Ultimate Limit State design to Eurocode 7 using numerical methods Part I: methodology and theory

Colin Smith & **Matthew Gilbert**, Department of Civil and Structural Engineering, University of Sheffield

(Part II will appear in the next edition of GE)

Summary

Assessment of the Ultimate Limit State (ULS) in Eurocode 7 is to be carried out using "Design Approach 1" in the UK. In most cases this involves two design checks, one which primarily involves an "action factor" approach (Design Approach 1, Combination 1, termed DA1/1) and one which primarily involves a "material factor" approach (Design Approach 1, Combination 2, termed DA1/2). The latter is generally straightforward to implement in numerical analysis procedures, but the former is potentially more challenging.

A survey of the current literature on Eurocode 7 indicates differences of opinion on how best to undertake DA1/1 checks (the same differences of opinion also apply to the "action/resistance factor" Design Approach 2, DA2, checks). This can lead to inconsistent application of Eurocode 7 when undertaking a numerical analysis, which in turn can lead to differences in the resulting design solutions.

In this two-part paper, a simple and consistent methodology for undertaking "action/resistance factor" design checks using numerical methods is proposed in Part I, while in Part II the methodology is used to develop a general-purpose design procedure which is then applied to a number of example problems.

Introduction

There exists a large body of literature in which Eurocode 7 (BSI 2004) is applied to geotechnical design problems using conventional "hand calculation" type methods. However, significantly less attention appears to have been paid to use of Eurocode 7 in conjunction with numerical analysis procedures. This paper seeks to address this for the Ultimate Limit State (ULS) assessment of structural and geotechnical failure (termed STR and GEO in Eurocode 7). Issues relevant to each of the three design approaches (DA) specified in the Eurocode will be considered. Each design approach may be broadly classified as follows:

- DA1 This approach requires two design combinations (DC) to be examined
- DC1 Factors on actions (loads) (DA1/1)
- DC2 Factors on material strength (DA1/2)
- DA2 Factors on actions and resistances
- DA3 Factors on actions and material strength.

In general only one of these three design approaches is permitted by the National Application document of each nation (in the UK, Design Approach 1, DA1 is to be used). Whichever approach is used, one of the Eurocode's strengths is that it provides a very general methodology which can be applied flexibly by engineers. This allows an appropriate margin of safety to be achieved in a design, while simultaneously ensuring that unreasonable or impossible modes of response in the accompanying mechanical analysis are not introduced.

However, this flexibility can also be viewed as a weakness. For example, a survey of the current literature on Eurocode 7 indicates differences of opinion as to when and how to introduce partial factors in DA1/1 and DA2. Much discussion on this issue may be found in the literature, such as Simpson (1997), Schuppener et al. (1998), Simpson & Driscoll (1998), Orr & Farrell (1999), Simpson (2000), Farrell & Orr (1998), Driscoll & Simpson (2001), Frank et al. (2004), Driscoll et al. (2005), Simpson (2007), Driscoll et al. (2008), Bond & Harris (2008).

When seeking a methodology to be used in conjunction with a generally applicable numerical analysis procedure, the ideal would be one that is "problem agnostic" (ie is the same irrespective of whether the problem involves evaluation of the stability of a foundation, a retaining wall, a slope etc.). Additionally, the methodology should, if possible, not require user intervention at intermediate stages during the calculations, and should provide a consistent, safe and mechanically reasonable assessment.

In this paper the authors:

- Examine the challenges and requirements for applying numerical analysis procedures to "material factor" and "action/resistance factor" type ULS checks. (The "action factor" approach will be considered as a special case of the "action/resistance" factor approach with a resistance factor of unity.)
- Propose a simple and consistent methodology which allows "action/resistance factor" type design approaches to be undertaken in conjunction with numerical analysis procedures. This builds on the methodology put forward in Smith & Gilbert (2010), where the salient issues involved in applying numerical analysis procedures to load and resistance factor design (LRFD) problems were briefly examined.

In Part II of the paper a simple framework for applying the above in the context of Eurocode 7 is described and then illustrated with a number of worked examples.

Ultimate Limit State assessment

In general, a given design solution will normally be inherently stable, and is by implication therefore not close to its ultimate limit state. In an ULS assessment the goal is to drive the system to collapse by some means (in a theoretical sense). The difference between the actual and ULS state can then be considered as a measure of Factor of Safety (FoS).

In many conventional analyses this process is carried out implicitly. For example, when designing a retaining wall, active or passive earth pressures are typically assumed to act on the wall.

However, when using a general numerical analysis procedure, this process must be carried out explicitly. Three main ways of explicitly driving a system to the ULS are listed in Table 1.

In many current numerical analysis procedures, increasing an existing load to drive the system to collapse (ie Method A in Table 1) is already an inherent feature. Thus a supplementary load factor, henceforth referred to as an "adequacy factor", λ_A, can be applied to one or more unfavourable loads, and the magnitude of λ_A required to achieve collapse can then be found using the numerical analysis procedure. Considering Method B in Table 1, it is alternatively possible to iteratively reduce soil strengths within a numerical analysis procedure, by a factor λ_B, until failure is achieved.

It should be noted that the actual collapse mode identified by an analysis will in general vary according to where λ is applied, and only when $\lambda = 1$ for each of the three methods can the collapse modes

»

Method	Description
A	Increase an existing load in the system
B	Reduce the soil strength
C	Impose an additional load (or group of loads) in the system

Table 1: Common ways of driving a system to the ULS

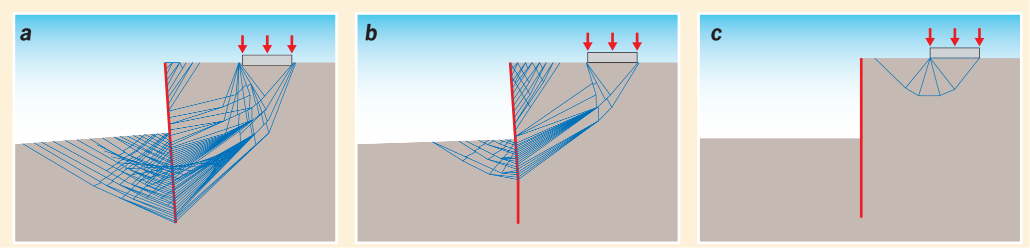

Figure 1: Three possible failure modes for a combined foundation and embedded retaining wall, dependent on specific soil and structural properties: (a) rotation of wall as rigid body, (b) formation of plastic hinge in wall, (c) local failure of foundation.

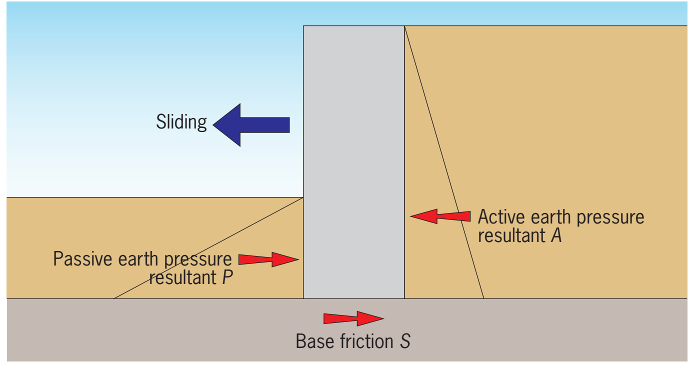

Figure 2: Conventional assumption made in the analysis of a gravity retaining wall against sliding.

be expected to match (assuming that the problem does not have several equally critical collapse modes).

It may be observed that the ULS is often assumed to be synonymous with plastic collapse, though according to the Eurocode definitions other types of ULS failure are also possible.

Since plastic limit analysis methods are commonly used by geotechnical engineers, the arguments developed in this paper are primarily presented in the context of plastic limit analysis. In particular, the Discontinuity Layout Optimisation (DLO) computational limit analysis procedure (Smith & Gilbert 2007), as implemented in the LimitState:GEO software (LimitState 2009), will be used to provide solutions to the example problems presented.

While solutions to ULS problems are generally more straightforward to obtain using computational limit analysis techniques, the basic methodology proposed is applicable to a broad range of numerical analysis tools. (Though there can be additional challenges in determining the ULS, for example when using finite element models employing complex constitutive soil models; these are not considered here.)

The role of partial factors

In Eurocode 7 it is stated that: "For each geotechnical design situation it shall be verified that no relevant limit state ... is exceeded" [EN 1997-1, 2.1(1)P]. To render the probability of a ULS occurring sufficiently small, the general approach in Eurocode 7 is to apply factors on uncertainties at their source in the calculation, rather than applying them to the whole calculation. Thus factors may be applied to one or more of the following characteristic parameters (characteristic values are defined in the Eurocode as "a cautious estimate of the value affecting the occurrence of the limit state" [EN 1997-1, 2.4.5.2(2)P], and are identified by subscript k): actions (F); action effects (E); material properties (X); resistances (R); geometrical parameters (a).

It is, of course, necessary to distinguish properly between these parameters before applying the partial factors, and unambiguous definitions of what is meant by an action, action effect and a resistance are proposed in Appendix I for use in a numerical analysis of a geotechnical stability problem.

Once the partial factors have been applied, design values (identified by subscript "d") are obtained, and are used in subsequent stability calculations. Factors to be applied to actions, action effects, material properties and resistances are given in Table 2.

Note that factors applied to geometric parameters will, for the sake of brevity, not be considered in this paper. To prevent limit state STR or GEO from occurring, design (ie factored) *actions* or *action effects* (denoted E_d) must be less than or equal to the corresponding design resistance (R_d) at the ultimate limit state, ie:

$$E_d \leq R_d \qquad (1)$$

Note that this key Eurocode equation is an inequality, and provides no measure of the degree of over-design.

It can be seen from Table 2 that in Eurocode 7 the numerical value of a given partial factor is dependent on the specific situation. e.g. the value of a partial factor depends on whether it is applied to an action / action effect, or to a resistance, and in the case of an action, whether this is permanent or variable and favourable or unfavourable.

Eurocode 7 with numerical analysis: observations

Material factor design approaches

"Material factor" design approaches (for example DA1/2 and DA3 in Eurocode 7) generally only require that parameters known in advance of an analysis are factored. These parameters are either material properties or externally applied *actions*. Factors of 1.0 are applied to *action effects* (as defined in in Appendix I) and resistances.

When used in conjunction with a general purpose numerical analysis procedure this type of approach has the significant advantage that the analysis will automatically identify the critical collapse mechanism, there being no need to consider manually a variety of prescribed potential failure modes, anticipated or otherwise.

For example, Figure 1 illustrates three different failure modes for a problem which involves a foundation and embedded retaining wall, the most critical of which would automatically be identified using a general numerical analysis procedure.

As has been mentioned earlier, in a general numerical analysis, the ULS may be achieved either through increasing an unfavourable load by a factor λ_A, (where this factor is applied as a multiplier) or by reducing soil strength across the model by a factor λ_B (where this factor is applied as a divisor). If after the analysis $\lambda_{A,B} \geq 1$ then the system can be taken to be inherently stable. Alternatively, if $\lambda_{A,B} < 1$ then the system is inherently unstable (where $\lambda_{A,B}$ denotes either λ_A or λ_B).

It is therefore reasonable to expect that any action effect (E_d) and resistance (R_d) pair within a system will satisfy $E_d \leq R_d$ when $\lambda_{A,B} \geq 1$ (although strictly speaking this is only true when plastic collapse governs the ULS).

Action factor and action/resistance factor design approaches

"Action factor" and "action/resistance factor" design approaches (such as DA1/1 and DA2 in Eurocode 7) typically require that *action effects* and *resistances*, considered at some internal location within the problem (for example at a soil/structure boundary), are factored. When using numerical analysis procedures, two challenges immediately present themselves:

- The first challenge can be illustrated by considering a conventional ULS check of a gravity retaining wall against sliding, as shown in Figure. 2.

Such a check is typically carried out by assuming that active and passive Rankine pressures act on opposite sides of the wall. These would be designated as an *action effect* and *resistance* respectively in this problem.

The design check (equation 1) therefore requires that:

$$\gamma_E A < \frac{S}{\gamma_{R,S}} + \frac{P}{\gamma_{R,P}} \qquad (2)$$

where the forces A, S and P are as indicated in Fig. 2, γ_E is the partial factor for *unfavourable permanent actions*, $\gamma_{R,S}$ is the partial factor for *sliding resistance* and $\gamma_{R,P}$ is the partial factor for *passive resistance*. This contrasts with a conventional computation of the factor of safety, as the ratio of resisting forces to disturbing forces: $R = (P + S)/A$.

The challenge when using an "action/resistance factor" design approach in conjunction with a numerical analysis procedure arises because the above system is only in equilibrium if the ratio of resisting forces to disturbing forces $R = 1$.

For example, if $R > 1$ then the passive earth pressure and base friction will exceed the active earth pressure. In practice equilibrium occurs because the stresses are in reality not limiting, failure, condition (ie the soil can be considered to be only mobilising part of its strength). However in a ULS plastic limit analysis both yield and equilibrium conditions must be satisfied, at least when an upper-bound limit analysis approach is being used.

- A second problem relates to the nature of the failure mode. In order to apply partial factors, the locations of action effects and resistances must be known.

This in turn requires advance knowledge of the collapse mode, which in turn requires that the analysis has already been performed. However, the form of collapse mode will depend on where the partial factors are applied, and a circular argument ensues. Only in simple cases, for example involving bearing capacity of a foundation or sliding failure of a retaining wall, will the positions of the actions and resistances remain unchanged during the analysis.

While the two challenges listed above could potentially be dealt with by using a sophisticated, perhaps iterative, analysis procedure, the first is inherently intractable and cannot be circumvented in a numerical analysis procedure without violating fundamental mechanical principles.

Note that for very simple problems (for example the bearing capacity of a surface footing) the first challenge does not arise since the factors are effectively applied at an interface between an externally applied load and the soil/structure domain. Therefore violation of internal equilibrium is not encountered.

However, when applied for example to retaining wall problems (see figure 2), where factors may be applied between the soil mass and the wall, the direct approach does not work and it is necessary to adopt an alternative procedure.

A proposed methodology for action/resistance factor design

Development of methodology

Consider again the problem shown in Figure 2. To induce the required active and passive pressures, and the base shear resistance on the boundaries of a real wall, it would be necessary to cause it to slide by introducing some external agent. The required ULS action effects and resistances then become available, partial factors can be applied and subsequently equation 1 evaluated.

It is suggested that this is the key to performing "action/resistance factor" design in conjunction with numerical analysis. A failure mode must first be proposed. This failure must be induced in the model by an external agent, and stability can then be evaluated. In a numerical analysis procedure the wall could be forced to move in a specific direction ("kinematic forcing") or the equilibrium relations could be modified in that direction ("equilibrium forcing"). In this paper it is suggested that the latter is preferable as it is in the same form as equation 1.

To achieve this it is necessary to apply a hypothetical unfavourable external force H parallel to the "equilibrium direction" to be checked, and to then increase this until failure occurs. For example, for a horizontal equilibrium check on a retaining wall, H would be applied in a direction expected to induce the anticipated failure (ie in the direction of the active pressure), with H passing through the centroid of the wall to avoid applying additional moments. This is a straightforward calculation for most numerical analysis procedures (and "natural" in the case of a numerical limit analysis approach).

Once failure has been achieved, appropriate "passive" and "active" zones are generated on either side of the wall, as assumed in a conventional hand analysis. It is then straightforward to determine the active force (A) and passive and base resistance forces (P and S) predicted by the analysis, and to incorporate these when evaluating equation 1. Note that the magnitude of the force H required to cause failure is not used when assessing safety. It is evident that this approach addresses both concerns raised previously, i.e. *actions* and *resistances* do not need to be defined in advance of the numerical analysis, and equilibrium is preserved at all times.

In simple terms the method can be considered analogous to "pushing" the construction in a range of directions.

The direction that requires the least effort to trigger instability is the critical direction and the magnitude of the "push" required relates to the margin of safety. A "small" push indicates a low margin of safety while a "large" push indicates a high margin of safety. "Large" and "small" are quantified by comparing them to the magnitudes of the other forces acting on the construction at failure.

The same basic methodology can also be used to assess internal structural stability, for example bending failure in a sheet pile wall, and a slightly more convenient "inverse-factor" method can in some cases alternatively be used. This approach is described in detail in Appendix II.

Parameter			Symbol	DA1/1	DA1/2	DA2	DA3
Action/	Permanent	Unfavourable	γ_G	1.35	1.0	1.35	1.0/1.35*
action effect		Favourable	γ_G	1.0	1.0	1.0	1.0
	Variable	Unfavourable	γ_Q	1.5	1.3	1.5	1.3/1.5*
		Favourable	γ_Q	0.0	0.0	0.0	0.0
Resistance			γ_R	1.0	1.0	1.1/1.4†	1.0
Soil parameters		c'	γ_c'	1.0	1.25	1.0	1.25
		tan ϕ'	$\gamma_{\tan\phi'}$	1.0	1.25	1.0	1.25
		c_u	γ_{c_u}	1.0	1.4	1.0	1.4

*DA3 values for actions are given for geotechnical/structural actions. † Factors on resistance depend on resistance type.

Table 2: Eurocode 7 partial factors for STR and GEO.

Application of proposed methodology: retaining wall example

Consider the retaining wall shown in Figure 3, for which a combined bearing/sliding failure is to be checked. Here it has been decided that equation 1 will be checked in an "equilibrium direction" 15° to the horizontal, so that the externally applied force (H) acts at an angle of 15° as indicated. Components of the self weight (W) and the resultant loads (A) and resistances (P and S) arising from the forced failure mechanism acting parallel to the "equilibrium direction" are given in Table 3 (overleaf). Fully frictional soil/wall interfaces are assumed.

The solution shown was identified using the LimitState: GEO software (LimitState, 2009) and can be used to illustrate how the loads (actions and action effects) and resistances acting on the wall can be determined by application of the hypothetical external force. Table 3 indicates that, when considering the factored actions and resistances in the direction of the external force, this wall is unsafe.

A number of observations on this approach can be made:

- The "equilibrium direction" needs to be stated in advance of an analysis to allow the externally applied force H to be applied correctly. Thus the "action/resistance factor" approach is inherently less flexible when used in conjunction with numerical analysis procedures than "material factor" approaches, in which the critical collapse mode can be determined automatically.

- Suitable locations need to be chosen to enable evaluation of *action effects* and *resistances* (for example interfaces between soil and structure). For investigation of a pure translational failure mode, H itself should be applied to the centroid of the structure to be assessed, to avoid applying an additional moment to the structure.

- In this method it is proposed that the numerical analysis is carried out using characteristic values of all actions (excepting variable actions, which will be considered in Part II of the paper). This ensures that the analysis is conducted without compromising fundamental principles of mechanics. Once the results from the analysis are available only those actions, action effects and resistances directly applied to the body (usually a » structure) are factored, and stability is then assessed. (At the stage of applying factors, equilibrium need not be enforced). It is considered in this calculation that these computed quantities embody all uncertainties associated with the design.

Note also that assessment of what constitutes a *favourable* and *unfavourable* action need only be considered in the sense of the "equilibrium direction"; application of this principle removes a key potential source of confusion for new users of Eurocode 7.

- It is proposed that equation 1 need only be checked in the "equilibrium direction" (ie the direction of application of *H*). It could be argued that it is pragmatic to consider equation 1 in a range of directions, which is not especially onerous to do. However, it is probably preferable to instead consider a range of anticipated equilibrium directions using the full procedure, as is standard practice, and to determine the critical direction. Note that in the preceding example the angle of 15° was used here purely for illustrative purposes, and in this specific case the most critical equilibrium direction for the wall is actually close to the horizontal.

- To test for overturning stability of a wall (for example), a hypothetical moment *M* should instead be applied to the wall to cause failure, with equation 1 then formulated in terms of moments.

- One challenge for the "action/resistance factor" approach is identification of suitable factors for combined and other non-standard failure modes. For example, at present Eurocode 7 DA2 specifies different factors for bearing, passive and sliding resistances, but it is unclear what factors should be used for mechanisms that cannot clearly be classified as involving bearing, passive or sliding failure.

- It is also possible to use the proposed methodology to compute a conventional FoS.

Discussion

Characteristic values in the mechanical analysis

When applying the proposed methodology it should be noted that all mechanical analyses are carried out using characteristic values. The main argument for this is that it ensures that physically unreasonable scenarios are not modelled. For example, the factoring of pore water pressures (which are regarded as actions) prior to performing a mechanical analysis is known to be problematic, and can for example sometimes lead to anomalous results when undertaking an effective stress analysis.

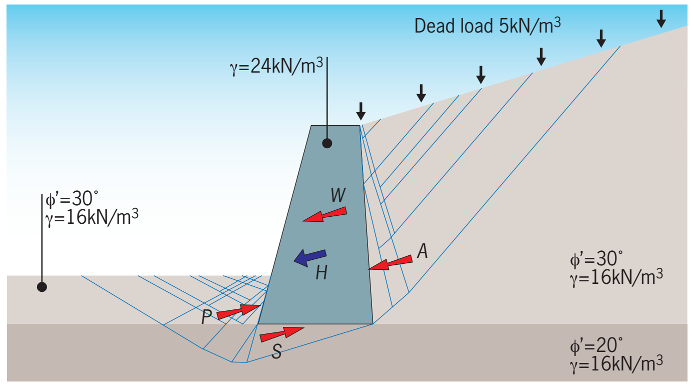

Figure 3: Design check against combined sliding and bearing capacity failure of a 4m high gravity retaining wall (translational mode).

Components of actions not aligned with the equilibrium direction

The proposed methodology requires that only those components of actions aligned with the "equilibrium direction" are factored. This appears to run counter to the generally held view that a whole action should be factored by the same amount. A simple example will be used to explore this issue further.

Figure 4 depicts a problem involving a weightless rectangular block resting on a horizontal rigid surface and subjected to a permanent load *P* inclined at angle α to the vertical. If the angle of friction of the interface between the block and rigid surface is ϕ', clearly the block will slide if $\alpha > \phi'$ and be stable if $\alpha < \phi'$. Stability may be assessed in a number of ways.

Design check 1 (direct approach): An engineer performing a Eurocode 7 design check, for example using DA1/1, would correctly surmise that the inclined load is a single action. Assuming $\phi' = 30°$, if $\alpha > 30°$ then *P* will be unfavourable with regard to sliding, and should be factored by 1.35. However, clearly any positive value of *P* will lead to sliding failure. If $\alpha < 30°$, then P will be favourable, and should be factored by 1.00. In this case *no* positive value of P will lead to sliding failure – ie the partial factors have had no effect on the assessment, and no margin of safety can be identified.

Design check 2 (using proposed methodology): Now consider a design check which uses the proposed methodology. In this case an "equilibrium direction" must first be prescribed, with the horizontal direction being the most obvious candidate.

The characteristic shearing resistance *T* on the block/surface interface is given by:

$$T_k = P_k \cos\alpha \tan\phi_k \qquad (3)$$

The horizontal component of P_k is always considered unfavourable (for $0° < \alpha < 90°$). Thus the requirement for safety (equation 1), after applying the action factors from DA1/1, becomes:

$$1.35 P_k \sin\alpha < T_k \qquad (4)$$

or

$$\tan\alpha < \frac{\tan\phi_k}{1.35} \qquad (5)$$

This is similar to the type of safety margin that would be prescribed by DA1/2, where a factor of 1.25 is applied to $\tan\phi_k$:

$$\tan\alpha < \frac{\tan\phi_k}{1.25} \qquad (6)$$

Now the above might be considered to violate the principle of factoring actions, since components of P are being factored differently, and if the uncertainty is in the magnitude of P then "all of P" should be factored simultaneously. However, the counter argument is that:

- As has already been demonstrated, no margin of safety can be identified using the "direct" approach, rendering it of little value.
- The "direct" approach involves factoring of actions at a different stage in the calculation to the factoring of action effects and resistances. It is suggested that it is logical to apply factors to actions, action effects and resistances at one stage of the calculation only.
- DA1/1 typically addresses limit state STR (which would involve subsequent analysis of the internal stresses within the block, identical using both methods) rather than GEO.
- Factoring *P* directly would mean a step change in its magnitude as its angle of inclination transitions from $\alpha < \phi$ to $\alpha > \phi$. Such step changes are generally undesirable in a safety assessment.
- The "direct approach" design check can in fact be carried out using the proposed methodology if the "equilibrium direction" is chosen to coincide with the direction of the inclined force *P*.
- The modelling of, for example, lateral water pressures on either side of an embedded wall is unaffected since they act in the same "equilibrium direction". However, upthrust beneath the wall might be factored differently.

The above issues arise principally because friction is involved in the problem, leading to the resistances and action effects in the problem being functions of applied actions.

Conclusions

1. Eurocode 7 employs "material factor", "action factor", and "action/resistance factor" type design approaches.

2. "Material factor" design approaches (for example DA1/2 and DA3 in Eurocode 7) can be used without difficulty in conjunction with general numerical analysis procedures, which will automatically identify the critical collapse mode. A "material factor" analysis can effectively be considered to emulate the fundamental Eurocode 7 stability test $E_d \leq R_d$ for action effect and resistance pairs at all locations within a given problem.

3. "Action factor" and "action/resistance factor" design approaches (such as DA1/1 and DA2 in Eurocode 7) are more challenging to undertake using numerical analysis procedures. This paper has proposed a simple and consistent methodology designed to address this. Using this methodology an analysis is carried out using a pre-specified "equilibrium direction", chosen according to the nature of the anticipated collapse mode (for example horizontal when sliding failure of a standard retaining wall is to be checked).

An additional disturbing force in this "equilibrium direction" is then applied, which causes a ULS collapse mode to be induced. The fundamental Eurocode 7 stability equation, $E_d \leq R_d$, is then also evaluated in this "equilibrium direction".

The requirement to have a pre-specified "equilibrium direction" means that this type of calculation cannot be used to automatically identify the critical collapse mode. Instead, each mode must be examined separately (as is done in conventional practice).

4. Internal structural stability may also be assessed directly using the "action/resistance" design approach. However, for common cases, such as sheet pile bending a more convenient equivalent "inverse-factor" method, similar to a "materials factor" approach, has been proposed.

Acknowledgements

All numerical analyses described herein were undertaken using LimitState:GEO version 2.0; see: www.limitstate.com/geo

References

Bond, A. & Harris, A. (2008), *Decoding Eurocode 7*, Taylor and Francis.

BSI (2004), BS EN 1997-1:2004 *Geotechnical design. General rules*.

Driscoll, R., Powell, J. & Scott, P. (2005), *A designers simple guide to BS EN 1997*, Technical report, Department for Communities and Local Government, London, www.communities.gov.uk/documents/planningandbuilding/pdf/153986.pdf.

Driscoll, R., Scott, P. & Powell, J. (2008), *EC7 - implications for UK practices. Eurocode 7 Geotechnical design*, CIRIA.

Driscoll, R. & Simpson, B. (2001), "EN1997 Eurocode 7: Geotechnical Design", *Proceedings of ICE, Civil Engineering* 144, 49–54.

Farrell, E. & Orr, T. (1998), "Safety of retaining walls with high water loadings when designed to Eurocode 7 using partial factors", *Ground Engineering* 31(10), 36–40.

Frank, R., Baudin, C., Driscoll, R., Kavvadas, M., Krebs Ovesen, N. & Orr, T. and Schuppener,B. (2004), *Designers' guide to EN 1997: Geotechnical design - Part I: General Rules*, ASCE, Thomas Telford Publishing, London.

LimitState (2009), *LimitState:GEO Manual VERSION 2.0*, September 2009 edn, LimitState Ltd.

Orr, T. & Farrell, E. (1999), *Geotechnical Design to Eurocode 7*, Springer-Verlag, London.

Schuppener, B., Walz, B., WeiBenbach, A. & Hock-Berghaus, K. (1998), "EC7 part I: a critical review and proposal for improvement: a German perspective", *Ground Engineering* 31(10), 32–35.

Simpson, B. (1997), "Theme lecture: Some questions about limit state design in geotechnical engineering, in *Proceedings of the Fourteenth International Conference on Soil Mechanics and Foundation Engineering'*, Vol. 4.

Simpson, B. (2000), "Partial factors: where to apply them?", in *LSD2000: International Workshop on Limit State Design in Geotechnical Engineering, Melbourne, Australia*.

Simpson, B. (2007), Approaches to ULS design – The merits of Design Approach 1 in Eurocode 7, in *ISGSR2007 First International Symposium on Geotechnical Safety & Risk*, Shanghai Tongji University, China.

Simpson, B. & Driscoll, R. (1998), *Eurocode 7, a commentary*, BRE.

Smith, C. C. & Gilbert, M. (2007), "Application of discontinuity layout optimization to plane plasticity problems," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 463, 2086* pp. 2461–2484.

Smith, C. & Gilbert, M. (2010), Advances in computational limit state analysis and design, in *GeoFlorida 2010: Advances in Analysis, Modeling Design (GSP 199)*, ASCE, p. 119.

		Value	Partial factor	Product
Action	W	41.3	1.35	55.8
Action effect	A	65.5	1.35	85.7
(kN/m)	sum	104.8		141.5
Resistances	P	29.2	(1.0)	29.2
(kN/m)	S	104.3	(1.0)	104.3
	sum	133.5		133.5
Outcome	$E_k < R_k$			$E_d > R_d$
	Safe			Unsafe

Table 3: Components of "actions" (self weight, W), "action effects" (active earth pressure, A) and "resistances" (passive earth pressure, P and base shear resistance S) arising from the forced failure mechanism shown in Figure 3, together with a check using DA1/1. The components are resolved in the chosen "equilibrium direction" (15° to the horizontal) and their type (action or resistance) determined according to their effect in this direction.

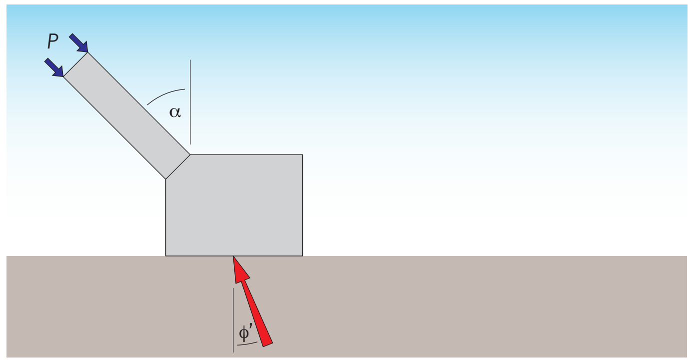

Figure 4: Design check against frictional sliding of a block subjected to an inclined load.

Appendix I: Definitions of Eurocode 7 actions, action effects and resistances

In order for partial factors to be correctly and consistently applied, it is important that what constitutes an *action*, *action effect*, and a *resistance* is clearly defined.

However Eurocode 7 itself is not fully clear on this matter, as evidenced by several different interpretations of these definitions in the literature. In this paper, the following are proposed as unambiguous definitions suitable for use in geotechnical numerical analysis:

Action *A (direct) action is a force or load whose value is independent of the collapse mechanism. Within a defined failure mechanism it may be either favourable (if it opposes collapse by consuming energy) or unfavourable (if it assists collapse by doing positive work within the mechanism).*

An example of an action is the dead weight of a foundation or an external structural load.

Variable loads almost always fall into this category. Static water pressures are also usually regarded as actions.

Action effect *The effects of actions can only be defined for a specific failure mechanism and are to be taken at a pre-specified location within a design problem. An action effect is derived from an "action effect model" which will typically involve actions and material strength and will act to drive the system to failure. The action effect should result in a net amount of positive work done in the collapse mechanism, and will always be unfavourable. Due to the involvement of material strength, an action effect is considered permanent.*

An example of an action effect is an active earth pressure. Note that there is an inconsistency here in Eurocode 7 where it is stated that earth pressures may be considered as actions (or specifically, *geotechnical actions*).

However, in a general numerical analysis earth pressures cannot be known until the critical collapse mode is identified. (For hand calculations, the use of closed form solutions such as Rankine's earth pressure equations can serve to mask this issue.)

Resistance *A resistance can only be defined for a specific failure mechanism. It must involve material strength in such a way that the mobilised strength opposes the specific failure mechanism being examined. The resistance should cause a net consumption of energy in the collapse mechanism. It should be determined using a "resistance model"and is taken at a pre-specified location within a design problem (as with action effects).*

An example of a resistance is bearing resistance for a footing or passive earth pressure for a retaining wall. Note that a resistance may also be a function of actions. These source actions may or may not oppose the failure mechanism. Driscoll et al. (2008) suggest that BS EN 1997-1 is rather ambiguous about the treatment of "favourable earth pressures", and argue that a passive pressure should be regarded as a *resistance* rather than a *fa vourable action*; the present authors agree and make the same assumption in this paper. This contrasts with some of the previous literature, for example Orr & Farrell (1999).

Note that the above definitions of *action effect* and *resistance* are given in the context of the geotechnical component of an analysis. For » internal structural design checks, it will, for example, typically be found that a resistance is simply a material property and is unaffected by the failure mechanism.

Appendix II: Assessment of internal structural stability

When checking the internal stability of a structure such as a sheet pile wall using an "action/resistance" approach, it is normally necessary to check for bending failure (and in some cases shear failure).

In this check, the *action effect/resistance* pair to be compared in equation 1 is taken, for example, as bending moment/plastic moment of resistance in a structure. To follow the proposed "action/resistance" analysis procedure correctly, it would be necessary to consider separately the stability of all cross-sections in the structure as follows. For any given cross-section:

- All quantities enter the calculation as characteristic values (with the exception of variable actions, which will be considered in Part II of the paper).
- The "equilibrium direction" is prescribed to be rotational, to initiate bending in the cross section.
- The hypothetical external action M applied is an unfavourable moment applied to the cross-section.

It is increased until the ULS is reached (assumed in this case to trigger the formation of a plastic hinge within the cross-section), i.e. the system is being perturbed by applying an additional moment to initiate collapse.

The magnitude of this moment is used to judge the stability of the structure. (A small additional moment is indicative of a low margin of safety whilst a large additional moment is indicative of a higher margin of safety).

- To undertake the design check in accordance with Eurocode 7, it is necessary to determine the actual characteristic moment (an *action effect*) E_k acting at the cross-section by considering the forces/moments other than M acting at the induced ULS.
- Equation 1 then takes the following form:

$$\gamma_G E_k \le \frac{M_{p,k}}{\gamma_M} \qquad (7)$$

where γ_M is the partial factor for the structural material (to be taken from the Structural Eurocodes).

This approach may at first sight seem cumbersome in that the check must be done separately at all cross-sections in e.g. a sheet pile wall. However, it is evident that the only quantity to be factored during the analysis is the *action effect* (ie the induced bending moment at the relevant cross section at the ULS), since the resistance is known in advance.

Thus an equivalent inverse factoring approach can be used to great advantage. This is similar to the approach recommended by Frank et al. (2004) in the context of spring models for retaining walls, and in effect turns the calculation into a "material factor" type calculation as follows:

1. In the numerical model, the plastic moment of resistance of the structural element Mp can be set to be equal to the design plastic moment of resistance divided by γ_G, ie

$$M_p = M_{p,d}/\gamma_G = M_{p,k}/(\gamma_M\gamma_G) \qquad (8)$$

2. The adequacy factor λ_A can then be applied to any unfavourable action within the problem, and providing

$$\lambda_A \ge 1.0 \qquad (9)$$

then the design is safe.

With this approach, the numerical method can automatically identify the critical structural collapse mechanism without recourse to multiple assessments.

It should be pointed out that equations 8 and 9 are generally only directly equivalent to equation 7 if the ULS involves formation of a single plastic hinge.

However, when multiple plastic hinges form the inverse factoring approach should always produce conservative results (*cf.* results obtained using the standard "action/resistance factor" approach).