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The Spiderwheel - a new apparatus to demonstrate energy conservation and moment of inertia.

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Abstract

Rotational and rolling motion often proves to be a difficult topic for undergraduate students in their mechanics courses. A number of demonstrations have been developed but often these rely on switching the rolling object with another in order to vary the mass or radius, and so the students see the *entire* rolling system change rather than the specific property in question. In this work a new apparatus, known as the spiderwheel, is described which allows large changes in mass distribution (and hence moment of inertia) without anything being added to or removed from the rolling body.

A quantitative analysis of the rolling spiderwheel allows students to determine the moment of inertia of the body and compare it with model systems, namely a point-mass (that is, a particle with non-zero mass rotating about a fixed axis), a solid cylinder and a thin rigid hoop. Despite the spiderwheel being a non-ideal system in that it has a complex geometry with less symmetry and multiple components compared to the aforementioned model systems, it is found that the simple point-mass model provides an excellent approximation.

Furthermore students in an undergraduate course were asked to predict the effect of moving the masses further from the axis of rotation, and the majority incorrectly predicted a faster roll down the ramp (in line with more comprehensive studies on these misconceptions[1]. The spiderwheel is a simple yet versatile model for visualising difficult concepts in rotational motion. Students can consolidate their understanding of these concepts by altering the parameters of the spiderwheel and directly observing the effects on rotational and translational velocity.

I. INTRODUCTION

In many undergraduate physics courses mechanics is taught early in the syllabus because much of it, in particular linear kinematics, is revision of what is taught in pre-university courses. However, conceptualising and understanding the moment of inertia often proves to be difficult even when considering simple symmetric systems. Rimoldini and Singh found that many students had difficulties with concepts in rotational and rolling motion, such as rotational energy and moment of inertia[1].

In order to address this point an earlier method for measuring centripetal force[2] was adapted to provide a benchtop demonstration of how the moment of inertia varies with distribution of mass along an axis[3]. Others have designed demonstrations that examine rolling motion of an iron sphere down a grooved track, with a photocell used to determine the final velocity of the sphere[4]. More recently others combined high-speed video techniques[5,6] to analyse the complete motion of solid and hollow cylinders down a ramp[7].

The motivation behind this work was to develop a system that visualises the effect of mass distribution on rolling motion. Whilst others have shown the effect of mass and object geometry in rolling motion by replacing the moving object[4–8], the system presented here follows the 'nothing added, nothing removed' principle. Students watching the demonstration can clearly see that the only change to the rolling object is a redistribution of mass. This is intended to address some of the observations made by Rimoldini and Singh in that students did not always recognise that I is a function of the mass distribution about an axis[1].

Using a simple ramp setup provides a means to discuss conservation of energy in a way that students are already familiar with (having typically been introduced to the block sliding down a frictionless ramp experiment in previous courses). This leads naturally to the discussion for rolling systems that have the same potential (gravitational) energy available to the system but is 'distributed' between translational and rotational motion; anecdotally students often hold the misconception that a larger mass distribution (i.e. when the mass of the system is distributed further from the axis of rotation) means a larger translational velocity at the end of the ramp.

II. METHODS

A simple yet adjustable system known as the spiderwheel was developed and built, the schematic of which can be seen in figure 1 and the complete system is shown in figure 2. The conceived design is a custom-built apparatus comprising of a central hollow metal tube with a rough rubber sheath (to prevent sliding), two plastic hubs with 4 metal spokes each with a movable aluminium mass (otherwise known as the 'grenade' of mass m_g = (0.67349 $\pm$ 0.00059) kg) held in place with a plastic screw. A hub and spoke section was fixed to each end of the central axle and the spokes were aligned with each other. The grenades were moved to different distances along the spokes but always in a symmetric manner - any asymmetry between grenade position on opposing hubs could cause the spiderwheel to turn and eventually collide with the ramp.

Timings were first done manually with a digital stop-watch, measuring the time between release and reaching a pre-determined point 1.050 meters down the ramp from the start point. The height and length of travel allow the angle of inclination to be determined, from which the resolved component of weight down the ramp was found and conservation of kinetic energy was used to find the final velocity.

Subsequently a simple switched circuit was introduced to start and stop a digital timer. In this case the spiderwheel was released from rest but allowed to travel 0.145 m before contacting the start switch and so the final velocity was found from the kinematic equations with a non-rest start. This method provides a more precise measure of the time taken for the spiderwheel to travel 1.050 m when measured by a single experimenter. The same equipment was demonstrated during a lecture the audience were asked to use their smart phones or similar devices to measure the time and the average result derived from audience participation was within 5% of the digital timer method.

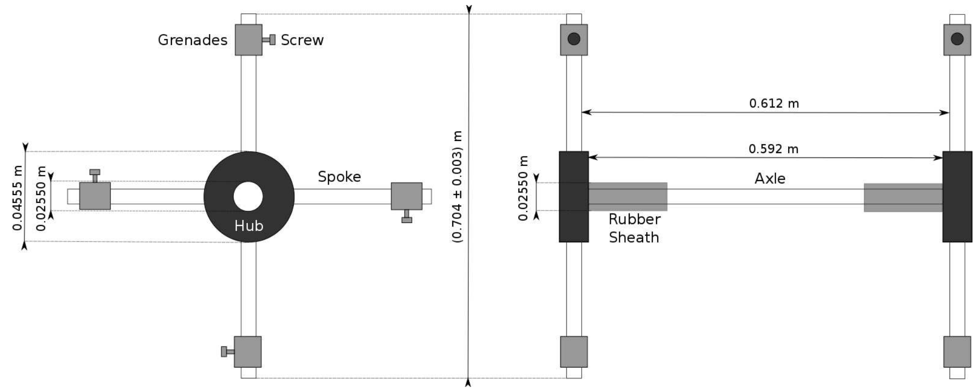

FIG. 1. Schematic of the spiderwheel system comprised of eight aluminium tubes fixed into plastic discs linked together by another aluminium tube with rubber sheathing adjacent to the discs.

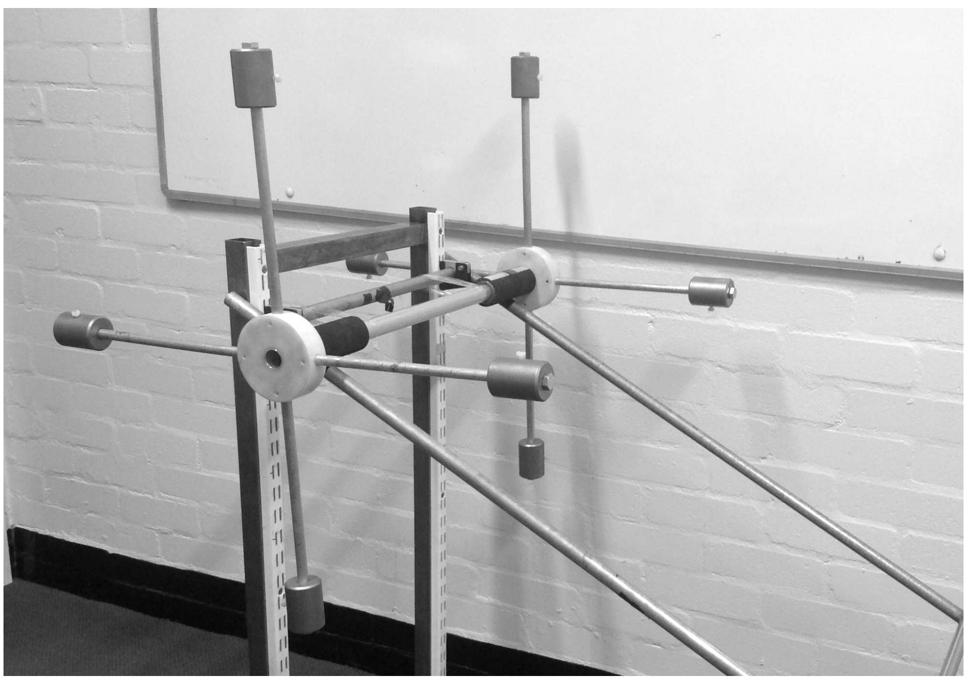

FIG. 2. Spiderwheel system setup. The brackets mounted to the ramp stand allow for easy adjustment and measurement of ramp height.

III. THEORY

For the convenience of the reader a summary of the theory describing an object rolling down a ramp without slipping is described in the following paragraphs, however a more detailed derivation can be found in any undergraduate mechanics textbook.

A. Rolling of simple symmetric systems

In the case of a mass sliding down an inclined frictionless surface the initial (potential) energy is converted into final kinetic energy at the end of the ramp. However for a rolling mass the same initial energy is distributed between translational (linear) and rotational kinetic energies. So for a rolling mass

$$mg\Delta h = \frac{1}{2}mv_{\mathrm{CM}}^2 + \frac{1}{2}I_{\mathrm{CM}}\omega^2 \tag{1}$$

where m is the mass of the rolling object, g is the acceleration due to gravity, Δh is the difference in height between the start and end points, v_{CM} is the linear centre of mass velocity, and ω is the angular velocity of the rolling mass. The moment of inertia I_{CM} is given by

$$I_{\mathrm{CM}} = cmr^2 \tag{2}$$

for a symmetric system of radius r, and the constant c depends on the geometric distribution of the mass. It can be shown that $c = 1$ for a rigid hoop and $c = 0.5$ for a solid cylinder, though more complicated mass distributions require complex modelling of the system which is beyond the scope of this work.

For circular or spherical object of radius r rolling down an inclined path the angular velocity can be expressed as $\omega = v/r$ and so equation 1 can be expressed as

$$v_{\mathrm{CM}}^2 = \frac{2g}{1+c}.\Delta h \tag{3}$$

B. Spiderwheel system

Unlike the simple rolling system described above, the spiderwheel has two different radii parameters: the distance from the centre of rotation to the movable masses, r, (equivalent to that in equation 2) and the radius of the central axle, R. This allows equation 3 to be rewritten in the following form that accounts for masses extending beyond the central axle,

$$v_{\mathrm{CM}}^2 = \frac{2g}{1+c\frac{R^2}{r^2}}.\Delta h \tag{4}$$

by noting that the angular velocity is now determined by the size of the axle (including rubber sheaths) and thus $\omega = v/R$. As R is a fixed parameter and r should be kept constant for different heights, the prefactor c that describes the shape of the mass distribution can be extract from a linear fit of v_{CM}^2 against Δh.

IV. RESULTS AND DISCUSSION

The time taken to roll a distance of L allowed the final velocity to be determined for different ramp heights. A straight line model was developed based upon equation 4 and fitting this model to the measured data allowed c to be determined, as shown in figure 3. This was repeated with the grenades at different positions from the axis of rotation, in the range $0.076\text{ m} \leq r \leq 0.306\text{ m}$.

In figure 4 the moment of inertia is determined using the values for c extracted from the fits of equation 4 in figure 3. These were compared to three idealised systems, the point mass + hub ($I = 8m_g r^2 + \frac{1}{2}m_h r_h^2$), thin rigid hoop ($I = Mr^2$) and solid cylinder ($I = \frac{1}{2}Mr^2$) where $M = 7.47$ kg is the total mass of the spiderwheel, $m_h = 2.081$ kg and $r_h = 0.0455$ m are the mass and radius of the hub respectively (it is assumed that the spiderwheel excluding grenades is approximated by a solid cylinder and as such the spokes are assumed to have negligible contribution).

Although the simplest approximations for the system are a poor model for the observed data it is found that the point mass plus solid cylinder hub model provides a good fit to the data ($\chi^2 = 0.675$, $n = 8$) when the first data point is excluded from the fit. This first point may lie outside the predications of the presented models as it corresponds to the grenades being in contact with the hub (minimum radius), and so a more complex model for a ridged cylinder would be needed but this is beyond the scope of elemantary courses. It should be noted that the additional term representing the moment of inertia from the hub provides little correction to the model. Only considering the grenades as point sources gives a fit with $\chi^2 = 1.11$.

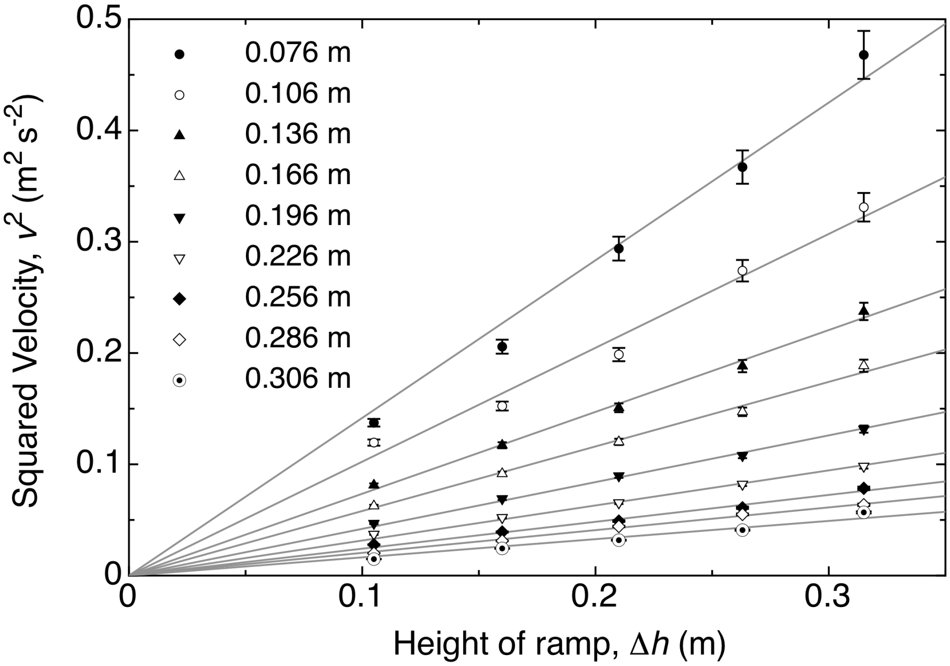

FIG. 3. Final velocities were calculated from the measured roll time down a predetermined ramp length inclined at an angle θ, with an acceleration $a = gsin\theta$. Straight lines represent the fits of equation 4 from which values of c can be determined.

A. Use as a demonstration

First year physics undergraduates are typically introduced to the concept of rotational motion immediately after content on linear dynamics, forces, and the conservation of energy and momentum. The spiderwheel demonstration was presented early in a course on rotational kinematics with the author, having spent the previous lectures discussing the concept of moment of inertia along with derivations of ideal cases.

For this demonstration the spiderwheel was set up with its grenades close to the hub and released down the ramp set to $\Delta h = 0.3$ m. Students observed the spiderwheel rolling down the ramp, after which the grenades were moved to the maximum distance from the axis of rotation. Whilst the demonstration was being reconfigured the students were asked whether the spiderwheel would go:

1. Faster?

2. Slower?

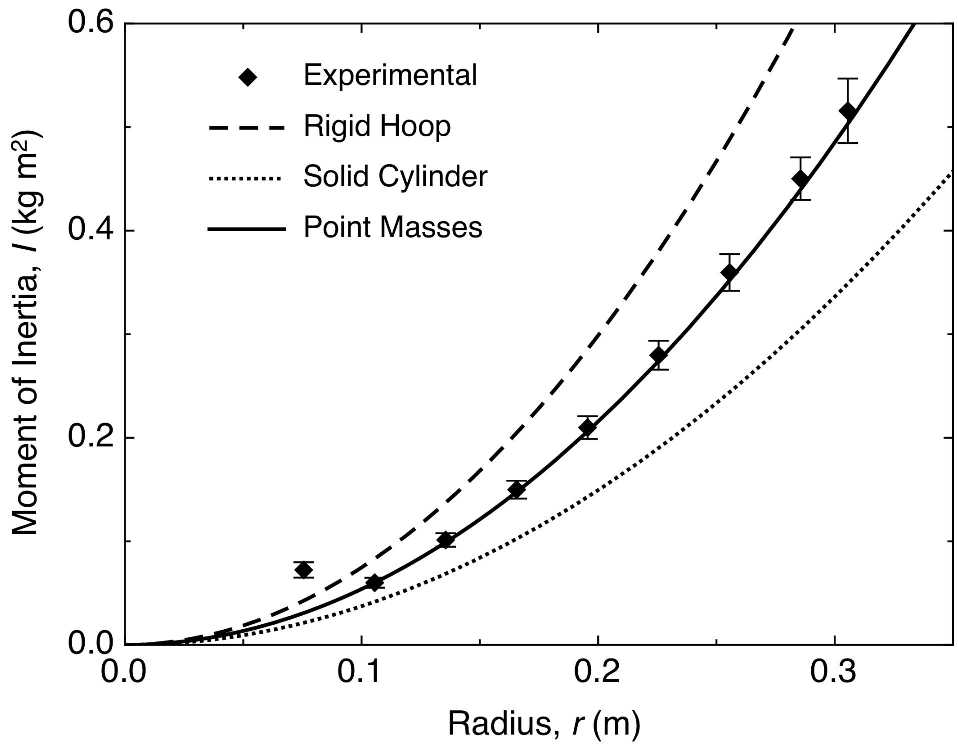

FIG. 4. Moment of inertia for the system calculated from c determined in figure 3. The model curves are for the ideal point masses (solid), thin rigid hoop (dashed) and uniform cylinder (dotted).

3. Same speed?

and were asked to respond using the TurningPoint audience response system (Turning Technologies, Belfast, UK). In a class of 122 the majority of students responded with 'Faster' (64%), with the most common explanation being that "it just seems right". Only 28% responded with the correct answer ('Slower'), and a minority of students opted for the 'Same speed' (8%).

These responses (which agree with other published work[1]) provided a starting point for the discussion on the importance of mass distribution in rotating systems, and the spiderwheel provides the additional benefit to other rolling systems in that the total mass and radius of rotation both remain constant.

As an interesting aside, some additional time at the end of the lecture was spent trying the spiderwheel in different mass distributions. The most interesting point from a pedagogical perspective was positioning the grenades as max-min-max-min on one hub and min-max-min-max on the other, such that each pair of opposing spokes had one grenade at maximum position and the other at minimum. This distribution gave a rolling time approximately equal to a fully symmetric distribution with masses halfway along the spokes (by rough measurement with digital watches or smartphone timers) as expected. The discussion following this emphasised to students that the mass distribution along the axis of rotation does not affect the rolling motion and so the different mass placements "average out".

V. CONCLUSION

Rolling motion is typically a difficult topic for undergraduate students to understand and whilst a number of experiments have been developed to help demonstrate this subject, the spiderwheel described here provides a novel system that allows students to explore the effect of mass redistribution in a closed system (i.e. no mass is added to or removed from the system). It also allows for quantitative measurements of the translational velocity to be made, from which the moment of inertia can be compared with the ideal cases derived in their lecture courses.

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