Neutrino mean free path in neutron matter with Brussels-Montreal Skyrme functionals

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Abstract. We calculate the neutrino mean free path in cold neutron matter with some modern Brussels-Montreal functionals. The three typical functionals used in this article produce quite different results implying a possible impact on the cooling mechanism of neutron stars.

1. Introduction

The recent Brussels-Montreal functionals [1] have been very successful in describing both ground state properties of finite nuclei and some astrophysical quantities as the equation of state [2]. These functionals fitted to essentially all the 2149 experimentally known masses are similar to the standard non-relativistic Skyrme effective interaction [3] excepting for the presence of two extra density- and momentum-dependent terms [4]. The latter have been introduced to allow for more flexibilities in reproducing at the same time the properties of nuclei and infinite nuclear matter, including the quality of the functionals against ferromagnetic instabilities in neutron matter [4]. In that respect to avoid such instabilities, the functionals BSk19, BSk20, BSk21 have been fitted to satisfy the Landau inequalities up to very high densities. Furthermore, it has been shown [2] that it was possible to use them for both finite nuclei and astrophysical applications [5, 6].

The neutrino emission is one of the most efficient mechanism to dissipate the energy accumulated during the formation of a neutron star [7]. Since the neutrinos cross different layers of nuclear matter and are scattered by neutrons, they lose energy, making the cooling mechanism less efficient. The neutrino mean free path (NMFP) is a fundamental ingredient for a proper understanding of the cooling process. In the present article, we investigate the NMFP with the 3 above-mentioned functionals.

The article is organized as follows: in Sec.2 we present the linear response formalism necessary for the determination of the response function. Then we show explicitly the influence of the response function on the NMFP in Sec.3 and finally draw our conclusions in Sec.4.

2. Linear response theory

In the following we briefly recall the formalism of the linear response (LR) theory in the case of pure neutron matter at zero temperature. For simplicity we adopt a system of natural units

$\hbar = c = 1$. In order to describe the response of a nuclear system to an external probe $\hat{Q}$, one usually introduces the so-called response function $\chi^{(S,M)}(\omega, \mathbf{q})$ per unit volume defined by :

$$\chi^{(S,M)}(\omega, \mathbf{q}) = \frac{1}{V} \sum_n |\langle n|\hat{Q}^{(S,M)}|0\rangle|^2 \left(\frac{1}{\omega - E_n + i\eta} + \frac{1}{-\omega - E_n + i\eta} \right), \qquad (1)$$

where S (M) is the spin (its projection along the z-axis). The variables ω and $\mathbf{q}$ represent the transferred energy and momentum, respectively. We refer to [8, 9, 10, 11] for a more detailed discussion on the LR theory with Skyrme functionals.

The BSk interactions differ from standard Skyrme [3] for the presence of some additional terms, v_{extra}, that are :

$$v_{extra} = \frac{1}{2}t_{1b}(1 + x_{1b}P_\sigma)\left(\mathbf{k}'^2\delta(\mathbf{r})\rho(r)^{\beta_1} + \rho(r)^{\beta_1}\delta(\mathbf{r})\mathbf{k}^2\right) + t_{2b}(1 + x_{2b}P_\sigma)\mathbf{k}'\rho(r)^{\beta_2}\delta(\mathbf{r})\mathbf{k}. \, (2)$$

These terms are the analogous of the terms proportional to t_1, x_1 and t_2, x_2 and thus they follow the same conventions concerning the momenta operators $\mathbf{k}, \mathbf{k}'$. Because of these terms, the expressions for the response functions given in ref. [11] have to be modified. The particular case $\beta_1 = \beta_2 = 1$ was considered in ref.[8]; the complete derivation will be presented in a forthcoming publication.
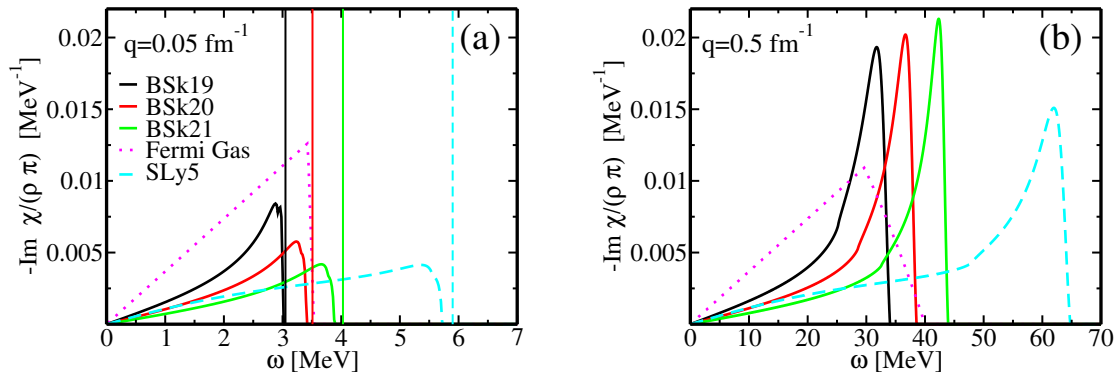

Figure 1. (Color online) Response function in the channel $S = 1, M = \pm 1$ calculated at $\rho = 0.16$ fm^{-3} for two different values of the transferred momentum and for the different functionals adopted in the present article. As a reference, the dotted line represents the response function of the unperturbed Fermi Gas.

In Fig.1, we show the response function at $\rho = 0.16$ fm^{-3} in the $S = 1, M = \pm 1$ channel for the three BSk functionals together with the functional SLy5 [3], commonly used for astrophysical applications, (given here as a reference) , and the Fermi Gas (FG) response function, calculated with the bare mass. We observe that the main effect of the interaction is related to the effective mass, and to a lesser extent to the incompressibility. In the two panels, we compare the case of two different values of the transferred momentum ($q = 0.05$ and 0.5 fm^{-1}). We observe, for the low exchanged momentum of panel (a), the appearance of a collective mode with zero width, the so-called zero-sound mode [12], which strongly suppresses the continuum part of the response function. At a larger transferred momentum (panel (b)), the zero sound mode disappears while the continuum is enhanced.
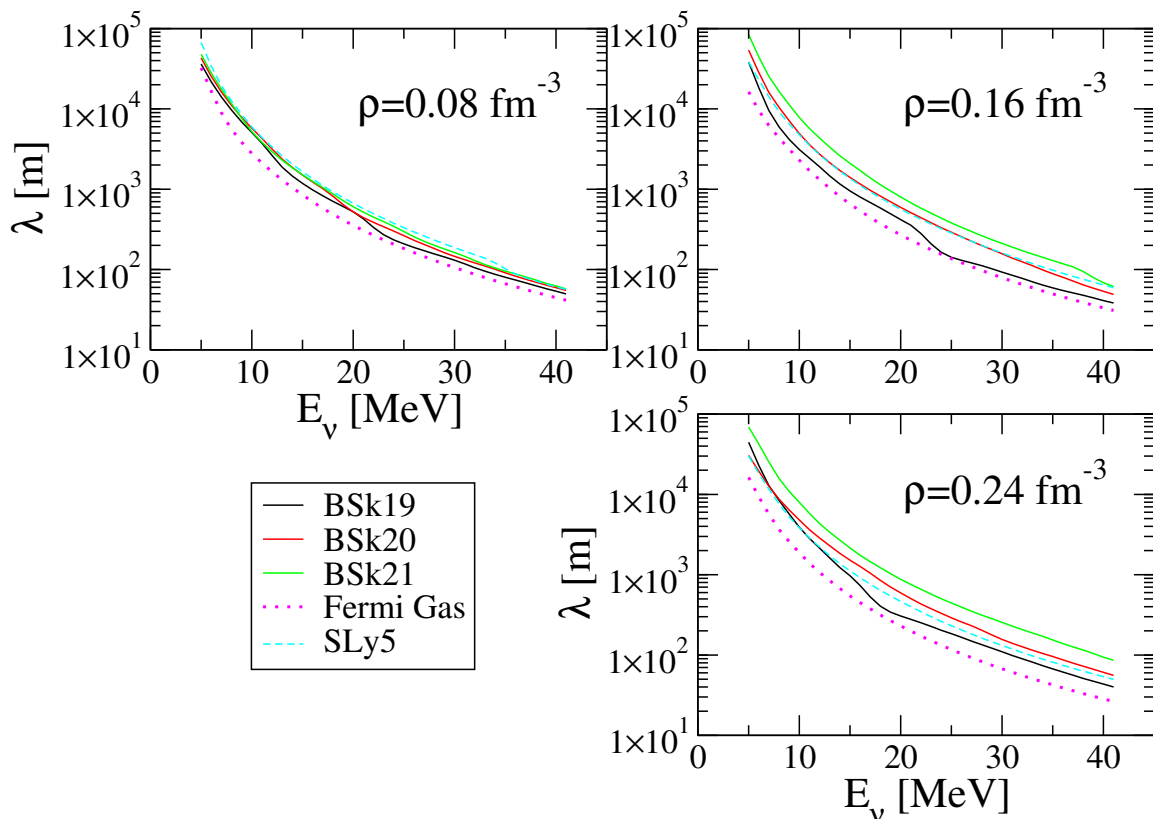
Figure 2. (Color online) Neutrino mean free path calculated for three different densities and for different functionals (see text for details).

3. Neutrino mean free path

The neutrino mean free path is defined as $\lambda = (\sigma\rho)^{-1}$, where σ is the total cross-section for the neutral current reaction $\nu + n \rightarrow \nu' + n'$. The differential cross-section, whose explicit expression is given in Eq.(21) of ref.[11], strongly depends on the response functions of all the different channels (S,M). However, as already illustrated in refs. [11, 13, 14], the cross section is dominated by the spin transverse response $R_T^A \equiv -Im\chi^{(S=1,M=\pm1)}(\omega, \mathbf{q})/(\pi\rho)$.

In Fig.2 we give an example of NMFP for the four functionals adopted here and for three different values of the density that are relevant for astrophysical calculations. We observe that at low density, the four functionals give a comparable value of λ. The differences among them actually start to emerge around the saturation density $\rho = 0.16$ fm^{-3}. In particular, it can be seen that BSk19 always gives the lowest value of λ and BSk21 the highest. These values are always larger that those obtained with the FG, *i.e.* without any residual interaction, thus showing explicitly that the nuclear medium always enhances the NMFP. This is however not a general result, as one can observe in Fig. 9 of ref. [11].

We also observe that SLy5 and BSk20 give very similar results concerning λ although the peaks of their response functions are located at quite different energies (Fig.1). However, when going from the differential cross section to the integrated one, kinematics reduce the integration range to [7] : $0 < \omega < E_\nu$ and $\omega < \mathbf{q} < |2E_\nu - \omega|$. As a consequence, the different response functions have to be compared only in this restricted range. In practice, since the maximum allowed value for the neutrino energy is $E_\nu = 40$ MeV, this corresponds to the low energy domain where SLy5 and BSk20 give actually similar results. Another important feature of Fig.2

is the presence of sudden drops in the NMFP as illustrated for BSk19 at $\rho = 0.16$ fm^{-3} and for $E_\nu \approx 22$ MeV. This rapid change occurs precisely for the values of ρ where the zero sound mode disappears and the response function suddenly increases. Such a phenomenon has been already studied in great details in ref.[15].

4. Conclusions

The calculation of the response functions within the linear response formalism has been extended in order to include the extra terms of the BSk19-21 functionals for the case of pure neutron matter at zero temperature. The resulting response functions have been used to calculate the neutrino mean free path. The preliminary results presented here exhibit some differences due to the parameterization itself (effective mass) and to general characteristic (zero sound). To obtain a more realistic result, it is however mandatory to include the temperature effects. In that case, the maximum transfer momentum between neutrino and nucleon can increase and one expects that it can eventually enter in regions with finite-size instabilities [16, 17, 18, 19]. Work along these lines is in progress.

Acknowledgments

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