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Finite Element Analysis of Progressive Degradation versus Failure Stress Criteria on Composite Damage Mechanics

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1. Introduction

It is well known that the engineering applications using composite materials is in constant growth, mainly because of the large strength/weight ratio that they provide. The modelling of these materials has been of interest for a long time, due to the experimental costs that can be saved by means of computer simulations. However, the mixed mode of failure in composite materials makes it a complicated task to deal with, resulting often in sophisticated damage models.

There have been numerous techniques proposed for the simulation or prediction of the failure of composites. Many of these techniques were integrated on analytical methods that were subsequently implemented on major simulation software packages or in-house finite element method programs. This is the case in failure models based on stress quadratic functionals, such as those by Tsai & Wu (1971), and implemented within ANSYS (Swanson, 2007) or by Hoffman (1967) and included within ABAQUS (Hibbit et al., 2007). Such functionals imply the disappearing of bearing capability to outstanding loads once the stress criteria are satisfied. From a strict numerical point of view, a finite element satisfying the criteria may potentially be removed from the mesh as it does not experience further loading. This possibility is available in major software packages such as LS-DYNA. The removal of a finite element frequently causes certain numerical oscillations when using explicit solvers. This may degenerate into instabilities and, hence, in divergence of the numerical procedure. A significant number of these criteria have been proposed in the last decades. For instance, the models by Tsai & Wu (1971), Hoffman (1967), Yamada and Sun (1978) or Puck and Schurmann (1998) amongst many others have been very popular. A worldwide assessment failure exercise (WWFE) of a number of these criteria is described in references (Hinton and Soden, 1998; Hinton et al., 2004). Also, Soden et al. (1998a) presented the result for fibre-reinforced composite laminates and their correlation to a set of shared-by-participants experimental data (Soden et al., 1998b). It is clear that considerable efforts have been done in the searching of a general criteria that may be applied in a wide range of problems. However, Daniel (2007) reveals discrepancies of up to 200-300% in the WWFE results shown by Soden et al. (1998a). Unawareness of the numerical consequences that carry the use of these criteria within a finite element method, such as instability and, finally, divergence of the numerical procedure , result in unrealistic solutions. On the other hand, different

computational techniques for modelling damage progressively were developed to adapt to the finite element methodology. The progressive damage causes the degradation of the stiffness in the damaged zone. Thus, a damaged finite element does not lose completely its loading bearing capacity but the latter is decreased inversely proportional to the degree of damage. The progressive damage models derive from the thermodynamical approaches proposed by Kachanov (1958) initially and, most famously acknowledged by Lemaitre (1992); Lemaitre and Chaboche (1990) and Chaboche (1981). Proposals in this field by Matzenmiller et al. (1995), Maimi et al. (2007a;b), Barbero & De Vivo (2001) or Schipperen (2001) have contributed to extend the number of techniques available for damage evolution on composite materials.

The progressive damage models are attractive as they are readily implemented either in major codes or in-house finite element programs. Nevertheless, these models have the uncertainty on when the onset of damage is reproduced. Some authors coupled it to stress criteria as initiation criteria for developing damage to solve this drawback. For instance, Lapczyk & Hurtado (2007) combined a progressive damage model with the stress criteria proposed by Hashin (1980) as a damage initiation criteria. The formulation is based on the fracture energy for representation of fibre failure and matrix failure. Hufenbach et al. (2004) have successfully shown how interactive criteria –combining progression and failure criteria– may be applied for the prediction of failure in textile reinforced composites assuming that they are formed by unidirectional layers. However, Cuntze & Freund (2004) state that the conditions of initiation of failure are not as relevant as the evolution of the stiffness degradation, due to the fact that its influence is decreasing with the damage progression. This is in agreement with other theories that defend the inelastic behaviour of a range of composite materials, (Barbero & Lonetti, 2002). Chow and Yang (1998) developed an inelastic model for the description of damage in composite laminates and its implementation into an incremental displacement-based Finite Element Method(*FEM*). The stress strain relationship is incorporated into a modified Newton-Raphson iterative method. More recently, Zobeiry et al. (Camanho et al., 2008) presented a progressive damage model with special attention to the nonlocal regularisation of the damage computations. A significant number of these last approaches are limited to plane stress models, such as those by, for example, Allen et al. (1987); Edlun and Volgers (2004); Harris et al. (1995); Hochard et al. (2001); Talreja (1987); Tan (1991) and McCartney (2003). In the best of knowledge, pioneering works on nonlinear behaviour of composites were developed by Chang and Chang (1987) and by Shahid and Chang (1995). Both works are dedicated to the analysis of composite plates. Lessard and Shokrieh (1995) state that two-dimensional analysis may produce sensibly different results as a consequence of the anisotropy induced by distinct modes of damage in the originally orthotropic composite. Nowadays, three dimensional models for laminates are readily implemented in computational techniques due to advances in computer power and programming facilities.

New techniques have been explored for assessing damage and, in some cases, healing on composite laminates. Such are the cases of the Virtual Crack Closure Technique (VCCT) or the use of cohesive elements –(interface elements)– on finite element procedures. Both of these techniques have links to the *Fracture Mechanics* field. VCCT was proposed by Rybicki and Kanninen (1977) and Rybicki et al. (1977) derived from the Irwint's theory (Irwin, 1948) for crack analysis. Xie & Biggers (2006) and Leski (2007) coded successfully VCCT within a finite element program. VCCT relies on the calculation of the J-integral without the restriction

of having excessive refinement of the mesh in the proximities of the crack tip which is also an advantage for computational saving. VCCT has the problem, like progressive damage models based in thermodynamical theory, of not having an initiation criteria for propagation of the fracture. The use of cohesive elements has been recently boosted. An excellent works by Camanho & Mathews (1999); Camanho et al. (2003) or Iannucci and Willows (2006) show a damage progression scheme combined with interface elements to couple the damage evolution with the mechanics of the fracture. An excellent review of these technique is provided by Wisnom (2010). Cohesive models are preferred to VCCT technique for a growing number of authors, such as Dugdale (1960), Xie & Waas (2006), Turon et al. (2007)), Tvergaard & Hutchinson (1996), Allen & Searcy (2000), or Cox & Yang (2006). Camanho et al. (2008) and also Hallet (1997) have used interface elements for the prediction of delamination on laminates.

In this chapter, finite element analysis is applied to laminates, and the formulation of the model is developed at lamina scale. The laminate is a stack of laminae of different, in general, fibre orientations. An explicit integration strategy for the finite element analysis is used due to the simplicity and robust convergence that provide[1]. The model is adapted in order to be included into an explicit *FEM* (see explicit formulation in Curiel Sosa et al. (2006) for implementation details) whereby the transient response may be conveniently simulated. This chapter is outlined as follows: firstly, a general discussion over damage modes is performed; secondly, the main theoretical aspects of the model are shown; thirdly, the computational algorithm, for implementation of the damage model as an individual module into an in-house *FEM* program as well as in major commercial software packages such as Abaqus or Ansys, is provided; and finally, a set of numerical examples including the low velocity impact on a composite laminate $[0, 90]_a$.

2. Composite damage modes

The damage in composites is generally represented by several modes: matrix cracking, matrix crushing, fibre kinking, fibre rupture or breakage. From a modelling point of view, one may find in the literature different choices of mixed modes of damage in composites. Hashin (1980) considered in his stress criteria four modes:

- matrix failure in tension and compression represented by a criterion that includes transverse-to-fibres stress and a combination of shear stresses.
- fibre-matrix disbonding as a function of the longitudinal stress and shear stresses.
- fibre failure in compression or tension, depending upon the limit values of the axial stresses.
- delamination.

Similar mixed modes criteria were used by Chang and Lessard (1991) which were coded by Ambur et al. (2004) as an *Abaqus* user subroutine for the reader interested in this type of computer implementations. They modelled the progression of the damage modes proposed by Hashin (1980) and applied it to the simulation of composite shells. No implementations to 3D solid elements were shown. Some authors, for example Curiel Sosa et al. (2008a); Curiel Sosa (2008b); Matzenmiller et al. (1995), take into account the following damage modes for modelling:

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[1] Stability must be satisfied, i.e. step-time can not trespass on the critical time step

- fibre rupture.
- fibre kinking.
- matrix cracking.
- matrix crushing.

In the progressive damage model presented in Section 3 a three dimensional element is used rather than shells and, hence, a general description of the modes is more appropriate under these premises. Delamination is in an upper material scale and is implicitly implemented in the general flow of the procedure as matrix cracking may eventually result in delamination. In this manner, inelasticity is integrated straightforward within the model. It is well-known from experimental evidence that the idealisation of composite behaviour as linear elastic is inadequate (Chang and Lessard, 1991; Xu, 1994) as inelastic deformations evolve not only due to micro-cracks at the micro-scale but also due to complex damage modes occurring at the macroscale, leading, eventually, to the total failure. Contrary to purely brittle materials, the fibre-reinforced composite material exhibits at some extent a softening behaviour preceding the total failure. This phenomenon, from a strictly thermodynamical point of view, is related to the dissipative (irreversible) process that rearranges the distribution of material properties due to the presence of damage.

3. Progressive damage modelling (PDM)

In this section, a progressive damage model for composites (PDM) is presented (Curiel Sosa et al., 2008a; Curiel Sosa, 2008b), the results of this model are then compared with the outcome of some stress failure criteria. The formulation concerning the implementation of the damage model is also presented below. In PDM, it is assumed that different damage modes develop simultaneously on the failed composite structure and that they can interact directly influencing each other. The approach presented may be framed within the continuum damage mechanics field. The damage variables represent the state of damage at any stage of deterioration. They are implicitly defined for what they may be considered internal state variables that can provide a quantifiable magnitude of the degradation of the composite material. In PDM, damage variables are referred to damage modes via superposition. Thus, a damage mode is represented for one or more than one damage variables with distinct weights as explained below. The adopted damage modes, generically denoted as γ, are modelled by means of a linear combination of growth functions Φ^γ and damage directors $\mathbf{v}^\gamma$ (Curiel Sosa et al., 2008a). In summary, PDM admits the modelling of different damage modes that can be integrated directly in the description of damage through Equation (6)which is explained on Section (4).

Thus, the damage state may be defined by a series of internal variables ω_{kj}, filling the diagonal damage tensor $\mathbf{D}$ (see Equation (1)) that represents the state of damage in the composite.

$$\hat{\boldsymbol{\sigma}} = \mathbf{D} \cdot \boldsymbol{\sigma} \tag{1}$$

where $\boldsymbol{\sigma}^T = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}]$ is an array formed by the stress components and $\hat{\boldsymbol{\sigma}}$ is the so-called effective stress array (Chaboche, 1981). The definition of tensors and properties is conducted in a local system of reference for the lamina. Variables and parameters numeric subscripts refer to the local lamina system of reference. Thus, axis 1 is pointed

in the longitudinal direction to the fibres whereas the other two axes, i.e. 2 and 3, are in perpendicular direction to fibres.

The damage tensor is built as a diagonal tensor and contains the damage internal variables ω_{kj}, see Equation (2). These are responsible for the degradation of the stiffness components.

$$diag\,(\mathbf{D}) \;=\; \left[\frac{1}{1-\omega_{11}},\; \frac{1}{1-\omega_{22}},\; \frac{1}{1-\omega_{33}},\; \frac{1}{1-\omega_{12}},\; \frac{1}{1-\omega_{23}},\; \frac{1}{1-\omega_{31}} \right] \tag{2}$$

The effective stresses $\hat{\sigma}$ are assumed to fulfil the strain equivalence principle (Lemaitre and Chaboche, 1990) resulting, eventually, in Equation (3).

$$\hat{\sigma} \;=\; \mathbf{C_0} \cdot \varepsilon \tag{3}$$

Inverting the damage tensor $\mathbf{D}$ and substituting Equation (1) into Equation (3), renders the stress-strain constitutive law, Equation (4).

$$\sigma \;=\; \mathbf{D}^{-1} \cdot \mathbf{C_0} \cdot \varepsilon \;=\; \mathbf{C}(\omega) \cdot \varepsilon \tag{4}$$

where $\mathbf{C_0}$ is the stiffness matrix. It should be noticed that the introduction of the degradation internal variables ω_{ij} yields a non-symmetric tensor $\mathbf{C}(\omega)$ (see Equation (5)). The matrices $\mathbf{A}$ and $\mathbf{B}$, defined in a local system of reference, are introduced in order to read $\mathbf{C}$ in a more compact manner.

$$\mathbf{A}(\omega) \;=\; \begin{bmatrix} \frac{(1-\omega_{11})(1-\nu_{23}\nu_{32})}{E_{22}E_{33}\Delta} & \frac{(1-\omega_{11})(\nu_{12}+\nu_{32}\nu_{13})}{E_{11}E_{33}\Delta} & \frac{(1-\omega_{11})(\nu_{13}+\nu_{12}\nu_{23})}{E_{11}E_{22}\Delta} \\ \frac{(1-\omega_{22})(\nu_{12}+\nu_{32}\nu_{13})}{E_{11}E_{33}\Delta} & \frac{(1-\omega_{22})(1-\nu_{13}\nu_{31})}{E_{11}E_{33}\Delta} & \frac{(1-\omega_{22})(\nu_{23}+\nu_{21}\nu_{13})}{E_{11}E_{22}\Delta} \\ \frac{(1-\omega_{33})(\nu_{13}+\nu_{12}\nu_{23})}{E_{11}E_{22}\Delta} & \frac{(1-\omega_{33})(\nu_{23}+\nu_{21}\nu_{13})}{E_{11}E_{22}\Delta} & \frac{(1-\omega_{33})(1-\nu_{12}\nu_{21})}{E_{11}E_{22}\Delta} \end{bmatrix}$$

where Δ is a determinant that depends upon the elastic properties,

$$\Delta \;=\; \frac{\left(1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13}\right)}{E_{11}E_{22}E_{33}}$$

$$\mathbf{B}(\omega) \;=\; \begin{bmatrix} (1-\omega_{12})G_{12} & 0 & 0 \\ 0 & (1-\omega_{23})G_{23} & 0 \\ 0 & 0 & (1-\omega_{31})G_{31} \end{bmatrix}$$

Note that the *'damaged'* stiffness tensor $\mathbf{C} \in \mathcal{R}^{6\times6}$ is built as follows,

$$\mathbf{C}(\omega) \;=\; \left[\begin{array}{c|c} \mathbf{A}(\omega) & \mathbf{O} \\ \hline \mathbf{O} & \mathbf{B}(\omega) \end{array} \right] \tag{5}$$

where $\mathbf{O} \in \mathcal{R}^{3\times3}$ is a matrix filled with zeros.

4. Definition of damage internal variables

The time variation of damage internal variables is defined as a linear combination of the Φ^{γ} growth functions and the damage directors (Equation (6)).

$$\dot{\omega} \;=\; \sum_{\gamma=1}^{nmodes} \Phi^{\gamma} \mathbf{v}^{\gamma} \tag{6}$$

In the above, γ denotes a mode of damage and *nmodes* denotes the total number of failure modes. The growth functions for each damage mode γ are computed through Equation (7).

$$\Phi^\gamma = < \nabla_\varepsilon g^\gamma, \dot{\varepsilon} >_+ \tag{7}$$

where ∇_ε is the strain gradient $\frac{\partial}{\partial \varepsilon}$ and $\dot{\varepsilon}$ the strain rate. $< \cdot >_+$ denotes the non-negative inner product accounting for the trespassing on the damage surface. The subscript $+$ indicates that the inner product vanishes for negative values. This ensures that there is no growth of damage if the damage surface is not reached. If the strain increment vector is pointing to the interior of the surface (for a generic damage mode γ) there is no progression of that particular damage mode. So a simple way to effectively computing this is to perform the nonnegative scalar product as represented in equation (7). In Equation (8), g^γ are the evolving damage surfaces in the strain space.

$$g^\gamma = \varepsilon^T \cdot \mathbf{G}^\gamma \cdot \varepsilon - c^\gamma \tag{8}$$

where c^γ is an empirical parameter defining the damage surface. The variations of these surfaces on the strain space result in Equation (9). It should be noticed that c^γ are not needed in the numerical scheme as Equation (9) is the one necessary for the computational procedure. In this manner, the number of experimental data, which are difficult, or even impossible with the current techniques to obtain, are sensibly reduced.

$$\nabla_\varepsilon g^\gamma = \varepsilon^T \cdot (\mathbf{G}^{\gamma T} + \mathbf{G}^\gamma) \tag{9}$$

After some algebra, $\mathbf{G}^\gamma$ second-order tensors are derived from Equation (3) and from the equivalence of the quadratic forms in stress and strain spaces given by Equation (10) (Curiel Sosa et al., 2008a).

$$\sigma^T \cdot \mathbf{F}^\gamma \cdot \sigma = \varepsilon^T \cdot \mathbf{G}^\gamma \cdot \varepsilon \tag{10}$$

$\mathbf{F}^\gamma$ are second-order tensors are derived from damage surfaces defined on the stress space. The modelling of the unitary damage directors $\mathbf{v}^\gamma$ is based upon the stiffness components that are degraded when a particular mode of damage occurs. For instance, fibre rupture $\mathbf{v}^{(1)}$ affects to the stiffness degradation in $(11), (12)$ and (31) directions,

$$\mathbf{v}^{(1)} = \left[\lambda_{11}^{(1)} \ 0 \ 0 \ \lambda_{12}^{(1)} \ 0 \ \lambda_{31}^{(1)} \right]^T$$

The weights λ_{ij}^γ may be estimated from experimental observations in a qualitative manner for the corresponding damage mode. This technique is still being researched to provide a more straightforward computational strategy that allows to update $\mathbf{v}^\gamma$ at every time step of the numerical procedure. Techniques such as *Inverse Modelling* or *Optimization* are also possible for a more efficient modelling of $\mathbf{v}^\gamma$. However, at present, no attempt of using these techniques is being made.

5. PDM algorithm

The PDM algorithm has been implemented into an in-house *FEM*. Additionally, it was coded within Abaqus^{TM} as a *vumat* subroutine. It is adapted for the majority of the commercial software packages based in the *explicit FEM*. The computation of stresses, performed by numerical integration, includes the constitutive law expressed by the model described (see the algorithm below). A loop over damage modes is performed for the computation of stress

at each quadrature point. This is gathered in step (I) below. The computational algorithm is briefly outlined as follows,

I. Loop over damage modes, for $\gamma = 1$ to $\gamma = nmodes$ do:

    i. Compute the *'damaged'* stiffness tensor: $\mathbf{C}(\boldsymbol{\omega})$.

    ii. Generate $\boldsymbol{F}^\gamma$. Note that the γ superscript may be treated as a third index which may provide clarity to the code.

    iii. Calculate: $\boldsymbol{G}^\gamma = \boldsymbol{C}^T \cdot \boldsymbol{F}^\gamma \cdot \boldsymbol{C}$. Each γ gives place to a distinct damage surface in the strain space g^γ, i.e. one for every damage mode (see Equation (8)). Calculation of g^γ is not required as $\boldsymbol{G}^\gamma$ is the only entity needed for the following steps.

    iv. Strain gradient of damage in strain space: $\nabla_\varepsilon g^\gamma = \varepsilon^T \cdot (\boldsymbol{G}^{\gamma T} + \boldsymbol{G}^\gamma)$.

    v. Growth of damage γ: $\Phi^\gamma = < \nabla_\varepsilon g^\gamma, \dot{\varepsilon} >_+$

    vi. Directional damage vector: $\mathbf{v}^\gamma$

II. For current gauss point, compute the damage internal variables array as a linear combination of damage directors and damage mode growth: $\dot{\boldsymbol{\omega}} = \sum_{k=1}^{nmodes} \Phi^\gamma \mathbf{v}^\gamma$.

6. Numerical examples

6.1 Tension and compression tests

In this section, tension and compression tests on a fibre reinforced composite are presented using the PDM algorithm. The lamina is formed by longitudinal glass fibres embedded firmly within an epoxy matrix. The material parameters for the tests are: Youngt's modulus $E_1 =$126GPa, $E_2 = E_3 =$11GPa, Poisson ratios $\nu_{12} = \nu_{13} =$0.28, $\nu_{23} = \nu_{32} =$0.4, $\nu_{21} = \nu_{31} =$0.024 normal strengths $X_{11} =$1950MPa, $X_{22} = X_{33} =$48MPa, and shear strengths $S_{12} = S_{31} =$79MPa. The load is a distributed force applied incrementally in parallel direction to the fibres up to complete failure, i.e. until the lamina can not withstand the load any longer. The load is applied over one of the sides of dimension $1\,m \times 0.1\,m$, whilst the opposite side is constrained in motion.

Charts represent the evolution of variables and parameters at the interior central point, i.e. centre of mass. Figure (1) displays the stress vs. strain relationship from the numerical simulation and comparison with the experimental data for the tension test. A misplacement can be observed in the slope corresponding to the linear elastic behaviour. However, this is not sensibly affecting the overall response and , in particular, the softening response in the nonlinear inelastic regime in which the proposed model is focused. A numerical quantification of each damage mode are the internal variables Φ^γ which represent the evolution of each damage mode. In other words, the evolution of Φ^γ is a numerical quantifiable representation of the damage mode γ allowing to know what are the magnitude of a particular damage mode and its relation to the remaining damage mode evolutions to be determined. It should be expected that the damage modes in tension would be fibre rupture and matrix cracking and, on the contrary, the damage modes that should evolve in compression would be fibre kinking and matrix crushing. This is what PDM detects efficiently as proved in Figure (2) for the tension test and, in Figure (3) for the compression test. This is an excellent characteristic of PDM as it permits the detection of the correct damage mode depending upon the stress state in corresponding region or domain. Thus, for example, Φ_2 corresponding to fibre kinking damage mode should be zero as this type of failure is not possible in a tension test. Fibre
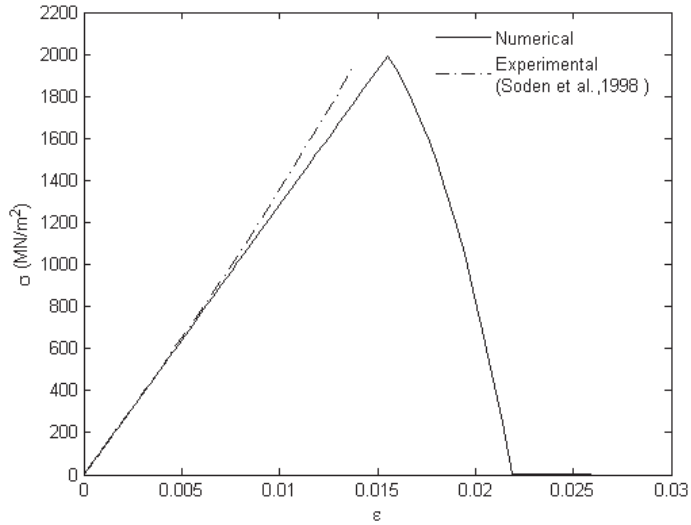
Fig. 1. Stress vs strain relationship in fibre direction for the tension test.
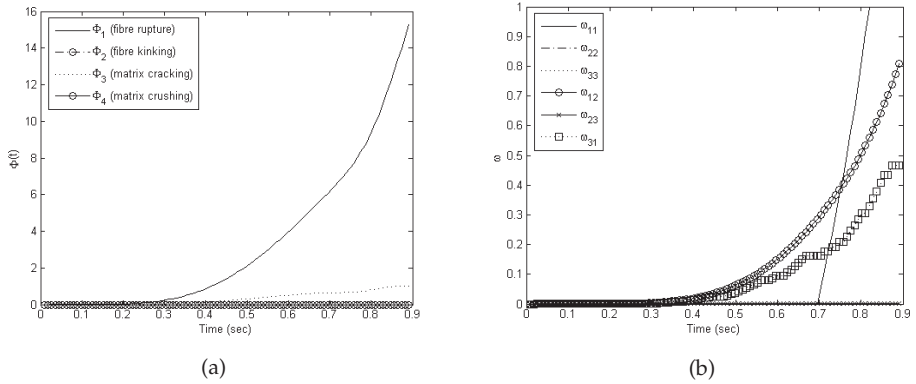

(a)

(b)

Fig. 2. Tension test –longitudinal direction to the fibres:(a) Evolution of each damage mode growth parameter. (b) Damage internal variables time evolution.
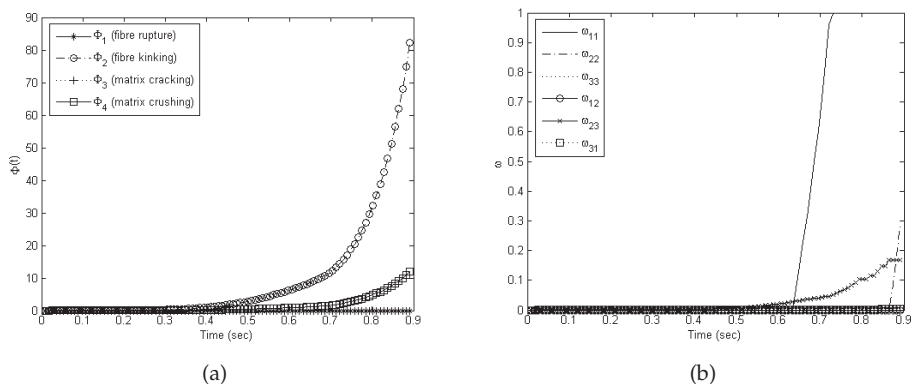
Fig. 3. Compression test –longitudinal direction to the fibres:(a) Evolution of each damage mode growth parameter. (b) Damage internal variables time evolution.

rupture or breakage and matrix cracking are the only modes expected in a tension test. This is in agreement with the computational model output, see Figure (2a). Also, the internal variables ω_{ij} shows how the different damage modes affect the degradation of the stiffness components (see Equation (4)). In Figure (2b), it is observed, as expected, that ω_{11} increases in an exponential manner. Eventually, it reached the maximum of 1 which is equivalent to complete failure.

In the compression test, the modes of damage obtained are fibre kinking and matrix crushing. This is in agreement with the mechanics of the composite as no other sort of damage should be observed in the centre of the sample in this test. As the failure is significantly affecting the longitudinal direction, again, the internal variable subjected to a higher rate of increment is ω_{11}, see Figure (3b).

6.2 Impact on $[0/90]_a$ laminate

In this section, the proposed model is tested by means of a well known three-dimensional example with clear matrix crushing and matrix cracking damage developments. This test consists of a low velocity impact –7.08ms^{-1}– on a laminate $[0/90]_a$ formed by 21 alternate laminae, see Figure 4, made of carbon fibres and epoxy resin. The composite obtained is a transversally isotropic fibre reinforced composite material with a volume fraction of 60%. Experimental tests were conducted by Hallet (1997) using a Hopkinson bar apparatus, see (Hou et al., 2000) for more details of the set-up of this experiment. Basically, the projectile is a titanium alloy rod with a diameter of 9.55 mm and a total length of 500 mm and its head is rounded in order to damp the vibrations. The mass of the projectile was calibrated to 260 g to strictly replicate the experiment.

The dimensions of the laminate were 2.6x85x85 mm^3 and it is supported by a steel ring with an inner diameter of 45 mm. In the experiments performed by Hallet (1997), the impact velocity was measured by infra-red timing gates just before the laminate was struck. C-scan and dye contrasts were used to detect damage after impact. The results of dye contrast test showed that the projectile impacted with an initial velocity of 7.08 ms^{-1} and, hence, this is the impact velocity that has been used in these numerical tests. In the experiments by Hallet (1997) as
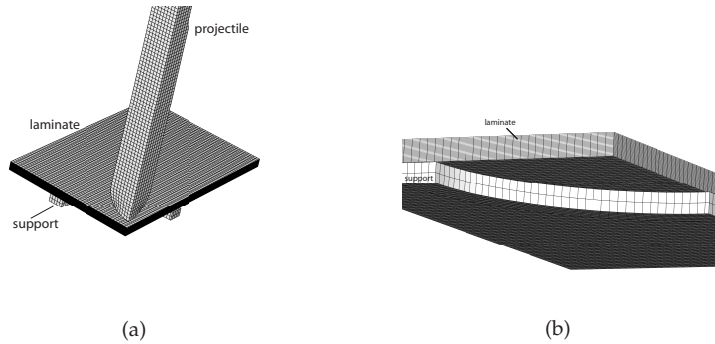
Fig. 4. Initial setup: (a) A quarter of the real configuration is used in the numerical tests thanks to the symmetry. (b) A closer snapshot of the laminate and the support.

well as in the numerical tests conducted by Hou et al. (2000), a matrix crushing zone was observed just beneath the contact region, i.e. in the through-thickness compression region under the projectile. The numerical result for matrix crushing region from the proposed model is depicted in Figure 5. It may be observed that the matrix crushing zones are located in agreement with the experimental results by Hallet (1997); Hou et al. (2000). The progressive development of those regions is more realistic than the result obtained using just stress failure criteria as depicted in Figure (6) which provides Boolean values for the damage variables without considering any progression of the damage. In the computational results using PDM, neither fibre rupture nor fibre kinking were developed as expected with that impact velocity. To turn off the damage modes according to physical reality is an excellent characteristic of PDM.

7. Conclusion

This chapter has provided an overview of the different techniques used for modelling damage in composites briefly showing the current state-of-art of the topic. Basically, from a computational point of view, there are two main trends:

- failure criteria which generally use a stress quadratic form.
- a progressive evolution of the damage.

The author's choice is the second one for two main reasons. Firstly, because failure criteria is creating Boolean values for deciding when a finite element is deleted or split which, in turn, cause numerical instabilities and, eventually, divergence in explicit finite element simulations. Secondly, the progression of damage, even if it is sudden, evolves sequentially in a microscopic scale which reasonably makes the progressive damage models more realistic. Following this second tendency, the author has presented a progressive damage model (PDM) for fibre reinforced composites. The approach is based in a directional computation and a progressive growth of of damage modes depending upon the stress state and strain rate amongst other variables. Moreover, the constitutive law is implicitly relying upon the strain rate, which makes the model suitable for a wide range of strain rate values including impact. The computation is, in general, intended for time stepping numerical methods and, in particular, for the *explicit FEM*. The PDM algorithm is offered for straightforward
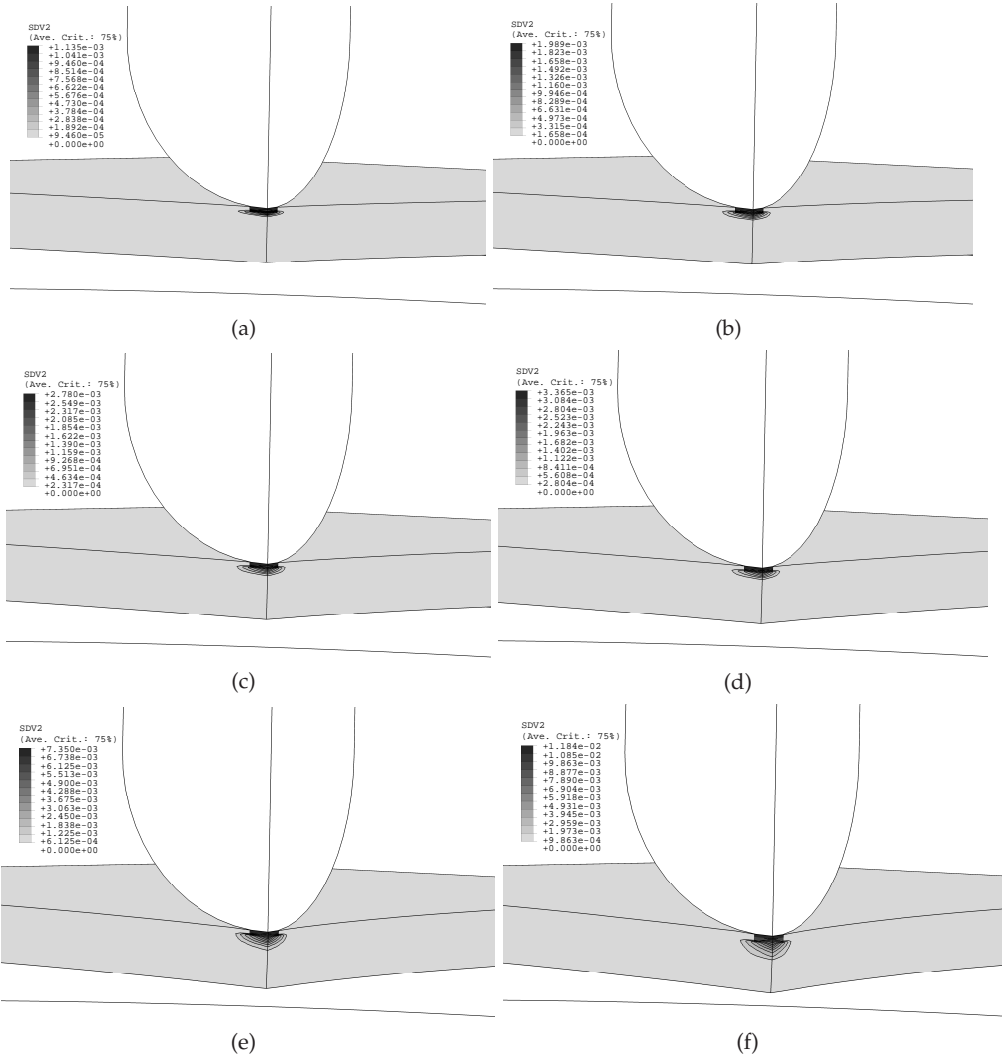
Fig. 5. Development of matrix crushing damaged zone and grades in the laminate when impacted at 7.08 $m\,s^{-1}$.

implementation in an explicit FEM code either commercial software package or in-house code. The outcome obtained by using PDM for tension and compression tests provide the expected progression of the damage variables, being able to determine the corresponding damage modes associated with each stress state and rate of strain, eventually leading to the expected behaviour. Furthermore, the computational results by using PDM for the low velocity impact on a laminate were in an excellent agreement with the experimental observations of matrix cracking and matrix crushing which, eventually, caused the delamination in those damaged regions of the laminate.
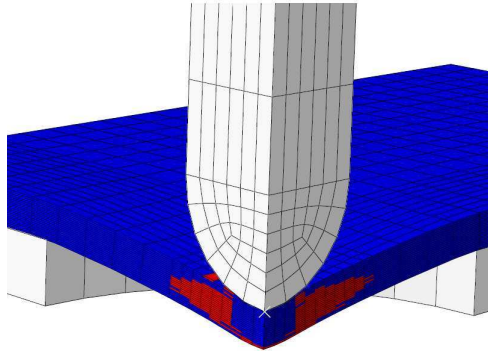
Fig. 6. Matrix cracking pattern using a classical failure criterium based on stress components. The elements in red fulfilled the criterium which means that they are not withstanding loads any longer.

Time = 0 s

Time = 0.286 ms

Time = 0.3072 ms

Time = 0.3389 ms
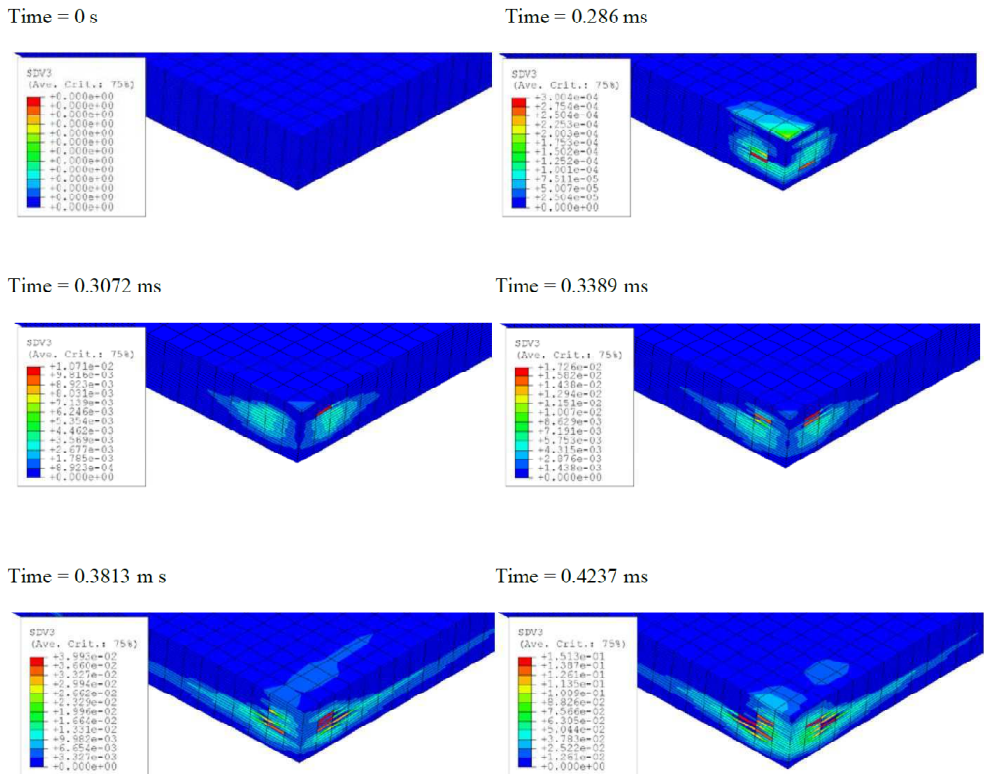

Time = 0.3813 m s

Time = 0.4237 ms

Fig. 7. Sequential matrix cracking pattern observed during simulation by explicit FEM using PDM. A time progressive evolution of this damage mode, which eventually turned into delamination, is observed in an excellent agreement with the experimental observations.

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