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# Reduction of Cogging Torque in Interior-Magnet Brushless Machines

Z. Q. Zhu, *Senior Member, IEEE*, S. Ruangsinchaiwanich, N. Schofield, and D. Howe

**Abstract**—An investigation into the cogging torque in a four-pole interior-magnet brushless machines having either a six-slot stator and a short-pitched nonoverlapping winding or a 12-slot stator and a full-pitched overlapping winding is described. It is shown by finite-element analyses and measurements that, by appropriately defining the pole-arc to pole-pitch ratio, the optimal pole-arc to pole-pitch ratio for minimum cogging torque, which has been derived for surface-mounted magnet machines, is equally applicable to interior-magnet machines.

**Index Terms**—Brushless machines, cogging torque, permanent-magnet machines, permanent magnets.

## I. INTRODUCTION

**C**OGGING torque results from the interaction of permanent magnet magnetomotive force (MMF) harmonics and air-gap permeance harmonics due to slotting. Generally, it is desirable to reduce the cogging torque since it does not contribute to the average torque and may cause speed ripple and vibration, particularly at light load and low speed. Numerous methods for reducing the cogging torque, such as skewing the stator slots and/or rotor magnets, shaping the magnets, optimizing the pole-arc to pole-pitch ratio of the magnets, employing a fractional number of slots per pole, etc., have been proposed [1]–[3]. While some cogging torque reduction techniques, e.g., skewing, are applicable to any type of permanent-magnet machine, most papers on this topic are generally restricted to machines having surface-mounted magnet rotors [1]–[5]. However, machines equipped with interior-magnet rotors generally have a higher torque density due to the saliency torque component and require less permanent magnet material. They also exhibit a higher demagnetization withstand capability, which is an important consideration under flux-weakening operation. However, relatively little has been reported on their cogging torque and its reduction.

This paper shows that the optimum pole-arc to pole-pitch ratio for achieving negligible cogging torque in surface-mounted magnet machines is applicable to interior-magnet machines. It describes an investigation into the cogging torque that results in a four-pole interior-magnet brushless machine having either six slots and a short-pitched nonoverlapping winding (i.e., concentrated coils) or 12 slots and a full-pitched overlapping winding, the rotor

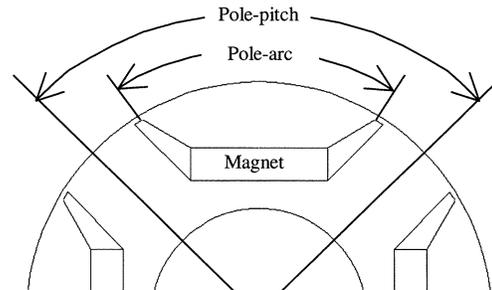


Fig. 1. Definition of pole-arc to pole-pitch ratio.

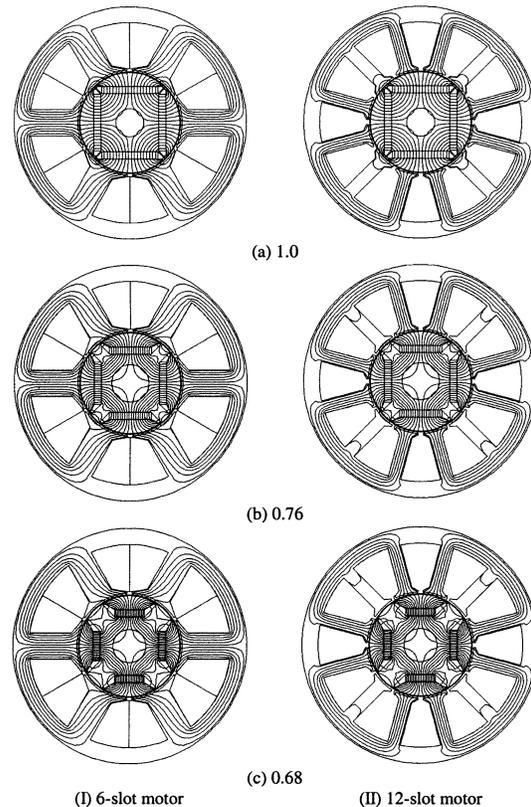


Fig. 2. Field distributions in four-pole interior-magnet brushless machines with pole-arc to pole-pitch ratio = 1.0, 0.76 and 0.68.

and stator diameters and the width of the slot openings being identical. The main machine parameters are stator outside diameter = 100 mm, active length = 50 mm, airgap length = 1 mm, rotor diameter = 43 mm, and width of stator slot openings = 2 mm. Sintered NdFeB magnets with a remanence of 1.2 T, a relative recoil permeability of 1.05, and a thickness of 3 mm are employed.

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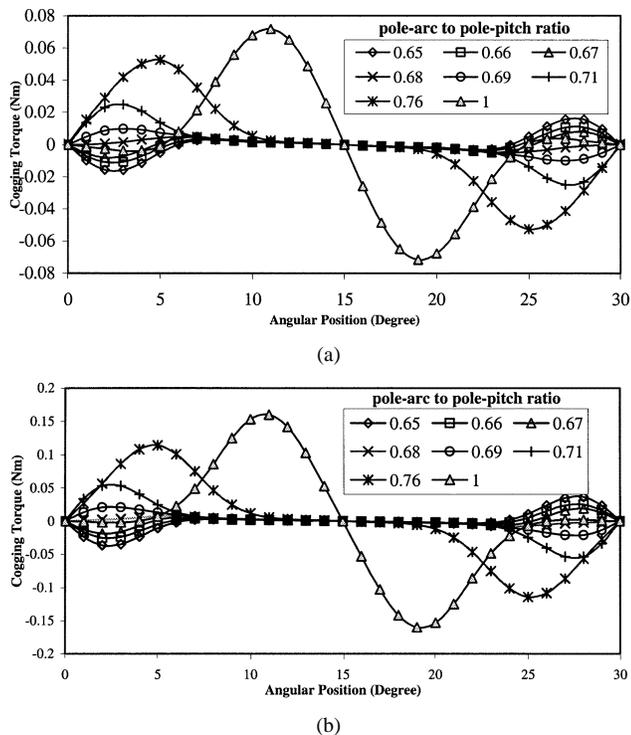


Fig. 3. Variation of cogging torque waveform with pole-arc to pole-pitch ratio. (a) 6-slot motor and (b) 12-slot motor.

II. FINITE- ELEMENT ANALYSIS

The finite element package, MEGA, is employed in the investigation and the Maxwell stress integration technique is used for the calculation of the cogging torque. Particular attention has been paid to the mesh discretization so as to ensure accurate cogging torque calculations [6].

In order to aid the discussion, the ‘‘pole-arc’’ of an interior-magnet rotor is defined as the arc, which is subtended between the rotor lamination bridges (see Fig. 1). This definition is equivalent to the magnet pole-arc in a surface-mounted magnet machine. The variation of the cogging torque with the ‘‘pole-arc’’ is investigated by finite element analysis. Fig. 2 shows the field distributions that result when the pole-arc to pole-pitch is equal to 1.0, 0.76, and 0.68, values of 1.0 and 0.68 resulting in the maximum and the minimum cogging torque, respectively, as will be evident from Figs. 3 and 4(a).

When flux leakage is neglected, the optimal magnet pole-arc to pole-pitch ratio for minimum cogging torque in a surface-mounted magnet machine can be determined analytically [2] from  $\alpha_p = (N_c/2p - k)/(N_c/2p)$ , where  $N_c$  is the least common multiple between the slot number  $N_s$  and the pole number  $2p$ , and  $k = 1, 2, 3, \dots (N_c/2p)$ . For both six-slot and 12-slot interior-magnet machines, the pole-arc to pole-pitch ratio for minimum cogging torque as deduced from finite element analyzes is 0.68, which compares with an analytically derived value of 0.667 for machines with a surface-magnet rotor when flux leakage is neglected. Fig. 4(a) shows the variation of the peak cogging torque for machines having the same magnet thickness, but with a varying magnet width and, hence, different magnet volumes and corresponds to the variation of the cogging torque waveforms with the pole-arc to

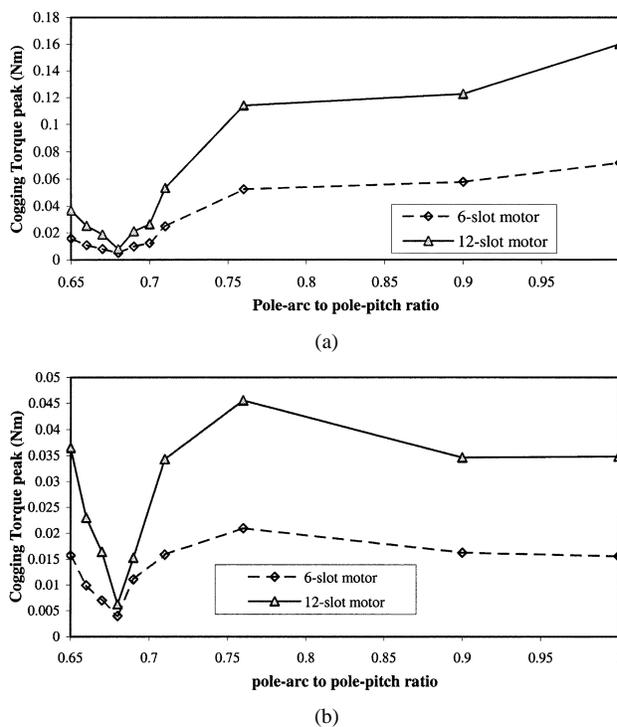


Fig. 4. Variation of peak cogging torque with pole-arc to pole-pitch ratio. (a) Varying magnet value and (b) constant magnet value.

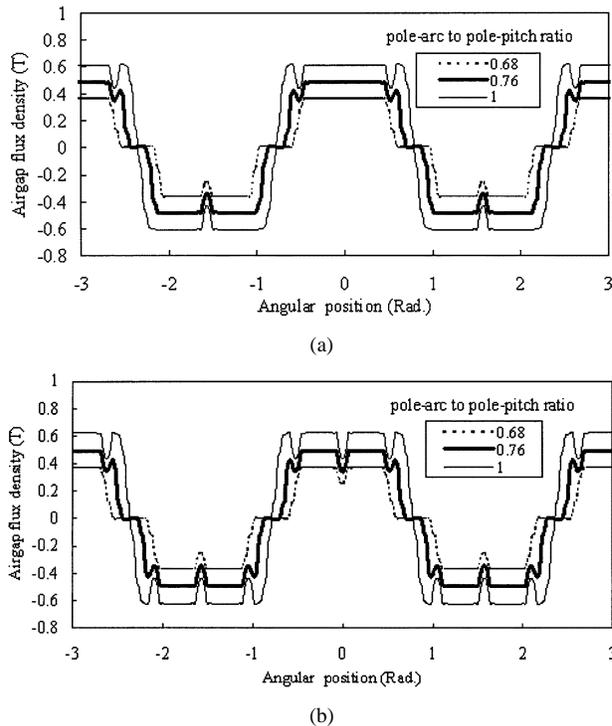


Fig. 5. Open-circuit air-gap field distributions in six-slot and 12-slot four-pole brushless machines. (a) 6-slot motor and (b) 12-slot motor.

pole-pitch ratio shown in Fig. 3. Different magnet widths result in different amplitudes of the air-gap flux density (see Fig. 5) and, hence, different levels of cogging torque. If the magnet width is maintained constant to correspond to a pole-arc to pole-pitch of 0.68, so as to maintain the magnet volume constant, the peak cogging torque varies as shown in Fig. 4(b).

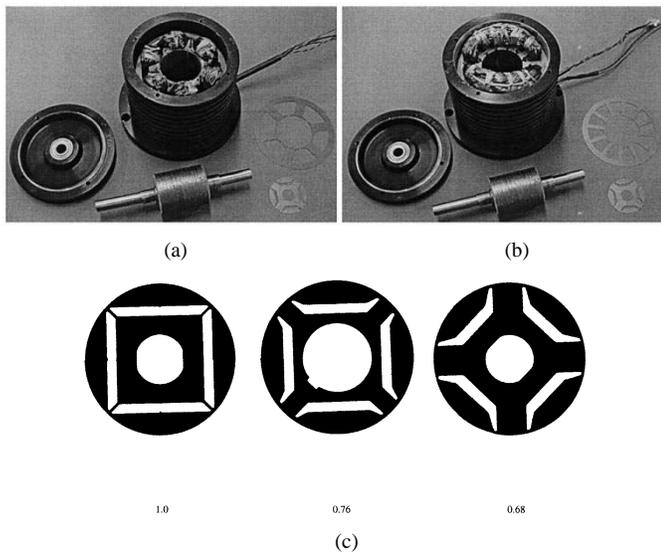


Fig. 6. Prototype machines. (a) Six-slot motor, (b) 12-slot motor, and (c) rotor laminations with different pole-arc to pole-pitch ratio.

As can be seen, the optimal pole-arc to pole-pitch ratio for minimum cogging torque remains unchanged.

### III. EXPERIMENTAL VALIDATION

Figs. 6(a) and (b) show prototype four-pole brushless machines having six slots and a short-pitched nonoverlapping winding and 12 slots and a full-pitched overlapping winding, respectively, while Fig. 6(c) shows rotor laminations with pole-arc to pole-pitch ratios of 1.0, 0.76, and 0.68. Fig. 7 compares predicted and measured cogging torque waveforms for the six- and 12-slot prototype machines when they are equipped with the three interior-magnet rotors. As can be seen, good agreement is achieved. Measurements confirm that the cogging torque is a minimum when the pole-arc to pole-pitch is 0.68. It is known that by employing a fractional slot number per pole, the cogging torque can be reduced [2]–[4]. A goodness factor was introduced in [2] and is defined as  $C_T = N_c/2pN_s$ . The larger the factor  $C_T$ , the smaller will be the resultant cogging torque. For four-pole machines,  $C_T = 0.5$  and  $0.25$  for the six- and 12-slot stators, respectively. Consequently, the cogging torque, which results in a machine with nonoverlapping windings, is approximately half of that for a similar machine having overlapping windings, as evidenced from Figs. 3 and 4. This can also be explained by the fact that, in the four-pole/12-slot machine, the number of slots per pole is three, while in the four-pole/six-slot machine, it is  $3/2$ , a fractional number. Thus, when half the edges of north and south poles in the four-pole/six-slot machine align with slot openings, the other edges align with the centers of teeth, whereas in the four-pole/12-slot machine, all the edges of the poles align with slot openings. Hence, the cogging torque in the six-slot machine with a nonoverlapping winding is approximately half of that in a similar 12-slot machine having overlapping windings.

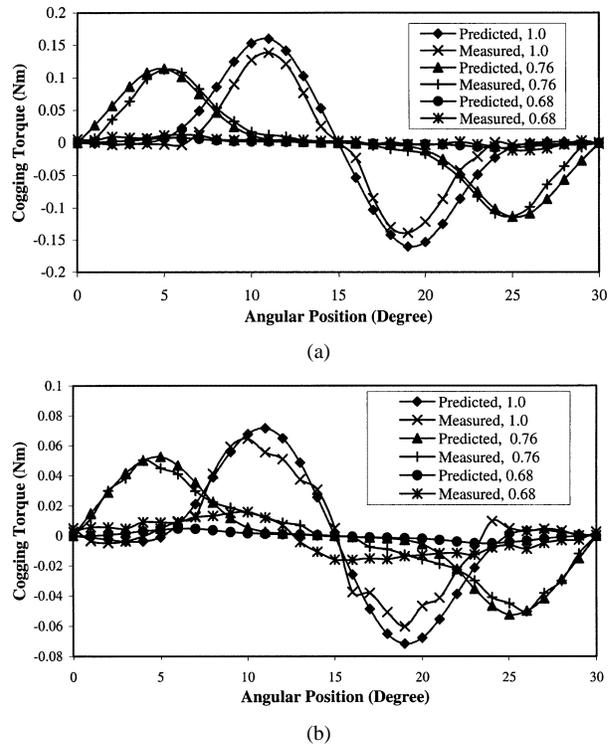


Fig. 7. Measured and predicted cogging torque waveforms for different pole-arc to pole-pitch ratios. (a) 6-slot motor and (b) 12-slot motor.

### IV. CONCLUSION

The cogging torque, which results in a four-pole interior-magnet brushless machine having either a six-slot stator with a short-pitched nonoverlapping winding or a 12-slot stator with a full-pitched overlapping winding, has been investigated by finite element analysis and validated by measurements. It has been shown that, by appropriately defining an effective pole-arc to pole-pitch ratio, the optimal ratio for minimum cogging torque, which was derived for surface-mounted magnet machines, is equally applicable to interior-magnet machines and that the cogging torque in machines with nonoverlapping windings is approximately half of that in similar machines having overlapping windings.

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