

This is a repository copy of *Toward an Automated Methodology for the Valuation of Reliability*.

White Rose Research Online URL for this paper: http://eprints.whiterose.ac.uk/91047/

Version: Accepted Version

Article:

Leahy, C, Batley, R orcid.org/0000-0002-2487-850X and Chen, H (2016) Toward an Automated Methodology for the Valuation of Reliability. Journal of Intelligent Transportation Systems: Technology, Planning, and Operations, 20 (4). pp. 334-344. ISSN 1547-2450

https://doi.org/10.1080/15472450.2015.1091736

(c) 2015, Taylor & Francis. This is an Accepted Manuscript of an article published by Taylor & Francis in Journal of Intelligent Transportation Systems: Technology, Planning, and Operations on 18/09/15 available online:

http://www.tandfonline.com/10.1080/15472450.2015.1091736

Reuse

Unless indicated otherwise, fulltext items are protected by copyright with all rights reserved. The copyright exception in section 29 of the Copyright, Designs and Patents Act 1988 allows the making of a single copy solely for the purpose of non-commercial research or private study within the limits of fair dealing. The publisher or other rights-holder may allow further reproduction and re-use of this version - refer to the White Rose Research Online record for this item. Where records identify the publisher as the copyright holder, users can verify any specific terms of use on the publisher's website.

Takedown

If you consider content in White Rose Research Online to be in breach of UK law, please notify us by emailing eprints@whiterose.ac.uk including the URL of the record and the reason for the withdrawal request.



Towards an Automated Methodology for the Valuation of Reliability

Author Information
Mr Christopher Leahy (Corresponding Author)
Email: ts08cl@leeds.ac.uk
Affiliation 1:
University of Leeds, Institute for Transport Studies, University Road, Leeds, LS2 9JT United
Kingdom
Dr Richard Batley
Email: r.p.batley@its.leeds.ac.uk
Affiliation 1:
University of Leeds, Leeds, United Kingdom
Dr Haibo Chen
Email: h.chen@its.leeds.ac.uk
Affiliation 1:

University of Leeds, Leeds, United Kingdom

Abstract

There is a significant body of research related to the valuation of reliability in transportation. Such work has tended to rely on the Stated Preference (SP) methodology where respondents are asked to make trade-offs between the mean and standard deviation of travel time. The literature has suggested that a Revealed Preference (RP) methodology may provide an alternative means of estimating a value of reliability. In this paper we show how emerging data sources reveal travellers' preferences and, in combination with traditional choice modelling methods, can be used to estimate a value of reliability. We illustrate this RP methodology using smart card data from the multi-modal public transport network of London, UK. We are able to estimate a useable 'Reliability Ratio' for three of four public transport modes modelled, and account for an unexpected result based upon the data available.

1.0 Introduction

There has been debate in the literature surrounding reliability in transportation and its formal definition. Many authors use the term "travel time variation" (TTV) to indicate that some measure of the width of the travel time distribution is actually what is being discussed. Although research has introduced alternative methods for understanding reliability (e.g. the scheduling approach of Small, 1982) it is this simple idea of the width of the travel time distribution that will focus the discussion that follows.

It is clear from previous research that TTV is an important factor for explaining agents' travel behaviour (Eddington, 2006), but its valuation remains an active research strand. It has been noted that a Revealed Preference (RP) methodology may be instructive in the valuation of TTV, but the situations where it can be effectively applied are rare (Bates et al, 2001), and consequently this area remains underdeveloped. Instead there has been a reliance on Stated Preference (SP) studies in order to value TTV (Ettema and Timmermans, 2006; Batley and Ibáñez, 2012; Börjesson et al, 2012). Meta-analysis of these SP studies (e.g. Li et al, 2010; Carrion and Levinson, 2012), as well as expert workshops (e.g. de Jong et al, 2009), have been only partially successful in clarifying the value of TTV. What we will demonstrate in the work that follows is that an RP approach, drawing upon existing discrete choice modelling methods, can be employed to elicit a valuation of TTV.

This development is made possible by datasets generated by computerised systems associated with the provision of transport services, where both aggregate route performance and traveller

behaviour are observed. Examples of such datasets include those collected from consumer satellite navigation units, automatic number plate recognition cameras, and public transport smart card usage. It is the latter that we will exploit to demonstrate application of our methodology.

Moreover, the specific contributions of our paper are as follows:

- 1. To develop a RP methodology for the valuation of TTV
- 2. To utilise automatically generated datasets to provide valuation evidence for TTV
- To demonstrate the use of such a dataset with existing discrete choice modelling techniques

The paper is arranged as follows. In section two we will explore the concept of TTV and its valuation, before examining the literature on valuing reliability using both RP and SP methods. In section three we set up a basic TTV model and outline the discrete choice model specifications relevant to the valuation of TTV. In section four we introduce an empirical dataset, outline the context of the research in relation to the smart card literature, and apply the data with choice modelling methods. In section five we discuss the modelling results and conclude by commenting on the methodology developed.

2.0 Background

2.1 Travel Time Variation (TTV)

Travel time variation in the context we use it here refers to the widely known phenomenon that undertaking a given trip will not always take the same amount of time. The exact nature of this

⁴ ACCEPTED MANUSCRIPT

aspect of travel is considered in detail in the literature, which often differentiates between recurrent variation and incident related delay (Bates et al, 2001). For the purposes of this paper we take a simple approach; we assume that every available travel option is associated with a Normal distribution of travel times, and furthermore can be accurately estimated and utilised in travellers' decision making processes. This approach therefore explicitly draws a relationship between the supply (or performance) of a transport system and the demand response from travellers; an idea which is formalised by the mean-variance approach. This was initially developed in the field of finance (Markowitz, 1959), but has become the most common framework for understanding TTV.

The prevalence of the mean-variance approach is partly due to the simplicity of modelling the marginal value of travel time alongside the marginal value of travel time variation. Adoption of this approach has resulted in the acceptance of the Reliability Ratio (RR) in value of reliability studies, which is given by the ratio of the marginal utility of time risk to the marginal utility of mean travel time:

$$RR = \frac{\frac{\partial U}{\partial \sigma}}{\int_{\delta \mu}} (1)$$

Where $\frac{\delta U}{\delta \mu}$ is the marginal utility of the mean travel time μ with respect to utility U, commonly expected to be negative; and $\frac{\partial U}{\partial \sigma}$ is the marginal utility of the standard deviation of travel time σ with respect to U, also expected to be negative. The value of RR is therefore usually expressed as

a positive value. It is a value for RR that is estimated by the RP methodology outlined in the remainder of the present paper.

2.2 The Stated Preference (SP) Approach to the Valuation of Travel Time Variation

The preferred method of estimating the reliability ratio to date has been via SP surveys where participants respond to a range of hypothetical travel options and are asked to make trade-offs between travel time, TTV and cost.

A key reason for the prevalence of SP studies in this research area is that, while RP studies are preferable in principle, practical situations where RP can be conducted successfully are rare (Bates et al, 2001). The research community has therefore been largely comfortable in accepting the SP method for valuing TTV, as evidenced by the number of studies performed (see the review papers of Li et al, 2010; Carrion and Levinson 2012). SP route choice experiments have yielded useful insights into the behaviour of travellers in relation to TTV; here we provide a brief overview of recent studies, as a basis for developing an alternative method for estimating the reliability ratio.

The SP study conducted in the much cited paper of Bates et al (2001) formed only part of the contribution of that paper to the wider TTV literature. However section 6 of that paper, focussed on the SP experiment warrants attention in its own right. Presentational issues related to prior studies were considered in some depth; a debate which has become very much more prescient in the TTV literature due to the complexity of the subject (for a later overview of the issues see Tseng et al, 2008). It also employed a discrete choice experiment for the estimation of the value

of reliability. The study did not publish a RR due to confidentiality issues but nevertheless confirmed TTV as highly valued by passengers.

The study of Hollander (2006) was focussed upon empirically estimating a RR using SP. This work is quoted widely in the literature due to the outlying nature of its reported RR. Hollander (2006) was an attempt to empirically compare the mean-variance approach to the alternative scheduling approach (Small, 1982), which was performed using SP and discrete choice modelling. This paper again considered the issue of survey presentation. Hollander's reported reliability ratio of 0.1 was well below most other estimates. A possible explanation for the result may be due to the survey being web-based, where respondent-researcher interaction was limited and the demographic profile of respondents in the sample was unrepresentative of the wider population.

What the aforementioned SP studies would seem to indicate is that a broad range of RR estimates exist. Further evidence of this phenomenon is available in the meta-analyses of Li et al (2010), Carrion and Levinson (2012) and Wardman and Batley (2014). The variation of RRs might be due to a number of situational factors; for example the demographic characteristics of respondents or mode availability. However the focus on presentational issues within the literature (of Bates et al, 2001; Hollander 2009 among others) would suggest that they play an important role in the estimation of accurate RRs, particularly given the complexity of the subject (Tseng et al, 2008).

In addition to these problems, we might also draw the reader to well documented drawbacks of hypothetical choice questionnaires, summarised in a recent review of travel time reliability

(Wardman and Batley, 2014). Wardman and Batley suggested that a strategic bias may be observed given the contentiousness of TTV or lateness to travellers, particularly when the purpose of the study is clear to the respondent. Furthermore we might add that the general difficulties related to SP of misapprehension, fatigue and boredom experienced by respondents might be exacerbated when dealing with the complexity of TTV issues.

Returning to the wide range of RR estimates found in the aforementioned meta-analyses, we now focus on practical shortcomings of the use of SP studies to estimate a RR; these being the cost, resource and knowledge required to conduct such studies. A possible alternative approach for practitioners might be to 'transfer' a pre-existing RR which was calculated under similar circumstances (i.e. mode, geographic location etc.), but it is clear that an appropriate RR will not be available for all situations. This does not take into account issues around context and fungibility (Orr et al, 2012), yet this is the current position of practice (see WebTAG Unit A1.3 for evidence of this from the UK). The reason for this situation is a tacit acknowledgement of the difficulty and resources required for conducting a unique choice experiment for each occasion that a RR is required.

It is clear then from the above section that accurate estimates of RRs are necessary, but that current methods to estimate it raise questions over their suitability. An alternative or complementary solution to the estimation of RRs might be appropriate; one which overcomes:

- how to make the appropriate choice of RR from the broad range of literature available
- issues of SP respondent understanding of TTV
- presentational issues specifically related to TTV SP surveys

- general problems related to SP (strategic/protest responses)
- cost/resource required to estimate a local RR

To address these issues we turn our attention to the less well research area of RP based estimates of the RR.

2.3 The Revealed Preference (RP) Approach

A Revealed Preference (RP) approach in the transportation context is based upon what actual traveller choices can tell us about their preferences — more specifically how they might trade-off between travel time, TTV and other attributes of the journey. An RP based methodology would overcome the issues highlighted in the previous section by incorporating participants' actual decision making choices, including imperfections such as habitual behaviour and partial information (Wardman and Batley, 2014). In overcoming the major deficiencies of SP we therefore consider that RP is to be preferred (as suggested by Bates et al, 2001) and could potentially elicit a more realistic RR. In addition we envisage an RP only approach would utilise readily available datasets, the analysis of which could potentially be automated. This approach could therefore provide a cost-effective, rapid, temporal/location specific method of estimating a RR.

The general concerns with an RP-only approach are that insufficient situations might exist in a dataset for effective estimation of parameters, as well as difficulty for the analyst in knowing the completeness of travellers' information (as opposed to SP contexts where it can be considered perfect). We also note the difficulty specific to mean-variance applications where travel time and standard deviation of travel time have been found to be correlated (Batley et al, 2008; de Jong et

al, 2009). An additional criticism of RP based methods might be the restriction to situations where appropriate data are available (Bates et al, 2001). However, as the present paper hopes to demonstrate, this will become less of a problem as automatic data collection systems become more prevalent.

The literature on RP based studies is limited, reflecting the difficulty of obtaining suitable and sufficient datasets. The fullest account of their use in the TTV context is provided by Carrion and Levinson (2012), from which we note the following key studies. Several RP studies have used data from the SR-91 highway in California where drivers were observed making a choice between a tolled express route and a standard un-tolled section of freeway. The most widely cited of these studies (Small et al, 2005), analysed RP and SP data linked to census income data (and other demographic data) using a mixed logit discrete choice model. Of interest is the rejection of mean and standard deviation in their utility function, replaced with non-parametric indicators. It is also of note that this model represented only a single mode (private car). The combined RP/SP model gave an overall RR of 1.1, but an RP only model estimated separately did not converge. Lam and Small (2001) was a closer attempt at an RP only methodology using SR-91 data loop detector and demographic data, and estimated a RR of 0.77 for males and 1.45 for females. Other work has attempted to estimate reliability models based upon mode choice: Bhat and Sardesai (2006) utilised an RP/SP methodology and incorporated public transport modes into their model, estimating a RR of 0.26. Börjesson (2008) specified a mixed-logit discrete choice model using RP and SP data and estimated an overall RR of 0.76; this paper concluded that there are systematic differences between RP and SP datasets, in that RP represents longer term adaptation by travellers to travel conditions.

Issues affecting these studies were the effort and resource required to estimate their models, since they involved one or more of the following activities: contacting travellers directly to obtain information about their route choices, matching SP records to observed number plates, making assumptions about missing data, and/or staff involved in the study driving the route repeatedly to estimate travel time distributions. None of the studies were able to observe both supply and demand aspects of the transport systems of interest without directly asking participants, and this need for interaction with travellers potentially introduced some of the negative issues with SP that we outlined in the previous section. The mode choice modelled in Börjesson (2008) was also reliant upon discrete departure times of the private car which may have confused traveller preferences between car (which can usually depart continuously), and bus which is subject to timetabled (discrete) departure times.

In the following section we address these issues by providing a methodology and framework for the use of automatically collected data in estimating RRs without the need for extensive surveying.

3.0 Modelling Framework

3.1 Choice and Risk

The phrase 'choice under uncertainty' is often used in the literature on travel time variation, however we do not use it here to avoid ambiguity. We instead use the phrase 'choice under risk' to denote the situation where the distribution of outcomes is known by the traveller – a key assumption as we progress with our methodology. Although perhaps unrealistic, this is a simplifying assumption which will allow a RR to be estimated. Further work (and an extended

temporal dimension to the data) could take into account learning or knowledge effects on the part of the traveller. We do comply with standard practice in the field in that we utilise the axioms of expected utility theory (see von Neumann and Morgenstern, 1947). It has been shown that this approach can provide results at odds with actual human behaviour (Kahnemann and Tversky, 1979), but nevertheless it is accepted in the transportation literature as a useful tool for understanding risk. In accepting von Neumann and Morgenstern's (vNM) expected utility framework, and implementing this through a discrete choice model, we assume that each of the travellers observed will have well defined (complete) and stochastically transitive preferences.

3.2 Mean-Variance

The mean-variance framework of travel time variation will be utilised as a means for modelling passengers' behavioural choices. The expected utility function of this framework takes the following general form:

$$EU = \beta_1 \mu + \beta_2 \sigma \tag{2}$$

Where μ is equal to mean travel time and σ is the standard deviation of travel time, and β_1 and β_2 are preference parameters for these variables. This representation of travel time risk has received criticism for being dependent on a Normal distribution of travel times which is rarely observed in reality (van Lint et al, 2008). Small et al (2005) recognised this issue and replaced the mean travel time with the median, and the standard deviation with the difference between the 90th percentile and the median. Both the standard mean-variance and this non-parametric variant will be tested in the empirical work that follows.

3.3 Data Requirements

The RP estimation of a reliability ratio will be based upon travellers' choice responses to observable supply side metrics of a transport system. A requirement of this method is a dataset which can provide estimates of mean and standard deviation of travel time and records of travellers' route choices. Such datasets are becoming more prevalent and can include car based trips (e.g. using ANPR systems) or public transport modes – with the latter being more straightforward to implement due to the fixed nature of routing.

The setting up of an experiment must consider situations where all possible routes between an origin-destination pair can be observed, and suitable sample sizes are available on each route. There must also be some means of knowing which route is taken – through intermediate observation or some indication at the origin or destination point.

3.4 Choice Model Specification

We will employ a discrete choice model to represent the route choice of the traveller in this context. In the empirical section that follows, we apply three well-known discrete choice model specifications, namely multinomial logit (MNL), mixed multinomial logit (MMNL) and crossnested logit (CNL) (a useful background to these specifications is provided by Train, 2009). The logit model is the simplest and most well known of these, where the probability of individual n choosing alternative i from choice set J ($i \in I$, where j = 1, ..., I) is given by:

$$P_{ni} = \frac{e^{\beta' x_{ni}}}{\sum_{j} e^{\beta' x_{nj}}} \tag{3}$$

Where the observed portion of utility for individual n and alternative j (V_{nj}) is linear in parameters and is therefore given by the term $\beta' x_{nj}$ (where x_{nj} is a vector of observed variables for individual n and alternative j).

The basic logit specification requires the condition of independence from irrelevant alternatives (IIA) which is often not satisfied in real-world applications due to correlation between alternatives. This correlation can be represented by either a mixture or nested choice model specification. Our use of a mixture specification is with the purpose of representing heterogeneity across travellers, in line with similar previous studies (Small et al, 2005; Bogers et al, 2007). An alternative use of the mixed logit form is to induce correlation between the error terms of similar alternatives, thereby representing a hierarchical or nested, structure; described as the error components logit interpretation (Batley et al, 2004). The MMNL model is estimated over a density of parameters, and therefore takes the general form:

$$P_{ni} = \int \left(\frac{e^{\beta' x_{ni}}}{\sum_{i} e^{\beta' x_{nj}}}\right) f(\beta) d\beta. \tag{4}$$

In the case of the MMNL model, the value of P_{ni} is a weighted value of the MNL formula estimated at different values of β . The weighting of β is given by the form of $f(\beta)$ (Train, 2009).

A nested structure of the choice problem can be represented within the choice modelling software by explicitly defining similar mode alternatives to be contained within a single nest. More generally we note that an alternative might have membership of more than one nest, meaning that a natural specification for the problem at hand is the cross-nested logit (CNL)

model specification. Here we provide the probability function for the Generalised Nested Logit (GNL) model, which can be seen as generalised modelling framework wherein CNL represents a special case. In particular, the restriction of dissimilarity parameter between modes in a single nest $k(\lambda_k)$ to be equal in equation 5 results in a CNL specification (Wen and Koppelman, 2001):

$$P_{ni} = \frac{\sum_{k=1}^{K} (\alpha_{ik} e^{V_{ni}})^{\frac{1}{\lambda_k}} \left[\sum_{j \in B_k} (\alpha_{jk} e^{V_{nj}})^{\frac{1}{\lambda_k}} \right]^{\lambda_{k-1}}}{\sum_{k=1}^{K} \left[\sum_{j \in B_k} (\alpha_{jk} e^{V_{nj}})^{\frac{1}{\lambda_k}} \right]^{\lambda_k}}.$$
 (5)

The subscript k represents and indexation of the nests in the model: k = 1, ..., K. B_k denotes a nest of alternatives. α_{ik} is the allocation parameter which signifies the proportion of membership of alternative i to nest k.

4.0 Empirical Application of the Model

4.1 Smart Card Data

In our example we utilised a public transport smart card dataset. Public transport smart cards provide a unique research opportunity in their own right (for an overview see Pelletier et al, 2011). Researchers have recognised the value of smart cards in observing travel behaviour (Choi et al, 2012) and monitoring system performance (Jang, 2010), but we are unaware of any smart card research which incorporates both supply and demand sides of a transport system.

The data available for this experiment were the trips made by a 5% sample of Oyster Card users on the Transport for London (TfL) public transport network during a single, non-school holiday month in 2011. The primary dataset contained details of each trip made, including an encrypted

user ID, mode, time, date and location of the beginning and ending for a trip on rail based modes. Bus based trips did not have a recorded end point and so were excluded from the RP choice modelling that follows. The impact of this is considered in the next section.

The initial dataset contained approximately 11 million Oyster transaction records. For the purposes of this demonstration we decided to concentrate our analysis on the AM peak period. To create the relevant dataset we removed non-AM peak journeys (beginning outside 07:00 – 10:00), weekend journeys, and all bus journeys. It was found that statistically significant differences existed between the travel conditions during each of the three peak hours. Models were therefore estimated over the AM peak as a whole and by each of the three peak hours. Disaggregation of the data down to the hour level did however reduce the number of OD pairs available.

4.2 Identification of Suitable Data

We were unable to utilise the full dataset for our modelling purposes as many trips were made without indication of route choice. Consequently it was necessary to identify the origin destination (OD) pairs where a mode choice was observable. This was done by creating an OD trip matrix specific to each of the modes available. A database relationship was created between each of these matrices to identify ODs where two or more modes were used with a count of sample of travellers at 15 or greater. ODs were excluded where one of the choices was dominated; i.e. where both the mean and standard deviation of travel time were lower for a given option. Unfortunately two modes featured in the sample were represented on only three OD pairs. Preliminary experiments showed that estimating parameters on such few choice situations

would lead to unreliable parameter estimates and therefore these two modes were removed. These processes resulted in 31 candidate OD pairs being available for the AM peak model estimation, giving a total of 4,140 usable records (an average of 134 trips per OD). A dataset of the three individual peak hours (7-10) was also created, containing a subset of these 31 OD pairs where the sample size was adequate in a single peak hour. Using this hour based dataset it would be possible to estimate models where the travel conditions were related to the hour of travel (as opposed to the whole AM peak period). This dataset consisted of 2,061 records.

The bus based smart card records did not include the time or location of a passenger alighting the vehicle. This presented a challenge for accurately estimating both bus performance and passenger behaviour. We therefore estimated choice model parameters based solely upon rail based modes, and omitted the bus data from model estimation. An additional concern was that there were no highway based modes in the study – including private car. We made the simplifying assumption that car would (notionally) exist in an independent 'non PT' nest, and therefore the estimates within the PT nest were unbiased. The omission of the bus mode was more problematic as this would likely cause bias within the PT nest.

To address this concern we created a list of all 62 origin and destination stations that were in the choice dataset. Returning to the full dataset of 11 million records, we were able to identify all travellers that interacted with these origins and destinations, including trips made using bus for all or part of the trip. We were then able to link transfers between bus and other modes (drawing upon the work of Seaborn et al, 2009), and thereby identify instances of travellers using bus for all or part of their trip on one of the 31 OD pairs. Some evidence was found of a small number of

travellers using bus between five of the ODs, however the numbers were small (1 or 2 travellers for each of these OD pairs). On reflection it was decided to proceed to the analysis stage with the full complement of 31 AM peak OD pairs. An alternative would have been to exclude the OD pairs where there was evidence of bus use, but much data would have been lost.

4.3 Data Analysis

To populate the mean-variance part of the choice model, we assigned an ID to each of the traveller records within the dataset based upon which OD they were travelling between. We assigned a further separate ID to each traveller based upon the mode of transport they utilised. Within this dataset, the use of three public transport modes was identified:

- Light Rail also referred to as the Docklands Light Railway, or DLR
- Metro also referred to as London Underground, or LU
- Heavy Rail standard heavy rail services, including London Overground. Referred to as
 'Rail' in the model specification

Some journeys consisted of two of these modes and so two further modes choices were created which were a composite of two of the other modes. Figure 1 demonstrates how each of the six modes was composed. The method of grouping suggested that there would be correlation between the modes when they contained a common method of transport. This supports use of a cross nested logit structure when modelling this data (multinomial logit and mixed multinomial logit models will also be estimated, in line with many similar studies in this field (Carrion and Levinson, 2012).

For any one OD pair, there were only two of the four modes available. We were able to calculate a sample mean travel time and standard deviation by using both OD and mode ID (denoted by subscript m) – i.e. each OD had two sets of mean-variance values; one for each of the public transport travel choices available to the traveller. We present the results of this process in Appendix 1 of the present paper. Each individual record, representing a single trip, has assigned to it the mean and standard deviation for the two options available.

A concern remained regarding the issue raised in the literature of correlation between mean and standard deviation of travel time (Batley et al, 2008; de Jong et al, 2009), however our analysis based upon the information in Appendix 1 suggested that this was not an issue in the present context.

5.0 Choice Modelling

The models in this section were estimated using Biogeme (Bierlaire, 2003). We utilised four choice model specifications: multinomial logit, mixed logit, cross nested logit and cross nested logit with mixture parameters. Preliminary work indicated significant differences between travel time and risk parameter estimates for each mode: therefore interaction terms between the mean-variance parameters and mode-based alternative specific constants (ASCs) are included where possible. We present results from the four separate model specifications for the AM peak time period. We also estimated these same model specifications with an alternative dataset where a mean and standard deviation term were calculated for each of the three peak hours as described in section 4.2. This process did not improve the results or model fit when compared to the overall AM Peak models and are not included further in the paper. Another alternative approach is the

specification of a percentile based utility function as described in section 3.2. When modelled, this specification did not offer an improved model fit or results. Therefore the results reported in Table 1 are related to the standard mean-variance specification where travel time and standard deviation are calculated for the entire peak period. The expected utility for individual n is therefore determined by the mode used and the OD pair:

$$EU_{n,OD,m} = \beta_1 \mu_{OD,m} + \beta_2 \sigma_{OD,m} + \varepsilon_n \tag{6}$$

Where $EU_{n,OD,m}$ is the expected utility for individual n based on their OD pair (OD) and mode choice (m). $\mu_{OD,m}$ and $\sigma_{OD,m}$ represent the mean and standard deviation of the chosen OD pair and mode combination. β_1 represents the marginal utility of mean travel time. Similarly, β_2 represents the marginal utility of travel time variation. The error term, ε , varies between individuals, denoted by subscript n.

When using a mixed model specification, the parameter estimates were specified to be Normally distributed; this was judged appropriate to allow for a proportion (but not all) of the travellers to exhibit behaviour that placed a positive marginal utility on travel time and risk. We utilised a 2-tailed t-test for significance of all parameter estimates.

5.1 AM Peak Model Estimates

The AM peak models were run using the non-dominated dataset containing 4,140 trip records. Alternative-specific constants (ASCs) were estimated in all cases, except for Metro (London Underground) fixed to zero as the reference case. We found that interaction terms between the

mode ASCs and mean and standard deviation improved the model results, and they were therefore included in the MNL and MMNL models. The inclusion of interaction terms within the CNL models caused issues, and they were not therefore retained. We report four key model specifications here:

- Model 1 was a basic MNL specification with three ASCs estimated for each of the modes other than London Underground shown in Figure 1. β_1 and β_2 were estimated for the marginal utilities of mean travel time and standard deviation respectively. Each of the ASCs was interacted with mean travel time and standard deviation, so as to give mode specific estimates of the first and second moments.
- Model 2 utilised a CNL structure as indicated in Figure 1. The α term defined in equation 5 was allowed to vary, and each hybrid mode could belong to two nests i.e. LU/DLR could belong to both 'LU' and 'DLR' nests. In preliminary testing, the inclusion of interaction terms caused issues with the ASCs and allocation parameters, and they were therefore omitted from the final model. The ASCs and β s were specified in the same manner as Model 1.
- Model 3 was a mixed MNL model, specified identically to Model 1 with the exception that the β estimate on mean travel time and standard deviation was allowed to vary to represent stochastic traveller preferences.
- Model 4 was a combination of Models 2 and 3; it included the CNL structure and nesting
 parameters of Model 2, and the mixture parameters for the marginal utilities of travel
 time and risk from Model 3. Preliminary estimation of Model 4 showed a similar issue to
 Model 2, in that the model would not converge when the ASCs and interaction terms

were specified. In common with Model 2, the interaction terms were therefore omitted. With reference to Table 1, the parameter estimates of Model 1 are statistically significant with the exception of the risk parameter for the heavy rail mode and the travel time parameter for the LU/DLR mode. There are significant differences between the modes all else equal, as indicated by the ASCs (defined in relation to London Underground only mode). Furthermore, the significance of most of the mean-variance interaction terms shows that passengers' attitudes towards travel time and risk vary across modes. We observe that the marginal utility of travel time is negative as expected across all modes. We also observe that the marginal utility of risk is negative on three of the four modes modelled. The model fit indicated by the adjusted R² (of 0.31) is acceptable.

The CNL specification in Model 2 results in only one parameter estimate for both travel time and travel time risk. Both are negative which suggests a rejection of the null hypothesis, however the t-statistic of 1.95 on the risk parameter means that strictly speaking the null cannot be rejected at the 5% level. The ASCs are all significant, indicating differences between the modes all else equal. The R² and final log-likelihood measures of model fit are lower than Model 1, indicating a poorer fit to the data.

Model 3 estimates significant parameters similar to those estimated in Model 1, where the risk parameter for Rail is found to be insignificantly different from the base case. The mixture parameter on the marginal utility of travel time is significantly different from zero, but the mixture parameter on travel time risk is not. The marginal utility of travel time is negative for all

four modes, and the risk parameter is negative on three of the four modes. The R^2 and log-likelihood measures indicate that the fit of Model 3 is an improvement over Models 1 and 2.

In Model 4 it is found that both travel time and travel time risk parameters are significant and negative. All ASCs are significantly different from zero. In contrast to Model 3, both mixture parameters are statistically significant at the 5% confidence level. The fit of this model to the data is however the poorest of the four models estimated.

Table 2 shows the reliability ratios (RRs) estimated. Single RRs were estimated from Models 2 and 4 as they did not contain interaction terms for each mode. Travel time and travel time risk are generally assumed to be viewed as a 'bad' by travellers. Therefore, negative parameter estimates for mean travel time and standard deviation are expected, resulting in a positively valued RR. We apply these criteria here and consider negative RRs and RRs calculated using positive marginal utilities of travel time and risk to be of less practical use.

Focussing on Models 1 and 3, valid RR values have been calculated for three of the four modes included in this study. These modes are LU, LU/DLR and Rail. The positive parameter estimate of risk on the LU/Rail mode meant that a negative (and unsatisfactory) RR was estimated for this mode. We use meta-analysis of Carrion and Levinson (2012) as a comparator for our results. They reported average RRs from 17 studies, with a range of 0.1 to 2.51 and a mean average of 1.09. The RR estimates for the LU mode are 0.54 and 0.35, which are low compared to the comparator value of 1.09, but are not unreasonable in relation to some estimates in the literature (e.g. Bhat and Sardesai, 2006). Estimates for the LU/DLR RR of 2.29 and 1.14 are reasonable in light of the comparison value. The estimates for the mode LU/Rail were negative across all four

models, as the parameter estimate of risk for this mode was always estimated as positive. Further investigation into this counter-intuitive result reveals that when the LU/Rail mode is available, travellers are more likely to use it when the standard deviation is higher on this mode when compared to the alternative. This is plotted for all twelve LU/Rail OD pairs, where a general positive relationship between the variables is indicative of risk seeking behaviour. It should be noted that these same travellers are also strongly averse to travel time.

Mode specific parameter estimates were not made by Models 2 and 4. This results in only one RR value estimated from each as shown in Table 2. Whilst acknowledging that the risk parameter in Model 2 is borderline significant (as shown in Table 1), the RR implied by this model was 0.40. Although low, this is acceptable in relation to the literature. The RR of 0.71 implied by Model 4 appears more in line with the literature, and demonstrates a degree of sensitivity of the RR to the model specification.

These model results provide evidence that a dataset such as that provided by a public transport smart card system can be used in the estimation of marginal utilities of travel time and travel time risk. Model 1 provides acceptable RR estimates for three of the four modes modelled, in the range 0.54 to 2.29. Model 3 also estimates three useable RR values between 0.25 and 1.14. Models 2 and 4, without interaction terms, estimated single RR values which could be used over all modes in an appraisal.

6.0 Conclusion

In this paper we have demonstrated a method of estimating a reliability ratio using revealed preference data. We initially outlined the benefit of undertaking the research; both in terms of the

limitations of the alternative stated preference method (general suitability, high survey cost) and the positive aspects of revealed preference (realism, low survey cost). We combined standard economic and choice modelling approaches to form the basis for introducing the possibility of estimating the reliability ratio from automatically collected data only. We then used one such data source, public transport smart card data, to demonstrate the method. We were able to estimate a realistic RR across three alternative modes during the AM peak period. The majority of these estimates were within an acceptable range when compared to the literature. A range of RR estimates in the literature highlights the importance of estimating a RR which is relevant to its application, an issue which the methodology outlined in this paper could provide a solution.

The unexpected result of a positive risk parameter on one of the modes was partially accounted for by the data; travellers were seemingly primarily motivated by travel time savings and not by travel time risk. In addition, a greater sample size and therefore an increased number of OD pairs in the choice dataset may assist with this issue. There is also the possibility that there are omitted

The assumption of perfect knowledge on the part of every passenger is also unrealistic. A development of the RP method would take into account variability in experience of passengers

variables from our model which bias the results. For example, station size or accessibility to

platforms may impact upon travellers mode choice, but this was not explicitly represented in the

model. The relatively small sample sizes at the OD level of aggregation meant that it was mainly

large stations that featured within the dataset, and a station size variable could not be easily

modelled. Such omitted variables are likely to be the reason for seemingly dominated choice

options which were nevertheless utilised by travellers.

on a route – this could be performed more accurately where the dataset covered a longer temporal duration than one month. In addition, future research might consider datasets that contain a cost variable so that monetary values of reliability can be estimated directly, thus overcoming the issue of fungibility when utilising a RR alongside separately estimated values of time (Orr et al, 2012).

We would recommend that further empirical research is conducted, where full population smart card datasets are collected to overcome issues with sample sizes and to include a wider range of modes and geographic locations. Additional data collection to take account of further variables should also improve the robustness of RP choice models.

Acknowledgements

The authors would like to thank both the ESRC White Rose Doctoral Training Centre and Transport for London for funding this research. We would also like to thank the reviewers for their comments on earlier versions of the paper. The work contained within this paper is the sole responsibility of the authors and does not reflect the views of the funding organisations or reviewers.

References

BATES, J., POLAK, J., JONES, P. & COOK, A. (2001). The valuation of reliability for personal travel. Transportation Research Part E: Logistics and Transportation Review, 37, 191-229.

BATLEY, R. P., TONER, J. P., & KNIGHT, M. J. (2004). A mixed logit model of UK household demand for alternative-fuel vehicles. Rivista Internazionale Di Economia Dei Trasporti. 31, 55-77.

BATLEY, R., GRANT-MULLER, S., NELLTHORP, J., DE JONG, G., WATLING, D., BATES, J., HESS, S. & POLAK, J. (2008). Multimodal Travel Time Variability. Institute for Transport Studies. Leeds, UK.

BATLEY, R. & IBÁÑEZ, J. N. (2012). Randomness in preference orderings, outcomes and attribute tastes: An application to journey time risk. Journal of Choice Modelling, 5, 157-175.

BIERLAIRE, M. (2003). BIOGEME: A free package for the estimation of discrete choice models, Proceedings of the 3rd Swiss Transportation Research Conference, Ascona, Switzerland.

BHAT, C. R. & SARDESAI, R. (2006). The impact of stop-making and travel time reliability on commute mode choice. Transportation Research Part B: Methodological, 40, 709-730.

BOGERS, E., VITI, F., HOOGENDOORN, S. & VAN ZUYLEN, H. (2007). Valuation of Different Types of Travel Time Reliability in Route Choice: Large-Scale Laboratory Experiment. Transportation Research Record: Journal of the Transportation Research Board, 1985, 162-170.

BÖRJESSON, M. (2008). Joint RP-SP data in a mixed logit analysis of trip timing decisions. Transportation Research Part E: Logistics and Transportation Review, 44 (6), 1025–1038.

BÖRJESSON, M., ELIASSON, J. & FRANKLIN, J. P. (2012). Valuations of travel time variability in scheduling versus mean–variance models. Transportation Research Part B: Methodological, 46, 855-873.

CARRION, C. & LEVINSON, D. (2012). Value of travel time reliability: A review of current evidence. Transportation Research Part A: Policy and Practice, 46, 720-741.

CHOI, J., LEE, Y., KIM, T. & SOHN, K. (2012). An analysis of Metro ridership at the station-to-station level in Seoul. Transportation, 39, 705-722.

DEPARTMENT FOR TRANSPORT, (2014). WebTAG Unit A1.3: User and Provider Impacts. London

ETTEMA, D. & TIMMERMANS, H. (2006). Costs of travel time uncertainty and benefits of travel time information: Conceptual model and numerical examples. Transportation Research Part C: Emerging Technologies, 14, 335-350.

HOLLANDER, Y. (2006). Direct versus indirect models for the effects of unreliability. Transportation Research Part A: Policy and Practice, 40, 699-711.

HOLLANDER, Y. (2009). What do we really know about travellers' response to unreliability? The International Choice Modelling Conference, Harrogate, UK

JANG, W. (2010). Travel Time and Transfer Analysis Using Transit Smart Card Data.Transportation Research Record: Journal of the Transportation Research Board, 2144, 142-149.

DE JONG, G., KOUWENHOVEN, M., KROES, E., RIETVELD, P. & WARFFEMIUS, P. (2009). Preliminary Monetary Values for the Reliability of Travel Times in Freight Transport. European Journal of Transport and Infrastructure Research, 9, 83-99.

EDDINGTON, R. (2006). The Eddington Transport Study. HM Treasury. London, UK.

KAHNEMANN, D. & TVERSKY, A. (1979). Prospect theory: An analysis of decision under risk. Econometrica: Journal of the Econometric Society, 263-291.

KNIGHT, F. H. (1921). Risk, Uncertainty, and Profit. Hart, Schaffner, and Marx Prize Essays, no. 31. Boston and New York: Houghton Mifflin.

LAM, T. C. & SMALL, K. A. (2001). The value of time and reliability: measurement from a value pricing experiment. Transportation Research Part E: Logistics and Transportation Review, 37, 231-251.

LI, Z., HENSHER, D. A. & ROSE, J. M. (2010). Willingness to pay for travel time reliability in passenger transport: A review and some new empirical evidence. Transportation Research Part E: Logistics and Transportation Review, 46, 384-403.

VAN LINT, J. W. C., VAN ZUYLEN, H. J. & TU, H. (2008). Travel time unreliability on freeways: Why measures based on variance tell only half the story. Transportation Research Part A: Policy and Practice, 42, 258-277.

ORR, S. HESS, S & SHELDON, R (2012) Consistency and fungibility of monetary valuations in transport: an empirical analysis of framing and mental accounting effects. Transportation Research Part A: Policy and Practice, 46 (10). 1507 - 1516.

PELLETIER, M.-P., TRÉPANIER, M. & MORENCY, C. (2011). Smart card data use in public transit: A literature review. Transportation Research Part C: Emerging Technologies, 19, 557-568.

SEABORN, C., ATTANUCCI, J. & WILSON, N. (2009). Analyzing Multimodal Public Transport Journeys in London with Smart Card Fare Payment Data. Transportation Research Record: Journal of the Transportation Research Board, 2121, 55-62.

SMALL, K. A. (1982). The Scheduling of Consumer Activities: Work Trips. The American Economic Review, 72, 467-479.

SMALL, K. A., WINSTON, C. & YAN, J. (2005). Uncovering the Distribution of Motorists' Preferences for Travel Time and Reliability. Econometrica, 73, 1367-1382.

TRAIN, K. E., (2009). Discrete choice methods with simulation. 2nd Edition, Cambridge University Press.

TSENG, Y.-Y., VERHOEF, E., DE JONG, G., KPOUWENHOVEN, M. & VAN DER HOORN, T. (2008). A pilot study into the perception of unreliability of travel times using in-depth interviews. Journal of Choice Modelling, 2, 8-28.

WARDMAN, M. & BATLEY, R. (2014). Travel time reliability: a review of late time valuations, elasticities and demand impacts in the passenger rail market in Great Britain. Transportation, 1-29.

WEN, C.-H. & KOPPELMAN, F. S. (2001). The generalized nested logit model. Transportation Research Part B: Methodological, 35, 627-641.

Appendix 1 – Observed traveller choices and mean-variance performance data for the AM peak OD pairs

				Mo	ode 1			Mode 2		
	Mod	Mode	Cou	Me	Medi	St	Cou	Me	Medi	St
	e 1	2	nt	an	an	Dev	nt	an	an	Dev
Arnos Grove_Canary		LU/D	62.0	55.		13.1	20.0	55.		
Wharf	LU	LR	0	48	57.00	9	0	55	55.00	7.04
Woodside Park_Canary		LU/D	52.0	55.			40.0	56.		
Wharf	LU	LR	0	44	55.00	5.07	0	00	55.50	3.65
		LU/D	142.	50.			22.0	52.		
Morden_Canary Wharf	LU	LR	00	35	49.00	6.04	0	91	52.00	4.16
East Finchley_Canary		LU/D	27.0	47.			24.0	49.		
Wharf	LU	LR	0	22	47.00	6.31	0	25	49.00	3.64
		LU/D	54.0	46.			26.0	46.		
Debden_Canary Wharf	LU	LR	0	35	45.00	5.19	0	77	47.50	3.51
Colliers Wood_Canary		LU/D	69.0	44.			19.0	49.		
Wharf	LU	LR	0	94	43.00	5.20	0	21	48.00	4.30
Hornchurch_Canary		LU/D	85.0	44.			23.0	43.		
Wharf	LU	LR	0	41	43.00	5.57	0	00	42.00	7.05

Hampstead_Canary		LU/D	30.0	44.			39.0	44.		
Wharf	LU	LR	0	17	43.00	4.96	0	92	45.00	3.25
Kentish Town_Canary		LU/D	19.0	38.			32.0	37.		
Wharf	LU	LR	0	95	39.00	3.76	0	38	36.50	4.21
Barkingside_Canary		LU/D	37.0	38.			15.0	41.		
Wharf	LU	LR	0	70	38.00	3.84	0	67	42.00	1.95
Buckhurst Hill_Canary		LU/D	119.	38.			15.0	41.		
Wharf	LU	LR	00	50	37.00	6.85	0	67	41.00	6.24
Shepherds		LU/D	96.0	37.			28.0	43.		
Bush_Canary Wharf	LU	LR	0	47	36.00	5.56	0	25	42.00	3.30
Clapham South_Canary		LU/D	355.	36.			43.0	41.		
Wharf	LU	LR	00	50	35.00	6.06	0	65	40.00	5.79
Gants Hill_Canary		LU/D	143.	33.			34.0	37.		
Wharf	LU	LR	00	21	32.00	5.44	0	21	37.00	4.26
South										
Woodford_Canary		LU/D	226.	31.			73.0	35.		
Wharf	LU	LR	00	78	31.00	5.16	0	08	34.00	4.37
Snaresbrook_Canary		LU/D	100.	31.			23.0	34.		
Wharf	LU	LR	00	50	30.00	6.21	0	35	33.00	5.84
Old Street_Canary		LU/D	71.0	23.			50.0	24.		
Wharf	LU	LR	0	03	22.00	4.50	0	58	24.00	3.60

		LU/D	179.	22.			34.0	26.		
Leyton_Canary Wharf	LU	LR	00	08	21.00	5.06	0	21	26.00	4.05
Ealing		LU/R	49.0	41.			35.0	42.		
Broadway_Kings Cross	LU	ail	0	67	41.00	8.17	0	86	42.00	5.36
		LU/R	25.0	30.		13.9	22.0	35.		
Paddington_Old Street	LU	ail	0	68	33.00	3	0	45	34.00	5.10
Walthamstow		LU/R	108.	32.			15.0	35.		
Central_Farringdon	LU	ail	00	79	32.00	4.53	0	40	36.00	4.42
Tooting		LU/R	280.	27.			16.0	28.		
Broadway_Victoria	LU	ail	00	00	26.00	5.35	0	75	28.50	2.91
Vauxhall_Canary		LU/R	37.0	31.			73.0	28.		
Wharf	LU	ail	0	14	30.00	5.20	0	67	27.00	5.89
Seven Sisters_Bethnal		LU/R	20.0	37.			23.0	26.		
Green	LU	ail	0	15	36.50	2.30	0	74	26.00	5.51
Harrow On The		LU/R	19.0	33.			34.0	21.		
Hill_Marylebone	LU	ail	0	89	33.00	4.54	0	21	19.50	5.04
			546.	13.			23.0	12.		
Brixton_Victoria	LU	Rail	00	73	13.00	2.53	0	13	10.00	4.73
		LU/R	68.0	28.			46.0	39.		
Richmond_Victoria	Rail	ail	0	91	28.00	6.28	0	15	39.00	4.59
Motspur Park_Victoria	Rail	LU/R	19.0	33.	32.00	5.97	20.0	38.	37.50	3.35

		ail	0	37			0	20		
Wimbledon_London		LU/R	16.0	38.			19.0	36.		
Bridge	Rail	ail	0	63	36.00	8.21	0	47	34.00	8.83
Earlsfield_London		LU/R	18.0	34.			22.0	31.		
Bridge	Rail	ail	0	61	35.00	3.22	0	23	30.00	5.75
Clapham										
Junction_London		LU/R	50.0	29.			111.	26.		
Bridge	Rail	ail	0	50	29.50	4.01	00	07	24.00	5.34

Abbreviations:

Mean – Sample mean travel time for OD and mode

St Dev - Sample standard deviation of travel time for OD and mode

LU – London Underground

DLR – Docklands Light Rail

Rail – Overground and/or National Rail

Table 1 – Parameter estimates and summary statistics of AM peak choice models

	Model	1 -	Model	2 -	Model 3	- MNL	Model 4	- CNL
	MNI	NL CNL			Mixt	Mixed		
Utility	Estimat	t-	Estimat	t-			Estimate	
Parameters	es	test	es	test	Estimates	t-test	S	t-test
ASC_LU	0.00		0.00		0.00		0.00	
ASC_LUDLR	0.85	2.8	-0.85	2.52	3.53	3.70	-0.90	-11.40
ASC_LURAIL	-5.05	9.0 8	-1.53	2.90	-8.10	-5.01	-1.68	-8.94
ASC_RAIL	-5.13	9.1 1	-3.34	14.3	-8.48	-5.05	-2.60	-10.94
B_RISK	-0.09	3.2	-0.10	1.95	-0.31	-2.15	-0.21	-4.47
B_RISK_LUDLR	-0.30	- 7.1	-	-	-0.73	-4.28	-	-

		4							
B_RISK_LURAI	0.44	5.7			0.38	2.28			
L		8	-	-			-	-	
B_RISK_RAIL	0.04	0.4			0.14	0.68			
		5	-	-			-	-	
		-		_					
B_TIME	-0.17	9.1	-0.25	2.87	-0.89	-3.61	-0.29	-4.21	
		3							
B_TIME_LUDL		-							
R	-0.01	1.1			-0.03	-2.26			
		4	-	-			-	-	
B_TIME_LURAI	0.06	3.5			0.14	3.17			
L		0	ı	1			1	-	
B_TIME_RAIL	0.12	7.7			0.21	4.46			
		8	-	-			-	-	
B_RISK_MIXTU	-	-			-0.15	-0.45	-0.23	-2.01	
RE			-	-					
B_TIME_MIXT	-	-			1.12	3.48	0.32	2.65	
URE			-	-					
Model									
Parameters									

-	-	1.00	0.00	-	-	1.00	0.00
		0.76	1.44			0.64	2.85
-	-	0.73	2.36	-	-	1.00	0.01
		0.25	0.47			0.36	1.58
		0.86	1.63			0.95	18.18
-	-	0.22	2.97	-	-	8.18	0.52
		0.15	0.28			0.05	1.03
4,140		4,140		4,140		4,140	
0.312		0.289		0.318		0.281	
1964.70		2028.26		-1944.63		-2049.98	
	4,140	4,140	0.76 0.73 0.25 0.86 0.22 0.15 4,140 4,140 0.312 0.289	- - 0.76 1.44 - - 0.73 2.36 - 0.25 0.47 - 0.86 1.63 - - 0.22 2.97 0.15 0.28 4,140 4,140 0.312 0.289 - - -	0.76 1.44 0.73 2.36 0.25 0.47 0.86 1.63 0.15 0.28 0.15 0.28 4,140 4,140 4,140 0.312 0.289 0.318 - - - - - - - - - - - - - -	0.76 1.44 - 0.73 2.36 - - 0.25 0.47 - - 0.86 1.63 - - - 0.15 0.28 - 0.15 0.28 - - 4,140 4,140 4,140 4,140 0.312 0.289 0.318 - - -1944.63	0.76 1.44 0.64 0.73 2.36 - - 1.00 0.25 0.47 0.36 0.95 0.86 1.63 0.95 - 8.18 0.15 0.28 0.05 0.05 4,140 4,140 4,140 4,140 0.312 0.289 0.318 0.281 -1944.63 -2049.98

Table 2 – Reliability ratios estimated by Models 1 to 4

	Model			Model		Model 3					Model 4			
	$\delta U/\delta$	σ	Relia bioney Ratio	δ^{U}/δ^{U}	σ	Relia biomy Ratio	δ^{U}/δ^{G}	$\delta U/\delta$	Relia µbility Ratio	δ^{U}/δ	σ	Relia bi v y Ratio		
LU	-0.09	- 0.	0.54	-0.10	- 0.	0.40	-0.31	-0.89	0.35	-0.21	- 0.	0.71		
/	0.40	17			25		1.04	0.01			29			
LU/	-0.40	-	2.29				-1.04	-0.91	1.14					
DLR		0. 17												
LU/	0.35	-	-3.11	-			0.07	-0.75	-0.10					
Rail	0.55	0.	3.11				0.07	0.75	0.10					
		11												
Rail	-0.09	-	1.72				-0.17	-0.68	0.25					
		0.												
		05												

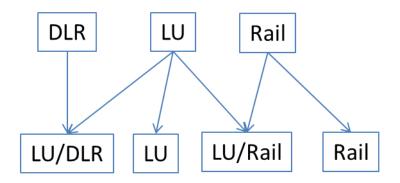


Figure 1 – Mapping of rail based modes to composite modes used in subsequent modelling

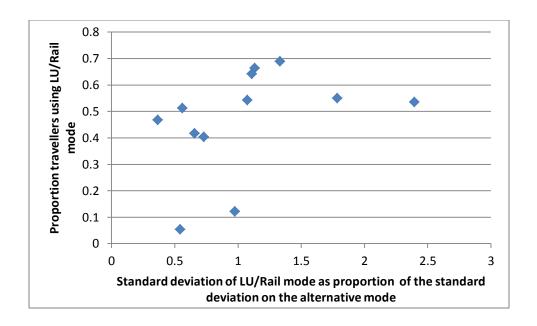


Figure 2 – OD plot of the standard deviation of LU/Rail mode against proportion using the LU/Rail mode