Modelling higher-order vagueness: columns, borderlines and boundaries

**Rosanna Keefe**

Department of Philosophy

45 Victoria Street

Sheffield S3 7QB

r.keefe@sheffield.ac.uk

Can we solve the paradoxes and challenges posed by the phenomena of vagueness and higher-order vagueness with a theory according to which first-order and higher-order vagueness coincide? Sorites paradoxes are compelling for vague predicates and yet, of course, yield absurd conclusions. Vague predicates have borderline cases – most people agree – but acknowledging those doesn’t go any significant way to solving the paradox. Moreover, the borderline cases themselves seem to form a category that itself lacks sharp boundaries and faces the problems of vagueness. This quickly yields the widely accepted thought that vague predicates (typically, always?) exhibit higher-order vagueness. The same predicates that have borderline cases will also have (or potentially have) cases that exhibit higher-orders of vagueness – borderline borderline cases etc. But higher-order vagueness introduces new problems such as a number of influential arguments attempting to show that higher-order vagueness is incoherent (Wright 1992, Fara 2003) or that it has an unexpected structure (Williamson 1999). And, anyway, the hierarchical structure of increasingly obscure categories that it seems to yield may seem a long way from the vague, unsettled phenomenon we are seeking to model.

With her *Columnar Higher-Order Vagueness*, Susanne Bobzien is offering us a new way to think about both vagueness and higher-order vagueness. [[1]](#footnote-1) It promises a way to avoid denying that higher-order vagueness exists while avoiding the problematic arguments from Wright and others. The hierarchical structure of a series of categories of decreasing size is given up in favour of a structure in which the categories corresponding to the different orders of vagueness are lined up in a column. Come one order of vagueness, come all, not just in the sense that if a predicate has one feature it has the other, but in the much stronger sense that any instantiation of one (in the application of the predicate to a particular object) is an instance of them all. Bobzien claims that not only does this solve the problems of higher-order vagueness, but also the problems with vagueness more generally, including the sorites paradox. Is this too good to be true? I will argue that it is.

**I**

*Columnar Higher-Order Vagueness and Borderline Cases.* Bobzien’s aim is to capture the structure of vagueness and borderlines. Central to this task is Bobzien’s C operator initially introduced as meaning ‘it is clear that’ and then further spelled out as ‘one can tell that’, so that CF*a* reads ‘one can tell that *a* is F’. This device is best understood in relation to a claim about a borderline case: *a* is borderline F iff it is not clear that *a* is F and not clear that *a* is not F, that is, ¬CF*a* ʌ ¬C¬F*a*. The use of an operator playing this kind of role is typical in discussion of vagueness, though it is sometimes a definitely or determinately operator, D, Def or ∆.

A typical ­line of argument leading to commitment to second order borderline cases goes as follows. Just as there is no clear point in a sorites series in which we suddenly shift from Fs to not-Fs (and thus there are borderline cases in which is not clear whether *a* is F), so there is no sudden shift to the borderline Fs, from the clear Fs. Higher-order vagueness is thus captured by iterating the C operator: a second-order borderline case of F is such that it is not clear whether or not it is borderline, so (substituting the statement that *a* is borderline F into the formula for being borderline), we get ¬C(¬CF*a* ʌ ¬C¬F*a*) ʌ ¬C¬(¬CF*a* ʌ ¬C¬F*a*). Adding a B operator, where BA is defined as ¬CA ʌ ¬C¬A, allows us to simplify and express second-order borderline cases of F as BBFx third-order ones as BBBFx etc.

An initial highly compelling principle within the logic of C is that CF*a* entails F*a*. If it is clear that *a* is F – or if *a* is definitely F etc. – then it must be that *a* *is* F: the operator is factive. Many other aspects of the logic are controversial however. It is quite natural to think of C as akin to a necessity operator and to appeal to a modal logic (containing (T) Cp → p, to reflect that C is factive). Different modal logics provide candidates that have been or may be defended on different grounds, where the logical behaviour of iterating operators differs between different systems. Williamson (1999) justifies the use of a modal logic by showing how the possible worlds framework is apt for the treatment of either an Epistemic View of vagueness (such as his own) or a supervaluationist one, with different precise interpretations of the vague language playing the role of the worlds. He then suggests that the analogue of the accessibility relation should be symmetric but not transitive, yielding the logic B. The details are not important for us here.[[2]](#footnote-2) I mention it to distinguish it from Bobzien’s approach, which does not start with the semantics and which leads to a different modal system, as we’ll see below.

Bobzien discusses, in order to reject, a hierarchical structure for higher-order vagueness. The tendency to see borderline Fs as falling *between* the (clear) Fs and the (clear) non-Fs naturally combines with seeing the first category of second-order borderline cases (borderline clear and borderline borderline) as falling between the (clearly) clearly Fs and the (clearly) not clearly Fs and thereon up through an endless hierarchy. The resulting structure of ever more refined categories has seemed unappealing to many, despite the lure of the above reasoning that appears to lead you to it.[[3]](#footnote-3) Bobzien offers an alternative where the iterations of the C (and B) operators don’t lead to smaller and smaller categories: the borderline cases are equally borderline borderline and borderline borderline borderline etc.; and the clear cases are equally clearly clear and clearly clearly clear etc. The structure is described as ‘columnar’ because the iteration of our operators line up in columns through a sorites series – the first column of clear (and clearly clear etc.) cases, followed by those that are borderline and simultaneously borderline at every order, with a final column of cases that are clearly not F and clearly clearly not F etc.

Bobzien models her structure with the modal logic S4M. In addition to some widely accepted principles governing C, (such as the axiom T, and K2 which captures the distributivity of C over ʌ), this contains the controversial axiom 4:

(4) CA → CCA

This axiom is analogous to the KK principle – that if you know then you know that you know – which has frequently been rejected in discussions of vagueness and higher-order vagueness.[[4]](#footnote-4) Here and elsewhere (especially her 2012), Bobzien has offered an interesting case for endorsing (4) and arguments against the prevalent assumption that it is incompatible with higher-order vagueness.

The S4 axiom alone does not deliver the columnar structure under discussion. This feature results from the addition of the axiom Bobzien labels (V), which, using the borderlineness operator, B, can be expressed as

(\*) BA → BBA

Bobzien shows that adding (V) to S4 is equivalent to adding the McKinsey axiom M to the modal logic S4, resulting in a system known (among other things) as S4M. With a vague predicate F and a characteristic sorites series running from Fs to not Fs, the described columnar structured follows, starting with the Fs that are clearly F and clearly clearly F and … CnF for all n. This is followed by items in the borderline column that are not only borderline F, but borderline borderline F, borderline borderline borderline F and … and BnF for all n. And in the final column are x for which Cn¬Fx is true for all n.

Columnar Higher-order Vagueness – CHOV – promises a number of appealing features, including responses to various arguments that higher-order vagueness is incoherent. In her paper in this volume, one of Bobzien’s main aims is to take the propositional modal logic she has defended elsewhere and develop and explore an extension to first-order logic. I will focus, however, on more general questions about the suitability of the general framework for capturing vagueness.

Since all borderline cases are also borderline borderline cases, Bobzien must deny that there are clear borderline cases.[[5]](#footnote-5) Before considering her reply to this observation taken as an objection to her theory, we can ask whether this consequence of her view is actually appealing in some ways, as Bobzien also suggests. For, settling on clear, agreed borderline cases of a typical vague predicate is strikingly difficult, so maybe we should not expect any cases to come out as *clearly* borderline. There is also some attraction in the thought that if we cannot tell that *a* is not borderline F (when we are in right circumstances to judge it), then it cannot be clearly F and should count as borderline: ‘what smacks of being borderline is borderline’ (2012, p.195). The uncertainty over whether *a* is a borderline case here is reflected in the fact it is also, according to CHOV, a borderline borderline case, so the above thought can be expressed as BBA → BA. There is a tension here, however. On the one hand, ‘what smacks of being borderline is borderline’ suggests a generous category of borderline cases. On the other hand, the theory declares that any case is at most a *borderline* case of a borderline case (and not even clearly that) and that even the statement that there are borderline cases is itself merely borderline. The lack of clear borderline cases is evidently bound up with the second half of this tension.

Bobzien attributes much of the allegedly erroneous thinking about borderline cases – including commitment to the hierarchy and the temptation to believe clear borderline cases – to adoption of the wrong one of two conceptions of borderline cases that she distinguishes. In general terms, we may often bundle together several features of being a borderline case of the categories F and G, including the inability to *decide* whether it is F or G and the idea that, as a borderline case it falls *between* the categories F and G. Bobzien criticises what she calls ‘in-between borderlineness’, which she regards as a prevalent conception and responsible for, among other things, the hierarchy of higher-order borderlines each nesting between other categories. Instead, she endorses ‘undecidability borderlineness’, where if *a* is borderline F/G, it is not possible to determine that it is F or to determine that it is G. She discusses her two conceptions at more length elsewhere (2013), but I just raise some brief queries about the discussion of this in the paper in this volume. I would argue that they are not solved by the previous longer discussion, but will not dissect that here.

Even if it is true, as seems plausible, that we sometimes fail to keep apart different putative features of borderline cases, the two conceptions may be neither exhaustive nor accurate as descriptions of our conceptions of borderlines. She summarises: ‘in-between borderlineness regarding categories CF and C¬F requires that it is possible to determine that [¬F*a* ʌ¬¬F*a*].’ Our classifying something as a borderline case would then involve us judging that we can determine that it yields a contradiction. But this is surely not integral to our standard conception of borderline cases. The tendency to see borderline cases as somehow falling between categories might be spelled out in terms of falling between *clear* cases either side of the boundary, for example, which does not face the same problems. And why think that we must be able to *determine* that the case falls between the categories if it is borderline? In short, I contend that we cannot diagnose the reluctance to deny that there are clear borderline cases in terms of the commitment to in-between-borderlineness as Bobzien explains it.

The argument she offers that attributes the tendency to believe in clear borderline cases to any particular account of borderline cases is anyway puzzling. She claims that in presenting *a* as a clear borderline case of F,

You must be able to distinguish *a* from the non-borderline F cases and the non-borderline ¬F cases. But this means, I maintain, that you have, perhaps inadvertently and at least temporarily, shifted to the above-described in-between borderlineness, or still another kind of borderlineness, and that you equivocate on ‘borderline’.

Why think this? Why not think that you can determine that you can’t determine that *a* is F (i.e. invoking her undecidability conception)? Perhaps, the thought is that if you cannot distinguish *a* from *b*, then you cannot determine that *a* is borderline if *b* is non-borderline. So if *a* is clearly borderline, you must be able to distinguish it from the clear Fs. But it is not clear to me why that is not compatible with the undecidability conception of borderlineness. Consider, for example, an advocate of Williamson’s Epistemic View. He will not regard borderline cases as falling between the Fs and the not-Fs (since they are each either F or not-F, though not knowably so). Yet, he can accommodate clear borderline cases, where we can tell that we can’t tell that *a* is F, which are distinguishable from non-borderline Fs, even if some borderline Fs aren’t distinguishable from non-borderline Fs. In short, Bobzien’s reply to the objection that there are clear borderline cases is not persuasive, and clear borderline cases look unproblematic on any plausible general conception of borderline cases.

The next three sections take up some broadly structural concerns about Columnar Higher-Order Vagueness.

**II**

*More categories that coincide.* As we have seen, with Columnar Higher-Order Vagueness, categories coincide within each column: borderline cases are also borderline borderline cases and cases of every order of borderline, while clear cases are clearly clear and clearn. As suggested above, it can be hard to identify a borderline case that is sufficiently widely agreed to be such – though this may be easier with some vague predicates than others and harder in some contexts than others – and the prospects of identifying a borderline borderline case or a third or higher-order borderline case are even more remote.[[6]](#footnote-6) Maybe this counts in favour of a view that is not committed to each of these as wholly distinct categories, but does it support the view that the categories coincide entirely?

Not only do borderline and borderline borderline cases coincide in the CHOV framework, but other pairs of categories coincide for which this equivalence seems even less plausible. For example, on the standard hierarchical conception of higher-order vagueness, there are two types of second-order borderline cases: those around the end of the borderline cases closest to the clear cases, (borderline clear Fs where we have ¬CCF*a* ʌ ¬C¬CF*a*), and those around the other end, closest to the clearly not-Fs (borderline clearly non-Fs where we have ¬C¬CF*a* ʌ ¬CC¬F*a*). In the columnar framework these two categories coincide, and both coincide with first-order borderline cases.[[7]](#footnote-7) This is another counterintuitive feature of the account. Even if we can’t produce a definitive clear first-order borderline case, we can typically distinguish between borderline clear Fs and the borderline clear not-Fs. Going through a sorites series, we will start with clear Fs and then come to cases where we aren’t sure whether to say they are still F or that they are borderline, but which we have no temptation to call clearly not-F (indeed, we would be likely to think that they are clearly not clearly not F). Similarly, continue further down the series, through most of the borderline cases, and we will hit cases that are on the borderline of the borderline and clearly not-F, categories, but are clearly not clearly F. Perhaps we won’t all agree on which cases fit each of these categories, but any individual’s categorisation through the sorites series will distinguish between these cases and there will typically be widespread agreement that something a well-placed observer is inclined to call borderline clearly red should definitely/clearly not also count as clearly orange. A plausible view of higher-order vagueness would require a theory to keep those types of second-order borderline cases apart.

**III**

*Modal principles (4) and (5).* Bobzien accepts the characteristic axiom of S4 for her C operator, but rejects the characteristic S5 one

(4) CA → CCA

(5) ¬CA → C¬CA

S5 is not an option for modelling higher-order vagueness. With S5, all the strings of modal operators and negations collapse down to a single modal operator with or without a negation. This leaves no room for higher-order borderline cases characterised by iterations of the operator. Indeed, second-order borderline cases would turn out to be incoherent. Consider a case on the borderline of CF and ¬CF, where ¬CCFx ʌ ¬C¬CFx. S5 principles ensure that the first conjunct is equivalent to ¬CFx,[[8]](#footnote-8) while the second conjunct is equivalent to CFx,[[9]](#footnote-9) so the case reduces to the contradictory ¬CFx ʌ CFx.

I will not directly address the question whether (4) is defensible here. It is rejected in Williamson’s work on vagueness (1994) and discussed more generally in his work on epistemology (2000 and elsewhere), on the grounds that it is not compatible with compelling ‘margin for error’ principles, according which if a subject is to know that p, they must believe that p in very similar cases (for example, cases in close proximity to each in a sorites series). Elsewhere, Bobzien hopes to preserve some kind of ‘margin for error’ principle for subjects, while retaining (4) (2010, p.7), but, again, I will not here try to settle whether this can be made to work. Instead, I ask whether the combination of endorsing (4) while denying (5) may be in tension or unjustified in this context. This will, of course, depend on the understanding of ‘B’ and ‘C’ and in this discussion I will draw on the ways she illuminates them in earlier papers. In *this* paper, she maintains that C is a technical term, so she may no longer be committed to such interpretations, but that makes even more urgent the concerns about relying on a technical term that I raise in Section V, below.

Epistemic logics with (4) but not (5) are explored in Hintikka’s 1962 and in discussions elsewhere. Even if our knowing p guarantees that we know that we know that p, (5) can fail, as can be seen when we consider cases in which we believe that p, and believe we know that p, but when in fact p is false. In those cases, we cannot know that p, but do not know that we don’t know. I will argue that counter-examples to (5) of this form are not available in the limited context in which our discussion of vagueness is at issue and where we are dealing with Bobzien’s C operator, rather than a more general ‘knows that’ operator. This does not rule out the possibility of other counter-examples, but I argue that it helps bring out the instability that I suggest infects a view endorsing (4) while rejecting (5).

The relevant kind of scenario to consider – in which a subject’s judgement as to whether *a* is F bears on questions of whether *a* is clearly F – is one where the subject is fully competent in the relevant respects and in an ideal position to make the judgement. This is the kind of scenario Bobzien draws on in supporting (4). However this kind of restriction is spelled out, it is needed for Bobzien’s picture and her notion of C and commitment to (4). In an earlier paper (2010) she introduced the notion of a CRISP – a competent, rational, informed speaker (for the classification in hand) – who is an idealised version of the normal user of vague natural language. This kind of idealised speaker will not just unknowingly get things wrong in her classification with the vague predicate. She may remain silent as to whether F*a* (endorsing neither ‘*a* is F’ nor its negation), but then she won’t be a counter-example to (5) in which she thinks she knows but in fact does not. Bobzien’s CRISPs leave a margin for error, thus they avoid making the claim that *a* is F in a case where they do not know whether or not *a* is F (2010, p.7). If I think I know that *a* is red when it is not, then I am not a competent informed speaker in relation to the judgement of whether *a* is red and my judgement does not have a bearing on whether *a* is clearly red, clearly not red or neither. So a CRISP is immune to the standard kind of counterexample to (5) as sketched above, where the subject does not know that *a* is F, but does not know that she does not know. A CRISP *cannot* believe she knows that F*a* when ¬*Fa* holds.

Now, it does not follow that if S, who is in fact a CRISP, cannot tell that *a* is F then S she can always tell that she can’t tell that *a* is F. For S can – and, according to this picture, will or at least should – doubt her competence; she may doubt whether she is, in fact, a CRISP and thus not be able to rule out the possibility that she merely thinks she knows that *a* is F when *a* may not actually be F. But why doesn’t such doubt about one’s competence also infect one’s ability to tell that one *can* tell? Just because S knows that p, and as a CRISP is thus guaranteed to be right, she may not know that she is a CRISP and thus may appropriately have some doubt over whether she actually knows that she knows. So, if the rejection of (5) turns on this kind of doubt about one’s competence, this should also cast doubt on the principle (4).

Since ‘what smacks of being borderline is borderline’ (Bobzien 2012, p.195), it looks like we should be able to tell that we have a borderline case quite easily by detecting doubt in its classification as F (or at least doubt of the right kind). At the very least, when something is widely agreed to be a borderline case and those in the best position to judge agree on this then surely we are in a position to know that it isn’t clearly clear etc. (Note that Bobzien recognises context-sensitivity – whether something is borderline can depend on context and comparison class – so we don’t have to be confident it would count as a borderline case in every context.) But this not so on Bobzien’s picture: in such cases, she is committed to maintaining that you cannot tell that it isn’t the case that, unbeknownst to everyone, you actually have a case that the competent speaker *can* tell is F. That is surely highly implausible. Moreover, were such a case possible, it would be natural to describe this as a case where even though that speaker can tell that *a* is F, he can’t tell that he can tell that *a* is F – after all, if he could tell that he could tell, he wouldn’t judge that he couldn’t – but, by (4), this characterisation of the case as ¬CCp will entail ¬Cp, so this description would be incoherent.[[10]](#footnote-10)

I suggest that the above considerations show that in constructing a notion of tellability fit to be employed in an account of C as intended, the defence of (4) but not (5) is, at best, highly unstable. Motivations for (4) carry over to (5), where having both would allow us to maintain that we can tell when something is borderline as well as when it is not. To the same end, we can consider Bobzien’s discussion of ‘self-revealing clarity’ (2012), which ‘shines through all higher-order levels’ and obeys (4). She maintains that ‘intuition supports this notion’ (pp. 194-5), because people judge that ‘it is clear that Tallulah is tall’ and ‘it is unclear whether it is clear that Tallulah is tall’ contradict. The force of such intuitions looks comparable to the force of the intuition that ‘it is doubtful whether Tallulah is tall’ contradicts ‘It is doubtful whether it is doubtful whether Tallulah is tall’, which would similarly support the incompatibility of ¬CF*a* and ¬C¬CF*a*, as yielded by the rejected principle (5). If clarity is self-revealing – because we are well placed to have confidence in our judgements of the kinds of classifications involving vague predicates in question – so, I suggest, is the kind of doubt characteristic of borderline cases, for which we are equally well placed to have confidence in our recognition of the fact that we’re *not* confident of a given classification.

**IV**

*Doubtful postulates and the interpretation of C.* In Bobzien’s section replying to common objections to CHOV, her Objection 5 offers an argument that appears to show that the theory is committed to two contradictory things – that (reading C as ‘it’s clear that’), it’s both clear that there are borderline cases of F (for vague F) and not clear that there are. The argument draws on a ‘basic assumption of CHOV’, that it is not clear that there are no borderline cases of F when F is susceptible to sorites. This is her (3) on p.?

(3) ¬C¬∃*x*BF*x*

She faults the argument by pointing out that it requires a further assumption, namely that basic assumption, (3), prefixed with the C operator (her (8)). The response is then to deny that required assumption, (8), thus committing the theory to her (11).[[11]](#footnote-11)

(11) ¬C¬C¬∃*x*BF*x*

Thus she is committed to (11) as well as (3). It is natural to describe this as a commitment to the surprising fact that it is a consequence of the theory that an assumption at the heart of the theory is not clearly true. Bobzien would not endorse *this* way of putting it, since she rejects the metalinguistic interpretation of C in terms of sentences being clearly true. (For example, CFx is to be read as ‘x is clearly F’ but not as ‘it is clearly true that x is F’.) But that just seems like a convenient way to put the equally striking claim that the theory is committed to ‘not clearly X’ when X is one of its own basic assumptions.

Bobzien notes that (3) is not a logical theorem of her chosen modal system. It is also, she says, ‘not part of what defines borderlineness’ (p.?). But it *is* (along with one other assumption) ‘the component of the theory that relates borderlineness to sorites sequences’ (p.?). From the outset, she presents the theory as the modal system plus these assumptions (p.?), and without them, the theory would have no resources to solve the sorites paradox or to explain our intuitions about the ‘seamless transition’ in sorites series. The assumption, note, is much weaker than the claim that there are borderline cases: it is merely the claim that it is not clear that there are *not*.

How bad it is to have a theory that is committed to this unclear status for its own central assumptions depends on how we understand ‘clearly’ or the C operator. If ¬CA were to mean that A is not determinate in the sense that it is either false or neither true nor false, then this looks particularly problematic: in that case, advocating a theory involving A would at the same time be advocating it as not true. This understanding of C (or D/Def as it is typically called) is the one naturally adopted by non-classical theories such as supervaluationism and degree theories, according to which borderline cases (for which ¬CA ʌ ¬C¬A) are neither true nor false. Now, Bobzien here adopts a ‘factive-cognitive’ interpretation of C instead, though ‘it is not thereby precluded that this interpretation is ultimately grounded in some other interpretation’ (p.?). She promises a theory that isn’t committed to bivalence, apparently maintaining a neutrality between different theories of first-order vagueness (e.g. p.?). I will return to this alleged neutrality in Section V: the absurdity of a theory that says of itself that it isn’t true is the first difficulty for the prospects of combining her theory with one that denies bivalence.

If ¬CA is compatible with the truth of A and means something more like ‘A is not knowable’, then the feature of the theory at issue amounts to saying that the theory declares that its basic assumptions cannot be known to be true, which looks a lot less problematic. It need not, in general, be an unacceptable consequence of a theory that by its own lights we do not and cannot know it to be true. Similarly, if employ the reading of C as ‘can tell that’, it does not look unreasonable to hold both that P is a postulate of the theory and that we cannot tell that P. But, even if we move towards an epistemic understanding of C in this way, the initial sense that this move is unobjectionable may turn on a simplistic rendering of that operator. After all, C is used in the characterisation of borderline cases: *a* is borderline F iff ¬CF*a* ʌ ¬C¬F*a*; ‘I take the meaning of ‘it is clear that’ to be specified in terms of borderlineness rather than the other way round’ (p?). So, it does not follow that ¬CA just because we do not know that A for some reason unconnected to its vagueness: if we have no idea about the colour of an inaccessible plant, that doesn’t make it a borderline case of ‘red’. The operator is intended to illuminate vagueness, not all kinds of ignorance. Declaring that the postulate, P is such that ¬CP is declaring it borderline status or false. And a theory that declares a key element of itself merely of borderline status is, at best, odd.

**V**

*Boundaries, Technical terms and Neutrality between Theories.* A non-vague predicate, F, draws sharp boundaries between the Fs and the not Fs: acknowledging borderline cases seems to allow us to avoid the division into two sharp categories. The need to avoid replacing the two-fold categories with three (equally sharp) ones is a typical part of the explanation of the need to accommodate higher-order vagueness.[[12]](#footnote-12) If CHOV replaces the hierarchy with three columns or categories of cases, isn’t that to miss the point of having higher-order vagueness? Is it not then committed to sharp boundaries and thus unsuitable as a theory of vagueness or higher-order vagueness?

To put it another way, according to CHOV, there is no instance of second-order vagueness that isn’t also an instance of first-order vagueness (and also third-order etc. up through the orders). If vagueness is higher-order vagueness in this way, then there’s no distinct higher-order vagueness, which suggests that the motivations for going beyond first-order vagueness cannot have been satisfied. It does not help to point out that those borderline cases F are equally second-order borderline Fs if those cases are still sharply-bounded.[[13]](#footnote-13)

Bobzien raises the sharp boundaries objection explicitly in her section replying to some common objections. Her reply there is to bite the bullet, defending the sharp boundaries between the three categories by appealing to the fact that her C operator is a technical term and distinct from natural language expressions such as ‘clearly F’. I am not sure exactly what the position on this operator amounts to here and the interpretation of C is not one of the main issues of her paper. But it will nonetheless be useful to consider whether regarding C as a technical term undermines the attempt to uncover the structure of vagueness and borderlineness, which will also bear on more general questions about the limitations on appealing to technical terms in one’s philosophical theory.

If the operator at the centre of one’s theory is a technical term, you may seem to thereby avoid objections regarding counter-intuitive features of that operator, since intuitions will be about natural language expressions. So, if a theory concerning vagueness uses an expression that draws sharp boundaries, then maybe that can still succeed in capturing the structure of vagueness. Bobzien’s C and B operators are, we are told, not to be identified with natural language expressions such as ‘clearly F’ and ‘borderline’ (p.?). Thus she replies to the objection that there are sharp boundaries on the CHOV picture by pointing out that it is sharp boundaries to these technical terms not the similar natural language terms to which the theory is committed. But is this a reasonable strategy?

Can Bobzien avoid the worry that she’s just then *stipulated* that S4M is the right logic and cannot be thought to have revealed the structure of (higher-order) vagueness and borderlines? If BA does not amount to A’s being *borderline*, then the logical structure of B doesn’t amount to the logical structure of borderlineness, and the pattern of iterations of B don’t correspond to the pattern of higher-order *borderline* cases. That leaves open the question what the real structure of borderline cases is and since vagueness is so closely tied to borderline cases, we have no reason to think that the theory illuminates vagueness at all or that it is any better in that respect than stipulating a Definitely operator resembling the natural language one, but which has no room for higher-order vagueness (e.g. one governed by an S5 logic).[[14]](#footnote-14) Bobzien may invoke her reply that her term ‘makes those boundaries indeterminable and has been selected for precisely that reason’, but ‘indeterminable’ must here equally be a technical term, and so may not be a significant feature even of the boundaries drawn by the technical terms; there is no reason to think that those boundaries are indeterminable in an intuitive sense not tied to the technical term.

We may not want to say that Bobzien needs to pin down exactly what her C and B operators amount to in our natural language terms or have proof that they correspond to our natural language terms. A theory may have illuminated the structure without having done this. In particular, it could introduce an operator and model it with a structure that has many features that might appeal to an account of vagueness, inviting the hypothesis that this *is* the structure of borderline cases and vagueness and that B can be read as ‘borderline’ in the ordinary sense. But this approach is not compatible with the attempt to defeat objections to structural features by falling back on the fact that the modelling operator is merely a technical term. If the hypothesis is correct, the corresponding natural language terms will share the relevant features of the technical terms and in Bobzien’s case this would include undesirable sharp boundaries to the fundamental categories, a last clear case in the sorites series etc.

That the C operator is a technical term is *more* problematic for Bobzien than it would be in other sorts of cases in which a C or D operator plays some role in a theory of vagueness, because of the very centrality of that operator to the theory. On Bobzien’s picture, the appeal to a technical term to do the work of modelling borderlineness does not build on a background theory of the semantics of vague predicates. By contrast, a degree theory of vagueness – to take one example – may maintain that truth comes in degrees, and that degree of truth falls off through the borderline cases, and this could leave scope for modelling the ‘definitely’ operator in different ways within that framework (most obviously as Dp iff p is degree 1 true, but other alternatives would be compatible with the framework).[[15]](#footnote-15) A degree theorist may make one choice about this and not dwell too long on whether that operator corresponds exactly with the natural language notion of ‘clearly’. But their account of the structure of vagueness, the solution to the sorites paradox and the semantic status of borderline cases does not then hinge on that technical term. A theory that revolves entirely around a ‘clearly’ or ‘definitely’ operator – as Bobzien’s does – cannot make progress towards any of those ends based on stipulation of the role of the technical term.

This brings us on to the neutrality between (or compatibility with) different theories of vagueness that Bobzien hopes to maintain. I suggested above that this neutrality is unsustainable and that she – like epistemicists – must build on classical logic with bivalence governing the vague language. Above, I argued this on the grounds that since key claims within the theory are unclear, this had better be compatible with them being true, as it would be on a bivalent epistemicist story. Further arguments here will briefly back up this suggestion, without exploring the, as yet undeveloped, details of the non-classical versions of Bobzien’s theory.

The specific modal logic S4M (and QS4M+BF+FIN) is defined in such a way that any theorem of classical logic is also a theorem of that logic. That does not yet commit us to Bivalence and/or epistemicism, as a theory such as Supervaluationism deviates from classical semantics while preserving all theorems of classical logic (see Fine 1975, Keefe 2000.) As it stands, it rules out most other non-classical positions on vagueness, such as a many-valued logic or an intuitionistic logic.[[16]](#footnote-16) But an analogous theory would build the alternative modal logics on an appropriate non-classical base, to which certain key principles such as (4) and (M) are added.

At the heart of many such non-classical theories of vagueness is the treatment of borderline case predications as something other than true or false (whether by falling in a truth-value gap or by taking some non-classical value). If we try to combine this with Bobzien’s C operator, seen as a technical term, there will be two different notions of borderline cases – those picked out in relation to her technical term, where we cannot tell whether or not something is the case, and those meeting the semantic condition (e.g. that are neither true nor false). Why should we expect these to coincide? And if they do coincide, the problem of a sharply bounded set of borderline cases looms again, for, then, if it is a problem on one conception, then it is also a problem on the other and appealing to C being a technical term will not help. On the other hand, if the two notions of borderline cases do not coincide, then issues about the structure of vagueness and higher-order vagueness will then track questions about the semantic condition, and the structure of C will not then be relevant to that, so of questionable interest. Note that this objection would also apply if her C operator wasn’t merely a technical term: it applies as long as it is picked out independently of the semantic borderline status. The prospects for combining CHOV with a non-classical theory of vagueness do not look promising.

**VI**

*Conclusion.* Where does that leave us in relation to the striking title claim that ‘vagueness is higher-order vagueness’? That depends on what is meant by higher-order vagueness. It is tempting to think that there is nothing significantly *higher-order* if the higher-order and the first-order necessarily coincide. I have argued that what is labelled higher-order vagueness in CHOV does not meet the needs for which higher-order vagueness was introduced. Bobzien’s conception of borderline cases is problematic and there are various unpalatable consequences regarding the structure imposed on them, such as the lack of clear borderline cases and the kinds of importantly different categories that are taken to coincide. I have suggested that the logic Bobzien advocates requires assumptions that cannot stably be maintained together and that are not compatible with a viable interpretation of the key C operator.

With her Columnar Higher-Order Vagueness, Bobzien has produced a very interesting new approach to higher-order vagueness, but in allowing all orders of vagueness only by letting them coincide with first-order vagueness, it does not succeed in providing the desired picture of boundarylessness at all levels. Building up from the first level with a hierarchy may yield problems, but building up in columns offers no significant advantage over not building up at all.[[17]](#footnote-17)

**REFERENCES**

Bobzien, Susanne 2010: ‘Higher-Order Vagueness, Radical Unclarity and Absolute Agnosticism’. *Philosophers’ Imprint* 10, pp. 1-30.

Bobzien, Susanne 2012: ‘If it’s Clear, then it’s Clear that it’s Clear, or is it? – Higher-order Vagueness and the S4 Axiom’. In: K.Ierodiakonou and B.Morison (eds), *Episteme, etc.*, Oxford: OUP, pp. 189-212.

Bobzien, Susanne 2013: ‘Higher-Order Vagueness and Borderline Nestings: A Persistent Confusion’. *Analytic Philosophy* 54, pp. 1-43.

Bobzien, Susanne 2015: ‘Columnar Higher-Order Vagueness or Vagueness is Higher-Order Vagueness’. *Aristotelian Society Supplementary Volume* 89.

Dorr, Cian (forthcoming): ‘How Vagueness Could Cut Out at any Order’. Review of Symbolic Logic.

Fara, Delia Graff 2003: ‘Gap Principles, Penumbral Consequence, and Infinitely Higher-Order Vagueness’. In: Jc Beall, (ed.) *Liars and Heaps*, Oxford: OUP, pp. 195-221. Originally published under the name ‘Delia Graff’.

Hintikka, Jaakko 1962: *Knowledge and Belief*. Ithaca: Cornell University Press.

Keefe, Rosanna 2000: *Theories of Vagueness*. Cambridge: Cambridge University Press.

Mahtani, Anna 2008: ‘Can Vagueness Cut Out at any Order?’. Australasian Journal of Philosophy 86, pp. 499 – 508.

Smith, Nicholas J.J. 2008: *Vagueness and Degrees of Truth*, Oxford University Press.

Soames, Scott 1999: *Understanding Truth*. New York: Oxford University Press.

Sorensen, Roy A. 2010: ‘Borderline Hermaphrodites: Higher-Order Vagueness by Example’. *Mind* 119, pp. 393-408.

[Williamson, Timothy](http://users.ox.ac.uk/~sfop0009/) 1994: *Vagueness.* London: Routledge.

[Williamson, Timothy](http://users.ox.ac.uk/~sfop0009/) 1999: ‘On the Structure of Higher-Order Vagueness’. *Mind* 108, pp. 127-142.

Williamson, Timothy 2000: *Knowledge and its Limits*. Oxford: Oxford University Press.

[Wright, Crispin](http://www.st-andrews.ac.uk/academic/philosophy/STAFF/wright.html) 1992: ‘Is Higher-Order Vagueness Coherent?’ *Analysis* 52, pp. 129-39.

[Wright, Crispin](http://www.st-andrews.ac.uk/academic/philosophy/STAFF/wright.html) 2001: ‘On Being in a Quandry’. Mind110, pp. 45-98.

Wright, Crispin 2010: ‘The Illusion of Higher-Order Vagueness’. In Richard Dietz and Sebastiano Moruzzi, *Cuts and Clouds*. Oxford: OUP, pp. 523-549.

1. Bobzien, this volume (2015); see also her 2010, 2012 and 2013. [↑](#footnote-ref-1)
2. Mahtani 2008 and Dorr (forthcoming) argue against the symmetry assumption, thus prompting a logic weaker than B. [↑](#footnote-ref-2)
3. See, e.g. Sainsbury 1990, Keefe 2000, pp.33-34, Wright (2010 and elsewhere). [↑](#footnote-ref-3)
4. E.g. Williamson 1994, p.228. [↑](#footnote-ref-4)
5. Suppose *a* is a clear borderline case of F, so CBF*a*. By (T), it follows that BF*a*. By (\*) above (her 2.9) it then follows that BBF*a*. But by the definition of B in terms of C (her 2.1), it follows that ¬CBF*a*, which contradicts the original assumption. Clear borderline cases are thus contradictory in the framework. [↑](#footnote-ref-5)
6. See, however, Sorensen 2010 who offers some nice purported cases of ‘higher-order vagueness by examples’. [↑](#footnote-ref-6)
7. One way to show that they coincide is to appeal to the result Bobzien cites in her 2013, that formulae starting with ¬C and having an odd number of negation signs (e.g. ¬CCF*a*, the first conjunct of the first type of case) are equivalent to ¬CF*a* and those starting with ¬C and having an even number of negation signs (e.g. the second conjunct, ¬C¬CF*a*) are equivalent to ¬C¬F*a*. Both types of second-order borderline case are thus provably equivalent to ¬CF*a* ʌ ¬C¬F*a*, that is a first-order borderline case. [↑](#footnote-ref-7)
8. From contraposition of (4). [↑](#footnote-ref-8)
9. From contraposition of (5) and double negation elimination. [↑](#footnote-ref-9)
10. This kind of case clearly relates to the first objection she discusses – that there *are* clear borderline cases. Her reply that the objection rests on shifting to the kind of ‘in-between borderlineness’ was questioned above. [↑](#footnote-ref-10)
11. The first part of the argument goes roughly as follows: from the basic assumption that it’s not clear that there are no borderline Fs and a theorem of the modal logic in question, it follows that there *are* borderline Fs. We can then use another modal principle of the theory – that if there are borderline Fs, then it isn’t clear there are borderline Fs – to conclude that it isn’t clear that there are borderline Fs. In her form of the (rejected) argument, we move from the conclusion that there are borderline cases to this claim prefixed with C, given the ‘tacit, and perfectly acceptable assumption that if one has just shown that A, one can tell that A’. This acceptable assumption is misapplied, however, if we have ‘shown’ that A from premises some of which are not clear, and so in this case we must deny the original assumption prefixed with C. [↑](#footnote-ref-11)
12. See, e.g., Williamson 1994, p.111 and Keefe 2000, p.31. [↑](#footnote-ref-12)
13. It might be tempting to respond that as there are no *clear* borderline cases according to CHOV, it is wrong to see the intermediate, non-clear, cases as first-order borderline cases, so that the boundary between the clear column and the intermediate category should *not* be described as a sharp boundary to the (first-order) borderline cases. But Bobzien’s characterisation of (first-order) borderline case doesn’t allow this, as a borderline F is one that is not clearly F and not clearly not-F, regardless of whether this is clearly true. [↑](#footnote-ref-13)
14. There may also be a question of whether she succeeds in capturing an plausible notion of ‘clear that’ or ‘tellability’ even if it isn’t suitable for modelling vagueness. I will not address this question here, but would expect to find that versions of Williamson’s arguments against luminosity cast doubt on this suggestion. [↑](#footnote-ref-14)
15. See, e.g., Keefe 2000, Chapter 4 on such theories of vagueness. [↑](#footnote-ref-15)
16. To cite a few of many possible examples, see, e.g., Soames 1999 for a three-valued logic, Smith 2008 on infinite-valued logics and Wright 2001 on intuitionistic logic. [↑](#footnote-ref-16)
17. I am very grateful to Stephen Bolton and Dominic Gregory for comments on an earlier draft of this paper. [↑](#footnote-ref-17)