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# Quantifying and decomposing the uncertainty in appraisal value of travel time savings

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## Abstract

This paper is concerned with computing interval estimates for appraisal values of travel time savings (VTTS) for non-work journeys. The paper has important conclusions relating to the benefits, in terms of uncertainty in appraisal VTTS, of resampling the base year VTTS and improving on the precision of the estimate of the GDP elasticity in the uprating equation. Importantly it is shown that the interval widths increase dramatically as VTTS is forecast further into the future. This has a significant modelling implication in that it is the uncertainty associated with the process of uprating base estimates of VTTS which results in the large interval width, rather than that associated with the base year VTTS estimate. This in turn implies a need to regularly resample the base VTTS, so as to avoid excessive temporal extrapolation of the base VTTS.

Keywords: Uncertainty, Value of Travel Time Savings, Appraisal

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## 1 Introduction

Given the prominence of travel time savings as a key source of benefit from transport investment schemes, there is a clear imperative to estimate the Value of Travel Time Savings (VTTS) as precisely as possible. However, in practical modelling and appraisal, there is rarely any analysis of the relationship between the precision of VTTS estimates and the outcomes arising from the appraisal. For example, transport appraisal guidance in the UK (WebTAG; Department for Transport (DfT), 2012) does not make such allowances; official UK estimates of VTTS documented in WebTAG Unit A1.3 are point estimates, and no account is taken of the sample distribution underpinning these estimates. This issue has recently come under high-profile scrutiny in the context of the business case for the proposed High Speed 2 (HS2) rail scheme in the UK, where the National Audit Office (NAO) recommended that: *'The Department (for Transport) and HS2 should recognise explicitly the uncertainty in the economic case by quoting ranges rather than a point estimate. The risks and uncertainty to the benefit-cost ratio have not been clearly stated'* (NAO, 2013 p12).

Partly in response to such critiques, the UK DfT has developed an 'Analytical Assurance Framework' (DfT, 2013), which endeavours to promote: *'the correct balance between robustness, timeliness, and cost, for the decision at hand'* (p4). The Framework encompasses three key dimensions, namely: 1) the potential for challenge to the analysis; 2) the risks of an error in the analysis and the uncertainty inherent in the analytical advice to the Permanent Secretary and Secretary of State; and 3) the degree to which error and/or uncertainty has been reduced. In the particular context of VTTS, DfT has recently employed the Framework to support decision-making in relation to the update of official UK appraisal values for 'non-work' journey purposes. These appraisal values derive from Willingness-to-Pay (WtP) evidence collected through Stated Preference (SP) surveys, and give rise to separate values for 'commuting' and 'other' non-work journeys, which are applied universally across all modes. It should be clarified that UK appraisal values for business journeys are based on the altogether different 'Cost Saving Approach' (CSA), which derives from wage rate data as opposed to WtP data<sup>1</sup>. The present paper focuses on the analysis of uncertainty inherent within non-work values, and the corresponding analysis of business values falls beyond our immediate scope.

Recognising that non-work appraisal values for 2014 and beyond are based on behavioural WtP values estimated in 1994 – i.e. some 20 years ago – the Department has undertaken two inter-related strands of work focussed around the following considerations:

1. The degree of error inherent within estimates of VTTS extrapolated over time from 1994, using the official model prescribed by DfT in WebTAG Unit A1.3.
2. The extent to which any error can be reduced through updating parameters in the model and/or through re-sampling the behavioural values altogether.

The present paper arises from research commissioned by DfT which seeks to inform the above considerations. The specific objectives of the paper are:

- a. To quantify the uncertainty associated with official UK estimates of non-work VTTS.
- b. Within these estimates, to identify key sources of uncertainty.

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<sup>1</sup> Current UK appraisal values of travel time savings for both non-work and business are documented in TAG Unit A1.3 (DfT 2014a).

Whilst we will focus on the precision of non-work VTTS estimates in the UK, it is important to acknowledge that the implications of our paper reach far beyond the UK, in the following respects. First, the formula used in the UK for estimating non-work VTTS has (in essence) been transferred to other countries, notably Switzerland and the Netherlands. Second, British VTTS estimates have, more generally, been seen as a key reference point for transport investment schemes, both across Europe (HEATCO, 2006) and beyond (World Bank, 2005). Third, even in countries that employ VTTS estimates based on other formulae, similar issues concerning the precision of estimates arise.

In general terms, there are two key dimensions to the analysis of uncertainty in VTTS estimates. First and foremost, we should consider the precision of base year estimates of VTTS, which typically arise from discrete choice modelling of WtP data. Second, given the 30-60 year economic life that typifies transport investment schemes, any imprecision in base year estimates of VTTS may be compounded as the scheme proceeds through its economic life. Within the non-work VTTS formula employed in the UK, the latter source of uncertainty is associated with the scaling of base year estimates by a GDP multiplier and associated GDP elasticity, which arise from a further econometric ‘meta’ model of SP and RP evidence on VTTS. As GDP moves away from the base year level, so the uncertainty in the estimate of the GDP elasticity is amplified within the overall non-work VTTS estimate; this in turn increases the error associated with the VTTS estimate.

In pursuit of the objectives outlined above, the paper is arranged as follows. Section 2 provides a brief summary of relevant academic literature concerning the analysis of uncertainty within transport models. Section 3 outlines the motivation for analysing uncertainty within estimates of non-work VTTS specifically. Section 4 develops an approach for computing the latter uncertainty, whilst Section 5 implements the approach and reports the resulting interval estimates. Section 6 considers the potential for improving the precision of non-work VTTS estimates by updating historical estimates of the GDP elasticity. Section 7 undertakes a set of ‘what if’ experiments to highlight the impact on the interval width of re-sampling and re-estimating the base VTTS, and Section 8 concludes.

## **2 The extant literature on analysing uncertainty within transport models**

Whilst practical modelling and appraisal often overlooks the uncertainty inherent within VTTS estimates, it should be acknowledged that there is an extant literature – albeit somewhat limited in size and scope – on the representation of uncertainty within transport models, especially in relation to travel demand forecasting. De Jong et al. (2007) provided a useful summary of 21 studies from the period 1980 to 2007, noting for each study:

- The type of uncertainty studied;
- Variables for which uncertainty is studied;
- Methods to quantify uncertainty;
- How uncertainty is expressed;
- Order of magnitude of uncertainty.

Complementing De Jong et al., Rasouli & Timmermans (2012) provided a similar summary of uncertainty inherent within 14 travel demand forecasting studies from the period 2002 to

2012. The studies identified by De Jong et al. and Rasouli & Timmermans are relevant to the present paper, to the extent that VTTS estimates were a source of uncertainty within a number of them and travel demand forecasts represent an important component of transport appraisal; however none of these studies focused on uncertainty in VTTS estimates specifically.

These prior studies offer various characterisations of the causes and effects of uncertainty, but the recent paper by Yang et al. (2013) outlined a simple but useful framework which can be applied to most studies of transport model uncertainty. More specifically, they distinguished between three steps in analysing such uncertainty, namely:

- i. Analysing the distributional characteristics of ‘input’ and ‘parameter’ uncertainty in the model, where the former reflects inherent uncertainty in model inputs, whilst the latter reflects uncertainty in parameters which could potentially be reduced by collecting more and/or better quality data.
- ii. Analysing the manner in which model input/parameter uncertainty is propagated by the model into model ‘output’ uncertainty.
- iii. Analysing the distributional characteristics of the output uncertainty, for given inputs and parameters.

Reconciling our present interest in the uncertainty of non-work VTTS estimates in the UK – as introduced in section 1 above – to Yang et al.’s framework, note that:

- The base year VTTS, GDP multiplier and GDP elasticity introduce sources of parameter uncertainty.
- Any uncertainty in base year VTTS is potentially propagated through the 30-60 year appraisal period, as a function of the GDP multiplier and GDP elasticity.

### **3 The motivation behind analysing uncertainty within VTTS estimates**

In 1994, Accent Marketing and Hague Consulting Group (AHCG) were commissioned by the UK Department for Transport (DfT) to undertake research to inform the update of official UK estimates of non-work VTTS, subsequently reporting their findings in 1999. DfT experienced difficulties in implementing AHCG’s findings, and therefore commissioned the Institute for Transport Studies (ITS), University of Leeds, together with John Bates Services, to help resolve some of the outstanding issues. This involved a substantial re-analysis of the AHCG data (Mackie et al., 2003), and the modelling approach developed and applied in the course of this re-analysis has since been adopted as standard in the UK (see TAG Unit A1.3; DfT (2014a)).

As noted above, official UK estimates of non-work VTTS arise from two econometric models, which contribute different insights. The first model estimates ‘behavioural’ values of travel time saving, whilst the second model facilitates the conversion of behavioural values into ‘appraisal’ values of travel time saving. This section will give the background to each model, and describe the manner in which the models are combined. It is not the intention of the paper to evaluate the suitability of this modelling approach, and the justification for each model will not therefore be discussed in any great detail.

### 3.1 Behavioural values

Re-analysing SP data collected by AHCG using discrete choice modelling methods, Mackie et al. (2003) discerned a significant relationship between VTTS and both income and journey cost (where the latter was employed as a proxy for journey distance), reporting the following preferred model<sup>2</sup>:

$$V = \left( \frac{\beta_T}{\beta_C} \right) \cdot \left( \frac{Y}{Y_0} \right)^{\eta_Y} \cdot \left( \frac{C}{C_0} \right)^{\eta_C} \quad (1)$$

where  $V$  is the non-work VTTS in pence per minute (ppm)<sup>3</sup>,  $Y$  is household income in £'000 pa,  $C$  is the cost of the 'current' journey in pence (both in 1994 prices),  $Y_0$  and  $C_0$  are 'reference' values of income and cost which take the fixed values 35 and 100 respectively, and  $\beta_T$ ,  $\beta_C$ ,  $\eta_Y$  and  $\eta_C$  are parameters to be estimated. The model was segmented by two types of non-work journeys, namely commuting and 'other' non-work.

### 3.2 Appraisal values

In order to translate behavioural values of time saving for non-work travel into values suitable for appraisal, three adjustments were made.

1. To facilitate interface with the primary segmentations (and bandings therein) of National Travel Survey (NTS) data (1995-2000), which considered travel distance but not travel cost, Mackie et al. constructed a 'bridge' between the two variables. This relationship assumed that cost per mile had an average value of 13.2 pence (1994 prices), and that cost and distance were linearly related. On this basis, the cost variables were re-interpreted as distance variables, such that:

$$V_{yd} = K \cdot \left( \frac{\beta_T}{\beta_C} \right) \cdot \left( \frac{Y_y}{Y_0} \right)^{\eta_Y} \cdot \left( \frac{D_d}{D_0} \right)^{\eta_C} \quad (2)$$

where  $V_{yd}$  is the non-work VTTS in pence per minute (ppm) for NTS income and distance bands  $y$  and  $d$  respectively;  $Y_y$  is household income in £'000 pa within NTS income band  $y$ , and  $Y_0$  is the associated 'reference' value, calculated to be  $Y_0 = 35$ ;  $D_d$  is the (inferred) distance of the 'current' journey in miles within NTS distance band  $d$ , and  $D_0$  is the associated 'reference' value, calculated to be  $D_0 =$

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<sup>2</sup> In passing, note that (1) was derived from discrete choice modelling of binary choice SP data, wherein the conditional indirect utility function was specified:

$$W_i = \beta_C \cdot \left( \frac{Y}{Y_0} \right)^{\eta_Y} \cdot \left( \frac{C}{C_0} \right)^{\eta_C} \cdot (C_i - C) + \beta_T \cdot (T_i - T) \quad \text{for } i = 1, 2$$

where  $W_i$  is conditional indirect utility for SP travel option  $i = 1, 2$ ;  $C$  is travel cost for the 'current' journey, and  $C_i$  is travel cost for option  $i = 1, 2$ ;  $T$  is travel time for the 'current' journey, and  $T_i$  is travel time for option  $i = 1, 2$ , and the remaining terms are defined as for (1).

<sup>3</sup> Note that separate values were calculated for commuting and 'other' non-work, but for presentational simplicity we do not index on this basis here.

100/13.2 = 7.58 miles (12.2 Km); and  $K$  is a further adjustment to account for price inflation. With regards to the latter adjustment, the AHCG data was in 1994 prices and 1994 income levels, and the  $K$  term thus facilitates adjustment for inflation from 1994 to the desired year (1997 in Mackie et al. (2003)), although this paper reports estimates in 2010 prices<sup>4</sup>. Separate adjustment for income growth is discussed below.

2. Employing (2), non-work VTTS estimates were computed for several distance and income combinations in NTS, and a base VTTS elicited as the weighted average:

$$\bar{V} = \frac{\sum_y \sum_d V_{yd} \cdot N_{yd} \cdot \bar{D}_d}{\sum_y \sum_d N_{yd} \cdot \bar{D}_d} \quad (3)$$

Where  $\bar{V}$  is the weighted average of non-work VTTS in pence per minute (ppm) across NTS income and distance bands  $y$  and  $d$  respectively;  $N_{yd}$  is the number of NTS journeys for the specified purpose (commuting or ‘other’ non-work, by all mechanised modes) within distance band  $d$  and household income band  $y$ ; and  $\bar{D}_d$  is the average distance for distance band  $d$  within NTS.

3. With reference to (1), the  $\eta_y$  term represents the elasticity of VTTS with respect to income, and captures any cross-sectional variation in VTTS across the SP data. In order to represent the influence of income growth over time, it was judged appropriate to introduce external evidence on the elasticity of VTTS with respect to GDP, drawn from meta-analysis of a wide range of RP and SP studies<sup>5</sup>. The econometric model of meta-data, which is summarised in Mackie et al (2003) and discussed in more detail in Wardman (2001), reported a GDP elasticity of 0.8. Applying this result to (3), the base year estimate of VTTS (for 1994) was updated using a logarithmic function as follows:

$$V_t^* = \bar{V} \cdot \left( \frac{GDP_t}{GDP_{1994}} \right)^{\varepsilon_{GDP}} \quad (4)$$

where  $\varepsilon_{GDP} = 0.8$ . In this way, we arrive finally at the definitive formula for estimating non-work VTTS, as recommended for appraisal purposes in official UK guidance.

## 4 Methodology

Having introduced the background to official methods for estimating non-work VTTS in the UK, we will now outline the methodology that we used to compute the statistical uncertainty surrounding estimates of (4). More specifically, we will describe a methodology to compute interval estimates, in the form of a confidence interval, associated with the point estimate of

<sup>4</sup> The estimate emerging from the [R] code later in this paper is in 1997 prices ( $K=1.1$  in that computation).

<sup>5</sup> Whilst not a time-series elasticity, this does give a sense of variation in VTTS with respect to travel conditions and external factors.

$V_t^*$ . In this regard – and drawing reference to Yang et al.’s (2013) framework outlined in section 2 above – four distinct sources of statistical uncertainty are relevant, namely.

- i. Parameter uncertainty arising from estimation of the discrete choice model (i.e. associated with the  $\beta_T$ ,  $\beta_C$ ,  $\eta_Y$  and  $\eta_C$  parameters in (1)); this introduces uncertainty to  $V_{yd}$  in (2).
- ii. Parameter uncertainty arising from estimation of the meta-model (i.e. associated with  $\varepsilon_{GDP}$  in (4)); this introduces uncertainty to  $V_t^*$  in (4).
- iii. Data uncertainty associated with the forecasting of future income levels; this introduces further uncertainty to  $V_t^*$  in (4).
- iv. Data uncertainty associated with the weighting of the base VTTS in (3); there is a degree of uncertainty as to whether the NTS sample is truly representative of the population.

The methodology outlined below will address the first two sources of uncertainty. Whilst any substantive analysis of the latter two sources of uncertainty falls beyond the remit of the present paper, section 5 will demonstrate that data uncertainty with respect to forecasting future income would serve only to amplify the results and policy conclusions from our analysis.

Before proceeding, we should acknowledge the potential for further – but unquantifiable – sources of uncertainty, in that the initial discrete choice model could have been mis-specified and/or (more likely) the data generating process could have been subject to a structural break since the base year of 1994. Whilst the former issue would simply call for re-specification of the discrete choice model, the latter issue would have more significant implications, necessitating the collection of new SP data. It is with such considerations in mind that national transport ministries, such as the UK DfT, intermittently review the case for spending public monies on updates to VTTS estimates. For present purposes, we will simplify matters by assuming that the discrete choice model was correctly specified in 1994, and that the specification continues to be defensible (indeed the latter assumption would seem to be (broadly<sup>6</sup>) supported by the Swiss and Dutch applications). By contrast, we will consider the scope for updating the GDP elasticity using more recent evidence.

The size and complexity of the problem provoked challenges in computing the statistical uncertainty of the non-work VTTS in (4). With reference to (3), estimates of non-work VTTS cover a large number of weighted non-linear functions; in total, there are 12 distance bands and 21 income bands, yielding 252 individual functions (i.e.  $d$  and  $y$  combinations) of the form (2). Furthermore, each such function is not independent of the others, given that they share the same parameter estimates.

For purposes of computing the standard error (SE) of the VTTS estimate (4), there are a range of statistical methods which could be employed. In choosing between methods, our starting point was to acknowledge that the only available inputs for the computation were the estimated parameters from the original SP model and the associated matrix of variances and

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<sup>6</sup> Both studies show slight deviations from (1), but the essence of the discrete choice model was the same.

co-variances<sup>7</sup>. This inevitably meant that the candidate statistical methods should utilise large sample (asymptotic) properties to provide valid intervals. Fortunately, this did not represent a significant constraint, since the sample sizes of the two contributing studies (the SP analysis and meta-analysis) were large (for example, the SP analysis generated 8038 and 4737 observations for the ‘other’ non-work and commuting journey purposes, respectively (Mackie et al., 2003 p18).

In order to compute confidence intervals for the problem at hand, we combined two well-established statistical methods, as follows:

- The Delta method; this utilises differential calculus to derive an expression for the variance of a specific function of estimated parameters. More specifically, the approach is to compute a first order Taylor series expansion of the variance (the square of the standard error) of a function around the estimated parameter value. This has the attractive property of converging to the true variance as the sample size increases.
- A simulation method attributed to Krinsky and Robb (1986) (K&R); this utilises the result that, for large samples, model parameter estimates are distributed Normally. In practical terms, each parameter is sampled from a multi-variate Normal distribution (utilising the estimated co-variance matrix), and the combination of parameters (i.e.  $\bar{V}$ , in the case of Stage 1 below) is then computed. This process is repeated many times ( $10^6$  times), and the computed  $\bar{V}$  are then ordered to form an empirical distribution, from which the standard error is calculated.

The two methods generally give similar results (e.g. see Krinsky and Robb (1991) and Greene (2012)); this is because both rely upon the large sampling result that the model parameter estimates are approximately Normally distributed. The choice between the two methods may, to a large extent, be influenced by the practicalities of the problem at hand. As outlined below, our own problem can be separated into two stages. For each stage, we utilise a different method (K&R for Stage 1 and the Delta method for Stage 2). The attraction of this formulation is that, by adopting different methods in the two stages, we can introduce much greater functionality in terms of allowing intervals to be computed for different years, for different assumptions on GDP growth, and for different GDP elasticities. The development of such scenarios would have been much more cumbersome if we had utilised K&R in both stages.

As noted above, both methods rely upon large sample approximations to the true parameter distribution in order to derive valid intervals. Whilst the underlying error distributions in the SP model (and indeed the meta-model) may not have been Normal, the maximum likelihood estimates of the model parameters have an (asymptotic) Normal distribution. This is based on the Central Limit Theorem applied to maximum likelihood estimates (see Greene, 2012, p554, Theorem 14.1 for a full discussion). Such reliance on large sample properties of estimators is commonplace in models estimated using maximum likelihood.

We justify a two stage process by recognising that the parameter estimates from the two econometric models (i.e. based on SP data (1), and based on meta-data (4)) are independent. This is because the sample data for each model are independent of each other. Importantly

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<sup>7</sup> The original SP data was not available to us, and we therefore dismissed the possibility of re-sampling using bootstrapping methods.

this allows for a two stage computation: first, uncertainty in  $\bar{V}$  is computed, and second, this is combined with uncertainty from the GDP elasticity to yield uncertainty in  $V_t^*$ .

#### 4.1 Stage 1: Computing the uncertainty of $\bar{V}$

With reference to (3),  $\bar{V}$  arises from a weighted average of non-work VTTS estimates across distance and income bands. This means that the actual base VTTS is a weighted summation of 252 non-linear functions. In principle this is not really a complication, but in practice most automated routines in off-the-shelf statistical packages are unable to consider such a convoluted relationship. The computation was therefore undertaken using bespoke code in the matrix programming language [R], and this is available from the first author on request.

Although it requires a relatively large amount of computing time, the K&R method was used to estimate the SE associated with the estimate of  $\bar{V}$ . The advantage of the K&R method is that it is easy to program, relative to deriving the associated expression required for the Delta method. The disadvantage is the need to re-run sampling simulations when parameters or levels of variables are changed (which limits sensitivity testing with respect to this element).

#### 4.2 Stage 2: Computing the uncertainty of $V_t^*$

Once the SE associated with the base non-work VTTS ( $\bar{V}$ ) had been computed, it was combined with the SE associated with the GDP elasticity ( $\varepsilon_{GDP}$ ) from the meta-analysis to form a confidence interval for the uprated VTTS. The latter SE was computed using the Delta method. Thus the K&R method was first used to compute the SE of the base non-work VTTS, followed by the Delta method to combine this with the uncertainty associated with the GDP elasticity. The Delta method can be used for Stage 2 since the expression for the overall SE is now relatively simple (unlike when computing the base VTTS). The advantage of using the Delta method for this part of the computation is that a closed form expression is available, thus allowing the user of this approach greater functionality vis-à-vis having to undertake further K&R runs in [R].

The SE for  $\varepsilon_{GDP}$  was reported by Mackie et al (2003) as 0.1646 and the SE for  $\bar{V}$  arose from the K&R simulations in Stage 1. GDP was assumed to be non-stochastic, i.e. with regards to point iii) at the outset of section 4, uncertainty associated with forecasting GDP was not taken into account.

Thus  $V_t^*$  in (4) comprises two sets of estimated parameters to which the Delta method is applied. Furthermore, because the two parameters derive from samples of data which are independent of each other (the meta-analysis sample is independent of the AHCG sample), there will be no correlation between the two parameter estimates. This simplifies the computation.

The Delta method proceeds by computing the (asymptotic) variance of  $V_t^*$  as:

$$\begin{aligned}
 asy.\text{var}(V_t^*) &= \left( \frac{\partial V_t^*}{\partial \bar{V}} \right)^2 \text{var}(\bar{V}) + \left( \frac{\partial V_t^*}{\partial \varepsilon_{GDP}} \right)^2 \text{var}(\varepsilon_{GDP}) \\
 asy.\text{var}(V_t^*) &= \left( \left( \frac{GDP_t}{GDP_{1994}} \right)^{\varepsilon_{GDP}} \right)^2 (SE(\bar{V}))^2 + \left( \bar{V} \cdot \ln \left( \frac{GDP_t}{GDP_{1994}} \right) \cdot \left( \frac{GDP_t}{GDP_{1994}} \right)^{\varepsilon_{GDP}} \right)^2 (SE(\varepsilon_{GDP}))^2
 \end{aligned} \tag{5}$$

The estimated SE of  $V_t^*$  is thus:

$$SE(V_t^*) = \sqrt{asy. \text{var}(V_t^*)}$$

The 95% Confidence Interval (CI) for  $V_t^*$  is then given as

$$95\% \text{ CI for } V_t^* = [V_t^* \pm 1.96 \cdot SE(V_t^*)] \quad (6)$$

## 5 Results

In this section, the findings on the width of interval estimates are described. Figure 1 presents 95% and 50% CIs for  $V_t^*$ , for the commuting journey purpose, from 1994 to 2075. Figure 2 presents the same interval for the 'other' non-work journey purpose. GDP was based on growth forecasts per head published in DfT (2012). The hypothetical appraisal period of 2015 to 2075 extends 81 years forward from the base year of 1994; this reflects the 60 year appraisal period which has become standard in UK-based appraisal of transport projects with indefinite lives (DfT, 2014b). The following two observations arise from Figures 1 and 2.

Firstly, as described in (4), the point estimate, i.e. the value actually used in appraisal, is increasing with GDP. Since forecasts of GDP are upward, the point estimate is increasing over time, even throughout the recent recession. In 1994, the central estimate for the commuting journey purpose is 9.39 pence per minute (ppm) (2010 prices), for 2015 it is 12.28 ppm, whilst in 2075 it is 32.69 ppm. For the 'other' non-work journey purpose, the 1994, 2015 and 2075 central estimates are 8.39, 10.97 and 29.21 ppm respectively.

Secondly, it is notable that the interval estimates for the earlier years are very tight relative to the intervals for later years. For example, the 1994 interval estimate for commuting is [8.35, 10.43] ppm and the associated SE is 0.53 ppm or 6% of the central estimate. In contrast, the interval estimate for 2075 is [15.85, 49.53] ppm and the associated SE is 8.59 ppm or 26% of the central estimate. Therefore, the interval increases over time both in absolute and proportionate terms.

This is a significant observation, suggesting that it is the growth in GDP away from the base that drives the increase in the width of the interval, and not the uncertainty within the GDP forecasts (since the latter is not quantified in this approach). In other words, uncertainty in the estimate of the GDP elasticity is amplified as the ratio of GDP in the year of interest to GDP in the base year increases. This can be seen in the expression for the variance of  $V_t^*$  in (5). In particular the term  $(\bar{V} \cdot \ln(GDP_t/GDP_{1994}) \cdot (GDP_t/GDP_{1994})^{\varepsilon_{GDP}})$  shows that the SE of  $V_t^*$  increases by a factor greater than proportional to GDP, and that this term is subsequently multiplied by the SE of the GDP elasticity. The width of the CI will therefore be sensitive to both the value of the point estimate and the error associated with the GDP elasticity, and this width will be amplified as the time period of interest becomes more distant from the base.

An important policy implication emerges from this analysis. It is not sufficient to provide a robust estimate of the base year non-work VTTS. Rather it is important to robustly estimate the growth factor (represented here by the GDP elasticity  $\varepsilon_{GDP}$ ) associated with the conversion of behavioural values into appraisal values. A more precise estimate of  $\varepsilon_{GDP}$  can

have substantial implications for the precision of the appraisal VTTS for years toward the end of the appraisal period. Given that the costs of projects tend to be front-loaded, with benefits (including travel time savings) appearing later in the appraisal horizon, it would seem beneficial to direct a reasonable research effort into precise VTTS estimates not just in the base year but also in future years i.e. through precise estimation of  $\varepsilon_{GDP}$ .

Indeed, the effect would be compounded if we were also to include uncertainty of the GDP forecast within the CIs (i.e. point iii in section 4). The upper and lower bounds of GDP forecasts naturally diverge as they extend into the future from the present day, and this would further contribute to the ‘fanning out’ of the CIs. Thus, in terms of absolute reductions in CIs at least, the benefits of re-sampling would become even more apparent.

This is not to say that re-sampling and re-modelling the base year non-work VTTS does not have benefits. Clearly this may (but not necessarily will) yield a more precise base estimate. Furthermore, re-sampling allows any unforeseen structural break that has occurred between the previous sampling date and the re-sampling date to be accounted for. However, the most immediate benefit indicated by this analysis is that re-sampling ‘resets’ the base year, meaning that that any future GDP value will be closer to the base year value than without re-sampling (assuming the upward trend).

Putting to one side the debate on whether to update or improve the model (this will be revisited in section 6), the analysis indicates that there exists substantial uncertainty as to the appraisal VTTS, even for years within the recommended 60 year appraisal period. Given the reliance of many transport schemes on time savings as a key source of benefit, it is likely that adopting high or low estimates would make material differences to the benefit-cost ratio of projects.

Arising from the above analysis, a natural question to ask is how can this information be integrated within an appraisal framework? This question provokes two responses.

Firstly, there is precedent in the appraisal framework in Britain for sensitivity testing of modelling assumptions (DfT, 2012b). Scheme promoters could be required to undertake a high and low VTTS sensitivity test.

Secondly, it should be recognised that whilst the 95% CI includes the actual value of  $V_t^*$  95 times out of a 100 in re-sampling, the region immediately around the central point estimate still represents the highest probability mass of the sampling statistic. Thus in probabilistic terms, there is a concentration of mass around the central estimate. This is important, since it indicates that the actual value of  $V_t^*$  is more likely to be located in proximity to the central estimate, and less likely to be located within the tails of the 95% CI. The latter proposition is reinforced by the 50% CI, which is reasonably proximate to the central point estimate. This observation, as well as the pragmatic desire to keep the high and low sensitivities within reasonable bounds, could encourage policymakers to set the high and low VTTS sensitivity levels at the 50% CI boundaries rather than at the 95% CI boundaries.

## **6 Incorporating best evidence on the GDP elasticity**

Abrantes and Wardman (2011) updated the meta-analysis (Wardman, 2001) that was used to generate the GDP elasticity  $\varepsilon_{GDP}$  in (4). This revised study included data from a substantial

number of studies not available in the original work and, as such, the updated GDP elasticity can be considered superior.

The revised point estimate was 0.9 (i.e. increased from 0.8 in the 2001 study), and embodied a smaller SE of 0.11 (as opposed to 0.16 in 2001). The implication for our estimated CI is shown for the commuting journey purpose in Figure 3, and for the 'other' non-work journey purpose in Figure 4. Table 1 summarises the existing and revised estimates. In addition to the central point estimate of non-work VTTS being greater for the revised estimate for each year, the smaller SE means that the CI is also narrower. The 1994 base year estimates are invariant to the GDP elasticity by construction. However, by 2075 (60 years from the hypothetical appraisal start year of 2015) the central estimate for the commuting journey purpose has increased to 38.20 ppm for the updated GDP elasticity (i.e. from the 2011 study), as compared with 32.69 ppm for the original GDP elasticity (i.e. from the 2001 study). The lower boundary has increased from 15.85 ppm to 24.71 ppm, whilst the upper bound has increased more modestly, to 51.70 ppm from 49.53 ppm. The net impact is to reduce the width of the CI, with the biggest gains realised at the lower bound (relative to the CI for the existing point estimate). This pattern is exactly the same for the 'other' non-work journey purpose (since both journey purposes are subject to the same GDP elasticities).

Moreover, by 2075 there remains a substantial degree of uncertainty concerning the estimate of non-work VTTS, but the revised GDP elasticity achieves a marked reduction in the width of the CI (e.g. 21% narrower for the 'other' non-work journey purpose).

**Table 1: Summary of 95% confidence interval estimates for the existing and revised GDP elasticities**

	1994	2015	2045	2075
<i>Commuting journey purpose</i>				
Pence per minute (2010 prices)				
Existing GDP elasticity				
Central estimate	9.39	12.28	19.69	32.69
Lower interval boundary	8.35	10.38	13.42	15.85
Upper interval boundary	10.43	14.18	25.96	49.53
Interval width	2.08	3.80	12.54	33.68
Updated GDP elasticity				
Central estimate	9.39	12.70	21.60	38.20
Lower interval boundary	8.35	11.02	16.68	24.71
Upper interval boundary	10.43	14.37	26.52	51.70
Interval width	2.08	3.36	9.84	26.99
<i>Percentage difference</i>	0%	-12%	-22%	-20%
	1994	2015	2045	2075
<i>Other non-work journey purpose</i>				
Existing GDP elasticity				
Central estimate	8.39	10.97	17.59	29.21
Lower interval boundary	7.70	9.48	12.15	14.32
Upper interval boundary	9.08	12.46	23.04	44.10
Interval width	1.37	2.97	10.89	29.77
Updated GDP elasticity				
Central estimate	8.39	11.35	19.30	34.14
Lower interval boundary	7.70	10.11	15.15	22.35
Upper interval boundary	9.08	12.58	23.45	45.92
Interval width	1.37	2.47	8.31	23.57
<i>Percentage difference</i>	0%	-17%	-24%	-21%

## 7 Sensitivity analysis to understand the decomposition of uncertainty and the implications for future investment in VTTS updates

Policy officials in the UK and elsewhere routinely face the challenge of allocating finite public monies to research and consultancy, whilst seeking to maximise the accuracy and precision of key planning parameters such as VTTS. Against this background, it is apparent that, whilst the previous section has demonstrated the clear analytical benefits of exploiting best evidence on the GDP elasticity, this does not obviate the need for policymakers to consider more significant questions such as:

- How frequently should the base VTTS be re-estimated?
- How much research and development expenditure should be committed to any such re-estimation?

Whilst it is outside the scope of the present paper to address these questions in any great detail, a simple ‘what if’ analysis using the existing model will offer useful insight.

This analysis begins by considering what the interval estimates for future year non-work VTTS would be if the base non-work VTTS were undertaken on 2015 rather than 1994 data. We assumed the same point estimate of non-work VTTS (2) as the original work, computed the base non-work VTTS (3) (as well as any inherent uncertainty), and uprated by GDP growth from 1994 to 2015 (4) (but considering any inherent uncertainty post-2015 only). In effect, this scenario considers the analytical benefits that would accrue if, for a given set of parameter estimates from (1), the analyst were able to ‘reset’ the extrapolation of appraisal values. Of course, if the parameter estimates from (1) were revised through re-sampling, and their precision improved, then this would introduce a second source of analytical benefit.

Finally, we computed 95% CIs for future year forecasts (2015 to 2075). These are shown (by the green line) in Figure 5 for the commuting journey purpose only. We restrict ourselves to this journey purpose, since the findings are the same for both journey purposes (and our calculations are in any case only illustrative).

We can see from Figure 5 that the interval widths are narrower post-2015. This is intuitive, since there is no longer a need to uprate (with uncertainty inherent within the uprating factor) between 1994 and 2015. What is more interesting is that there is a greater absolute decrease in the interval width for the later years relative to the earlier years. In particular, the 95% CI for 2075 (60 years from 2015) is 39% narrower for the re-sampled 2015 data relative to the original 1994 data, a decrease in uncertainty of 13.71 ppm. This compares to a decrease of 5.9 ppm for 2045. The overall impact of re-sampling is to achieve a substantial increase in precision for appraisal non-work VTTS estimates, especially towards the end of the 60 year appraisal period.

This finding is useful in itself for informing research and development policy. It implies that there is a need to regularly re-sample the base VTTS. However, research and development budgets are often heavily constrained and it is therefore useful to understand the trade-off between frequency of re-sampling (the above issue) and the level of expenditure assigned to each such re-sampling. We characterise the latter issue as relating to the precision of estimating the base non-work VTTS, which tends to be a function of sample size.

In addition to considering the scenario where the base non-work VTTS is re-sampled in 2015 but with the same precision as 1994, Figure 5 also shows (by way of the red line) the scenario where the base non-work VTTS is re-sampled in 2015 but with an increase in precision by a factor of three (i.e. the SE associated with the 2015 estimate is one third of the previously described 2015 reference<sup>8</sup>). This comparison reveals that in the years immediately following the new base of 2015, appraisal values are considerably more precise. For example, in 2025 the 95% CI for non-work VTTS has a width of 1.51 ppm for the 2015 base (red line), as compared with 2.73 ppm for the reference 2015 case (green line). However in later years, the difference becomes very small. In 2075 the interval widths are 19.83 and 20.51 ppm respectively, a difference of only 3.4%.

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<sup>8</sup> Improvements in the precision of behavioural estimates could be realised through increases in sample size and/or through the exploitation of ‘efficient’ SP design methods (which were not available in 1994).

## 8 Conclusion

This paper has developed confidence intervals (CIs) for non-work appraisal values of travel time savings (VTTS). The paper has decomposed uncertainty into two independent parts; firstly, that which arises from estimating non-work VTTS in the base year, and secondly, that which arises from applying the GDP elasticity to uprate the base non-work VTTS estimate to the relevant appraisal year. The base year non-work VTTS was found to be estimated precisely relative to the uncertainty that manifests further into the forecasting period.

This gives rise to an interesting conclusion. The production of robust VTTS estimates for use in appraisal requires more than precisely estimating VTTS in the base year, since the precision of parameters used for uprating is a crucial determinant of the degree of uncertainty inherent within the overall VTTS for a given appraisal year. It has furthermore been demonstrated that, by exploiting best evidence on the GDP elasticity, the CIs for appraisal VTTS can be reduced, with the biggest percentage and absolute reductions pertaining to values further into the forecasting period.

That said, forecasting 81 years into the future (from the base year) is undeniably difficult, and even though the base year VTTS is the minor contributor to overall uncertainty, re-estimating the base year VTTS has the effect of re-setting the forecasting period. Whilst acknowledging the constraints on research and development budgets, we would advocate regular re-sampling of VTTS. Such re-sampling could however be modest in scale (and expense), since the benefit of precision in the base year VTTS estimate is considerably outweighed by any imprecision introduced when uprating over time.

An important caveat is that the estimated base year VTTS is assumed to be an unbiased estimator of the actual base year VTTS. Any economy in the data collection underpinning re-sampling of VTTS should not therefore overlook the need for the base model to be correctly specified and appropriately estimated.

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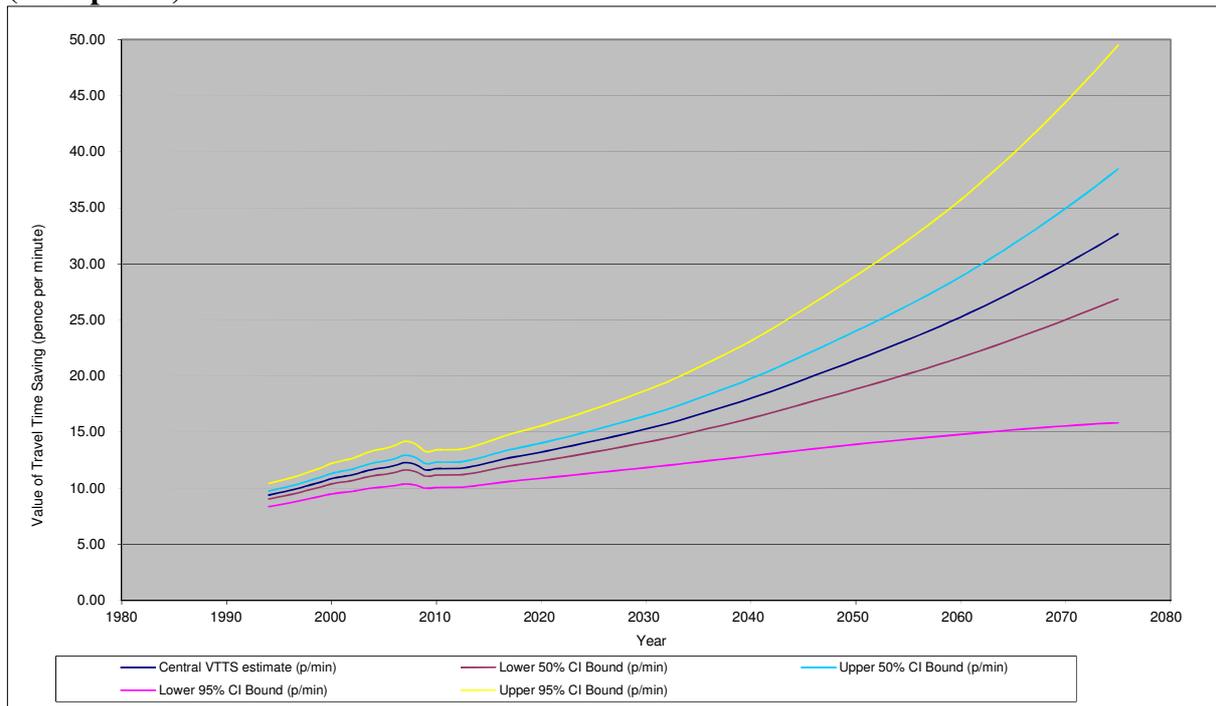
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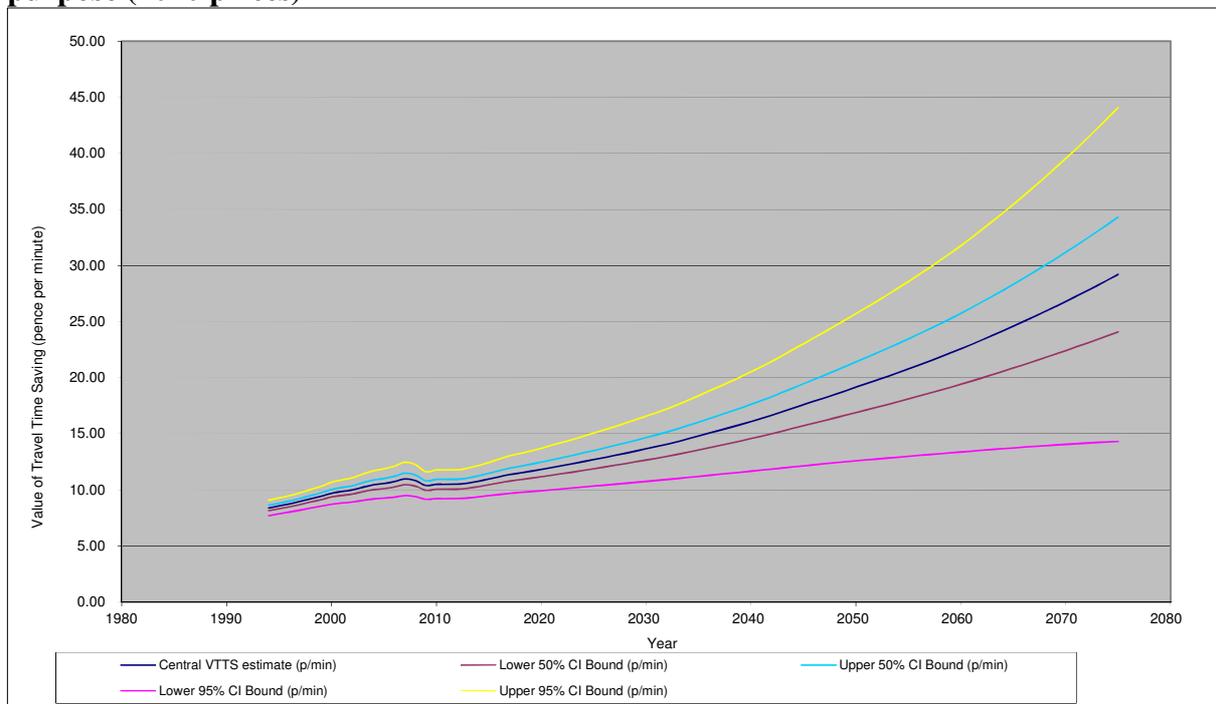
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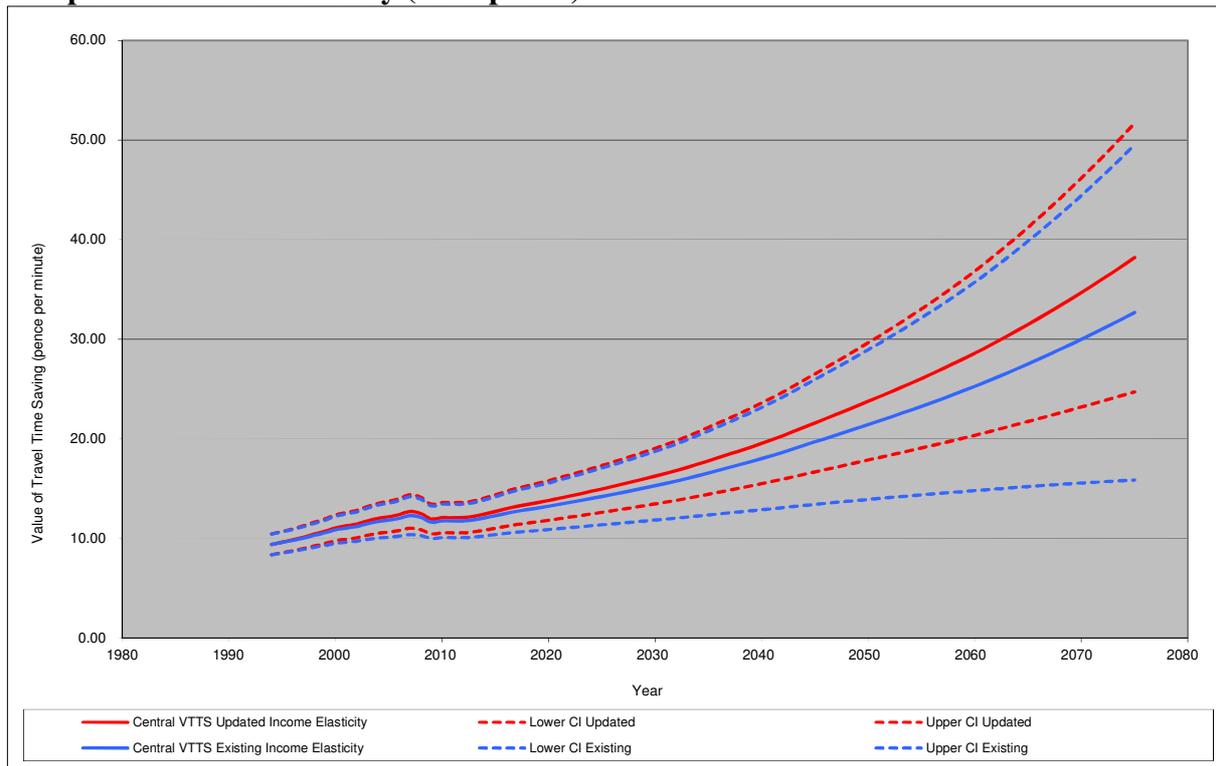
**Figure 1: 95% and 50% confidence intervals for VTTS for the commuting trip purpose (2010 prices)**



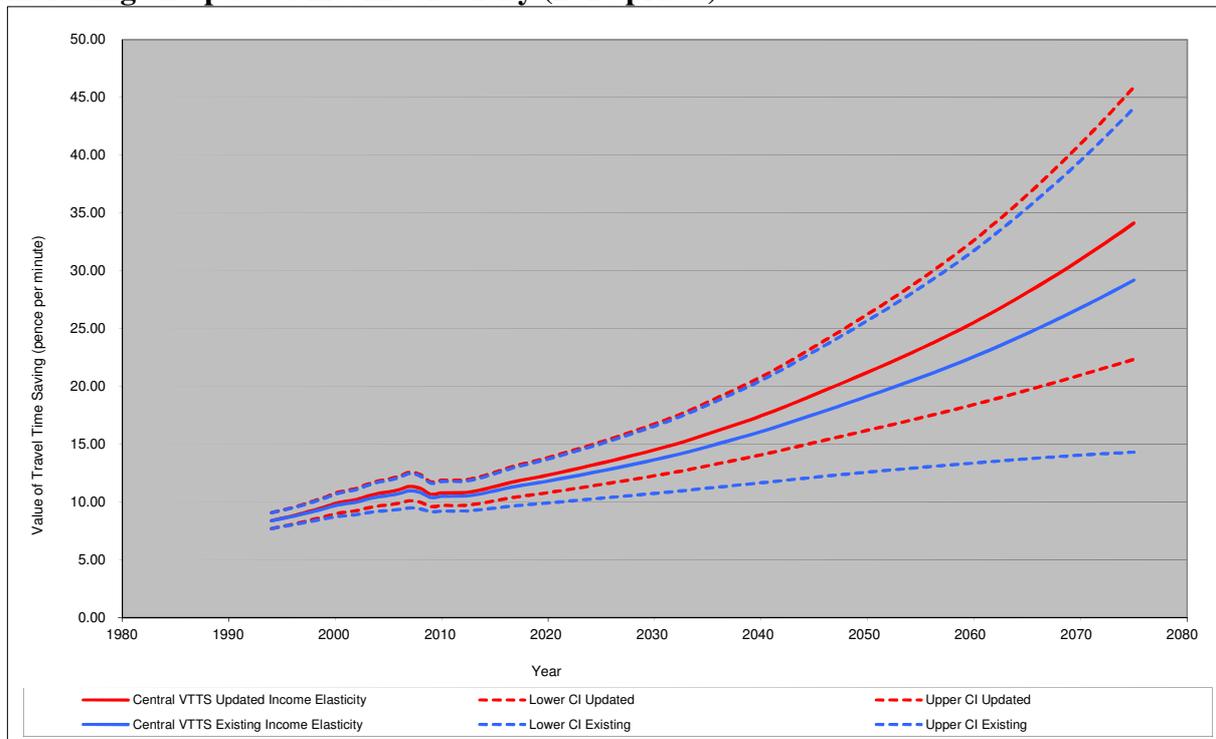
**Figure 2: 95% and 50% confidence intervals for VTTS for the 'other' non-work trip purpose (2010 prices)**



**Figure 3: 95% confidence intervals for VTTS for the commuting trip purpose including an updated income elasticity (2010 prices)**



**Figure 4: 95% confidence intervals for the 'other' non-work trip making purpose including an updated income elasticity (2010 prices)**



**Figure 5: 95% confidence intervals for VTTS for the commuting trip purpose including ‘what if’ analysis scenarios (2010 prices)**

