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MODELING AND SIMULATION OF TAWAF AND SA'YEE: A SURVEY OF RECENT WORK IN THE FIELD

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ABSTRACT

Between 2002 and 2012 the number of pilgrims taking part in the 5-day *hajj* (the annual pilgrimage to Mecca) rose dramatically from 1.9m to 3.2m, before stabilizing at around 2 million following the introduction of new quotas in 2013. The gathering together of so many people has obvious crowd-safety implications, ranging from stampedes and protests to pickpocketing and disease control, and there is an obvious need for models and simulations of the relevant crowd behaviours. Moreover, the regular occurrence of the event, the size and diveristy of the crowds involved, and the amount of freely available information make this an excellent case study for the study of crowd behaviour. We survey recent attempts to model the key *hajj* rituals of *tawaf* (during which pilgrims collectively circumambulate the *Ka'aba* seven times) and *sa'yee* (running or walking seven times between two nearby hills), and highlight ways in which some of the limitations of these studies may be overcome in future work.

INTRODUCTION

One of the five pillars of Islam is the 5-day *hajj*, or annual pilgrimage to Mecca. Each adult Muslim is expected to perform the *hajj* at least once during their lifetime, unless prevented by health or financial considerations. To put this in perspective, in 2010 Muslims constituted over 23% of the global population, or more than 1.6 billion people, and this number is forecast to rise to around 2.2 billion by 2030 (Grim and Karim 2011). Clearly, therefore, the number of visitors to Mecca during the *hajj* is considerable, and can be expected to rise still further in the coming decades (pilgrimages at other times of year are referred to as *umrah*, and similar considerations apply). This has obvious implications for crowd control and safety, and many attempts have consequently been made to model and simulate the relevant crowd behaviours. In this paper we survey recent work in this area, and highlight ways in which some of the limitations of these studies may be overcome in future work.

THE TAWAF AND SA'YEE RITUALS

The *hajj*, or annual pilgrimage to Mecca, involves the performance of several interlinked rituals, of which two of the most crowd-intensive are *tawaf* and *sa'yee*. Both rituals are performed on specific days each year at Islam's most sacred mosque, the *Masjid al-Haram*, in Mecca. This is a large complex which currently covers some 356,800 square metres, and work in underway to expand this to 1.1m square metres by 2016, by which time the mosque will be capable of housing 2.5m worshippers (Cheruppa 2015).
During *tawaf* ("circling"), each pilgrim circumabulates the *Ka'aba*, a cuboid building at the centre of the mosque, seven times in an anticlockwise direction. The area around the *Ka'aba* where this is carried out is called the *Mataf*. This is followed by *sa'yee* ("ritual walking"), during which pilgrims run or walk seven times along a corridor joining the nearby hills of Safa and Marwah. The corridor and hills are incorporated within the mosque, in an area called the *mas'a* gallery. The requirement that *tawaf* and *sa'yee* be performed as part of a coordinated series of rituals during the *hajj* leads to significant restrictions in both space and time, which in turn create high crowd densities throughout the *tawaf* ritual (Fig. 1).

THE CHALLENGE

There are many properties of the *tawaf* ritual that make its simulation difficult yet very interesting for the research field of crowd simulation, including complex flows of motion, changeable velocities, high densities and heterogeneous populations (Curtis et al. 2011). According to Janajrah and Virk (2014) the most significant factors include:

- the geometry of the *Mataf* area (Fig. 1);
- the location of key historical and ritual attractions;
- certain behaviours adopted by pilgrims, including chain-like movements and clustering,

and we may also include

- the size of the crowd gathered, containing in particular a high number of disabled individuals, as well as family and peer groups;
- the annual recurrence of the event, which allows us to make and test simulation hypotheses, and gather further evidence in the real world.

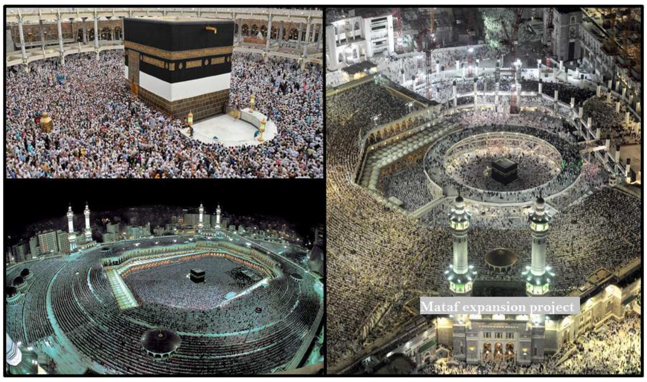

Figure 1: Three views of *tawaf* taking place within the *Mataf* region before (left) and after (right) the expansion project (Saudi Press Agency)

In particular, congestion at bottlenecks can cause pilgrims to move in conflicting directions simultaneously, giving a high risk of collision. This is further complicated by the heterogeneous nature of the population, since different groups of pilgrims move in different ways and by different means (quickly vs slowly; individuals vs groups; foot vs wheelchair). The development of appropriate models is essential to help identify triggers for key behaviors within dense crowds, e.g., trampling, falling, collisions, pushing, etc. In particular, the ability to simulate groups of people within the crowd is important, because such groups can act as obstacles in their own right.

Although we do not address it here, it is worth also noting the problem of *validation* – how do we know whether a simulation gives a reliable approximation to real-world behaviours? Current research relies on analysis of video footage, so as to identify real-world parameters against which simulations could be compared (Dridi 2014), e.g., paths followed by family members can be expected to cluster more than those of strangers. This task has been greatly simplified by the advent of modern technologies. For example, Koshak and Fouda (2008) used a variety of Geographic Information Systems (GIS) and Global Positioning Systems (GPS) to monitor the movement of pedestrians performing *tawaf*, and then used tracking-analysis software to analyze and visualize pilgrim movement patterns. As well as providing historical data for benchmark purposes, this kind of approach can be seen as a possible alternative to simulation, since it helps reveal real-world patterns that can inform future enhancements to the urban and architectural environment.

THE STUDIES

During the *tawaf* ritual, a large number of individuals move as members of an extensive crowd. It is natural, therefore, to break the system into two parts, one representing high-level crowd dynamics, and the other representing individual deviations from the norm. Curtis et al. (2011) proposed using a finite-state machine for high-level behavioural modelling, combined with an agent-level collision avoidance strategy. This approach allows the system to be calibrated by matching the overall distribution of simulated pedestrian velocities to observed values, while agents can be configured to exhibit behaviours like pausing to perform sub-rituals, entering and leaving specific floors in the *Masjid al-Haram*, joining and leaving queues, and circling the *Ka'aba*. Their model behaves well even in the presence of excessive crowd densities, and many of the simulation outcomes matched those observed when watching *tawaf* performing by actual people. In addition, the system is general enough to allow a wide range of distinct behaviours to be introduced in future studies, such as agents representing different combinations of age and gender. Nonetheless, the approach has important limitations. For example, because the underlying focus in most crowd simulations is on individuals moving within the assembled crowd, it is not easy to discover what might happen if pilgrims gravitate together in small groups, as for example during *tawaf*. Such considerations are of paramount importance when modelling disaster evacuation scenarios, since group dynamics can seriously affect individual reasoning. Moreover, it is not enough simply to match overall velocity distributions, since these vary with the pilgrims' location within the *Mataf* region, individual capabilities and group affiliations. Instead, we suggest that detailed velocity profiles need to be compiled for each of the main landmarks within the *Masjid al-Haram*, and for each type of pilgrim modeled, along with his or her group type. This leads to another approach to simulating crowd dynamics, which is to rely entirely on agent-based modelling (ABM). Rather than imposing high-level dynamics on the crowd, the focus is on modelling individuals and groups of individuals; running enough instances of crowd members then generates crowd behaviours as 'emergent phenomena'. The basic geometry of the *Masjid al-Haram* in this case can be modelled by treating walls, barriers, attractions, etc., as agents in their own right, with specific ways of interacting with pilgrim-agents (Karmakharm et al. 2010); other known navigation techniques can also be utilised. Khan and McLeod (2012) used this technique to investigate the effects of three parameters – the *Masjid al-Haram* courtyard layout, pilgrim crowd properties, and the management preferences of the *hajj* authorities – on the safety (lack of collisions), health (spread of disease), satisfaction (calm environment) and throughput of pilgrims. This work provided many recommendations and insights for the crowd-management priorities during the *hajj*, and could be further by the development of an appropriate crowd simulator capable of modelling large-scale experiments, capturing the key behavioral metrics and features of the system.

Mulyana and Gunawan (2010) adopted a similar approach, simulating pilgrims as *intelligent agents* (Fig. 2), and again found that the simulation of the *hajj* crowd matched the practical performance of pilgrims during *tawaf* and *sa'yee*. Their system has the additional advantage that it can be used to help train pilgrims before they travel to the *Masjid al-Haram* (other models can also be used for this purpose; see, e.g. (Yasin et al. 2012)). Although the additional computational requirements involved in simulating intelligence mean that pilgrim numbers currently have to be kept relatively low in the simulations, such problems could be alleviated in part by applying quantization within a virtual environment equipped with kinetic data structures and developing better collision-detection algorithms.

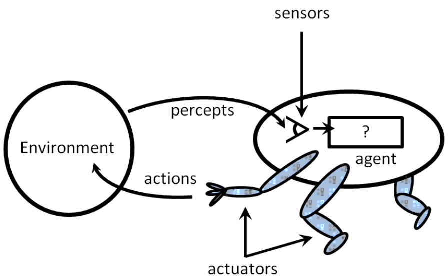

Figure 2: Intelligent Agents (Russell and Norvig 2009)

The activities of *tawaf* have also been modeled by Haghighati and Hassan (2013) using the discrete-event simulation tool ARENA. Pilgrims were represented as discrete units which repeatedly queue to enter and flow within the system, eventually leaving it after seven iterations (Fig. 3). Many factors were investigated within the model, including the behavior of switching throughout the *tawaf*, the availability of space, group sizes and inter-arrival times between pilgrims. Their results suggest that a key factor affecting crowd densification is the practice by pilgrims of lane-switching during performance of *tawaf*, and that this also reduced the efficiency of queueing strategies; this led them to suggest the use during *tawaf* of scheduling, spiral paths and clear separation. Indeed, their work suggests that organizing the *tawaf* to be performed along specific routes, and scheduling how pilgrims enter the *Mataf* region, will help prevent congestion and maintain crowd densities at sustainable levels, thereby generating a better experience for pilgrims.

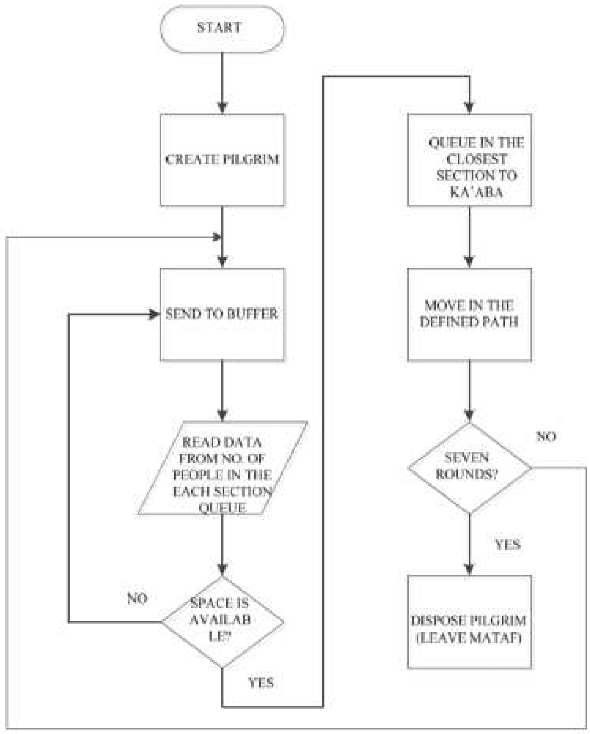

Figure 3: Modelling the behaviour of individual pilgrims (Haghighati and Hassan 2013)

Another approach giving realistic simulations is to employ concepts from virtual physics (Fig 4). Kim et al. (2015) treat pilgrims within the crowd as objects interacting with obstacles via physical forces, while enabling agents to anticipate collisions and take appropriate action to avoid them. Their approach reproduces several emergent properties and allows the simulation of a few thousand agents. While the agent numbers are small relative to the crowd sizes required for *hajj* simulations, the increasing efficiency of high performance computing resources offers hope that their work can soon be expanded to include systems involving millions of agents.

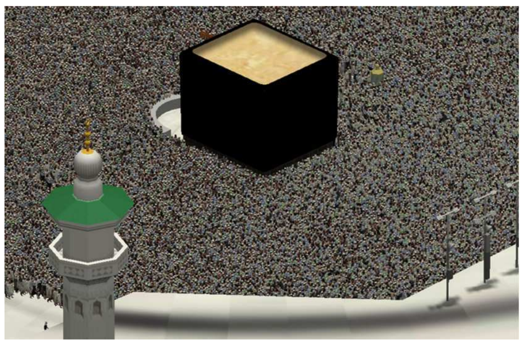

Figure 4: Simulation of *tawaf* by Kim et al. (2015)

Zainuddin and her colleagues have produced a series of useful studies in recent years. Zainuddin et al. (2009; 2010) used SimWalk, a software system that allows the design and analysis of flows through pedestrian areas, to model crowds operating under the Social Force Model (SFM), a complex approach based on the use of second-order differential equations to represent the forces acting on individuals. Their simulation allowed them to test, for example, a proposal (Al-Haboubi and Selim 1997) for the construction of a spiral path, showing that the use of such a path would allow circumambulation to be completed in around 21 minutes as opposed to 35 minutes without it. However, the study is limited by various built-in assumptions. For example, pilgrims are assumed always to follow the shortest possible path to complete their task, and the problem of crowd congestion at entrances is neglected. This latter problem is taken up in (Zainuddin and Aik 2012) which uses a Response Surface Methodology (Box and Wilson 1951) in which the *Mataf* courtyard is modelled as a grid of cellular automata which pass pilgrims from one grid location to the next according to current conditions. Their results suggest that the RSM approach is extremely promising, and although it is computationally expensive we believe it warrants further investigation.

Another study to investigate the effects of imposing spiralling motions on the crowd is that of Shuaibu et al. (2013), who considered the relative efficiency of circular motion during *tawaf*, as opposed to spiralling inwards for four laps along a specific spiral trajectory and outwards for the remaining three. Spiralling was once again found to be beneficial, since it reduced the simulated *tawaf* completion time from 60.1 minutes to 38.0, while simultaneously reducing crowd densities from 8.8 to 4.2 people per square metre. However, their work does not yet take account of certain key factors during the performance of *tawaf*, such as queueing and waiting at various traditional locations, and it also assumes that all motion occurs in a single 2-dimensional plane. As with other simulations discussed above, another significant factor is the relatively low number of agents modelled.

Although the approaches we have considered so far focus mainly on *tawaf*, Sakellariou et al. (2014) recently proposed ways to analyze, simulate and specify the performance of the crowd during *sa'yee*. The approach adopted is to develop an abstract generic state-based model of crowd behaviour based on X-machines, a kind of extended finite state machine with applications ranging from the modelling of biosystems (Gheorghe et al. 2001) to the development of novel testing strategies for distributed hybrid systems (Stannett and Gheorghe 2015). Taking *sa'yee* as exemplar, they show how models in their framework can be mapped into existing agent-based systems, in this case NetLogo (Fig. 5). The paper is also unusual in that it explicitly addresses the validation issue, and provides a rigorous approach for extracting data, parameters and behavioural patterns from video data. Moreover, the simulations described in the paper include as many as 70000 agents. It is worth noting, however, that the X-machine approach is particularly well adapted for use with the high-performance FLAME GPU agent-based modelling framework (Richmond and Romano 2011), and we envisage that adapting their approach to this framework would allow agent populations to be raised enough to make full-scale modelling of *tawaf* feasible.

```
to-report state-def-of-persons
  report (list

      state "Entering"
      # x-func "follower?" goto "MoveToLeader"
      ...
      end-state

      state "Walking"
      # x-func "reachExitSafa" goto "AtExit"
      # x-func "reachExitMarwah" goto "AtExit"
      # x-func "leaderFar" goto "MoveToLeader"
      # x-func "leaderFarTurn" goto "MoveToLeader"
      # x-func "followerFarMove" goto "WaitingForGroup"
      # x-func "followersFar" goto "WaitingForGroup"
      # x-func "moveToExit" goto "Walking"
      # otherwise do "nothing" goto "Walking"
      end-state

      (... more states ...)

      state "Exiting"
      # x-func "reachedEnd" goto "Exiting"
      # x-func "leavingCorridor" goto "Exiting"'
      end-state

      )
  end
```

Figure 5: Coding of an X-machine in NetLogo (Sakellariou et al. 2014)

Just as virtual physics has provided valuable insights for crowd simulation, so some researchers have turned to biology for inspiration. In contrast to other approaches based on individuals attempting to find the best route through a system, de Lima Bicho et al. (2012) simulate the competition for space between moving agents. Originally developed as a way to model the branching architecture of trees, the space colonization algorithm they use generates several realistic crowd behaviours, even though agents only observe free space rather than one another. These include (and hence potentially explain) several characteristic behaviours that are taken as starting points for other approaches, including collision avoidance, lane formation, and the relationship between velocity and crowd density. While this is certainly a very promising approach, the authors are careful to note some current limitations; as with other approaches, validation remains a problem, and the focus on individuals means that groups are neglected.

A problem we have repeatedly identified above is the inability of systems to handle enough agents to make life-size modelling of *tawaf* possible. Nonetheless, Narain et al. (2009) have demonstrated a technique that may overcome this problem in the long term. Their approach is to represent the crowd simultaneously as both a collection of discrete agents, and a single continuous system to which a novel *unilateral incompressibility constraint* is applied. This allows them to model large-scale behaviours with relative ease, making it possible to simulate crowd populations of several hundred thousand on a standard desk-top computer. While this is not quite large enough to model life-size *tawaf* populations, the increasing power of computer hardware again suggests that this will soon be possible. The authors also note a number of other limitations relating to the algorithms they use. For example, when two groups of agents approach each other, they will not react until their actually encounter one another – there is no in-built capability for groups to plan ahead and take early action against collisions.

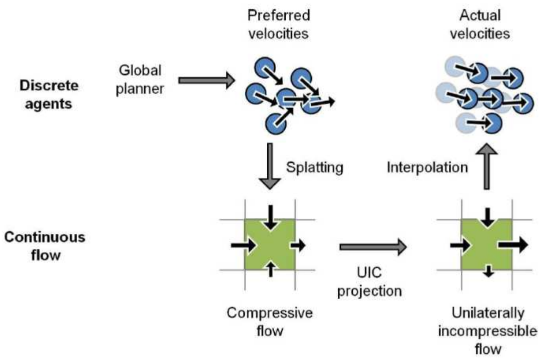

Figure 6: The dual-representation approach suggested by Narain et al. (2009)

DISCUSSION

As we have seen, the need to model crowd behaviours during *tawaf* and *sa'yee* is pressing, and many techniques have been proposed. In general these focus on *tawaf*, since this involves large crowds of people entering a confined space via various uncoordinated entrances (Fig. 7), although a few studies have looked at *sa'yee* as part of a general analysis of crowd modelling strategies.

In each case the goal is to avoid collisions between pilgrims, to minimize the time it takes to complete each ritual, and to maximize crowd safety and well-being. The papers we have surveyed show a wide range of different ways to achieve these aims. While some researchers impose experimental data on the simulation, others seek to develop models in which the required properties 'emerge'. Similarly, while some authors impose properties like collision avoidance on agent behaviour, others seek models that have collision avoidance itself as an emergent property. An important issue that is neglected in many of the studies is the existence of family and friendship groups within the crowd, and

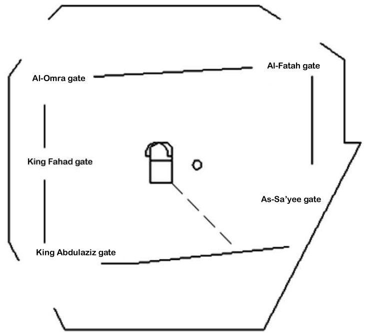

Figure 7: Entrances to the *Mataf* region (Zainuddin et al. 2010)

work is urgently needed to update the studies accordingly, since the existence of such groups can lead both to dynamic new obstacles during crowd evacuation situations, and to unexpected goal formation on the part of individual agents (e.g., parents may head towards a source of danger in search of children, rather than attempting to flee as would otherwise be the case). In this regard, it is essential that pilgrims be modeled not simply as automata, but as agents equipped with both intelligence and emotional states (Kefalas et al. 2012). Ultimately, however, all of these studies rely on the provision of accurate real-world data for validation purposes, together with the ability to run simulations that are large enough to make comparisons with reality meaningful. In this regard, we note that none of the models surveyed here successfully produce simulations with agent populations in the millions, as would be required for a life-size simulation of *tawaf*. Nonetheless, the increasing availability of cheap computational power, coupled with new modelling frameworks like FLAME GPU, offer hope that viable full-scale simulations will be within reach in the very near future. In the mean time, it is clear that further research is warranted, so as to combine the best features of each of the studies cited above.

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Table 1: Summary of Findings

Study	**Techniques**	**Model**	**Focus**	**Notes**
Narain et al. (2009)	CFD	Unilateral incompressibility Constraint; Lagrangian and Eulerian methods	*tawaf* and other densely crowded areas	Dual-representation, discrete agents overlaid on continuous high-level dynamics. Excellent performance, handles several hundred thousand agents easily.
Zainuddin et al. (2009; 2010)	ABS	Social-Force Model (SimWalk)	*tawaf*	Uses SimWalk and the Social Force Model, shows that construction of a spiral pathway can significantly increase throughput for *tawaf*. Neglects crowd congestion at entrances.
Mulyana and Gunawan (2010)	ABS	Agent based	*tawaf*, *sa'yee*, *Jamarat*	Models pilgrims as intelligent agents, and can be used to help train pilgrims prior to arrival. Computationally intensive, relatively few agents.
Curtis et al. (2011)	ABS	FSM, RVO	*tawaf*	Finite state machine model of crowd dynamics, pilgrims modelled as agents. Behaves well with excessive densities. Simulates 7 key behaviors. Can be extended to include more complex agents.
de Lima Bicho et al. (2012)	ABS	FSM, RVO	*tawaf*	Uses ideas from biology. Agents compete for space. Generates properties that are taken as starting points by other studies.
Khan and McLeod (2012)	ABS, CA	micro-level Behavioral algorithm	*tawaf*	Considers three parameters (courtyard layout, crowd properties, management preferences) and their effect on safety, health, satisfaction and throughput.
Zainuddin and Aik (2012)	CA	Response Surface Methodology (RSM)	*tawaf*	Extends earlier work by including congestion. Based on Response Surface Methodology. Computationally expensive, but produces excellent results.
Shuaibu et al. (2013)	Mixed Models	Social-Force model, Rule-Based model, Cellular-Automata	*tawaf*	Shows that spiral motion is more efficient than undirected circular motion during *tawaf*, but assumes all motion occurs in a 2-dimensional plane.
Haghighati and Hassan (2013)	ABS	Discrete-event system	*tawaf*	Shows the significance of lane-switching on crowd throughput.
Sakellariou et al. (2014)	ABS	X-Machines, NetLogo	*sa'yee*	Develops a generic abstract approach and applies it to *sa'yee*. Uses X-machines and NetLogo. Could benefit from translation to FLAME GPU.
Kim et al. (2015)	ABS	Velocity Based And FSM	*tawaf*	Uses ideas from physics. Allows simulation of a few thousand agents.

Key: ABS = Agent-Based System; CA = Cellular Automaton; CFD = Computational Fluid Dynamics; FSM = Finite State Machine; RVO = Reciprocal Velocity Obstacles

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