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Synthesis of Coupling Matrix for Lossy Filter Networks

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Abstract — A generalized method for the synthesis of lossy microwave filters is given in this paper. An equation on the polynomials of S parameters is given to replace the power conservation. When the polynomials of S parameters satisfy a given condition, it is guaranteed that the admittance parameters as well as the coupling matrix (CM) can be derived from the S parameters. Two special cases are discussed for solving the refection function from a prescribed transfer function. In the first case, F_{11} (the numerator of S_{11}) equals to F_{22} (the numerator of S_{22}). This is the case that is equivalent to the even/odd mode analysis but is extended to be applied for asymmetric filter responses. In the second case, the loss distribution among a filter network is given. A method of iteration is applied to derive the CM with the prescribed loss distribution. The method is an extension to the conventional method of predistortion with non-uniform resonator Qs and lossy invertors.

Index Terms — Lossy synthesis, filter synthesis, predistortion.

I. INTRODUCTION

Generally speaking, there are two problems in the synthesis of lossy CMs: the determination of lossy transfer functions that can be realized by lossy networks and the derivation of lossy circuits based on the given responses. The lossy synthesis methods reviewed earlier solve these problems partially and are only applicable within certain constants. The method of predistortion [1] provides the denominator polynomial of lossy transfer functions with uniform resonator losses. The method given [2] determines lossy circuits based on even and odd modes analysis. The filter networks realized either have dissipations only at the input and output resonators or have complex cross couplings which are difficult to implement. In [3][4], the even and odd mode analysis which is originally only valid for symmetrical responses is extended to the case when $S_{11}=S_{22}$. However, the lossy network synthesized requires the use of hybrids to combine sub-networks. The method in [5] is based on a very specific type of lossy responses for which the characteristic polynomials are multiplied by certain constants so that the lossless synthesis method is still valid.

A generalized lossy synthesis technique is presented in this paper. The method can (1) find the reflection function from the transfer function when unitary condition is not satisfied; (2) derive the expressions for the complex Y parameters and (3) synthesize the lossy CM with prescribed loss distribution. The method is based on a condition set for the polynomials of S parameters which replaces the use of power conservation in the lossless case and it is guaranteed that the admittance parameters and corresponding CMs can be derived.

Two special cases are given for solving the refection function with a prescribed transfer function. In the first case, F_{11} the numerator of S_{11} equals to F_{22} the numerator of S_{22}. The method is equivalent to the even and odd mode analysis for asymmetric filter responses. Since the networks are transversal arrays which have a parallel connection of the even and odd mode sub-networks, they can be transformed to any realizable configurations. In the second case, loss distributions are given. An method of iteration is applied so that the synthesized CM has the prescribed loss distribution. The method is equivalent to an extension of conventional method of predistortion with non-uniform resonator *Qs* and lossy invertors.

The lossy synthesis method provided is capable of synthesizing lossy networks with prescribed non-uniform *Qs*. The application of this method is found in the implementation of filter networks consisting of two different kinds of resonators. Filter with both dielectric and coaxial resonators is used in to provide improved spurious [6]. Various examples of synthesizing CMs are given in this section to illustrate the design processes. A 6^{th} degree filter with TM dielectric and coaxial resonators is modeled in HFSS and simulated.

II. THEORY

A. Lossy Characteristics

For the derivation of S parameters in this paper, the rational polynomial expressions in (1) are used. S_{21} is the general Chebyshev response or other filter characteristics. The lossy synthesis method presented can be applied to any lossy insertion loss functions in the form of rational polynomials. In this paper, as dissipations included in filter networks introduce rounding at bandedge and thus deteriorate the filter's performance, an insertion loss which is the same as the lossless one multiplied by a constant smaller than one as shown in (2) is used to illustrate the synthesis process because it maintains the selectivity of the filter network. S_{21}' represents the lossy one and k_{21} determines the insertion loss level. Using the polynomial expression, the denominator polynomial is the same as in the lossless case and the numerator polynomial is given in (3).

$$S_{11} = \frac{F_{11}(s)}{E(s)},\ S_{21} = \frac{P(s)}{E(s)},\ S_{22} = \frac{F_{22}(s)}{E(s)} \tag{1}$$

$$S_{21}' = k_{21} S_{21} \tag{2}$$

$$P(s)' = k_{21}P(s) \tag{3}$$

B. S to Y Transformation

In [7], the transformation between S parameters and admittance parameters is given. For a two-port network with unit reference impedance, the transformation could be simplified as in (4). These Y parameters are rational polynomials and the degree of the denominator is *2N*.

$$\begin{aligned} Y_{11} &= \frac{(E - F_{11})(E + F_{22}) + p^2}{(E + F_{11})(E + F_{22}) - p^2} \\ Y_{21} &= \frac{-2pE}{(E + F_{11})(E + F_{22}) - p^2} \end{aligned} \tag{4}$$

For the N^{th} degree S parameters to be realizable by an N^{th} degree filter network, the admittance parameters in (4) must also be a rational polynomial of degree N. This can only be achieved when the condition given in (5) is satisfied while E_x is an N^{th} degree polynomial. Substituting E_x into (4), the Y parameters can be expressed as in (6) which are rational polynomials of degree N and can be used for the synthesis of CM. As a result, the condition given in (5) for the polynomials of S parameters must be satisfied so that the response can be realized by an N^{th} degree network. This requirement is general for both lossless and lossy cases.

$$F_{11}(s)\cdot F_{22}(s) - P(s)\cdot P(s) = E(s)\cdot E_x(s) \tag{5}$$

$$\begin{aligned} Y_{11} &= \frac{Y_{11n}}{Y_d} = \frac{E(s) - F_{11}(s) + F_{22}(s) - E_x}{E(s) + F_{11}(s) + F_{22}(s) + E_x} \\ Y_{21} &= \frac{Y_{21n}}{Y_d} = \frac{-2P(s)}{E(s) + F_{11}(s) + F_{22}(s) + E_x} \end{aligned} \tag{6}$$

For lossless networks, according to the condition of power conservation, E_x is the complex conjugate of E. However, it is unknown for lossy networks. Using the lossy characteristics of (2), the condition is modified to (7). In this paper, two different cases are discussed for solving (7) in the following sections.

$$F_{11}(s)'\cdot F_{22}(s)' - k_{21}^2 P(s)'\cdot P(s)' = E(s)'\cdot E_x(s)' \tag{7}$$

III. Case I: $F_{11}=F_{22}$

When $F_{22}=F_{11}$, or more generally as in (8) in which k_{11} is a constant, (9) can be derived from (7). The two terms in (9) which are combinations of F_{11} and P provides the roots of E. Given the expressions for P and E, F_{11} can be solved through a set of linear equations as in (10).

$$F_{22}(s)' = k_{11}F_{11}(s)' \tag{8}$$

$$\left(\sqrt{k_{11}}F_{11}(s)' + k_{21}P(s)'\right)\left(\sqrt{k_{11}}F_{11}(s)' - k_{21}P(s)'\right) = E(s)'\cdot E_x(s)' \tag{9}$$

$$\begin{aligned} \left(\sqrt{k_{11}}F_{11}(s)' + k_{21}P(s)'\right)_{s=rei} &= 0 \\ \left(\sqrt{k_{11}}F_{11}(s)' - k_{21}P(s)'\right)_{s=rej} &= 0 \end{aligned} \tag{10}$$

This method of deriving the reflection function is general that the filter response doesn't need to be symmetric. Based on the synthesis procedure, the even and odd mode analysis of [2] and [3] which is originally used for symmetric networks could be applied to asymmetric networks when the response satisfies the conditions in (7) and (8).

IV. Case II: Given Loss Distribution

Solution to (8) could also be found when polynomial E_x is given. Losses of a filter network can be modeled in a modified CM with complex resonators and invertors. For any CM regardless of the configuration, it could be transform back to the transversal array so the admittance parameters for the network can be found directly. Then the response of the network could be derived by the transformation of Y parameters to S parameters given in (11) in which where Y_x is a polynomial defined in (12).

Polynomial expressions for S parameters are found here directly from admittance parameters without matrix inversions. The condition in (12) is equivalent to the one in (7) and this can be proved by polynomial substitutions.

$$\begin{aligned} F_{11}(s) &= Y_d - Y_{11n} + Y_{22n} - Y_x \\ E(s) &= Y_d + Y_{11n} + Y_{22n} + Y_x \\ E_x(s) &= Y_d - Y_{11n} - Y_{22n} + Y_x \end{aligned} \tag{11}$$

$$Y_{11n}Y_{22n} - Y_{21n}^2 = Y_d Y_x \tag{12}$$

A method of iteration is introduced for solving this problem. The initial values are chosen to be the polynomial E_x of the lossless network. For lossy network, E_x' and P' are updated in each iteration and are denoted as E_{xi} and P_i. Using this E_{xi}, P_i and E, the polynomial for F_{11i} and F_{22i} and a lossy coupling matrix M_t can be found. A new lossy coupling matrix M_i could be defined by adding the prescribed loss distribution to the real part of CM M_t. For this lossy network M_i, the new polynomials E_{xi+1} and P_{i+1} can be found by the method given in the last section. And this procedure is applied iteratively until the loss of the synthesized coupling matrix is the same to the prescribed ones under a degree of precision.

1) Given E and loss distribution δ which is a imaginary matrix with its diagonal elements representing the dissipation of each resonator and the off-diagonal ones representing the loss of invertors according to (13).
2) The initial values for E_{xi} and Pi are equivalent to the ones in the lossless case.
3) Derive F_{11i} and F_{22i} using E_{xi}, P_i and E using (7)
4) Synthesize lossy coupling matrix M_t according to [8].
5) Derive E_{xi+1} and E_{xi+1} from lossy M_i which is equivalent to $real(M_t)+\Delta$ according to (11).

6) go back to 3) if the imaginary part of M_t is not close enough to δ.

$$\delta = \frac{f_0}{BW \cdot Q} \qquad (13)$$

In step 2), the zeros of F_{11i} and F_{22i} no longer lie on the imaginary axis as in the lossless case. In step 4), the coupling matrix might have loss distribution different from the given one. For example, when only resonator loss is concerned, the M_i might contain complex invertors. However, the loss distribution will converge to the prescribed one after about 10 iterations.

V. Design Example

The example is a 6th degree filter with general Chebyshev response. The filter is symmetric with four transmission zeros at 1.6j, -1.6j, 2.4j and -2.4j in the lowpass domain. The centre frequency is 2GHz and the bandwidth is 0.12 GHz. The filter is realized by two coaxial resonators at the input and output with Q of 3200 and four TM_{01} dielectric resonators with Q of 2500. The original and designed CMs are compared in Table I. The EM model and the simulated results are shown in Fig.1.

TABLE I

Coupling Matrix Mt in the First and Last Iterations

M_{S1}	0.9957	M_{56}	0.8309	M_{S1}	0.6608	M_{56}	1.0458
M_{12}	0.8309	M_{6L}	0.9957	M_{12}	0.7055	M_{6L}	1.2272
M_{23}	0.5823	M_{25}	-0.1521	M_{23}	0.5548	M_{25}	-0.1595
M_{34}	0.6971	M_{16}	0.0147	M_{34}	0.7067	M_{16}	0.0174
M_{45}	0.5823			M_{45}	0.6353		

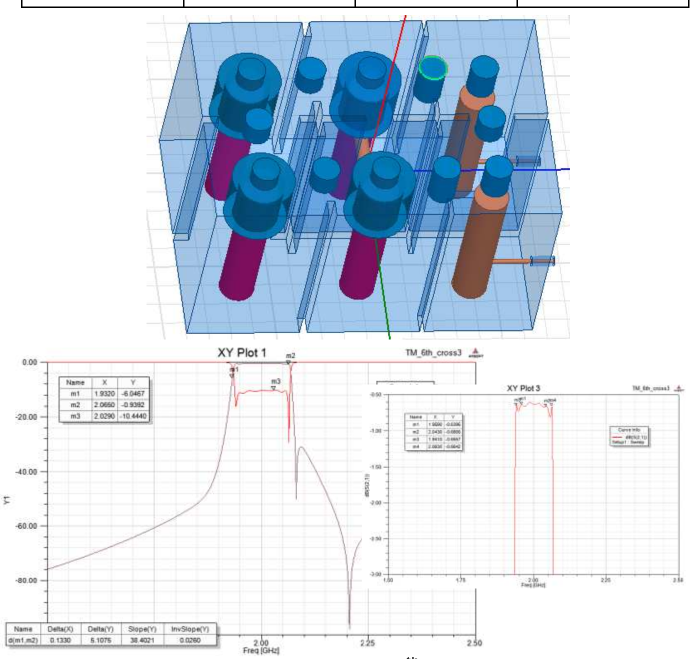

Fig. 1. EM model and response of the 6th order mixed coaxial and TM mode dielectric filter

V. Conclusion

The lossy synthesis method is an extension to the lossless coupling matrix synthesis given in [8] which first derives the admittance parameters from the characteristic functions using the equation of power conservation, then generate a canonical CM that can be transformed to required configurations. The lossy synthesis method presented is based on a new condition for the lossy characteristic polynomials to replace the power conservation. When the lossy transfer function is defined, solutions to the characteristic polynomials are found under two conditions. In the first one, S_{11} equals to kS_{22} where k is a constant. This an even and odd mode of analysis applied to asymmetric filter responses. In the second one, the loss distribution is given and thus is a non-uniform predistortion.

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