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The Life Cycle of a Debris Particle

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Debris particles exist in most lubrication systems; they are frequently responsible for the early failure of tribological machine elements. Particles come from the surrounding environment, or may be generated within the machine components. As the lubricant circulates, these particles get flushed into the machine elements. Contact pressures are high and oil films are small, so that the relatively large particles damage even the hardest gear, bearing, or cam surface. This damage can lead to contact fatigue or wear, and thus premature failure of the whole machine. Further, one failure can result in the generation of further wear debris, often in very great quantities, that then can have a knock-on effect in other parts of the lubricated system.

This paper gives an overview of important features of the life cycle of a debris particle; entrainment of debris particles into a contact, resulting surface damage, shortened component life, and debris particle procreation by fatigue and wear. The debris life cycle coincides with the early mortality of the machine element. The methods by which component life, under particulate contaminated conditions, can be determined are reviewed.

1. INTRODUCTION

It is an unfortunate fact that industrial oil supplies are contaminated with debris particles. Depending on the nature of the oil supply, contamination levels vary from around 10^{-1} to 10^1 g/l. A motor car oil system contains something like one teaspoon of debris particles in the sump. This may not sound much, but particle sizes are small, so this corresponds to millions of individual particles, each of which has the possibility of causing damage to lubricated machine elements.

Most of the analysis to determine oil film thickness in machine elements assumes a clean lubricant. The predicted oil films are very thin and usually much thinner than the size of debris particles. This means that should they find their way to a lubricated contact then they are likely to cause some kind of surface damage. This leads to wear or hastens the onset of contact fatigue. The debris in an oil supply can reduce the life of the component considerably. In this paper this process is described in terms of the debris life cycle. From birth when the particle finds its way into an oil

supply, through its early years as it is entrained into machine element contacts, adult life where it causes damage, procreation by wear and fatigue, and then when it passes the rest of its life in a filter or the bottom of a sump.

2. PARTICLE GENERATION

Debris particles can be classified according to their source (see figure 1). They may be entrained into an oil system from the environment, or generated from within the machine. The former category is most likely to be minerals or handled products, such as sand, coal, or earth. The size of these sorts of particles vary greatly, from sub-micron to millimetre; whilst the shapes tend to be blocky and angular. Probably the nature of circulating lubricant system is such that only those less than around $100\mu\text{m}$ will find their way to machine components (larger particles will settle quicker in a sump). However, grease lubricated bearings on a mining conveyor belt may see much larger particles.

Particles may also be generated from within a machine or engine [1]. Components where there is not full lubricant separation will produce wear particles, either by abrasion (where the asperities of one surface plough material out of another) or by adhesion (where particles from one surface are plucked out by adhesion to the other).

		Classification	Examples
external contaminants	environmental		dust, airborne particles
	handled materials		coal dust, sand, railway track ballast
	pre-installed		grinding grits, weld spatter, oil can rust, swarf
internal contaminants	process generated		combustion products - soot
	fatigue debris		bearing spalls, gear tooth pits
	wear debris		cutting slithers, filmy particles,
	scuffing debris		whitemetal from a seized journal
	component breakdown debris		fracture fragment, broken strainer, hose flexing

Figure 1. Classification of debris particles found in lubrication systems.

The presence of debris in a contact can also result in wear and the production of yet more debris particles. The nature of these wear processes take place on an asperity scale so the debris particles are usually less than 100µm or so in size. Contact fatigue failure can also cause debris formation. Crack growth leads to the formation of pits and spalled material. The materials of these kinds of particles are those which are commonly used to manufacture machine elements such as; steel, brass/bronzes, PTFE, nylon etc. They are likely to occur in a wide variety of shapes (thin film/foil, shavings, ribbons, flakes, and chunks) but generally they will be ductile in nature.

There is a large body of work on the identification and classification of wear debris particles as an aid to machine condition monitoring and fault detection (see for example [1]).

A further form of generated debris particles, unique to combustion engines, is soot from the combustion process. Soot particles are small (sub-micron) hard amorphous carbon particles and can

occur in high concentrations (up to around 8% by mass) especially in heavily used engine oils. Figure 2 shows the some pictures of some typical entrained and self-generated debris particles.

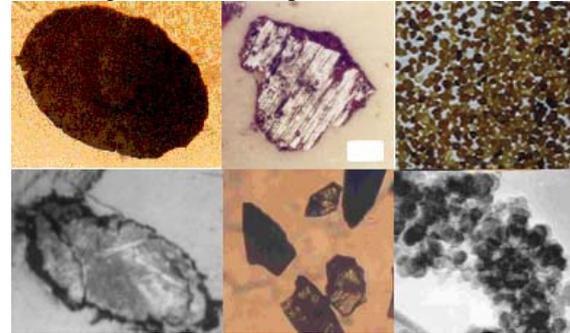


Figure 2. Micrographs of typical debris particles (top left to bottom right: 150µm iron platelet, 2mm fatigue chunk, 1mm sand, spall fragment [1], 60µm silicon carbide grinding grit, 50nm soot).

3. ENTRAINMENT INTO MACHINE ELEMENT CONTACTS

There is plenty of evidence that debris particles reduce the life of machine elements, and there are several models of the relationships. However, there is little work on how particles become entrained into the machine element in the first place. This is likely to be strongly dependent on the nature of the lubricant supply system. A sealed-for-life system will have a different entrainment mechanism than a circulating supply. Most of the debris particles found in a supply system will be denser than the lubricant and so will rapidly drop out of suspension. The actual population of circulating particles will be lower than that present in the system. However, given that some particles must be present in the component feed stream, there is little work done on how many of these enter into a contact (or indeed whether a machine element can be designed to naturally reject particles).

3.1. Indent Counting Experiments

Counting the number of surface dents on a rolling element has been used [2] to determine the proportion of particles in a lubricant supply that enter a contact. A simple ball on flat rolling contact

was used with low levels of contamination with diamond particles (which would not fracture and so lead to multiple indentations). The surface dents created by each particle were counted and related back to the number of particles to have entered. Figure 3 shows the entry ratio (the concentration of particles in the contact divided by that in the bulk) against particle size/film thickness ratio for several speeds.

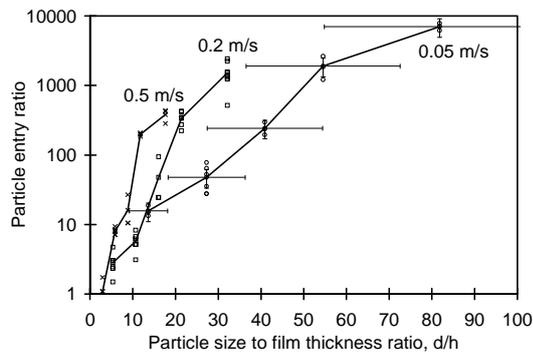


Figure 3. The entrainment of particles into a rolling point contact [2]. The contact can act to concentrate particles.

The results demonstrate that a contact can have a significant debris concentrating effect. This is because the frictional entrainment forces on a particle are much greater than the fluid drag forces. As soon as a particle is caught in the entry zone of a contact it is drawn into the contact by the rolling elements whilst the surrounding oil is squeezed out. Smaller particles feel the entrainment forces much further down the entry zone and so are entrained in fewer numbers.

3.2. Fluid Forces, Friction, and Entrainment

The balance between fluid drag forces and frictional entrainment forces has been modelled in detail by Nikas [3]. He defined trajectories of particles that were drawn into the contact or rejected from it (figure 4 shows an example plot). He demonstrated that lower slide roll ratios increased the probability of entrainment. He also showed that the thickness of the oil film has relatively little effect (as the entrainment activity all occurs in the contact entry zone). Smaller particles

can approach close to the contact and so are more affected by the side and back flow.

There is also a possibility that particles can accumulate in the entry region to a contact. At higher slide-roll ratios particles can rotate in the inlet region (rather than be drawn into it). Nikas [3] observed this in his modelling; as did Sliney [4] and Wan & Spikes [5] both studying the entrainment of suspended MoS₂ particles under mixed rolling and sliding conditions.

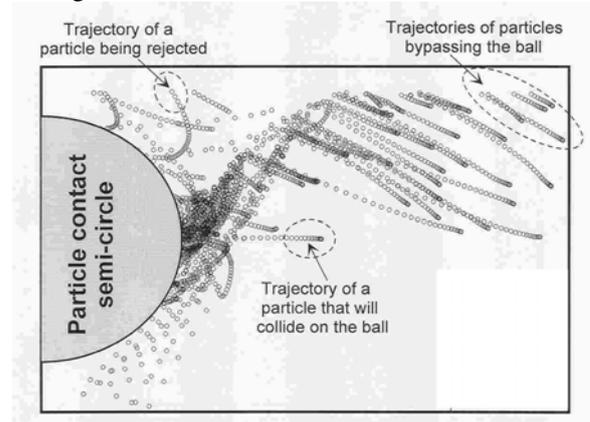


Figure 4. Modelled 20µm diameter particle trajectories at a mixed rolling/sliding point contact (from [3]).

This is significant because a build up of particles in a contact can cause blockage and resulting oil starvation. It is possible this could be a mechanism for several failures associated with soot contaminated lubricants in automotive engines (since the particles themselves are not particularly large or harmful). Failure is by starvation rather than by direct debris damage.

The author has been unable to find any studies of entrainment into other kinds (i.e. non-conformal) of machine element. It is likely that similar principles apply. The particles travel with the fluid flow until they collide and are trapped by solid surfaces meeting and then are drawn into the conjunction.

4. PARTICLE BEHAVIOUR

4.1. Particle Deformation and Fracture

Many debris particles will be larger than the oil film thickness in a machine element (this is certainly true for elastohydrodynamic contacts, less so for hydrodynamic contacts). Therefore once a particle is entrained into the contact, either it is reduced in size, or the contact elements are pushed apart. The former implies that the contact elements carry the load whilst the latter implies that the particles themselves carry the bulk of the load.

For most contaminated lubricant supply systems the former will be the case. The particles which find their way into a contact will be, at any one time, few. And the load on a machine element is high enough such that they will deform and crush the debris particles. Dwyer-Joyce et al. [6] defined various mechanisms by which a particle might be accommodated in a point contact.

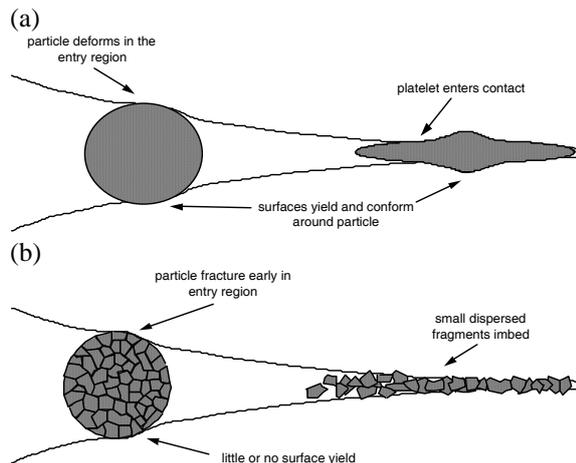


Figure 5. Schematic representation of debris particle deformation in a rolling/sliding point contact: (a) ductile particles are rolled into platelets, whilst (b) brittle particles are crushed to fragments.

Figure 5 schematically demonstrates how ductile particles are squashed into platelets and brittle particles are crushed into fragments. This process happens in the entry region to a contact and so it is the deformed platelet or crushed fragments

that pass into the contact. The surface damage to the rolling elements will depend on these particle deformation mechanisms. Harder particles will deform to a thicker platelet and cause smaller deeper indents. Tougher particles will crush to larger fragments causing bigger dents.

4.2. Optical Studies of Particle Behaviour

The technique of optical elastohydrodynamic lubrication (EHL) provides a useful way of studying particle behaviour in and around a contact [5,7]. A steel ball is loaded in pure rolling against a glass disk with a semi-reflective coating. The contact is fed with lubricant mixed with debris particles. A pattern of interference fringes shows the thickness and extent of the oil film formed. High-speed video or short duration flash photography can be used to capture images of particle entrainment and deformation [7]. The photographs included as figure 6 show ductile debris platelets and sand particles that have deformed and crushed when entrained into the contact.

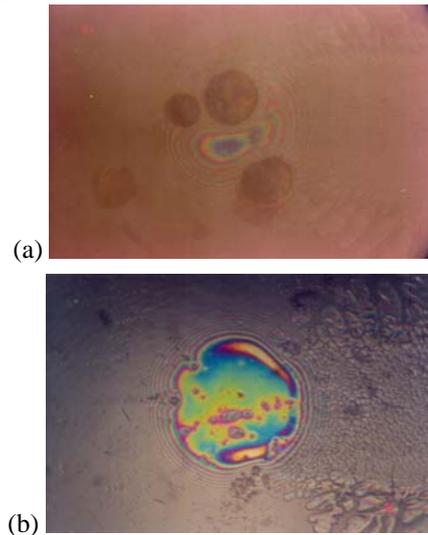


Figure 6. Photographs of (a) copper debris particles and (b) sand particles in a rolling EHL contact.

The colour fringes surrounding the particles remain largely unchanged. The particles themselves have little effect on the oil film, i.e. they do not

support significant load (unless they are present in very high concentration).

5. SURFACE DAMAGE

When the debris particle passes through the contact it will cause some damage to the bearing surface. The nature of the deformation process described above dominates the nature of the surface damage. Ductile particles cause smooth rounded, relatively shallow indents, whilst brittle particles cause deep steep sided dents. Figure 7 shows maps of surfaces (recorded using a stylus profilometer) dented by steel particles and silicon carbide.

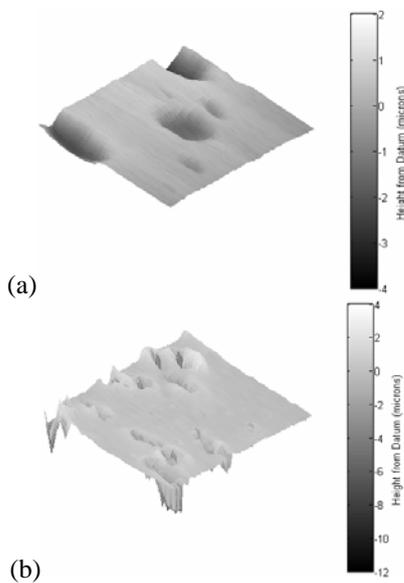


Figure 7. Surface map of a bearing steel roller damaged by (a) low carbon steel (b) silicon carbide, particles in the lubricant. Measurement patch size 0.5mm x 1mm.

Hamer et al. [8] developed a model to predict the size of the ductile particle dents by assuming the particle extrudes plastically between the closing surfaces. Figure 8 shows the outcome in the form of a damage map. The map shows regions of plastic indentation of the bearing surfaces by different size debris particles.

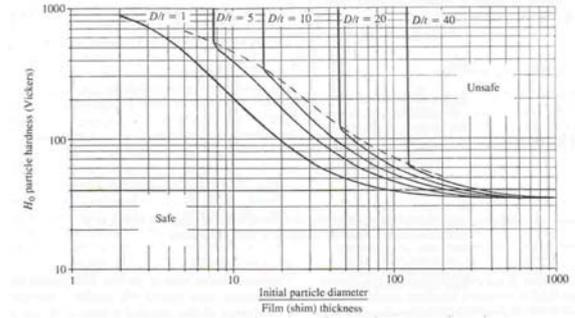


Figure 8 – relationship between particle size/shape/hardness and dent size (from [8]).

Predicting the scale of surface damage caused by brittle particles is more difficult. One approach is to determine the size of the fracture fragments that get entrained into, and survive passage through, the contact. These will essentially embed rigidly in the bearing surfaces and so the damage will be of a similar size to the particle size. The size of the fracture fragment will depend on its fracture toughness; it is assumed that the particle cannot be crushed below its critical crack size, a_{cc} in the contract stress field so [7]:

$$a_{cc} = \frac{1}{\pi} \left(\frac{3K_{IC}}{(1-2\nu)H} \right)^2$$

The calculation assumes that the debris particle is subjected to a pressure distribution with a peak value equal to the hardness of the counterface material. Figure 9 is a plot of this relation with some measured fragment sizes superimposed.

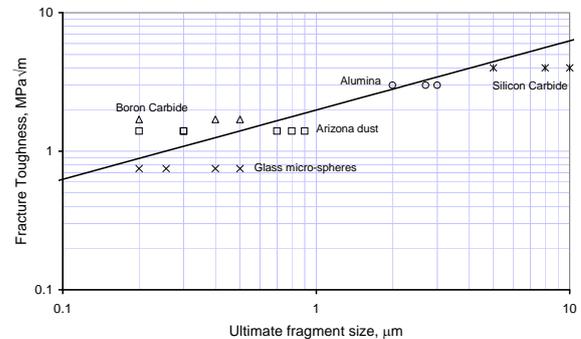


Figure 9. The ultimate fracture fragment size for a brittle particle crushed by a rolling element contact.

Table 1 shows some typical dent sizes for various types of particles entrained into an EHL contacts for both ductile and brittle type debris. In the case of the brittle materials the width of the dent is controlled by the mass of fragments that indent the surface in a clump.

Debris material	Particle size, μm	Dent depth, μm	Dent width, μm
Copper [6]	58-63 μm	0.5-1.0	150-200
Gearbox debris [10]	various	0.6-4.8	50-360
Low C steel [6]	58-63 μm	1.0-2.0	150-200
M50 high C steel [9]	32-40 μm	0.3-4	50-80
Arizona dust [6]	0-100 μm	0.6-1.0	50
Glass beads [6]	60 μm	0.2-0.3	30-40
Alumina [6]	58-63 μm	0.5-1.0	40-75

Table 1. The scale of damage caused by various types of debris particles in an EHL contact (high carbon rolling bearing steel elements).

6. ROLLING CONTACT FATIGUE

6.1. Debris in Rolling Bearings

One of the prime concerns for rolling bearing manufacturers is fatigue failure caused by lubricant debris. Figure 10 shows a photograph of a fatigue spall initiated at a debris dent. These spalls initiate at either the trailing or leading edge of the indent, depending on the direction of traction.



Figure 10. A fatigue spall initiated at a debris dent on a rolling bearing raceway.

The life of the bearing is greatly reduced by the presence of debris. Figure 11, extracted from the pioneering work of Sayles and MacPherson [11] shows the effect of filtering out these particles on bearing life; coarser filtration reduces both the bearing life (by up 7 times) and the scatter.

Loewenthal et al. [12] observed similar results with life improvements by factors of 2 to 3 when using ultra-clean $3\mu\text{m}$ filtration.

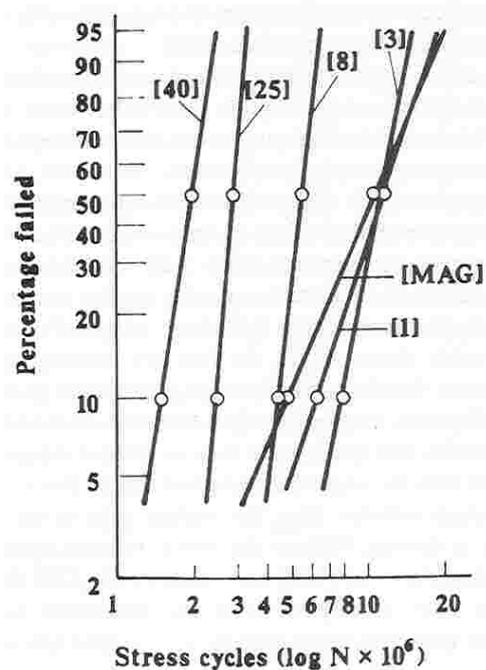


Figure 11. Fatigue life improvement obtained by lubricant filtration (the number in square brackets refers to the filter size in microns) from [11].

6.2. Twin Disk Simulations

A suitable experimental approach is the use of a twin-disk simulator. The disk surfaces are artificially damaged (usually using a hardness indenter) and then run under lubricated slide/roll conditions [13,14,15,16,17].

Figure 12 shows a sequence of photographs for a dent initiating at a Rockwell indent run in oil [14]. The dents are first plastically flattened (in this case the yield stress of the steel is $\sim 900\text{MPa}$, significantly less than a bearing steel, so the plastic deformation is extensive), and then a crack initiates from the trailing edge. Typically these dented surfaces fail in $10^5 - 10^8$ cycles depending on the dent size/shape, load, speed, traction, and materials.

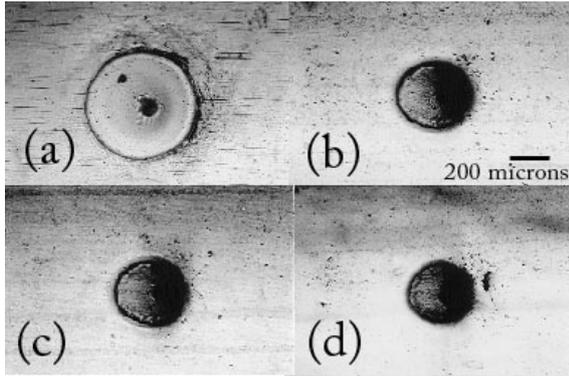


Figure 12. Progression of damage around a conical dent. (a) $N = 0$; (b) $N = 131k$; (c) $N = 465k$; (d) $N = 866k$ cycles. (1% sliding, $p_0 = 1500$ MPa, oil lubrication, rolling and traction directions both right to left).

Wedevan and Cusano [18] used optical elastohydrodynamics to study the effect of such dent on the oil film formation. They observed local thin film and corresponding pressure peaks at the dent edges. The trailing edge of the dent (where the entrainment shape is converging) showed the highest pressure rise.

6.3. Modelling Fatigue Life of Dented Surfaces

The effect of dents on fatigue life has been modelled by several authors. The procedure is generally to determine the stress field around the indent by using either a numerical contact model [1] for dry contact, or a numerical EHL solver [16,20,21,22]. Figure 13 shows some sample results for dent passing through a rolling-sliding contact [16]. The stress concentration at the dent edges is shown.

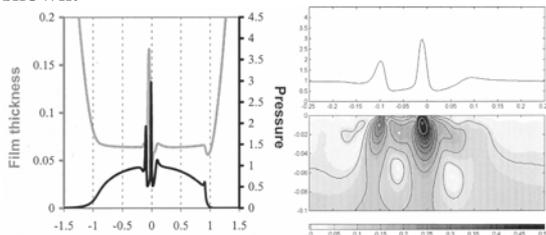


Figure 13. Sample results from a numerical EHL solution for a dented surface [16]. The film

thickness, contact pressure, and sub-surface shear stress are shown as a dent passes through the contact (slide roll ratio +3%).

The stress field is required at different relative positions of the contact with respect to the dent (i.e. a time varying stress field). This is then used in conjunction with a contact fatigue life model. The Ioannides and Harris [23] model has proved particularly useful because in this approach the fatigue failure probability is presented as an integral of a sub-surface stress amplitude field [22].

7. WEAR BY DEBRIS PARTICLES

7.1. Two & Three Body Abrasion

A trapped debris particle can cause wear of the contact surfaces. In a pure rolling contact, little wear takes place and the surfaces are dented. However, in a fully sliding contact, or a mixed rolling sliding contact the particle is subjected to relative motion. One surface slides across the other and the particle is trapped in between.

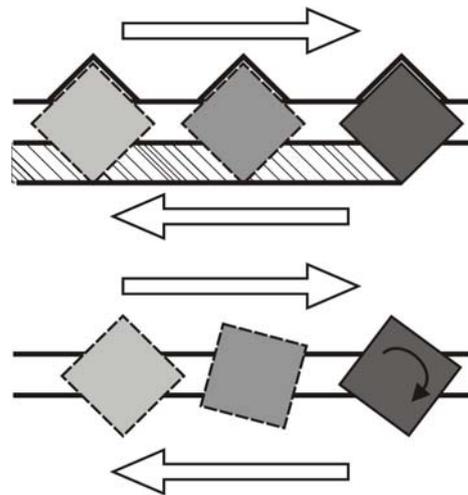


Figure 14. Schematic representation of two-body (top) and three-body (bottom) abrasive wear mechanisms.

Wear can occur by two mechanisms. Firstly the particle may remain stuck (or imbedded) on one surface while it scratches the other. This is known as two-body wear. The particle imbeds in the softer surface and scratches the harder. Alternatively the particle may roll or tumble through the contact causing a series of indents on both surfaces. This is known as three-body abrasion, and is less damaging to the surfaces (see figure 14).

Figure 15 shows a sequence of wear tracks from five tests using diamond abrasives in a lubricated ball on disk contact. The particle size to film thickness ratio has been varied (by using various grades of abrasive particles). Smaller particles roll through the contact (three body abrasion). As the particles get larger, with respect to the oil film they are more likely to adhere to the harder ball and scratch the disk (two body abrasion).

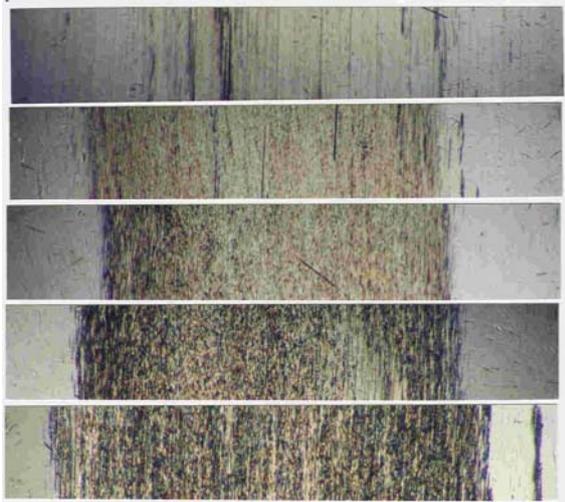


Figure 15. Sequence of wear tracks from five tests using diamond abrasives; particle size to film thickness ratio (from top to bottom) $D/h = 1$, $D/h = 2$, $D/h = 3$, $D/h = 6$, $D/h = 9$.

The particle size/film thickness ratio [24,25], particle shape [26], and hardness of the contact elements [27] all control whether the particle rolls or slides through the contact. The latter paper includes a useful map to distinguish regimes of two- and three-body abrasion was developed using results obtained from a ball cratering machine.

7.2. Modelling Wear by Debris Particles

One approach for the modelling of wear by hard particles is to determine the geometrical overlap between the particle and the contacting surface elements [24,27]. This has been done for the specific case of ball bearing wear by small rigid debris particles (for example the crushed fragments of brittle particles) in the oil [28]. The wear volume is determined as the sum of the material removal actions caused by each particle as it ploughs or tumbles through the contact.

$$V = \sum_{i=1}^n A_i d_i f_i$$

where n is the number of particles entrained into the contact, A is the area of the cross section with which they intrude into the rolling element surface, f is the proportion of that area that is removed as wear, and d is the distance they slide through the contact. Empirical coefficients were used to determine entrainment, and material removal factors. Sample results, shown in figure 16, demonstrate that the model gives an order of magnitude agreement with experimental data. This simplified approach neglects interactions between particles, the formation of groves on the surface, and any roughness effects.

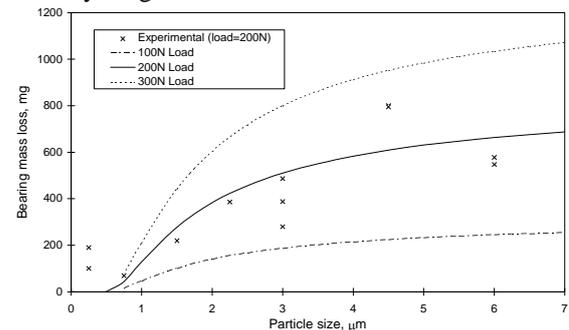


Figure 16. Bearing (6203 ball bearing) mass loss against particle size. Results of the model are compared with some experimental data (10 g/l diamond particles, 700k cycles).

8. CONCLUSION

The life of a debris particle begins at its creation by a wear or fatigue process, or when it is drawn into a lubrication system from the environment. The size and shapes of typical debris particles has been

well studied and provides a useful indicator for machine condition monitoring.

Debris particles have been shown to enter lubricated contacts (such as gears, rolling bearings, and journals) and under certain conditions the contact can significantly concentrate such particles. The particles are deformed or fractured by passage through the contact, and cause damage to the contacting element surfaces.

Determination of the relationship between surface dents and fatigue has been approached both experimentally and numerically. The effects are clear; the dent causes localised high stresses and fatigue cracking initiates at the dent edge.

If there is relative sliding between the elements then abrasive wear takes place. Particles have been shown to either roll or plough through the contact. The summation of many such small abrasive actions leads to gross material removal and eventual (if gradual) component failure.

The presence of debris is inevitable. The processes whereby these particles lead to fatigue, wear, and component failure have been well studied. Frequent oil change and continuous filtration ameliorate the effects of debris but at a cost. Perhaps future research could profitably be aimed at designing machine element for improved tolerance to debris attack; either by surface treatment, lubricant additives, or design to minimise particle entrainment.

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