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ON THE INVERSION OF UNIFORM RANK
MULTIVARIABLE SYSTEMS

by

D. H. Owens B.Sc., A.R.C.S., Ph.D., A.F.I.M.A., C.Eng., M.I.E.E.

Department of Control Engineering,
University of Sheffield,
Mappin Street,
Sheffield S1 3JD.

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Abstract

A simple numerical procedure is presented for the inversion of a system $S(A,B,C)$ satisfying the constraints $CA^{i-1}B = 0$, $1 \leq i \leq k - 1$, and $|CA^{k-1}B| \neq 0$ for some $k \geq 1$. The results generalize the work of Kouvaritakis for the case of $k = 1$.

The inversion of the m-input-m-output strictly proper, linear, time-invariant system $S(A,B,C)$ of state dimension n is of great theoretical and practical interest^(1,2). Although general techniques are available^(3,4), great simplification in computational procedures are possible in special cases⁽⁵⁾ of practical interest. The purpose of this note is to extend these results to the case of a $m \times m$ square system of uniform rank k (see, for example, ref. 2). That is a system satisfying the relations,

$$\begin{aligned} CA^{i-1} B &= 0 & 1 \leq i \leq k-1 \\ |CA^{k-1} B| &\neq 0 \end{aligned} \quad (1)$$

for some integer $k \geq 1$. In this case it is trivially verified that the inverse of the system transfer function matrix $G(s) = C(sI_n - A)^{-1} B$ takes the form

$$G^{-1}(s) = s^k A_0 + s^{k-1} A_1 + \dots + s A_{k-1} + A_k + A_0 H(s) \quad (2)$$

where $H(s)$ is strictly proper and $|A_0| \neq 0$. The calculation of the inverse system hence reduces to the calculation of A_0, A_1, \dots, A_k and $H(s)$.

Without loss of generality⁽⁷⁾ suppose that $S(A,B,C)$ has the special form

$$\begin{aligned} A &= \begin{pmatrix} 0 & I_m & 0 & \dots & 0 & 0 \\ 0 & 0 & I_m & & \cdot & \cdot \\ \cdot & & & & \cdot & \cdot \\ \cdot & & & & 0 & 0 \\ \cdot & & & & & \\ \cdot & & & 0 & I_m & 0 \\ \cdot & & & & & \\ -A_0^{-1} A_k & \dots & -A_0^{-1} A_2 & -A_0^{-1} A_1 & -C_2 \\ B_2 & 0 & \dots & 0 & 0 & A_2 \end{pmatrix} \\ B &= \begin{pmatrix} 0_{(k-1)m \times m} \\ A_0^{-1} \\ 0_{(n-km) \times m} \end{pmatrix} & C &= [I_m \quad 0] \end{aligned} \quad (3)$$

by choice of state of the form $x^T(t) = [y^T(t), \dot{y}^T(t), \dots, y^{(k-1)T}(t), z^T(t)]$.

It is easily verified that, in this basis,

$$H(s) = C_2 (sI_{n-km} - A_2)^{-1} B_2 \quad (4)$$

$$C A^{i-1} = \begin{bmatrix} 0 & I_m & 0 \end{bmatrix}, \quad 1 \leq i \leq k \quad (5)$$

$$C A^k = - [A_0^{-1} A_k, \dots, A_0^{-1} A_1, C_2] \quad (6)$$

$$C A^{k-1} B = A_0^{-1} \quad (7)$$

Following recent results ⁽⁶⁾, it is convenient to define the full rank ⁽⁷⁾ matrices

$$C_k = \begin{bmatrix} C \\ C A \\ \vdots \\ C A^{k-1} \end{bmatrix}, \quad B_k = [B, AB, \dots, A^{k-1} B] \quad (8)$$

In particular, in the defined basis, these matrices take the form

$$C_k = \begin{bmatrix} I_{km} & 0 \end{bmatrix}, \quad B_k = \begin{bmatrix} C_k & B_k \\ 0 \end{bmatrix} \quad (9)$$

and hence, by combination with (6) and (7), the relation

$$[A_k \ A_{k-1} \ \dots \ A_2 \ A_1] = - (C A^{k-1} B)^{-1} C A^k B_k (C_k B_k)^{-1} \quad (10)$$

together with (7) defines the matrices A_0, A_1, \dots, A_k uniquely. In fact, the right-hand side of (10) and $C A^{k-1} B$ are independent of basis and hence can be used directly without the need to transform to the special form defined by (3).

Consider now the construction ⁽⁶⁾ of full rank $n \times (n-km)$ and $(n-km) \times n$ annihilators M_k and N_k respectively, satisfying the relations

$$C_k M_k = 0, \quad N_k B_k = 0, \quad N_k M_k = I_{n-km} \quad (11)$$

In the defined basis, we can always choose (eqn. (5)) $M_k^T = \begin{bmatrix} 0 & I_{n-km} \end{bmatrix} = N_k$

when a simple calculation yields the relations

$$C_2 = - C A^k M_k, \quad B_2 = N_k A^k B (C A^{k-1} B)^{-1} \\ A_2 = N_k A M_k \quad (12)$$

and hence, using (4),

$$H(s) = - C A^k M_k (s I_{n-km} - N_k A M_k)^{-1} N_k A^k B (C A^{k-1} B)^{-1} \quad (13)$$

Again this relation is basis independent and independent of the precise choice of N_k and M_k satisfying equation (11) as can be verified by noting that transformations of the form $A \rightarrow T^{-1} A T$, $B \rightarrow T^{-1} B$, $C \rightarrow C T$ 'induce' the 'transformations' $N_k \rightarrow L N_k T$, $M_k \rightarrow T^{-1} M_k L^{-1}$ where the matrix L is nonsingular.

In summary, we have proved the following basis independent result

Theorem

Let $S(A,B,C)$ have uniform rank k and define full rank matrices C_k and B_k by equation (8). The system inverse transfer function matrix then takes the form given in equation (2) with

$$A_0 = (C A^{k-1} B)^{-1} \quad (14)$$

$$[A_k, A_{k-1}, \dots, A_1] = -(C A^{k-1} B)^{-1} C A^k B_k (C_k B_k)^{-1} \quad (15)$$

and

$$H(s) = - C A^k M_k (s I_{n-km} - N_k A M_k)^{-1} N_k A^k B (C A^{k-1} B)^{-1} \quad (16)$$

where the annihilators N_k, M_k are any solutions of the relations

$$C_k M_k = 0, \quad N_k B_k = 0, \quad N_k M_k = I_{km} \quad (17)$$

This result reduces to previous work⁽⁵⁾ in the case of $k=1$. It also has the same overall structure of previous work with the simple operations involved being easily implemented on a digital computer.

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