium supporting tube of the 9-slot, 10-pole machine are 1302.4 W and 180.1 W, respectively, which are 7.1 and 14.5 times greater than 184.5 W and 12.4 W in the magnets and support tube of the 18-slot, 10-pole machine, respectively.

Table 5 compares the average eddy current losses in the magnets and supporting tube of the 9-slot, 10-pole and 18-slot, 10-pole machines at the rated speed and rated current. The losses in the magnets and supporting tube of the 9-slot, 10-pole machine are 6.9 and 14.1 times greater than those of the proposed 18-slot, 10-pole machine. Figure 10 shows the eddy current density distributions in the two machines at the mover position of  $\pi/4$  when operating at the rated speed and current. It is seen that the peak eddy current density in the 9-slot machine is  $\sim 2.2$  times greater than that of the proposed machine. Indeed, the mover losses in 9-slot, 10-pole machine at the rated operation are excessive under the given cooling condition. To reduce the losses, the magnets have to be circumferentially segmented into 16 pieces so that the total mover loss can be limited below 300W, and the mover temperature below 175 °C. In contrast, with

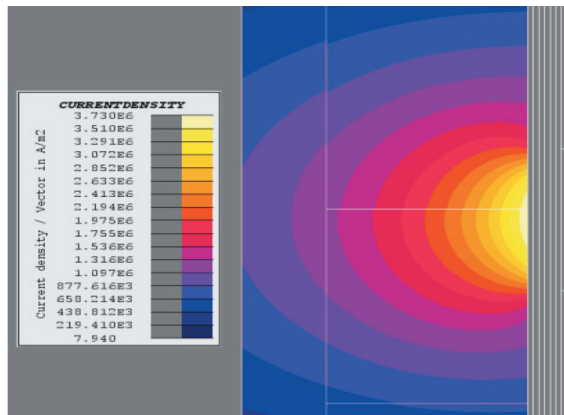
the proposed 18-slot, 10-pole design, the mover temperature is less than  $150^{\circ}\text{C}$ , without any segmentation of magnets.

Table 5. Comparison of eddy current losses

	18s10p	9s10p
Eddy current loss in magnets (W)	115.04	797.96
Eddy current loss in tube (W)	7.92	111.97



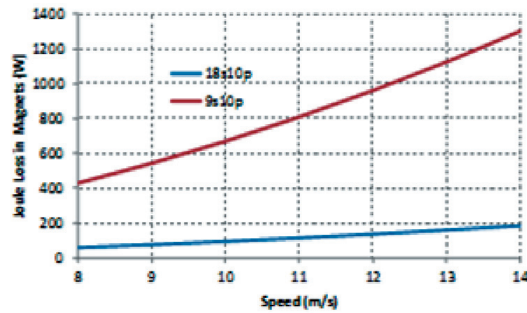
a) 18-slot, 10-pole



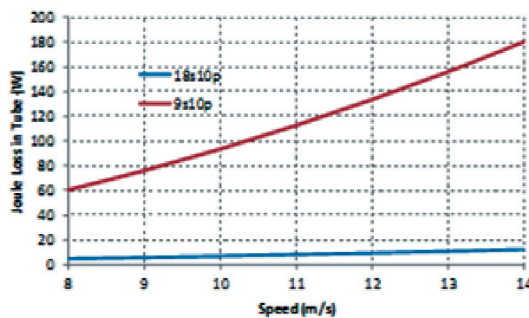
b) 9s10p

Fig. 10. Eddy current density distribution in magnets and tube at mover position of  $\pi/4$  when operating under the rated condition

Figure 11 compares eddy current loss variations of the two machines with mover speed at the rated current. The advantages of the proposed machine in reducing mover losses is evident.



a) in magnets



b) in supporting tube

Fig. 11. Variations of mover eddy current losses with speed

## 5. Conclusion

New winding configurations for fractional-slot tubular PM machines with low space harmonics have been described, and the merits and performance of the proposed winding configurations have been evaluated for an 18-slot, 10-pole fractional slot tubular PM machine. It has been shown that all even space harmonics in the machine are eliminated, and consequently, mover eddy current loss is significantly reduced. Compared with a 9-slot, 10-pole design variant, the proposed machine reduces the eddy current losses in magnets and supporting tube by 7 and 14 times, respectively. The impact of this reduction on managing the mover temperature is very significant.

## References

- [1] Wang J., Wang W., Atallah K., *A Linear Permanent-Magnet Motor for Active Vehicle Suspension*. IEEE Trans. on Vehicular Technology 60(1): 55-63 (2011).
- [2] Wang J., et al., *Design and Experimental Verification of a Linear Permanent Magnet Generator for a Free-Piston Energy Converter*. IEEE Trans. on Energy Conversion 22(2): 299-306 (2007).
- [3] Wang J., Howe D., Lin Z., *Design Optimization of Short-Stroke Single-Phase Tubular Permanent-Magnet Motor for Refrigeration Applications*. IEEE Trans. on Industrial Electronics 57(1): 327-334 (2010).

- [4] Galea M, et al. *Considerations for the design of a tubular motor for an aerospace application*. Proc. of Electrical Machines and Systems (ICEMS), 2011 International Conference on (2011).
- [5] Wang J., Wang W., Atallah K., Howe D., *Demagnetization Assessment for 3-phase Tubular Brushless Permanent Magnet Machines*. IEEE Trans. on Magnetics 44(9): 2195-2203(2008).
- [6] Wang J., *An Electric Machine*. UK Patent, 1221635.4, Filed on Nov. 30 (2012).
- [7] Wang J., Patel V., Wang, W., *Fractional-slot Per Phase, Per Pole Permanent Magnet Machines with Low Space Harmonic Contents*. IEEE Trans. Magnetics 50(1): 8200209 (2014).
- [8] Wang J., “Fractional-slot Tubular Permanent Magnet Machines with Low Space Harmonics”, Proc. of Linear Drives and Industry Applications (LDIA2015), Aachen, Germany July, 2015.
- [9] Wang J., Atallah K., Zhu Z. Q., Howe D., *Modular 3-phase permanent magnet brushless machines for in-wheel applications*. IEEE Trans. on Vehicular Technology 57(5): 2714-2720 (2008).
- [10] Wang J., Howe D., *Tubular modular permanent magnet machine equipped with quasi-Halbach magnetized magnets – Parts I and II*. IEEE Trans. Magnetics. 41(9): 2470-2489 (2005).