Proceedings Paper:

Weng, J., Liu, W., Hu, H. et al. (2 more authors) (2015) A Mathematical Formulation for MIMO Channel Map. In: Antennas & Propagation Conference (LAPC), 2015 Loughborough. 2015 Loughborough Antennas & Propagation Conference (LAPC), 02-03 Nov 2015, Loughborough, UK. IEEE , pp. 1-5.

https://doi.org/10.1109/LAPC.2015.7366128

A Mathematical Formulation for MIMO Channel Map

Jialai Weng, Wuling Liu, Haonan Hu, Xiaoli Chu, Jie Zhang
Department of Electronic and Electrical Engineering
The University of Sheffield, Sheffield, S1 3JD, UK
Email: jialai.weng@shef.ac.uk; wuling.liu@shef.ac.uk; hhu8@shef.ac.uk; x.chu@shef.ac.uk; jie.zhang@sheffield.ac.uk

***Abstract*—Channel map has been widely used as an effective tool in network planning and deployment. However, there is a lack of mathematical formulation of the channel map as a tool for multiple-input multiple-output (MIMO) networks planning. In this work, we define and construct a MIMO channel map based on the extension of a mathematical formulation for the conventional single antenna channel map. We demonstrate the effectiveness of the constructed MIMO channel map in planning a distributed MIMO channel network through a numerical example. The results show that location optimisation in distributed MIMO systems gains significantly from an abstract formulation of the channel map.**

***Index Terms*—MIMO, Channel Map, Channel Model**

I. Introduction

Wireless network node location optimisation has been an efficient way to improve the performance of the wireless networks. In cellular wireless network planning, the locations of the base stations are optimised to achieve the requirements of the network design targets under the constraints of the network deployment.

In order to achieve the target of optimal base station deployment, a signal coverage map over the deployment space is essential. Various candidate locations in the map are compared and the optimal one is chosen. Such a coverage map is widely used in the network planning practice. Many researches have been devoted in building such a coverage map for the network planning purpose. Pioneering work [1] partitioned the map into areas and adopted empirical model to predict the path loss contour plot as an path loss map. The work [2] followed this method and extended to an outdoor-indoor transient situation. To accurately predict the channel information requires significant amount of computation, therefore many research works devoted to computer-based simulation. Early works such as the WiSE tools by Bell Labs [3] and CINDOOR [4] were specially designed for designing and planning indoor networks. The method of building such channel maps is mainly deterministic channel propagation model, which contains roughly two major modelling methods: FDTD related models and ray based models. The work of the [5] first used the name of channel map and proposed a ray tracing method for building the channel map. The work [6] proposed an efficient computational electromagnetics method for building the channel map. The channel map has long been widely used as an essential tool in network design. However, it still lacks a rigorous mathematical formulation, which limits the further development of the channel map as an essential tool in network planning, especially for MIMO network planning.

The name 'channel map' and 'channel model' are used interchangeably in the channel modelling community. This stems from the fact that the electromagnetic simulation based channel models outputs a channel modelling result in a form of channel map, such as [6], [7]. However, according to the definition of channel model [8], we point out that the concept of channel map is different from channel model. The construction of channel maps relies on the channel models. Various channel models provide the basis for building a channel map. One of the purposes of the paper is to give a rigorous definition of the channel map.

With the application of the modern advance wireless transmission techniques, such as MIMO system, the network performance is largely benefited from those advanced techniques. These advance techniques are widely deployed in the current and future generation wireless networks, such as HetNet. Meanwhile, to plan networks equipped with these advance techniques, network planning tools face the challenges of considering these advanced techniques in the planning process. Many challenges stem from a lack of rigorous formulation for the channel map. Although the channel map has been widely used as a tool in network planning, a mathematical formulation for the channel map has been missing. This limits the development and functionality of the channel map as a tool in advanced network planning, such as MIMO network planning.

The purpose of the paper is to give a mathematical formulation to the channel map and based on this formulation to further develop the channel map tools for MIMO network planning and optimisation. We first give a mathematical definition of the widely-used single antenna channel map. Based on this definition, we further propose a method of constructing channel maps for MIMO networks based on the conventional channel maps. We also gives a numerical example to demonstrate the effectiveness of the MIMO channel map in distributed MIMO network design.

The rest of the paper is as follows: Section II gives the definition of channel map for both SISO and MIMO channel. Section III gives a numerical example to illustrate the construction of a channel map for distributed MIMO network in a typical network planning scenario. Section IV concludes the work.

978-1-4799-8943-0/15/$31.00 ©2015 IEEE

II. Mathematical Formulation of Channel Map

A. Single Antenna Channel Map

In channel map, the receiver location information in the map is modelled as a location set. Each potential receiver location is specified by its Cartesian coordinates. Therefore the receiver locations in the map is given as a set $\mathcal{L} \subset \mathbb{R}^3$ denotes the region where the network is deployed. In the channel map, it is a common practice to plot the channel information over all the potential receiver locations, with a fixed source location. Then the *receiver location* set is given as the location set $\mathcal{L}$ subtracted the source locations $\mathcal{S}$. It is denoted as:

$$\mathcal{M} = \mathcal{L} \backslash \mathcal{S} \tag{1}$$

where $\mathcal{S}$ denotes the source location set. The number of the potential receiver location is the cardinality of this set $|\mathcal{M}|$.

Under the assumption that the channel is a narrow band static channel, the channel information set is given as the set $\mathbb{C}$ of complex numbers.

The channel map maps the location set $\mathcal{L}$ to the channel set $\mathbb{C}$. The wireless channel can be modelled as the electromagnetic field in the coverage space. In electromagnetic channel modelling, the channel value at a location is the electric field value of the electromagnetic field [9], [10]. Thus we can write:

$$H(l) = E(l) \tag{2}$$

where $E(l)$ is the electric field at the location l and with the source radiating in the source domain $\mathcal{S}$, and $H(l)$ represents the channel information at the location l. Following the above definitions, we can give a definition of the single antenna channel map

Definition 1. *(Single Antenna Channel Map) A single antenna channel map is defined as the set of channel map functions such that give the mapping from the physical location set $\mathcal{M}$ to the channel information set $\mathbb{C}$.*

Thus, we can write the channel map as a function:

$$f : \mathcal{M} \to \mathbb{C} \quad \text{where} \quad f(l) = H(l) \tag{3}$$

where $H(s, l)$ is the channel information at the location l.

This definition gives a mathematical formulation of the widely used single antenna channel map in network planning. The channel map describes the channel information over the location map.

The channel map function characterises the channel information over the design space. The channel map function at each potential location is determined by the physics of the propagation. Therefore each map function can be written in a form of Green function [11] as:

$$H(l) = \int\limits_{\mathcal{S}} G(p, l) J(p) dp \tag{4}$$

where $G(p, l)$ represents the Green function with a source at the location p and the field value at l; $J(p)$ denotes the source at p; dp is the infinitesimally small points of the source; the integral is over the source current domain $\mathcal{S}$. In practice numerical electromagnetic simulation tools can be used to solve the channel map [6], [7].

B. MIMO Channel Map

For MIMO channel map, the multiple transmitters of the MIMO channel are assumed to be determined at fixed locations. The objective of the MIMO channel map as in the single antenna channel map is to draw a channel information map over the potential receiver locations.

Based on the formulation of the single antenna channel map, we have a vector form of channel map for MIMO systems. In MIMO the channel information is to track each sub-channel between each transmitter and receiver pair. Each element in the channel map vector is a single antenna channel map generated by one single transmitter. Thus we define the channel map vector as:

Definition 2. *(Channel Map Vector) A channel map vector is defined as a vector of n channel maps with the same location set $\mathcal{M}$. It is written as:*

$$(f_1(\mathcal{M}), \ldots, f_i(\mathcal{M}), \ldots, f_n(\mathcal{M})) \tag{5}$$

where f_i is the i-th channel map.

For the MIMO system, the potential multiple receiver locations form a set of receiver location vector

$$\mathcal{L}_v = \{(l_1, l_2, ...l_m) | l_j \in \mathcal{M}_j \quad \text{for all} \quad 1 \leq j \leq m\}. \tag{6}$$

where $\mathcal{M}_j$ is the potential receiver location set for the j-th map. Each element in the set $\mathcal{L}_v$ is a receiver location vector, identifying the multiple receiver locations. By applying the receiver location vectors to the channel map vector in (2), we obtain the MIMO channel matrices.

The channel information for MIMO systems is a complex matrix and the set of channel information can be written as $\mathbb{C}^{m \times n}$. The channel map function is also written in a matrix form. Each element in the matrix corresponds to a transmitter-receiver pair in the channel matrix.

$$\mathbf{H}(\mathbf{L}) = \begin{bmatrix} H_{1,1}(l_1) & H_{1,2}(l_1) & \cdots & H_{1,m}(l_1) \\ H_{2,1}(l_2) & H_{2,2}(l_2) & \cdots & H_{2,m}(l_2) \\ \vdots & \vdots & \ddots & \vdots \\ H_{n,1}(l_n) & H_{t,2}(l_n) & \cdots & H_{n,m}(l_n) \end{bmatrix} \tag{7}$$

Like in the single antenna channel map, the channel information can also be represented by the electric field values in a matrix form:

$$\mathbf{H}(\mathbf{L}) = \mathbf{E}(\mathbf{L}) \tag{8}$$

Similar as in the single antenna channel map definition, we define the MIMO channel map as the mapping functions from the receiver location vector set to the channel matrix set.

Definition 3. *(MIMO Channel Map) The MIMO channel map is defined as the set of functions that map the receiver location vector set $\mathcal{L}_v$ to the channel matrix set $\mathbf{H}(\mathbf{L})$ as:*

$$\mathbf{F} \colon \mathbf{L}_v \to \mathbf{H}(\mathbf{L}) \quad \textit{where} \quad \mathbf{F}(\mathbf{L}) = \mathbf{H}(\mathbf{L}) \tag{9}$$

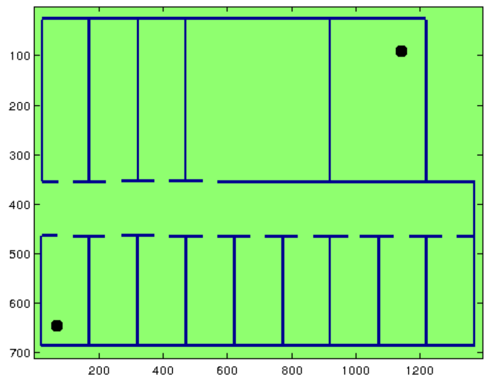

Fig. 1: The Transmitter Antenna Locations

According to the representation of the electrical field value, the electrical field value $\mathbf{E}(\mathbf{L})$ can also be written in a matrix form in (10):

This definition gives a mathematical formulation of the MIMO channel map. Compared to the single antenna channel map, the MIMO channel map is a matrix-valued function . Such a mathematical formulation has the advantage of representing the abstract location vectors, which is infeasible in physical location map. This feature overcomes the limitation of representing channel information over the 2D or 3D physical map and offers an abstract channel map representing channel information over abstract location vectors. We will utilise this feature in an example of planning distributed MIMO systems in Section III.

III. Numerical Examples

In this section, we give an example of the MIMO channel map to demonstrate its application in distributed MIMO systems. Following the definition in Section II, we build a distributed MIMO channel map for a typical indoor network deployment scenario. The simulation tool for single channel map construction is the computer simulation tool presented in [12].

The simulation scenario is a typical office environment. The floor map is shown in Figure 1. The environment comprises walls, doors, windows and ceilings as a typical indoor network planning scenario. The frequency is set to be 2.4GHz. The bandwidth is set to be 15kHz as one single carrier bandwidth in the long term evolution (LTE) networks. We deploy a 2×2 distributed MIMO system in the environment.

The 2 transmitter antennas are deployed in a distributed way. For the reason of demonstration, we only choose one set of locations to deploy the 2 distributed transmitter antennas. The transmitter locations are marked in Figure 1. The simulated channel amplitude maps and channel phase maps are shown in Figure 2 and Figure 3, respectively.

Although distributed receiver antennas are still rare in mobile terminals, it is a potential technique for high data rate

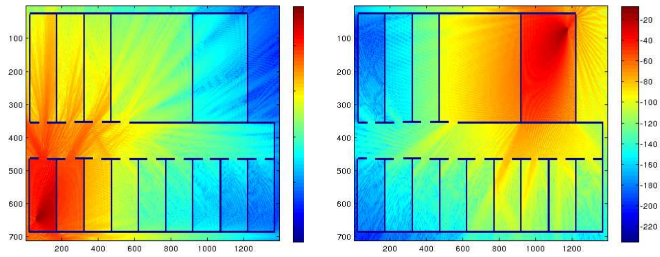

Fig. 2: Channel Amplitude Map Vector

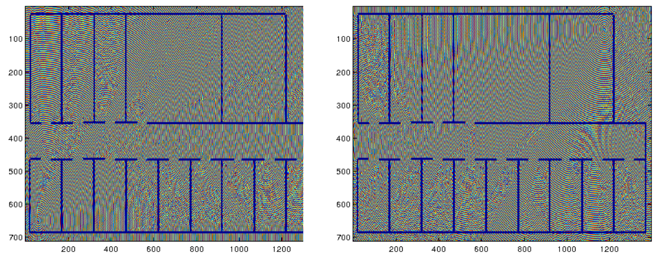

Fig. 3: Channel Phase Map Vector

backhaul connection. It also has the potential to be implemented in a form of cooperative networks [13]. Following the definition in Section II, we first construct the channel map vector. The channel amplitude map vector generated by the simulation tool with transmitters at 2 locations is shown in Figure 2. The channel phase map vector is shown in Figure 3. We arrange the 4 channel maps to a form of channel vector. The mathematical form of the channel map vector follows Definition 2. The matrix form is given in (7).

The total number of location points in the map is 988320. The number of the total potential location vectors is $C^2_{988320} \sim 4.8839 \times 10^{11}$. It costs high computational resource to search the whole possible combinations of the receiver locations. We sample the 988320 receiver locations to choose 2000 locations as the candidate locations. Thus, the search space reduces to $C^2_{2000} = 1999000$. We generate the complete set of the receiver location vectors. Each element in the set is then applied to the channel map vector to identify the channel value as:

$$\mathbf{H}(\mathbf{L}) = \begin{bmatrix} f_1(l_1) & f_1(l_2) \\ f_2(l_1) & f_2(l_2) \end{bmatrix} \tag{11}$$

A. Channel Capacity Optimisation

In this part, we optimise the network performance by choosing the receiver locations which achieves the maximum channel capacity.

We look up the channel matrix values in the channel map corresponding to all the receiver location vectors. The result is a table mapping from the receiver location vector to the corresponding channel matrix. We then use the channel matrix to calculate the MIMO channel capacity, according to the

$$\mathbf{E}(\mathbf{S},\mathbf{L}) = \int \begin{bmatrix} G_{1,1}(s_1,l_1) & G_{1,2}(s_2,l_1) & \cdots & G_{1,m}(s_m,l_1) \\ G_{2,1}(s_1,l_2) & G_{2,2}(s_2,l_2) & \cdots & G_{2,m}(s_m,l_2) \\ \vdots & \vdots & \ddots & \vdots \\ G_{n,1}(s_1,l_n) & G_{t,2}(s_2,l_n) & \cdots & G_{n,m}(s_m,l_n) \end{bmatrix} \begin{bmatrix} J_1(p_1) & 0 & \cdots & 0 \\ 0 & J_2(p_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_n(p_n) \end{bmatrix} d\mathbf{p} \tag{10}$$

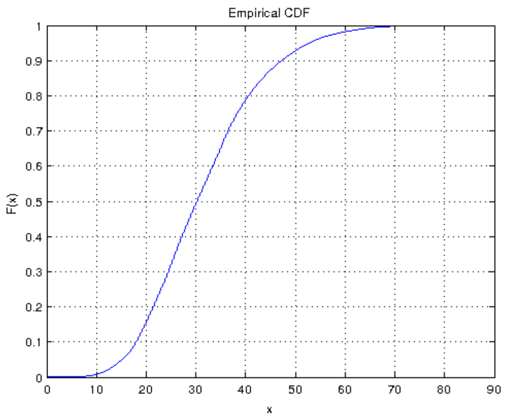

Fig. 4: The CDF of Capacity in Distributed MIMO

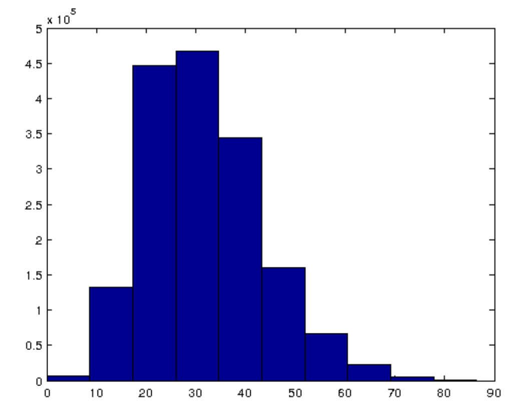

Fig. 5: The PDF of Capacity in Distributed MIMO

formula:

$$C = \log_2 \det(I + snrHH^*) \tag{12}$$

The cumulative distribution function (CDF) of the resulting channel capacity is shown in Figure 4. The probability density function (pdf) of the capacity is shown in Figure 5. The statistics of the resulting capacity values are summarised in Table I. The mean value of the capacity is 31.5006 bits/s/Hz and the maximum value is 86.4459 bits/s/Hz. The optimal receiver antenna locations that achieve the maximum capacity are indicated in Figure 6.

TABLE I: Statistics of the Capacity Values

Maximum	Minimum	Mean	Standard Deviation
86.4459	0.0144	31.5006	11.5412

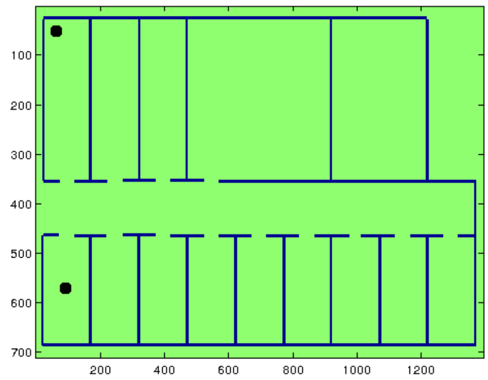

Fig. 6: The Optimal Locations for Receiver Antennas

TABLE II: Statistics of the Error Rate

Maximum	Minimum	Mean	Standard Deviation
0.5161	0.0261	0.4906	0.0321

The gain of optimally designed receiver location from an arbitrarily random choice of receiver locations significant in this case. We can see that the majority of the receiver locations supports a capacity near the mean value of from 20 to 40 bits/s/Hz range. The optimal capacity value offers a nearly 3 times gain from these mostly likely receiver locations by random choice. This shows that the a significant capacity gain can be achieved by carefully choosing the locations of the distributed antennas.

B. Error Rate Optimisation

By using the channel map generated for the distributed MIMO system, we also study the choice of receiver locations on error rate performance. We adopt the Alamouti block space time code [14] to be used in the distributed MIMO system in our simulation. The signal-to-noise-ratio (SNR) is set to be 10dB. We simulate the MIMO system using all the candidate channels from the candidate receiver locations. The CDF and the PDF of the error rates at all the candidate locations are given in Figure 7 and Figure 8, respectively.

The statistics of the error rate are given in Table II. We see that by choosing the optimal locations of the receiver locations we can achieve the optimal error rate 0.0261 while the mean error rate is about 0.5. The receiver antenna locations achieving this optimal error rate are indicated in Figure 9. This result shows that we can achieve good error rate performance even the the majority of the signal coverage is poor, by choosing the optimal receiver locations.

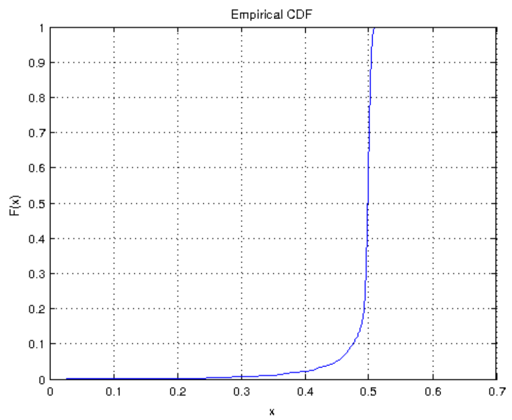

Fig. 7: The CDF of Error Rate in the Distributed MIMO

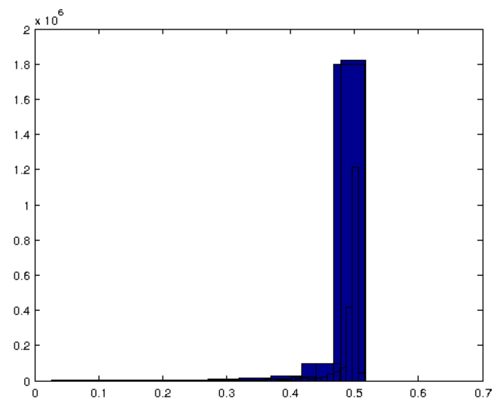

Fig. 8: The PDF of Error Rate in the Distributed MIMO

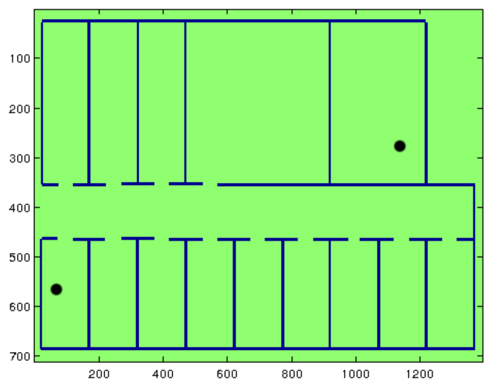

Fig. 9: The Optimal Receiver Antenna Locations

IV. CONCLUSION

In this paper, we give a mathematical formulation to the widely-used channel map tool in network planning. Following the formulation we also propose a method for constructing MIMO channel maps using existing single antenna channel map tools. The MIMO channel map extends the conventional channel map tools to advanced MIMO network planning applications. A numerical example is given to demonstrate the construction and application of the channel maps in MIMO network planning. The results show that the MIMO channel maps are an effective tool in MIMO network planning. Significant gains in both spectral efficiency and error rate performance are achievable by using the channel map tool to carefully plan the locations of the MIMO network nodes.

REFERENCES

[1] S. Seidel and T. Rappaport, "914 MHz path loss prediction models for indoor wireless communications in multifloored buildings," *Antennas and Propagation, IEEE Transactions on*, vol. 40, no. 2, pp. 207–217, Feb 1992.

[2] G. Durgin, T. Rappaport, and H. Xu, "Radio path loss and penetration loss measurements in and around homes and trees at 5.85 GHz," in *Antennas and Propagation Society International Symposium, 1998. IEEE*, vol. 2, June 1998, pp. 618–621 vol.2.

[3] S. Fortune, D. Gay, B. Kernighan, O. Landron, R. Valenzuela, and M. Wright, "Wise design of indoor wireless systems: practical computation and optimization," *Computational Science Engineering, IEEE*, vol. 2, no. 1, pp. 58–68, Spring 1995.

[4] R. Torres, L. Valle, M. Domingo, and M. Diez, "Cindoor: an engineering tool for planning and design of wireless systems in enclosed spaces," *Antennas and Propagation Magazine, IEEE*, vol. 41, no. 4, pp. 11–22, Sep 1999.

[5] H. Suzuki and A. Mohan, "Measurement and prediction of high spatial resolution indoor radio channel characteristic map," *Vehicular Technology, IEEE Transactions on*, vol. 49, no. 4, pp. 1321–1333, Jul 2000.

[6] J.-M. Gorce, K. Jaffres-Runser, and G. de la Roche, "Deterministic approach for fast simulations of indoor radio wave propagation," *Antennas and Propagation, IEEE Transactions on*, vol. 55, no. 3, pp. 938–948, March 2007.

[7] P. Bernardi, R. Cicchetti, and O. Testa, "An accurate utd model for the analysis of complex indoor radio environments in microwave WLAN systems," *Antennas and Propagation, IEEE Transactions on*, vol. 52, no. 6, pp. 1509–1520, 2004.

[8] T. Sarkar, Z. Ji, K. Kim, A. Medouri, and M. Salazar-Palma, "A survey of various propagation models for mobile communication," *Antennas and Propagation Magazine, IEEE*, vol. 45, no. 3, pp. 51–82, June 2003.

[9] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, ser. Wiley series in telecommunications. Cambridge University Press, 2005. [Online]. Available: http://books.google.co.uk/books?id=66XBb5tZX6EC

[10] T. Sarkar, S. Burintramart, N. Yllmazerl, Y. Zhang, A. Del, M. Salazar-Palma, M. Lagunas, E. Mokole, and M. Wicks, "A look at the concept of channel capacity from a maxwellian viewpoint," *Antennas and Propagation Magazine, IEEE*, vol. 50, no. 3, pp. 21–50, June 2008.

[11] J. D. Jackson, *Classical Electrodynamics Third Edition*, 3rd ed. Wiley, 1998.

[12] J.-M. Gorce, K. Jaffres-Runser, and G. De La Roche, "Deterministic approach for fast simulations of indoor radio wave propagation," *Antennas and Propagation, IEEE Transactions on*, vol. 55, no. 3, pp. 938–948, 2007.

[13] G. Kramer, I. Marić, and R. D. Yates, "Cooperative communications," *Foundations and Trends® in Networking*, vol. 1, no. 3, pp. 271–425, 2006.

[14] S. Alamouti, "A simple transmit diversity technique for wireless communications," *Selected Areas in Communications, IEEE Journal on*, vol. 16, no. 8, pp. 1451–1458, Oct 1998.