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Relative Necessity Reformulated

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Abstract This paper discusses some serious difficulties for what we shall call the standard account of various kinds of relative necessity, according to which any given kind of relative necessity may be defined by a strict conditional - necessarily, if C then p - where C is a suitable constant proposition, such as a conjunction of physical laws. We argue, with the help of Humberstone (*Reports on Mathematical Logic*, *31*, 33–421, 1981), that the standard account has several unpalatable consequences. We argue that Humberstone's alternative account has certain disadvantages, and offer another - considerably simpler - solution.

Keywords Absolute · Humberstone · Logical · Necessity · Relative · Two-dimensionalism

1 Introduction

Attributions of necessity and possibility are often qualified. We may assert, not that something is necessary or possible *simpliciter*, but that it is logically necessary or possible, or physically, or mathematically so, for example. It is also natural and plausible to suppose that at least some of these kinds of necessity are not absolute, but relative, in the sense, roughly, that what is said to be necessary is not being said to be necessary outright or without qualification, but only on—or relative to—certain

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propositions taken as assumptions, or otherwise held fixed.[1] Thus on one common view, physical necessity is a matter of following from the laws of physics, and physical possibility is compatibility with them. Varying the body of laws, or other propositions, gives us other forms of relative necessity.

How, in more precise terms, should the idea that a kind of necessity (possibility) is relative be explained? In broadest terms, it seems that the most promising approach will involve taking some kind of non-relative, or absolute, modality as one's starting point, and explaining other, relative kinds as in some way relativizations of that basic kind. But how should this be done? What kind of necessity and possibility should be taken as the starting point, and what exactly is it for another kind of necessity and possibility to be a relativized form of that kind?

Interest in these questions may be prompted, and answers to them shaped, by at least two quite distinct considerations. Our goal may simply be to achieve a better understanding of the contrast between absolute and merely relative forms of necessity and possibility. This may go with a view to the effect that some kinds of necessity are absolute, and others merely relative, but it need not. One might still seek to elucidate the contrast, even if one thought it empty on one side. For one might suppose that while, if one kind of necessity is to be explained as a relativized form of another, the second kind must not itself be a relativized form of the first, it does not follow that the more basic kind must itself be absolute. To be sure, it might be thought that unless at least one kind of necessity is absolute, we shall be involved in an infinite regress. But it is at least not obvious that any such regress must be both infinite and vicious.[2]

We may be more ambitious. Our hope may be that, by showing that an ostensibly large and varied range of kinds of relative necessity can be exhibited as restrictions or relativizations of a single underlying kind of necessity—logical necessity, perhaps—we can achieve a conceptual reduction of ostensibly different kinds of modality to a single kind. Here we shall simply observe that this is a further, independent aim. As we shall see in due course, there is room for serious doubt whether it can be accomplished. But even if it cannot, that need not preclude an account of relative modalities that casts light on the contrast between absolute and merely relative necessities, and so answers to the first goal. Our aim in this paper is to cater to the first goal, without committing ourselves to the second.

We shall proceed as follows. First, we introduce the standard account of relative necessity, and discuss some putative problems for it, most notably propounded

[1]This contrast goes back a long way. For example, Aristotle distinguishes what is necessary outright from what is necessary only relative to certain assumptions (see, for example, [2], An.Pr.30b32-33, De Int.19a25-27; the distinction is implicit in De Soph.El. 166a22-30).

[2]In [4], Hartry Field writes: 'In this discussion, I have avoided taking a stand on whether even logical necessity should be viewed as 'absolute' necessity. One view, to which I am attracted, is to reject the whole notion of 'absolute' necessity as unintelligible. Another view, also with some attractions, regards the notion as intelligible but regards the only things that are absolutely necessary as logic and matters of definition'. (p.237, fn.8) Field does not here countenance the possibility of a third view, which allows that the notion is intelligible, but denies that any form of necessity is absolute. Such a position appears comparable to that to which Quine is apparently committed at the end of 'Two Dogmas'—that the notion of analytic truth is intelligible (because one can explain it as 'true and immune to revision on empirical grounds'), but empty.

by Lloyd Humberstone. Having resolved some of those problems, we move on to discuss Humberstone's diagnosis and solution to the remainder, and highlight some shortcomings with his approach. We then explore some alternative remedies, ultimately unearthing a much deeper problem with the standard account and some of its amended versions. In response, we offer our own proposed account, and discuss some of its consequences, technical and philosophical. We close with a summary of our findings.

2 Relative Necessity: The Standard Account

The standard account defines each kind of relative necessity by means of a necessitated or strict conditional, whose antecedent is a propositional constant for the body of assumptions relative to which the consequent is asserted to be necessary. Thus in a now classic treatment, Timothy Smiley wrote:

> If we define $\mathbf{O}A$ as $\mathbf{L}(T \supset A)$ then to assert $\mathbf{O}A$ is to assert that T strictly implies A or that A is necessary relative to T. Since the pattern of the definition is independent of the particular interpretation that may be put on T we can say that to the extent that the standard alethic modal systems embody the idea of absolute or logical necessity, the corresponding **O**-systems embody the idea of relative necessity—necessity relative to an arbitrary proposition or body of propositions. They should therefore be appropriate for the formalisation of any modal notion that can be analysed in terms of relative necessity. [21] p.113.[3]

This kind of formulation of relative necessity has been quite widely endorsed in subsequent work.[4] The standard account takes the kind of absolute necessity in terms of which different forms of relative necessity are to be explained to be logical necessity, so that it is relatively necessary that p just when, as a matter of logical necessity, C materially implies p, where C is a proposition of a certain kind—briefly, $\Box(C \rightarrow p)$); and it is relatively possible that p just when p's truth is not ruled out by C, i.e. when it is not the case that, as a matter of logical necessity, C implies not-p—briefly, $\neg\Box(C \rightarrow \neg p)$), or equivalently $\Diamond(C \wedge p)$.

Before considering those features which are genuinely problematic, it is worth briefly surveying some more or less obvious peculiarities of the notion of relative necessity, as formulated on the standard account. First, in the absence of any restriction upon the choice of C, each proposition p will be relatively necessary in an indefinite number of ways or senses. Any conjunction of propositions which numbers p as one of its conjuncts strictly implies p, as does p itself. Forms of relative necessity come cheap, and most of them are entirely without interest. This is not—or not obviously—a crippling drawback. A defender of the standard account can reply

[3] Smiley uses '**L**' as a necessity operator, more usually written '$\Box$'. See also [1] and [12].

[4] Gideon Rosen, for example, writes, 'given any proposition, ϕ, we can always introduce a 'restricted necessity operator' by means of a formula of the form $\Box_\phi P =_{def} \Box(\phi \rightarrow P)$.' [20], p.33. See also [8], p.93.

that it simply underlines the point that we should not expect interesting, non-trivial kinds of relative necessity to result unless the choice of C is restricted in interesting ways—to the laws of physics, say, or those of mathematics.

Second, since every logical necessity is strictly implied by any proposition whatever, every logical truth is C-necessary—necessary relative to C—no matter how C is chosen. But this, again, is not problematic. In particular, the fact that what is logically necessary is automatically relatively necessary in any sense you care to specify is in no tension with the plausible claim that logical necessity is absolute. The important contrast is not with relative necessity, but with merely relative necessity, where it is merely relatively necessary that p if it is, say, C-necessary that p but absolutely possible that $\neg p$.

Thirdly, since C always strictly implies itself, it automatically counts as C-necessary. So on the standard account, assuming that physical necessity, say, is to be analysed as a form of relative necessity, the laws of physics themselves automatically—and so, it would seem, trivially—qualify as physically necessary. But while physical necessity may not be absolute, it may be felt that there is more to the kind of necessity attaching to the laws of physics than their mere self-strict-implication.[5] This may be felt to be a more serious objection and we shall have a little more to say about it below. First, we should see why the standard account appears to be in far deeper trouble, for reasons to which Lloyd Humberstone drew attention over three decades ago.[6]

3 Humberstone's Problems

Humberstone raises the following problems for the standard account.[7]

3.1 Modal Collapse

One might suppose that there are several distinct kinds of relative necessity, and that many of them are factive, in the sense that, where $\Box_C$ is our relative necessity operator, $\Box_C p \rightarrow p$, for every p. In other words, the characteristic axiom of the quite weak modal logic T holds for $\Box_C$. In particular, one would expect any kind of alethic necessity operator to be factive.[8] For example, it might be held that both biological and physical necessity are two distinct kinds of necessity which are both relative and factive. However, Humberstone argues that, on very modest assumptions about the logic of the absolute modality operator in terms of which, on the standard account, they

[5] See, for example, [6], p.266

[6] See [10], also [11]

[7] In a footnote of [10, p.34], Humberstone acknowledges Kit Fine as first noticing these problems.

[8] Alethic modalities are standardly taken to be those which concern 'modes' of truth, in contrast with epistemic and doxastic modalities, concerned with knowledge and belief, and deontic modalities, concerned with obligatoriness and permissibility. Factivity does not define alethic modalities, since epistemic necessity is also factive. The collapsing argument is intended to apply to all factive modalities—so that it is epistemically necessary that p iff it is physically necessary that p, for example.

are to be defined, there is at most one factive kind of relative necessity. The modest assumption is that the logic of $\Box$ is at least as strong as K—the weakest normal modal logic. Let $\Box_{C_1} A$ and $\Box_{C_2} A$ be defined by $\Box(C_1 \rightarrow A)$ and $\Box(C_2 \rightarrow A)$ respectively. Then, Humberstone claims, it is readily proved that $\Box_{C_1} A$ and $\Box_{C_2} A$ are equivalent.

(1)	$\Box(C_1 \rightarrow p) \rightarrow p$	T-axiom for $\Box_{C_1}$
(2)	$\Box(C_2 \rightarrow p) \rightarrow p$	T-axiom for $\Box_{C_2}$
(3)	$\Box(C_1 \rightarrow C_1) \rightarrow C_1$	(1)$\times C_1/p$
(4)	$\Box(C_2 \rightarrow C_2) \rightarrow C_2$	(2)$\times\ C_2/p$
(5)	$\Box(C_1 \rightarrow C_1)$	propositional logic $\times$ Necessitation Rule
(6)	$\Box(C_2 \rightarrow C_2)$	propositional logic $\times$ Necessitation Rule
(7)	C_1	(3), (5) $\times$ modus ponens
(8)	C_2	(4), (6) $\times$ modus ponens
(9)	$\Box(C_1 \leftrightarrow C_2)$	(7), (8) $\times$ propositional logic, Necessitation Rule
(10)	$\Box(C_1 \rightarrow q) \leftrightarrow \Box(C_2 \rightarrow q)$	(9) $\times$ obvious reasoning in K
(11)	$\Box_{C_1} q \leftrightarrow \Box_{C_2} q$	(10) $\times$ Def.$\Box C_1$, Def.$\Box C_2$

The key steps in this proof purport to show that when any relative modality $\Box_C$ defined in the standard way satisfies the T axioms, one can prove C. By Necessitation, $\Box C$, so that for any p, $\Box_C p$ entails $\Box p$. But $\Box p$ obviously entails $\Box_C p$.[9] So relative modalities satisfying the T axioms collapse, not only into each other, but into logical modalities—relativization is a waste of time.

3.2 Unwanted Interactions: Imposing S4

It seems reasonable to suppose that our absolute necessity operator, $\Box$, satisfies the S4 axiom. If $\Box$ does obey the S4 axiom, every relative necessity operator $\Box_C$ must do so as well. The proof is simple. It is a theorem of S4 that $\Box A \rightarrow \Box(B \rightarrow \Box A)$. An instance of this theorem, substituting $C \rightarrow p$ for A and C for B is $\Box(C \rightarrow p) \rightarrow \Box(C \rightarrow \Box(C \rightarrow p))$. But this is simply the S4 axiom for a relative necessity operator $\Box_C$ defined by $\Box_C p =_{def} \Box(C \rightarrow p)$. So, if we assume that our absolute necessity operator, $\Box$, satisfies the S4 axiom, it immediately follows that our relative necessity operator, $\Box_C$, does as well, i.e. $\Box_C p \rightarrow \Box_C \Box_C p$. Yet it may seem both odd and implausible to suggest that if it is, say, physically necessary that p, then it is physically necessary that it is physically necessary that p. More generally, we should expect to be able to define kinds of relative necessity in terms of $\Box$ which do not satisfy the characteristic S4 principles. The standard account is either overly restrictive, or defective, in only allowing one to define kinds of relative necessity that satisfy the S4 axiom.

Second, assuming $\Box$ is S4-like, one can show in a similar way that, where $\Box_{C_1}$ and $\Box_{C_2}$ are any two different forms of relative necessity defined in the standard way, the bizarre 'mixed' S4 principle $\Box_{C_1} p \rightarrow \Box_{C_2} \Box_{C_1} p$ holds. Again, it is a theorem of

[9] Cf [11], p.50

S4 that $\Box A \rightarrow \Box(B \rightarrow \Box A)$, so in particular: $\Box(q \rightarrow p) \rightarrow \Box(r \rightarrow \Box(q \rightarrow p))$. Hence for any C_i: $\Box(C_1 \rightarrow p) \rightarrow \Box(C_i \rightarrow \Box(C_1 \rightarrow p))$. Yet it appears quite implausible that if something is, say, physically necessary, then it is biologically (or morally, and so on) necessary that it is physically necessary. To give another example, one might define drawer-necessity relative to truths about items in NN's top desk drawer. For example, it is drawer-necessary (but not, say, physically necessary) that all the pencils in NN's drawer are blunt. However, the consequences of the standard account include that if it is drawer-necessary that p, then it is physically necessary that it is drawer-necessary that p (and also that if it is physically necessary that p, then it is drawer-necessary that it is physically necessary that p). But surely both results are absurd. It is not a matter of the laws of physics that NN keeps only blunt pencils in his top desk drawer. Nor should it be a matter of drawer-necessity what does or doesn't follow from the laws of physics.

3.3 Out of the Frying Pan?

To the modal collapse proof as it stands, there is a fairly obvious objection. The problem concerns the application of Necessitation at step 9. This rule allows us to to necessitate a proposition only if it has been established as a theorem, and if $\Box$ expresses logical necessity, the proposition to be necessitated must be a theorem of *logic*. But in this case, the proposition to be necessitated, viz. $C_1 \leftrightarrow C_2$, is no theorem of logic, since it depends upon 1) and 2), which are non-logical axioms of a system for relative necessity. Consider 1): its inner antecedent, C_1, will be some non-logical proposition. For example, if we are considering an analysis of physical necessity, it will be a proposition we take to be a physical law, or perhaps a conjunction of such laws. Thus it might be, say, the proposition that force = mass $\times$ acceleration, so that 1) will assert that if the proposition that p is strictly implied by the proposition that $f = m \times a$, then (it is true that) p. True or not, this is evidently no theorem of logic. It would seem, then, that the crucial step of Necessitation is illicit, so that the proof does not, after all, lead to modal collapse.

A similar flaw occurs in other relevant proofs. In his original paper, Humberstone argues that the standard account is afflicted by two other problems. First, on the assumption of a relative necessity operator $\Box_C$ satisfying the K, T and S4 axioms, one can show that the S4 axiom must also be satisfied by $\Box$. However, this relies on the modal collapse argument to establish the crucial result, $\Box C$, and so is also invalid.[10] Second, he claims that, on the assumptions that one relative necessity operator $\Box_{C_1}$ satisfies the K axioms, the axioms B and D, but not T, while another $\Box_{C_2}$ satisfies the K axioms and also the T axiom, we can prove that $\Box_{C_1}$ must satisfy the T axiom

[10]The purported proof was supposed to go as follows. By the argument above, $\Box C$. But for any p, we have $\Box p \rightarrow \Box(C \rightarrow p)$, and by hypothesis, $\Box(C \rightarrow p) \rightarrow \Box(C \rightarrow \Box(C \rightarrow p))$. But from these it follows that $\Box p \rightarrow \Box(C \rightarrow \Box(C \rightarrow p))$. But since, by the K axiom for $\Box$, $\Box(C \rightarrow \Box(C \rightarrow p)) \rightarrow (\Box C \rightarrow \Box\Box(C \rightarrow p))$, it follows that $\Box p \rightarrow (\Box C \rightarrow \Box\Box(C \rightarrow p))$. And since $\Box\Box(C \rightarrow p) \rightarrow \Box(\Box C \rightarrow \Box p)$ and $\Box(\Box C \rightarrow \Box p) \rightarrow (\Box\Box C \rightarrow \Box\Box p)$,it follows that $\Box p \rightarrow (\Box C \rightarrow (\Box\Box C \rightarrow \Box\Box p))$. But we can permute and detach the second and third antecedents, so that $\Box p \rightarrow \Box\Box p$. Thus, the assumption of a relative necessity operator $\Box_C$ satisfying the K, T and S4 axioms forces the S4 axiom for $\Box$.

after all. We have chosen not to discuss this problem in detail not only because it is a rather less intuitive combination of modal properties, but also because the proof fails for the same reason as that of problem 1: illicit use of Necessitation.

Is the standard account thus vindicated? No. Whilst some of Humberstone's problems, in the form presented, can be dissolved, those of the unwanted imposition of S4, and 'mixed' S4, remain. Hence, there is still good reason to explore alternatives to the standard account. Moreover, in so doing we must take care to ensure that any alternative does not accidentally revive those problems now deemed solved. More importantly, in developing an alternative account, we will discover a far deeper problem for the standard account.

4 Humberstone's Solution

Although, as we have observed, some of Humberstone's problems may be set aside as relying on illicit steps of Necessitation, others remain. We wish now to consider his proposed solution to them. To understand it, it is useful to review his diagnosis of the source of the problems he takes to be fatal to the standard account.

He writes:

> [T]he difficulties we have become entangled with result from making substitutions too generally. To see this, let us consider what happens semantically with the idea of relative necessity. We are given an arbitrary modal operator and asked to code it up as some combination of a fixed operator '$\Box$' and a propositional constant. But a propositional constant, if it is to be of the same category as the usual propositional variables, is (in effect) assigned a set of worlds in the Kripke semantics whereas the operators we are trying to use these constants to express correspond instead to binary (accessibility) relations between worlds: thus there is bound to be a loss of information in the translation. ([10], p.36. See also [11], p.51)

His thought, in other words, seems to be as follows: modal operators are quantifiers over worlds, and in particular, relative modal operators are restricted world quantifiers. Thus whereas an absolute operator, $\Box$, quantifies unrestrictedly, so that $\Box p$ says that p is true at every world, a relative necessity operator, $\Box_C$, quantifies only restrictedly, so that $\Box_C p$ says that p is true at each of a restricted range of worlds. The question is: how is this restriction to be captured? The standard account seeks to capture it by relativizing to a propositional constant—a constant which expresses a proposition true at just those worlds in the restricted range intended. Instead of interpreting the relative necessity claim as that p is true at each of a restricted range of worlds, it interprets it as the claim that the conditional $C \rightarrow p$, where C is the propositional constant, is true at all worlds. The trouble with this, Humberstone thinks, is that it loses vital information about the distinctive accessibility relation in terms of which a relative modal operator needs to be understood. What distinguishes one kind of relative necessity from another, he thinks, is that each corresponds to a different accessibility relation. Thus what is physically necessary at a given world is what holds true at every world that is physically possible relative to that world, whereas

what is epistemically necessary is what holds true, rather, at every world that is epistemically possible with respect to that world. These are quite different relations, but the standard account does not do justice to their difference.

In more detail,

> if R is a binary relation on some set W, and S is a subset of W, we define the range-restriction of R to S symbolized R_S thus: xR_Sy iff xRy and $y \in S$. ... translating (or defining) $\mathbf{O}A$ as $\Box(C \rightarrow A)$ amounts to taking the accessibility relation for "**O**" to be the range-restriction of that for "$\Box$" to the set of worlds at which C holds. Thus instead of being able to cope with an arbitrary collection of modal operators, we are forced to deal only with collections whose accessibility relations are range-restrictions of a single relation, and it is this circumstance which underlies the difficulties ... ([10], p.36)

In terms of this diagnosis, one can, he points out, give a straightforward explanation of the difficulties. Thus defining the relative necessity operators $\Box_{C_1}$ and $\Box_{C_2}$ by: $\Box_{C_1}A$ iff $\Box(C_1 \rightarrow A)$ and $\Box_{C_2}A$ iff $\Box(C_2 \rightarrow A)$ amounts to taking their associated accessibility relations R_1 and R_2 to be range-restrictions of the underlying accessibility relation R associated with $\Box$, so that they are just sub-relations of R with the same domain W but (proper) subsets S_1 and S_2 of R's range W as their ranges. But then the supposition that $\Box_{C_1}$ and $\Box_{C_2}$ both obey the T principle is just the supposition that R_1 and R_2 are both reflexive—whence, since each has domain W, it must have the *whole* of W as its range after all. So the supposition of reflexivity *undoes* the range-restriction—R_1 and R_2 both end up with the *same* range as R, and hence as each other, and since they share the same domain, they are just the same relation by different names. Hence collapse. Similar explanations, he suggests, may be given for the other difficulties. See [10], pp.36–7.

It does not, of course, follow from the fact that some of Humberstone's arguments rely on illicit Necessitation steps that there are no formally correct arguments to the same conclusions; so it does not straightforwardly follow that there must be something amiss with his purported diagnosis. But in fact there is a questionable assumption on which it rests. For if the addition of the T-schemes for $\Box_{C_1}$ and $\Box_{C_2}$ is to enforce reflexivity on the their corresponding accessibility relations $R_1$and R_2,Humberstone must assume that those schemes are *logically valid*. The assumption of validity, as opposed to mere truth at a world, is crucial. $\Box p \rightarrow p$ could be *true at* a world w *without* the accessibility relation being reflexive—for we might have wRw but $\neg w'Rw'$. $v(\Box p \rightarrow p, w) = 1$ requires that if $v(\Box p, w) = 1$ then $v(p, w) = 1$ also, and since wRw, the truth of this antecedent requires the truth of this consequent. Further, we can suppose that $v(\Box p, w') = 1$ while $v(p, w') = 0$—there is no incompatibility here, since the truth of this antecedent does *not* require $v(p, w') = 1$, since $\neg w'Rw'$. What cannot be the case is that $v(\Box p \rightarrow p, w) = 1$ is true for *all* w, but R is not reflexive. But the assumption that the T-schemes are logically valid is simply the model-theoretic counterpart of the equally problematic proof-theoretic assumption—that $\Box_{C_1}p \rightarrow p$ and $\Box_{C_2}p \rightarrow p$ are logical axioms—which, as we have seen, vitiates the formal proofs for problems 1 and 3.

Prescinding from our reservations, Humberstone's proposed remedy should come as no surprise given his diagnosis—since the propositional constants central to the

standard treatment lose essential information about the distinctive accessibility relations which characterize different kinds of relative necessity, they must be replaced by a new type of constant which encodes the lost information. As he puts it, 'the constants must do some of the "relational" work themselves' (p.37). To this end, he assumes a two-dimensional framework, in which formulae, including the new type of 'relational' constants, are evaluated with respect not to single worlds, but pairs of worlds:

> [W]e want, for A a formula and "$\models$" a truth-relation determined by a model $\langle W, R, V\rangle$, to make sense of not the usual $\models_x A$ but rather $\models^x_y A$ (where $x, y \in W$), so that when it comes to evaluating one of our special constants, which I shall now write as $\boldsymbol{R}$ instead of C, to emphasize the relationality, we can say: $\models^x_y \boldsymbol{R}$ iff xRy. ([10], p.37)

The idea is that a given such $\boldsymbol{R}$ is to be true with respect to the pair of worlds x and y just when x bears the relevant accessibility relation, R, to y. Thus

> If we have in mind a formalization of physical necessity, we might read "$\models^x_y \boldsymbol{R}$" as "the laws of x are true in y" ([10], p. 38)

Using constants of this new type—semantically interpreted as 'dipropositions'—Humberstone's revised definitions of relative necessity operators have the same surface form as in the standard account. That is, where **O** is a relative necessity operator, we have

$$\mathbf{O}A =_{def} \Box(\boldsymbol{R} \rightarrow A)$$

But, crucially, dipropositional constants are not substitutable for normal propositional variables:

> The propositional variables really do range over propositions, but the sentential constants $\boldsymbol{R_i}$ cannot be substituted for them because the latter are not propositional constants. In the terminology of [Humberstone (1981)], they are semantically interpreted not as propositions but as dipropositions—sets of (ordered) pairs of worlds. ([11], p. 53)

Clearly, since each of problems depends upon substitution of the relevant propositional constant C_i for a propositional variable, this restriction effectively blocks all of them. Crucially, this restriction blocks the problems that, on closer examination, are still valid and hence still pressing. In particular, the schema for the S4 axiom for a relative necessity operator is:

(S4)$\Box(\boldsymbol{R} \rightarrow A) \rightarrow \Box(\boldsymbol{R} \rightarrow \Box(\boldsymbol{R} \rightarrow A))$, provided $\boldsymbol{R}$ does not occur in A.

5 Shortcomings

Humberstone's solution comes at a price. In this section we highlight two disadvantages, which seem to us sufficiently serious to warrant looking for an alternative solution. The first disadvantage concerns the modal logic of the underlying absolute necessity operator, $\Box$. It is usual, and it seems to us overwhelmingly plausible, to

take $\Box$ to express *logical* necessity. And it is, further, commonly supposed, and again very plausible, that the modal logic of $\Box$, interpreted as expressing logical necessity, should be S5.[11] But Humberstone cannot both take his absolute necessity operator $\Box$ to express logical necessity and agree that logical necessity satisfies the S5 principles. It is less than totally clear which claim he means to deny. Noting Smiley's suggestion that we should take absolute necessity to be logical necessity, he comments that this is

> a suggestion which is not entirely easy to evaluate. Most people believe that logical necessity satisfies at least the T axiom and the S4 axiom; however, neither $\Box A \rightarrow A$ nor $\Box A \rightarrow \Box\Box A$ is valid for arbitrary A, though both are valid for $\boldsymbol{R}$-free A. ... it may be held that since "$\Box$" does not satisfy all instances of the familiar schemata, it cannot be regarded as expressing logical necessity. This matter cannot be settled here. ([10], p.40)

Since denying that the T and S4 axioms hold unrestrictedly for logical necessity is hardly an attractive option, it may seem that Humberstone's best course is to accept that his absolute necessity operator does not express logical necessity. But this, too, has its disadvantages. Setting aside the absence of any plausible alternative candidate, it is independently plausible that the kind of necessity in terms of which various forms of relative necessary are to be explained *should* be logical necessity. Further, if that rôle is assigned to some other kind of (absolute) necessity, logical necessity would, if not simply a restriction of that kind of necessity, have to be treated as a form of relative necessity.[12] But it is unclear how logical necessity could be merely relative, if logical necessity is stronger than the kind of necessity of which it is supposed to be a relativization (as it would be, if it obeys unrestricted T and S4, and plausibly S5, axioms). And in any case, even if the absolute necessity is not identified with logical necessity, it may be argued that its modal logic should be S5.[13] All told, if there is an alternative formulation of relative necessity that can avoid making difficult claims about logical necessity—either that it does not satisfy the S5 principles, or that it is not absolute necessity—so much the better. Our aim below is to offer such a formulation.[14]

A second, and in our view more fundamental, disadvantage of Humberstone's solution is that it is inextricably reliant on the assumption that modal thought and talk is to be understood as, and analysed in terms of, thought and talk about possible

[11] An argument for the claim that logical necessity is the strongest kind of necessity is given in [19]. For further defence, see [8] and [9], ch.2.

[12] One kind of necessity is a (proper) restriction of another if necessities of the first kind are a (proper) subclass of necessities of the second kind, whereas when one kind is a relativization of another, the relation is reversed, i.e. necessities of the second kind are a (proper) subclass of necessities of the first. For example, one might hold that logical necessity is a restriction of metaphysical necessity. But given its peculiarities, it would not be plausible to take Humberstone's absolute necessity to be metaphysical necessity.

[13] For argument in support of the claim that the logic of absolute necessity is S5, and that logical necessity is a species of absolute necessity, see [9], chs.4, 5.4. Timothy Williamson argues, along quite different lines, that the logic of metaphysical necessity is S5 in [22], ch.3.3.

[14] See [14] for an axiomatization of Humberstone's proposed two-dimensionalist logic of absolute necessity and a detailed discussion of its logical properties.

worlds. Our concern is not simply that the proposed solution presupposes the two-dimensional framework, which in turn appears to require acceptance of a plurality of worlds. Nor is the complaint that the solution requires acceptance of some form of extreme realism about worlds, such as Lewis's—for there is no reason to suppose that it does so. The point is rather that, while there is no denying the enormous utility of possible world semantics in model-theoretic treatments of modal systems, it is one thing to hold that the truth-conditions of modal propositions can be usefully *modelled* in terms of systems of worlds, and quite another to claim that such propositions are fundamentally propositions *about* worlds and the relations between them—that understanding such propositions requires understanding them as making claims about worlds, and relations between them. The standard account, for all its faults, made no controversial demands on the metaphysics of modality, and took no stand on disputed questions about the nature and basis of necessity. This, it seems to us, is a virtue which a better account should preserve. Humberstone's proposal, by contrast, requires us to accept that modal propositions—or at least propositions asserting relative necessity—are really propositions about relations between worlds. This comes out most forcefully in the new dipropositional constants. Humberstone does not spell out just what, under his proposed analysis, we are saying when we assert, for example, that such-and-such is physically possible, or physically necessary—how, exactly, $\boldsymbol{R}$ is to be understood, when it is the constant for physical necessity. The closest he comes to doing so is his suggestion that $\models^{x}_{y} \boldsymbol{R}$ might be read as "the laws of x are true in y". This does not tell us how to interpret $\boldsymbol{R}$ in the context $\Box(\boldsymbol{R} \rightarrow A)$, but it seems that there will be no alternative but to construe it as asserting something about worlds which are physically accessible from, or physically possible relative to, a given world. What is lacking is a way to account for our understanding of claims about relative necessity which does not require us to understand them as claims about relations between worlds. There may be some world-free way to construe dipropositions, but we take the onus to be with our opponent to offer such a construal.

Humberstone's is not the only possible two-dimensional treatment.[15] Alternative two-dimensionalist solutions may be able to avoid these disadvantages. But it is, we think, of interest to see whether a satisfactory solution can be developed which does *not* draw on a two-dimensional framework, and this will be our course in what follows.

6 Remedies

6.1 Adding a Conjunct

Before we present our preferred alternative treatment of relative modalities, it will be useful briefly to discuss another remedy which has been proposed. The added conjunct stategy, in its simplest form, consists in expanding the definiens for $\Box_C p$

[15] A rather different two-dimensionalist treatment of relative necessity is proposed in [7], a paper to which Humberstone does refer [10, p. 40] For a brief discussion, see Appendix.

by adding the propositional constant C as a conjunct.[16,17] That is, for any form of relative modality for which the T axiom is to hold, we define:

$$\Box_C p =_{def} C \wedge \Box(C \rightarrow p)$$

Whilst this strategy blocks Humberstone's modal collapse argument, that argument itself collapses anyway, so that there is no need for any further measure to block it. More importantly, this added-conjunct strategy does nothing to solve Humberstone's S4 problem, as the reader may easily verify. If $\Box$ satisfies the S4 axiom, so must any form of relative necessity—adding a conjunct does not block this result, because the S4 axiom for $\Box_{C_i}$ under the revised definition, i.e.

$$(C_1 \wedge \Box(C_1 \rightarrow p)) \rightarrow (C_1 \wedge (\Box(C_1 \rightarrow (C_1 \wedge \Box(C_1 \rightarrow p)))))$$

is fairly obviously still a theorem of S4 for $\Box$.[18] The 'mixed' S4 problem, however, is now solved.[19]

6.2 A Better Diagnosis

We are now in a position to draw two crucial lessons from the failings of the standard account, and attempts to remedy those failings. In brief, (1) crucial information has been lost, but (2) that information must be reintroduced in a suitably general form, if the account of relative necessity is to have any plausible application.

First, then, independently of these logical shortcomings, the additional conjunct strategy suffers from another defect. Recall Humberstone's complaint that the standard account founders because it loses vital information. We agree with Humberstone on this point—but not on what vital information is lost. Here is our alternative diagnosis. When we adopt the standard account of a relative modality—physical necessity, say—we proceed as follows: 'Let C be a conjunction of the laws of physics. Then

[16] A version of this strategy is mentioned by Steven Kuhn, who writes: 'Other remedies [i.e. other than Humberstone's own proposal] may also be possible. Wlodzimierz Rabinowicz, in correspondence, attributes to Lars Bergstrom the idea that, when $\mathbf{O}A \supset A$ obtains, the reduction of relative necessity should be given by $\boldsymbol{O}A = \Box(L \supset A) \,\&\, L$ rather than $\boldsymbol{O}A = \Box(L \supset A)$.' ([14], note 2)

[17] Clearly this proposal is appropriate only for factive kinds of relative necessity. The effect of adding C as a separate conjunct is that $\Box_C p$ always implies p. However, as we discuss briefly in Section 8, there are other problems to be considered for treating non-factive necessities.

[18] If the main antecedent is true at w but the main consequent false there, then since C_1 is a conjunct in the main antecedent (and so likewise true at w), it must be the right conjunct that is false at w. This requires w' with C_1 true and $\Box(C_1 \rightarrow p)$ false, and the latter requires w'' with $C_1 \rightarrow p$ false. But this is impossible—for since we are in S4 for $\Box$, w'' must be accessible from w, so that $C_1 \rightarrow p$ must be true at w''. So the whole conditional is valid, and thus a theorem by completeness.

[19] The foregoing model-theoretic argument depends upon the identity of the left conjuncts in the main antecedent and consequent. Thus:

$$(C_1 \wedge \Box(C_1 \rightarrow p)) \rightarrow (C_2 \wedge (\Box(C_2 \rightarrow (C_1 \wedge \Box(C_1 \rightarrow p)))))$$

is not a theorem of S4. There is a simple one world counter-model with C_1 and p both true, but C_2 false. Thus the additional conjunct strategy does block the last part of problem 2.

to say that it is physically necessary that p is just to say that p is a logical consequence of C—in symbols: $\Box(C \rightarrow p)$.' The key point here is that, in adopting this formalization, we simply *leave it to be understood* that C is a conjunction of physical laws—nothing in our *definiens actually records* that that is so. So that information *is* lost. And it is, surely, vital information. Nothing explicit in the *definiens* distinguishes between a strict conditional purporting to express the *physical* necessity of its consequent, and one which purports to express some *other* kind of relative necessity.

One moral we might draw from this is that a better definition needs explicitly to record the relevant information about the status of the antecedent C. The simple added-conjunct strategy does no better in this respect than the standard account. An obvious way to remedy this particular shortcoming would be to employ an additional operator, π say, which might, in case we are seeking to define physical necessity, be read as 'it is a law of physics that ...'.[20] Amending the simple added-conjunct analysis to:

$$\text{It is physically necessary that } p \qquad \text{iff} \qquad \pi(C) \wedge \Box(C \rightarrow p)$$

not only restores the lost information, but actually provides a solution to the S4 problem. This analysis preserves what is good about the added-conjunct strategy—blocking the second 'mixed' version of the S4 problem—and it deals with the simpler S4 problem, which eluded the simpler strategy. For now, the S4 principle for $\Box_{C1}$ is:

$$(\pi(C_1) \wedge \Box(C_1 \rightarrow p)) \rightarrow (\pi(C_1) \wedge (\Box(C_1 \rightarrow (\pi(C_1) \wedge \Box(C_1 \rightarrow p)))))$$

and a simple calculation reveals that this is no theorem of S4 for $\Box$. Crucially, while antecedent and consequent here share the same left conjunct, $\pi(C_1)$, this conjunct is not necessitated—it is allowed that the laws of physics might, logically, have been otherwise. Consequently, there is a simple two world counter-model in which $\pi(C_1)$ is true along with C_1 and p at w, but $\pi(C_1)$ is false at w', even though C_1 is true there. The counter-model exploits the fact that being true is necessary, but not sufficient, for being a law of physics—so that there are possible circumstances in which the propositions which are the actual laws of physics would still be true, but only accidentally, and not as a matter of physical law.

This brings us to our second crucial point. The additional conjunct strategy, whether in its simple or revised form, remains afflicted with a defect *which was all along sufficient to justify rejecting the standard account*, quite independently of the problems with which we have been occupied, but which appears to us, remarkably, to have escaped notice in previous discussion. As the use of C and $\pi(C)$ makes plain, each of these accounts simply *assumes* that we are able actually to state, say, the laws of physics; neither provides an analysis we could advance, unless were we actually able to do so. At least, this is so, on the charitable assumption that the intention

[20] Typically what is physically necessary is a consequence of several laws of physics, not just one. In order to postpone discussion of some complications best left aside *pro tem*, we shall assume—as seems not unreasonable—that any conjunction of laws of physics counts as a law of physics. For discussion of this and other complications, see below, Section 7.

behind these accounts is that what is physically necessary should be what is required by the *true* laws of physics, and not just by what we *take* to be the laws. We do not, of course, mean to deny that it may on occasion reasonably be claimed that something is physically necessary, or physically impossible. Someone who makes such a claim may believe, on good grounds, that such-and-such propositions are laws of physics, and that they require, or rule out, the truth of this or that further proposition. Or, without being confident about any specific candidates to be laws of physics, she may believe that *whatever precisely* the relevant laws are, they will be such as to require, or rule out, the truth of certain propositions—such as that a human being should move by its own unaided effort at 200mph. Our point, at bottom, is that someone may *understand* the claim that, or the question whether, such-and-such is a *physical necessity* or (im)possibility *without knowing*, or claiming to know, *what the laws of physics are*, even approximately. Whatever the question is, that such a person is considering, it cannot be the question whether certain *specified* propositions are laws of physics which strictly imply that such-and-such. Of course, she may put her question this way: 'Do the laws of physics require that such-and-such?'. But her question need not concern certain specific (candidate) laws—it can still be a perfectly general question: 'Are there physical laws which require that such-and-such?'

6.3 Existential Generalization

There is an obvious way to do justice to this point—replace our propositional constant by a variable and existentially generalize through the position it occupies. That is, we should define:

$$\text{It is physically necessary that } p \qquad \text{iff} \qquad \exists q(\pi(q) \wedge \Box(q \rightarrow p))$$

where, as suggested above, $\pi(q)$ may abbreviate something like 'It is a law of physics that q'. It is not assumed that the laws are known; hence $\pi(q)$ could be read as saying that q is one of the actual physical laws, *whatever they are*, or *may be*—but *not* 'whatever they could have been.'[21]

For any kind of relative necessity, then, our proposal is to define it in line with the following schema:

$$\text{It is } \Phi\text{-necessary that } p \qquad \text{iff} \qquad \exists q(\Phi(q) \wedge \Box(q \rightarrow p))$$

where, $\Phi(q)$ abbreviates 'It is a Φ-proposition that q'. The condition captured by "Φ" may be more or less interesting, more or less restrictive. We will continue to use the example of the laws of physics, i.e. $\pi(q)$, as a representative, and philosophically interesting, example.

Such a proposal captures quite naturally, or so it seems to us, the main idea of relative necessity. It introduces explicitly the information lost by the standard account. Not only does it specify the kind of proposition to which the necessity is relative, e.g.

[21] We are understanding the claim that it is physically necessary that p so that to make this claim is not to deny that the laws of physics could have been otherwise (in such a way as not to require that p). In other words, 'whatever they are, or may be' is to be given an epistemic reading, not a metaphysical one.

laws of physics, but also does so in a plausibly general way. If one claims, for example, that it is necessary relative to the laws of physics that p, one need not be claiming that p is necessary relative to some particular laws that one is able state, but only that there are some such laws that necessitate p. So what might at first glance appear to be an unnecessarily complex formulation, as contrasted with the standard account, is in fact simply recording explicitly the information that was all along assumed by the standard account, and in a suitably general way.

Is this proposal subject to any of the problems Humberstone took to afflict the standard analysis? Setting aside the arguments which we saw to be flawed by faulty Necessitation steps, it is easy to see that the arguments central to the remaining problems—those involving unwanted imposition of S4-like properties—no longer go through, since they both depend upon substituting a special propositional constant or constants for propositional variables in the S4 theorems $\Box(q \rightarrow p) \rightarrow \Box(q \rightarrow \Box(q \rightarrow p))$ and $\Box(q \rightarrow p) \rightarrow \Box(r \rightarrow \Box(q \rightarrow p))$, both of which are derivable in S4 from the so-called paradox of strict implication $\Box A \rightarrow \Box(B \rightarrow A)$, and we no longer have any such propositional constant(s) to substitute.

Might there be some other route by which some or all of these problems could be re-instated? In the case of Humberstone's first problem, the question is whether, when a T-axiom governing $\Box_\pi$, defined as now proposed, is added to an underlying modal system, say K or T, to obtain a system S, we can derive $\Box p$ from $\Box_\pi p$. The distinctive axiom is $\exists q(\pi(q) \wedge \Box(q \rightarrow p)) \rightarrow p$. With this available, we could argue:

1	(1)	$\exists q(\pi(q) \wedge \Box(q \rightarrow p))$	
1	(2)	p	by T-axiom for $\Box_\pi$

But to get $\Box p$, we would need to apply the Rule of Necessitation or a rule of $\Box$-introduction. We cannot use the Rule of Necessitation, as this only permits us to necessitate theorems, but p is no theorem. And we cannot use the $\Box$-introduction rule, as this allows us to necessitate a formula only if it depends only on suitably modal assumptions, and (1) is not suitably modal. There is no other obvious strategy for a derivation, and we are confident there isn't one to be found. Proving this—assuming we are right—is a more substantial task than we can undertake here. But we can at least sketch how we think things would go.

Giving a model to show that $\exists q(\pi(q) \wedge \Box(q \rightarrow p)) \nvdash_S \Box p$ would require giving a semantics for the system S, and that will require, inter alia, a semantics for propositional quantification. Perhaps the most obvious method is (following [5]), to interpret propositions as sets of worlds, and take propositional variables to range over subsets of the set of worlds of the model. We would need to show that S is sound with respect to the semantics, i.e. that $\Gamma \vdash_S A$ only if $\Gamma \vDash_S A$. As well as dealing with propositional quantifiers, the semantics will include a clause governing the π-operator (and more generally, for any relative necessity system, a clause governing the Φ-operator). We may presume that this will ensure that $\pi(p)$ is true at a world w only if p is so: the laws of physics are, apart from anything else, true.

Since $\exists q(\pi(q) \wedge \Box(q \rightarrow p)) \rightarrow p$ is an axiom of S, we require this formula to come out true in every S-model. To see, informally, why it will do so, let $\mathcal{M}$ be any S-model. $\mathcal{M}$ will have a set of worlds W as its principal domain, so that propositional variables range over subsets of W. If $\exists q(\pi(q) \wedge \Box(q \rightarrow p)) \rightarrow p$ is to evaluate as

true at each world w, it must be that $\exists q(\pi(q) \wedge \Box(q \rightarrow p))$ is false at w or p is true at w. If p is true at w, so is $\exists q(\pi(q) \wedge \Box(q \rightarrow p)) \rightarrow p$, just as required. So suppose p is false at w. Then we require that $\exists q(\pi(q) \wedge \Box(q \rightarrow p))$ be false at w. In effect, a propositionally quantified formula $\exists p A(p)$ will be true in a world in a model iff $A(p)$ is true at that world for some replacement of the variable p by some propositional constant p_0. So $\exists q(\pi(q) \wedge \Box(q \rightarrow p))$ is false at w iff $(\pi(q_0) \wedge \Box(q_0 \rightarrow p))$ is always false at w, no matter how q_0 is chosen. Pick any q_0. We can suppose that q_0 is true or false at w. If q_0 is true at w,then since p is false at w, $q_0 \rightarrow p$ is false at w, so that $\Box(q_0 \rightarrow p)$ is false and hence $(\pi(q_0) \wedge \Box(q_0 \rightarrow p))$ is false as required. If q_0 is false at w, then given our assumption about the clause for π, $\pi(q_0)$ will likewise be false at w, so that again $(\pi(q_0) \wedge \Box(q_0 \rightarrow p))$ is false as required.

To see that $\exists q(\pi(q) \wedge \Box(q \rightarrow p)) \nvdash_S \Box p$, we shall need to show that in some S-model, for some w, $\exists q(\pi(q) \wedge \Box(q \rightarrow p))$ is true at w while $\Box p$ is false at w. Intuitively, this is clearly possible. Suppose $\Box p$ is false at w. For $\exists q(\pi(q) \wedge \Box(q \rightarrow p))$ to be true at w, we require that $\pi(q) \wedge \Box(q \rightarrow p)$ is true at w for some choice of q. Since we are taking π to be factive, q must be true at w. And $\Box(q \rightarrow p)$ must be true at w. This means that p must be true at w (we are assuming S includes the T-axiom for plain $\Box$, so that the accessibility relation is reflexive). But this is entirely consistent with p and q being false at w' for some w' accessible from w, as required for truth of $\Box(q \rightarrow p)$ and falsehood of $\Box p$ at w.

We are reasonably confident that with further work, this informal sketch can be turned into a rigorous model-theoretic proof, and that we shall be able to show, along similar lines, that our proposed definition of relative necessity does not succumb to any of the other problems discussed in Section 3.

7 Complications and Refinements

The system S, lightly sketched in 6.3, clearly calls for more rigorous formulation and development. We cannot undertake a full-dress presentation here. Our purpose in this section is rather to draw attention to some signficant aspects of the system we intend, and to deal with some complications noted in Section 6.2 (see fn. 20). Here we have benefitted from very helpful discussion of an earlier version with David Makinson and some observations made by an anonymous referee, to whom we are much indebted. The complications, and in some cases, refinements, concern the following facts. (i) $\Box p \rightarrow \Box_\Phi p$, i.e. the principle that logical necessity implies relative necessity, which Makinson calls *Down*, is not a theorem of S. (ii) The converse of *Down*, $\Box_\Phi p \rightarrow \Box p$, which Makinson calls *Up*, is not a theorem of either S or S + *Down*. (iii) S appears to lack the *conjunction property*, i.e. that $\Box_\Phi A, \Box_\Phi B \vdash \Box_\Phi(A \wedge B)$. We give a brief statement of the logical facts, then comment on the issues to which they give rise.

(i) $S \nvdash Down$

Let our relative necessity operator be $\Box_\Phi$, defined as $\exists q(\Phi(q) \wedge \Box(q \rightarrow p))$. Interpret Φ so that $v(\Phi A, w) = 0$ for all $w \in W$. Then $v(\exists q(\Phi q, w) = 0$ for all $w \in W$. Hence $v(\exists q(\Phi q \wedge \Box(q \rightarrow A)), w) = 0$ for all $w \in W$, i.e.

$v(\Box_\Phi A, w) = 0$ for all $w \in W$. So, vacuously, $v(\Box_\Phi A \rightarrow A, w) = 1$ for all $w \in W$. That is, the T-schema holds with this interpretation of Φ.

Now let $\top$ be any tautology, so that $v(\top, w) = 1$ for all $w \in W$. Hence $v(\Box\top, w) = 1$ for all $w \in W$. But $v(\Box_\Phi\top, w) = v(\exists q(\Phi q \wedge \Box(q \rightarrow \top)), w) = 0$ for all $w \in W$. Hence $v(\Box\top \rightarrow \Box_\Phi\top), w) = 0$ for all $w \in W$. That is, *Down* is not derivable in *S*.

(ii) *S* (*S*+*Down*) $\nvdash$ *Up*

Let $W = \{w_1, w_2\}$ and $R = W \times W = \{\langle w_1, w_1\rangle, \langle w_1, w_2\rangle, \langle w_2, w_1\rangle, \langle w_2, w_2\rangle\}$ so that R is an equivalence relation on W. Interpret Φ so that $v(\Phi A, w) = 1$ iff $v(A, w) = 1$ for all $w \in W$.

It follows that $v(\Box_\Phi A, w) = 1$ iff $v(A, w) = 1$ for all $w \in W$. It immediately follows from this last that $v(\Box_\Phi A \rightarrow A, w) = 1$ for all $w \in W$—since if $v(\Box_\Phi A, w) = 1$, then $v(A, w) = 1$ for all $w \in W$. So the T-schema is validated by this model. But this model falsifies *Up*, if we stipulate (as we may), for some p, that $v(p, w_1) = 1$ but $v(p, w_2) = 0$.

(iii) $\Box_\Phi A, \Box_\Phi B \nvdash \Box_\Phi(A \wedge B)$

Under our definition, the premises are $\exists q(\Phi(q) \wedge \Box(q \rightarrow A)$ and $\exists q(\Phi(q) \wedge \Box(q \rightarrow B)$. As far as these premises go, there need be no *single* proposition q such that $\Phi(q)$ which strictly implies both A and B. So we appear unable to infer $\exists q(\Phi(q) \wedge \Box(q \rightarrow (A \wedge B))$, as required for the conjunction property.

7.1 Comments on (i), (ii) and (iii).

For reasons which will rapidly become apparent, it makes best sense to begin with point (iii).

7.2 The Conjunction Property

What, in essence, appears to block the derivation of $\Box_\Phi(A \wedge B)$ (i.e. $\exists q(\Phi(q) \wedge \Box(q \rightarrow (A \wedge B))))$ from $\Box_\Phi A$ and $\Box_\Phi B$ (i.e. $\exists q(\Phi(q) \wedge \Box(q \rightarrow A))$ and $\exists q(\Phi(q) \wedge \Box(q \rightarrow B)))$ is the fact that we cannot infer from the premises that there is a *single* proposition r such that $\Phi(r) \wedge \Box(r \rightarrow (A \wedge B))$. As our anonymous referee points out, this problem would be solved if we could generally infer $\Phi(A \wedge B)$ from $\Phi(A)$ and $\Phi(B)$, since we could then conjoin the possibly distinct propositions q_i, q_j such that $\Phi(q_i) \wedge \Box(q_i \rightarrow A)$ and $\Phi(q_j) \wedge \Box(q_j \rightarrow B)$, whose existence is guaranteed by the premises, to obtain $\Phi(q_i \wedge q_j) \wedge \Box((q_i \wedge q_j) \rightarrow (A \wedge B))$, whence $\exists q(\Phi(q) \wedge \Box(q \rightarrow (A \wedge B)))$. As a matter of fact, we make just this assumption for our operator π above (see footnote 20). However, an objection to this remedy, also proposed by our referee, is that it re-opens and reinforces the concern, to be discussed in section 8, that a Φ-operator is really nothing but a thinly disguised duplication of the relative necessity operator which we are seeking to define. In that case, rather than working via the conjunction property for Φ, we might as well directly stipulate that $(\Box_\Phi A \wedge \Box_\Phi B) \rightarrow \Box_\Phi(A \wedge B)$.

The problem raised is a special case of a wider issue, i.e., whether relative necessity operators, according to our proposed formulation, are closed under logical consequence, that is, if $A_1, ..., A_n \vdash B$ then $\Box_\Phi A_1, ..., \Box_\Phi A_n \vdash \Box_\Phi B$ for $n \geq 1$[22]. We need a solution that does not simply beg the question. And indeed, there is a simple revision of our proposal from which closure under logical consequence follows, without the complication of needing to specify and justify a conjunction property for Φ-operators.

Instead of defining $\Box_\Phi A$ by a simple existential quantification $\exists q(\Phi(q) \wedge \Box(q \rightarrow A))$,we may simply use a finite string of quantifiers:

$$\Box_\Phi A =_{def} \exists q_1, ..., \exists q_n(\Phi(q_1) \wedge ... \wedge \Phi(q_n) \wedge \Box(q_1 \wedge ... \wedge q_n \rightarrow A))$$

Clearly we obtain closure under logical consequence now, without needing to insist that a conjunction of Φ-propositions must itself be counted as a Φ-proposition.[23]

7.3 Down

Makinson's own view is that the failure of *Down* is a defect—without it, the system is 'rather weak'. On the contrary, we take there to be good reason why we should not have *Down*. *Down* is, obviously, closure under logical consequence for the limiting case where $n = 0$. It is bound to fail if, as seems reasonable, we allow that the laws of physics, say, might have been otherwise, and that it is a contingent matter whether there are any such laws at all, for then $\Box B$ may be true, but $\Box_\Phi B$ (i.e. $\exists q_1, ..., \exists q_n(\Phi(q_1) \wedge ... \wedge \Phi(q_n) \wedge \Box(q_1 \wedge ... \wedge q_n \rightarrow B))$) false, because there are no true Φ-propositions. Of course, under the hypothesis that there is at least one true Φ-proposition, *Down* will hold; where there are, say, some laws of physics, their logical consequences will, trivially include all the logical necessities, which will harmlessly qualify as physical necessities.

That *Down should* be validated is, no doubt, just what one would think, if one thinks about relative necesity in essentially world-terms: that is, so that what is *physically* necessary, for instance, is essentially just what is true throughout a *restricted* range of all *logically* possible worlds. For then, since anything logically necessary (i.e. true throughout the whole unrestricted range) must be true throughout any restriction of it, it must also be physically necessary. Our answer to this is that it is

[22] We should not expect closure when $n = 0$, for reasons to be given in our discussion of *Down*.

[23] As a referee for this journal observed, this refinement, in allowing for variation over natural numbers serving as indices, and in using suspension dots, results in a right hand side that is no longer a formula of a simple extension of the language of ordinary modal logic by Φ and propositional quantifiers. A fuller treatment of our proposal would need to settle on a formal treatment of these extra devices.

As our referee points out, we might accommodate numerical indices by enriching the language to include explicit quantification over the natural numbers; or we might instead take our formulation as shorthand for an infinite set of axioms of the form: $\exists q_1 ... \exists q_n(\Phi(q_1) \wedge ... \wedge \Phi(q_n) \wedge \Box(q_1 \wedge ... \wedge q_n \rightarrow A)) \rightarrow \Box_\Phi A$. We have no aversion to enriching the language by adding quantification over the natural numbers, if that is necessary for current purposes, nor is it obvious to us what the shortcomings, if any, would be in opting for an axiom scheme in place of a fully explicit definition. We shall not attempt to adjudicate between these, and perhaps other, alternatives here.

just a mistake to think of forms of relative necessity as fundamentally to be understood in world terms. If we drop that prejudice, then it can seem entirely natural and correct to characterize or define a form of relative necessity in such a way that logical necessity does not automatically ensure relative necessity.

7.4 $S(S + Down) \nvdash Up$

Evidently this is a welcome result from our point of view; Makinson's proof confirms what we claim in 6.3.

8 Further Issues

In this closing section, we anticipate and respond to an objection to our preferred account, and draw attention to a limitation on its range of application.

The objection focuses on the sentential operators, such as π which play a key rôle in our definition scheme. These operators serve to demarcate propositions to which specific kinds of relative necessity and possibility are relative. Because we wish to avoid assuming that we are able to actually state explicitly the relevant propositions, we need to characterize them in general terms, as propositions of a certain kind. Thus, as we suggested, $\pi(q)$ might be read as abbreviating something along the lines of 'it is a law of physics that q', where this is to be understood as making a non-specific reference to the laws of physics, whatever they are, rather than to what we happen presently to take to be laws. Put bluntly, the objection says that by making essential use of operators so understood, the account simply gives up on the reductive explanatory aspiration which informs the original Anderson-Kanger-Smiley project. For what drove that project—and what gave it its interest—was the prospect of showing that, contrary to appearances, we do not need to recognize a great variety of *independent* notions of necessity, because we can explain each relative form of necessity using just a *single* 'absolute' notion (probably logical necessity). This explanatory aim is completely undermined by our appeal to laws of physics, for example, because the notion of a law of physics itself involves the idea of physical necessity.[24]

A first point to be made in response to this objection is that, whatever force it may possess, it does not tell *selectively* against the explanation of relative necessity we have proposed. For essentially the same objection could be brought against Humberstone's two-dimensionalist proposal. Of course, he makes no use of an *operator* comparable to our π operator. But for each distinct kind of relative necessity, his proposal will require a distinctive dipropositional constant, $\boldsymbol{R}$, and this will need to be explained. The only kind of explanation he suggests is that we might understand $\models^{x}_{y}\boldsymbol{R}$ as 'the laws of x are true in y'. As it stands, of course, this is hopelessly vague—there are many different kinds of laws, and he presumably does not mean that

[24]This objection, or something close to to it, was put to one of us in conversation by Lloyd Humberstone.

they should *all* hold. What he intends, presumably, is that two-dimensionally understood, the diproposition will say, for example, that *x*'s *physical* laws are true in *y*. So his proposal, too, essentially involves an appeal to the notion of physical law, and so is no less vulnerable than ours to the envisaged objection. Thus if it should prove impossible satisfactorily to characterize physical laws without appeal to the notion of physical necessity, his account will be no better placed to subserve the reductionist aims of the standard account than our own.[25] It is true enough that the objection does not apply to the standard account itself. But as we have seen, there are good reasons to explore alternatives. Moreover, it seems inescapable that the deficiencies of the standard account have their source in its suppression of vital information. It is quite unclear how the requisite information could be incorporated without making use, in the analysis of physical necessity, say, of the notion of a physical law, or something near enough equivalent to it.

Second, even if it is granted that the notion of a law of physics, say, involves an implicit appeal to the notion of physical necessity or possibility, it is not clear that this need be objectionable. Whether it is so or not depends upon what the proposal sets out to accomplish. As we observed at the outset, one might hope that an analysis of relative necessity would subserve a reductionist programme. That is, one might hope to show that, contrary to appearances, there is no need to recognize a variety of independent kinds of necessity—physical, mathematical, etc.—by showing that each of these ostensibly different kinds of modality may be fully explained using just one single kind of modality, such as logical modality. If this were one's aim, then the objection, if sound, would indeed be fatal. However, as we saw, there is another, no less important, aim one might have in pursuing analyses of forms of relative necessity—that of achieving an improved understanding of the contrast between merely relative and absolute kinds of necessity. The achievement of that aim is in no way compromised by the irreducibility—if such it is—of the various kinds of putatively relative necessity to a single absolute necessity.

Third, the claim that the notion of physical law cannot be understood without bringing in that of physical necessity may be challenged. There are various ways in which one might attempt to characterize laws of physics that make no overt appeal to the notion of physical necessity. One might characterize laws of physics without any appeal to (familiar) modal notions at all. For example, Maudlin [18] takes lawhood to be a primitive status, and indeed proposes a definition of physical modality in terms of laws.[26] Or, more commonly, one might characterize laws of physics making use of some modal notion *other than* physical necessity. For example, Lewis's "best deductive system" account of laws of nature arguably only draws on the logical necessity built into the notion of a deductive system (see Lewis [15–17]). Or so-called "necessitarians" take the laws of physics to be *metaphysically* necessary

[25] The same point applies equally to the alternative 2-D analysis of relative necessity proposed by van Fraassen in [7], discussed briefly in the Appendix below.

[26] 'My own proposal is simple: laws of nature ought to be accepted as ontologically primitive. ... Laws are the patterns that nature respects; to say what is physically possible is to say what the constraint of those patterns allows.' [18, p. 15]

(see [3] for a representative summary). These views would avoid the troublesome circularity of defining physical necessity in terms of physical laws, in turn defined in terms of physical necessity. The latter would of course introduce a further kind of modality—metaphysical—to be treated as relative or absolute, but the objection presently under consideration would be avoided. The success of this response to the objection depends upon the success of one of these alternative accounts of the laws of physics, but we will not be able to adjudicate on that matter here. It is certainly not obvious that these will all prove to involve a more or less thickly disguised appeal to the notion of physical necessity, and so to be unavailable as independent characterization of laws of physics.[27]

Our claim in this paper is that our proposal is the preferable treatment of *alethic* kinds of relative necessity, such as physical necessity or mathematical necessity. There are potential problems for applying it to non-alethic relative modalities. Whether these problems can be overcome, we leave as further work to be carried out elsewhere. But we will briefly survey two key difficulties for treating non-alethic relative necessities.

The first difficulty arises from the relation between relative necessity and logical necessity. On the standard account, since every logical necessity is strictly implied by any proposition whatever, every logical truth is C-necessary—necessary relative to C—no matter how C is chosen. Similarly, although, as we have seen, *Down* does not in general hold for our alternative account, if it is C-necessary that p just when $\exists q(C(q) \wedge \Box(q \rightarrow p))$, then, *so long as the existence condition is fulfilled* (there is a C-proposition), every logical truth will be C-necessary. This seems to be a reasonable result for alethic necessities: it would certainly be strange to claim that, although it is logically necessary that p, it is nevertheless possible, relative to existing physical laws, that $\neg p$. However, it has implausible consequences for non-alethic necessities for which, intuitively, even if some relevant C-propositions exist, it is not always the case that if $\Box p$, then $\Box_C p$, such as epistemic or deontic modalities. Consider kinds of necessity defined relative to a conjunction of known propositions, or a conjunction of moral precepts. The current proposal for treating these necessities would yield the result that any logical truth is thereby epistemically necessary and morally necessary. However, it seems that we should leave room for the *epistemic* possibility that a proposition whose truth-value is as yet undecided should turn out to be false, even if in fact it is a logical truth. It also seems wrong to take logical truth to be a matter of moral obligation—we might not think that the world would be a *morally* worse place if a contradiction were true in it. Indeed, one might think, to the contrary, that if 'ought' expresses moral obligatoriness, 'It ought to be the case that p' implies that it is at least metaphysically (and so logically) possible that $\neg p$.

A different kind of problem arises from kinds of relative necessity where one might expect to find inconsistent q, although $C(q)$, such as doxastic and legal modalities. One might expect that some conjunctions of beliefs are inconsistent, or that some conjunctions of laws of a given state are inconsistent. However, for any C-necessity defined in terms of an inconsistent proposition (i.e. where the existential

[27] One of us is more sanguine about the prospects for the reductive project than the other.

condition is fulfilled by an inconsistent proposition), this would have the unfortunate result that *everything* would be C-necessary, given the inference rule that everything follows from a contradiction. But again, just because the statute books are likely to contain strict inconsistencies, does not mean that everything is legally required. That would be quite bizarre.[28]

9 Conclusion

Our leading question has been: How should we best formulate claims of relative necessity? We have considered several answers

First, we reviewed the standard account: $\Box_\Phi =_{df.} \Box(C \rightarrow p)$. This, we argued, falls foul of the S4 and mixed S4 logical problems as presented by Humberstone. Moreover, it omits important information about the nature of the proposition (i.e. C) to which the necessity is relative. Further, and crucially, the account assumes that, in making a claim of relative necessity, one is able to state all of the relevant propositions; for example, to make a claim of physical necessity in accordance with the standard account, one would have to be able to state the laws of physics. But such an assumption is too demanding—whilst we may be unable to *answer* the question whether it is physically necessary that p without some knowledge of the laws of physics, no such knowledge is required merely to understand the question; nor, accordingly, should it be presupposed by a good explanation of claims about physical necessity. These reasons led us to explore alternatives to the standard account.

Second, then, we considered Humberstone's two-dimensional alternative. We had two main concerns with this approach: first, it is, we argued, unclear how best to interpret the $\Box$-operator of absolute necessity, without making unpalatable claims about the logical properties of logical necessity; and, second, it appeared to us implausibly to require not just that claims about relative necessity can be *modelled* in terms of worlds, but that they are in fact to be understood as claims about worlds.

We then considered a series of amendments to the standard account. The simplest of these consists in adding the proposition C itself as an extra conjunct in the analysans. This does indeed block the mixed S4 problem, but the simple S4 problem remains. So also does the crucial problem of understanding: the account still requires that one be able to, for example, state the laws of physics in order to make a claim of physical necessity. Adding a more complex conjunct, including the information of what kind of proposition is involved (e.g. specifying that C is a law of physics), solves the remaining S4 problem, but does nothing to resolve the problem of understanding, for the proposed analysans is still one which we can offer only if we are able to state the relevant laws (e.g. the laws of physics).

We then introduced our own, preferred, account.

$$\Box_\Phi A =_{df.} \exists q_1, ..., \exists q_n(\Phi(q_1) \wedge ... \wedge \Phi(q_n) \wedge \Box(q_1 \wedge ... \wedge q_n \rightarrow A))$$

[28] This kind of problem is presented, and a solution offered, in [13].

This captures the information lost by the standard account and the simple added conjunct account—that it is Φ-propositions relative to which things are necessary—but, in contrast with the more refined additional conjunct account, it does so in a suitably general way. The proposal avoids the logical problems (imposing S4, etc.), and does so without any intrusive use of worlds semantics in the analysis. There is no longer the problem that one must (be able to) state the Φ-propositions if one is to give the analysis—the analysans involves only the non-specific requirement that there be some propositions of that kind. Finally, we aired some technical and philosophical issues that may arise for our proposal. We have not had space here to resolve every issue, but we hope to have shown that pursuit of this account for relative necessity is at the very least a promising alternative to what, in our view, are some rather less promising accounts.[29]

Appendix: An Alternative 2-Dimensional Solution?

In an article to which Humberstone refers ([7]) Bas van Fraassen proposes a rather different two-dimensional analysis of relative (e.g. physical) necessity. The analysis replaces the strict conditional $\Box(A \rightarrow B)$ by a two-dimensional conditional $A \Rightarrow B$ (van Fraassen uses $\rightarrow$, but this risks confusion with our use of the same symbol for the truth-functional conditional), defined as follows: $[A \Rightarrow B](\alpha) = \{\beta : [A](\beta) \subseteq [B](\alpha)\}$. Sentences within square brackets denote the propositions they express, which are identified with the sets of worlds at which they are true. So this says that the proposition expressed by $A \Rightarrow B$ at world α is the set of worlds β such that the proposition A expresses at β is included in the proposition B expresses at α. In other words, what $A \Rightarrow B$ says at α is true at β iff every world at which what A says at β is true is a world at which what B says at α is true. This contrasts with the strict conditional, which in the two-dimensional setting is true at α iff every world at which the proposition expressed by its antecedent at α is true is one at which the proposition expressed by its consequent at α is true. The conditions coincide when the world of evaluation is the same as the world of utterance, but diverge when they diverge. 'It is physically necessary that A' is then defined as $R \Rightarrow A$ where R is a special constant which does double duty, both expressing 'the appropriate relation of relative physical necessity' and the corresponding proposition, defined $[R](\alpha) = \{\beta : \alpha R \beta\}$, i.e. the set of worlds physically possible relative to α (op. cit.,

[29] We are grateful to Fabrice Correia for extensive discussion of the development of the core proposal; to Lloyd Humberstone for discussion of some of the central ideas in this paper; to Bas van Fraassen for extensive and instructive correspondence about his 1977 paper; to David Makinson for detailed and thoughtful technical suggestions; to the participants in a modality workshop in Nottingham and members of a King's College London work-in-progress group for discussion of earlier incarnations, particularly Ian Rumfitt; and to two anonymous referees for this journal for their comments and suggestions.

p.82). Van Fraassen tells us that R—the *law sentence*, as he calls it—'may say that the laws of α hold, or that they are laws, or that they are the only laws.'

Van Fraassen's leading idea is that the relative character of physical necessity is best understood indexically. We can approach this as follows. When we hold that it is physically necessary that p, we commit ourselves to the claim that, even if things had been different in a whole host of ways, barring some (miraculous) suspension of the laws of physics, it would still have to be the case that p. We are not just claiming that, given the fact that things are the way they actually are (right down to the last detail), it has to be that p; we are allowing that circumstances might have varied in all sorts of ways, and claiming that even so, the laws of physics require that p. We can understand van Fraassen's new conditional as designed to capture this. It seeks to do so by allowing the context for the antecedent to vary in any ways consistent with its still saying something true, but keeping the actual context for the consequent fixed. Thus we might read $R \Rightarrow A$ as something like 'In any circumstances in which the laws of physics hold it will be that A'—we can do so, if we may read $A \Rightarrow B$ as 'Any circumstances in which A, as said in those circumstances, would evaluate as true, is one in which B, as said here and now, would also be true'. In sum, the idea is to give a way of understanding claims about physical necessity which captures the fact that they make a claim whose truth has a high degree of independence from the actual circumstances in which they are made—so long as the physical laws hold, then although things can differ in all sorts of other ways, it will still be that p. Obviously, when we envisage different circumstances here, we are restricting attention to different circumstances in which the laws of physics *as they are* would *not* be altered—that is, we are keeping the *actual* laws, not talking about (more radically divergent) circumstances in which there would be *different* physical laws. That is, we are using 'the laws of physics' *rigidly*. The same is true of our own account—we require $\pi(q)$ to say that q is a statement of the laws of physics *as they are*—we aren't claiming (falsely) that no matter what the laws of physics *might be*, they strictly imply p.

The obvious questions are: Does this account avoid the kind of problems which beset the standard account? Is it otherwise satisfactory?

As far as we have been able to see, the answer to the first is that it does. In van Fraassen's system, validity can plausibly be understood in one of two ways. A strong requirement would be that validity requires that $\forall\alpha\forall\beta\, Tr([A](\alpha), \beta)$, i.e., A is valid just when, for any worlds α, β, what A says at α is true at β. A weaker requirement would be that $\forall\beta\, Tr([A](\alpha), \beta)$ when α is taken to be the real world, i.e., A is valid just when what A says at the actual world is true at any world β. Whichever notion of validity one operates under, it appears that, due to the unconventional behaviour of van Fraassen's conditional, neither $A \Rightarrow (B \Rightarrow A)$, nor the especially relevant instance for the S4 problem, $(B \Rightarrow A) \Rightarrow (B \Rightarrow (B \Rightarrow A))$, is valid. Hence, van Fraassen's system appears not to be subject to the S4 problems that afflict the standard account.[30]

As regards the second question, we have some doubts. A first point is that $A \Rightarrow A$ is not a law of van Fraassen's logic. For since R is indexical, there may be a world,

[30]The reader may amuse herself by verifying these claims.

other than the actual world, such that what R says there is true at some world at which what R says at the actual world is not true. Whence, on the proposed semantics for the conditional, $R \Rightarrow R$ is not valid.[31] But the law of identity, 'If A then A', is often taken to be fundamental, and has as strong a claim to be definitive of the conditional as the other principles which van Fraassen mentions (*op. cit.*, p.82) as 'earmarks' of the concept. It is true that he views it as a 'cluster concept', so that some may go missing without destroying a connective's claim to be a conditional (or implication connective—van Fraassen makes no distinction here). It remains a serious cost, and one that we, at least, are reluctant to incur.[32]

A further concern is that the account may suffer from the same drawback as we found in Humberstone's, i.e. that it does not just exploit the two dimensional framework in the model theory, but builds talk of worlds into the very content of claims about, say, physical necessity. The key question here is how the special propositional constants, such as van Fraassen's R, are to be understood. That there is unwanted worlds content is certainly suggested by his own proposed readings of R, which 'may say that the laws of α hold, or that they are laws, or that they are the only laws.' It is not obvious that the world variable is dispensable without unsuiting R for its purpose. That van Fraassen thinks, to the contrary, that there is no essential reference to worlds is perhaps suggested by his official agnosticism about them ('The items in the models, such as possible worlds, I regard with a suspension of disbelief, as similar to the ropes and pulleys, and little billiard balls that were introduced in nineteenth-century physics', p.74). But if, more generally, we ask ourselves: 'What is the *parameter* with respect to which talk of physical necessity is supposed to be *indexical*?' it is not clear that the answer can be anything other than 'Possible Worlds'.

Perhaps these doubts can be answered. If so, there is another, and better, two-dimensional solution to Humberstone's problems. We see nothing inimical to our own proposal in this. For it would remain the case that if we are right, those problems can be solved more simply and economically, without any recourse to the two-dimensional framework.

[31] There will be other casualities. To mention only the most obvious: $\not\models A \wedge A \Rightarrow A, \not\models A \Rightarrow A \wedge A, \not\models A \vee A \Rightarrow A, \not\models A \Rightarrow A \vee A$.

[32] In fairness to van Fraassen, we should point out that he observes (in correspondence), that $A \Rightarrow A$ *will* be a thereom of the logic we may label $\mathcal{L}_{simp}$, which has as theorems those sentences which are *true simpliciter* at every world, i.e. true at α if understood as uttered in α, for every world α. But it will not be a theorem of the logic $\mathcal{L}_{univ}$, which has as theorems those sentences which, understood as uttered in *any* world, are true at *every* world. He appears to be sceptical, not just about the claims of either of these notions and their logics to capture the traditional notion of logical necessity, but about whether there is a good notion to be captured. Here we can only record our disagreement with him on the last question. Of course, if the meanings of words, including logical words, are allowed to vary with world of utterance, then it must be doubtful that there are any *sentences* which, as uttered at any world, are true at every world. What is of far greater interest is whether, holding the meanings of relevant words fixed, there are sentences which express propositions which are absolutely necessary—true at absolutely all worlds. It seems to us that 'If A then A' can be, and often is, used to express a proposition, stronger than a merely material conditional, which is absolutely necessarily true, and is so in virtue of the nature of the conditional alone, and so has a good title to be regarded as logically necessary.

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