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LINEAR MULTIPASS PROCESSES: A TWO-DIMENSIONAL INTERPRETATION

by

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Abstract

A large class of discrete multipass processes can be viewed as 2-D systems of the form proposed by Roesser. It is shown that the criteria for stability along the pass is equivalent to Shanks condition for 2-D BIBO stability. The interplay of ideas between the two disciplines should make possible the emergence of a coherent feedback control and systems theory for multipass processes.

1. Introduction

In a number of recent publications, Edwards[1-3] has examined the stability of multipass processes. Such processes are characterized by repetitive operations that can be illustrated by consideration of machining operations where the material or workpiece involved is processed by a sequence of passes of the processing tool. Assuming the pass length α to be constant, the output vector $y_k(x)$, $0 \leq x \leq \alpha$ (x being the independent spatial or temporal variable), generated on the kth pass acts as a forcing function on the next pass and hence contributes to the dynamics of the new output $y_{k+1}(x)$, $0 \leq x \leq \alpha$.

Edwards approach[1-3] to stability analysis is to convert the system into an infinite length single-pass process described by a differential/algebraic delay system and to apply standard scalar inverse-Nyquist stability criteria. The validity of this approach has been discussed by Owens[4,5] who has expressed concern that this approach neglects the initial conditions on each pass. An abstract functional analytic model for linear multipass processes was proposed and the distinct concepts of uniform asymptotic stability and stability along the pass were introduced and necessary and sufficient conditions derived in each case in terms of conditions on the system operator.

In this paper it is shown that a large class of discrete multipass processes can be viewed from the viewpoint of the well-developed theory of 2-dimensional systems which provides an alternative approach to systems stability analysis. The basis of the paper is a demonstration that a general model of linear, discrete time-invariant multipass processes are special cases of the state space model for

2-D systems suggested by Roesser[6]. The main result is that Owens criterion[4] for stability along the pass is equivalent, in this case, to a well-known 2-D stability criteria. Not only does this result release many recently derived stability tests for 2-D systems for application to multipass processes, it suggests the exciting possibility that the recently developed concepts of controllability, observability and minimality of 2-D systems could form the basis for the development of a control theory for multipass processes.

2. Two-dimensional Systems[16]: A review of basic concepts

2.1 Input/output Description

A 2-D linear shift invariant system can be described by the convolution of the input u(m,n) and the impulse response function h(m,n). In the current work interest will be restricted to scalar systems with input-output relationships of the recursive form

$$y(m,n) = \sum_{k=0}^{K} \sum_{\ell=0}^{L} a(k,\ell)u(m-k,n-\ell)$$

$$- \sum_{\substack{i=0 \\ (i,j)\neq(0,0)}}^{I} \sum_{j=0}^{J} b(i,j)y(m-i,n-j) \qquad \ldots(1)$$

This difference equation in fact describes a form of 2-D digital filter referred to as a quarter-plane filter[7]. It is also termed spatially causal over the quadrant $(m,n)>(0,0)$ as $y(m,n)$ depends only on input and output variables at points $(i,j) \leq (m,n)$.

Taking the 2-D z-transform of eqn (1) leads to the transfer function relating y to u of the form

$$H(z_1,z_2) = \frac{A(z_1,z_2)}{B(z_1,z_2)} \qquad \ldots(2)$$

where
$$A(z_1,z_2) = \sum_{k=0}^{K} \sum_{\ell=0}^{L} a(k,\ell) z_1^{k} z_2^{\ell}$$

$$B(z_1,z_2) = \sum_{i=0}^{I} \sum_{j=0}^{J} b(i,j) z_1^{i} z_2^{j} \qquad \ldots(3)$$

and, for notational simplicity, we have taken $b(0,0) = 1$. The reader should note that, in contrast to normal control practice, we are adopting the 2-D system convention of regarding z_1 and z_2 as <u>backward</u> shifts.

Expanding H as a power series yields

$$H(z_1,z_2) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h(m,n) z_1^{m} z_2^{n} \qquad \ldots(4)$$

and the system is said to be bounded-input-bounded-output (BIBO) stable if, and only if,

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} |h(m,n)| < +\infty \qquad \ldots(5)$$

If A and B are mutually prime and H has no nonesssential singularities of the second kind[7] (ie there exists no $(\bar{z}_1,\bar{z}_2)$ such that $A(\bar{z}_1,\bar{z}_2) = B(\bar{z}_1,\bar{z}_2) = 0$) then a convenient test for BIBO stability has been presented by Shanks[8]

$$B(z_1,z_2) \neq 0 \qquad \text{for} \qquad |z_1| \leq 1 \;,\; |z_2| \leq 1 \qquad \ldots(6)$$

To check this condition is computationally very involved but can be simplified by application of the results of Huang[9] who has shown that equation (6) is equivalent to

$$\text{(a)} \quad B(z_1,0) \neq 0 \quad \forall \quad |z_1| \leq 1$$

$$\text{(b)} \quad B(z_1,z_2) \neq 0 \quad \forall \quad |z_1| = 1 \ , \ |z_2| \leq 1 \qquad \ldots(7)$$

Even these conditions have been simplified and generalized[10] and have formed the basis of a Nyquist-like stability test[11] for 2-D systems. They are also interchangeable in terms of z_1 and z_2.

2.2 State-space Model for 2-D Systems

Roesser[6] has presented a 2-D state-space model for systems recursive in the positive quadrant of the form (using Roessers original notation)

$$x_h(i+1,j) = A_1 x_h(i,j) + A_2 x_v(i,j) + B_1 u(i,j) \qquad \ldots(8)$$

$$x_v(i,j+1) = A_3 x_h(i,j) + A_4 x_v(i,j) + B_2 u(i,j) \qquad \ldots(9)$$

$$y(i,j) = C_1 x_h(i,j) + C_2 x_v(i,j) + Du(i,j) \qquad \ldots(10)$$

where i,j are positive integer valued horizontal and vertical coordinates, $x_h \in R^{n_1}$, $x_v \in R^{n_2}$ are vectors which propagate information in the horizontal and vertical directions respectively, $u \in R^{\ell}$ and $y \in R^m$ are vector inputs and outputs respectively and A_1, A_2, A_3, A_4, B_1, B_2, C_1, C_2 and D are matrices of the appropriate dimensions.

Applying the 2-D z-transform to eqns (8)-(10) gives the following 2-D transfer function matrix

$$H(z_1,z_2) = [C_1,C_2] \begin{pmatrix} z_1^{-1} I_{n_1} - A_1 & , \ -A_2 \\ -A_3 & z_2^{-1} I_{n_2} - A_4 \end{pmatrix}^{-1} \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} + D \qquad \ldots(11)$$

which can be computed by known algorithms[12]. Applying the ideas of section 2.1 to each element in turn, the BIBO stability of the system (as expressed by the Shanks or Huang test) is dependent on the roots of the characteristic polynomial

$$\rho(z_1,z_2) = \begin{vmatrix} I_{n_1}-z_1A_1 & -z_1A_2 \\ -z_2A_3 & I_{n_2}-z_2A_4 \end{vmatrix} \qquad \ldots(12)$$

which (applying Schurs formula[13]) can be expressed in the alternative form

$$\rho(z_1,z_2) = |I_{n_1}-z_1A_1|\cdot|I_{n_2}-z_2A_4-z_1z_2A_3(I_{n_1}-z_1A_1)^{-1}A_2| \quad \ldots(13)$$

We now prove the following theorem:

Theorem 1:

The Shanks (or, equivalently, the Huang) stability test for the 2-D Roesser model is equivalent to the conditions

(a) A_1 is a stability matrix

(b) A_4 is a stability matrix

(c) All eigenvalues of the transfer function matrix

$$Q(z_1^{-1}) \triangleq A_4 + A_3(z_1^{-1}I_{n_1}-A_1)^{-1}A_2 \qquad \ldots(14)$$

with $|z_1| = 1$ lie in the interior of the unit circle in the complex plane.

Proof

Applying the Huang test (7)(a) to $\rho(z_1,z_2)$ requires that $\rho(z_1,0) = |I_{n_1}-z_1A_1| \neq 0$ for $|z_1| \leq 1$ and, by interchanging the roles of z_1 and z_2, $\rho(0,z_2) = |I_{n_2}-z_2A_4| \neq 0$ for $|z_2| \leq 1$. It follows that A_1 and A_4 are stability matrices and, using (13), condition (7)(b) reduces to

$$T(z_1,z_2) \triangleq |I_{n_2} - z_2Q(z_1^{-1})| \neq 0 \;, \quad |z_1| = 1 \;, \; |z_2| \leq 1 \qquad \ldots(15)$$

It follows immediately that all eigenvalues of $Q(z_1^{-1})$, $|z_1| = 1$ lie in the interior of the unit circle in the complex plane. Conversely suppose that (a)-(c) above hold. We see immediately that $\rho(z_1,0) \neq 0$ for all $|z_1| \leq 1$ and hence that (7)(a) is valid. Also $\rho(z_1,z_2) \equiv \rho(z_1,0)T(z_1,z_2) \neq 0$ for $|z_1| = 1$, $|z_2| \leq 1$ as the eigenvalues of $I-z_2Q(z_1^{-1})$ are non-zero in this domain.

This theorem is, in its own right, a new form of stability test for the Roesser model. Its primary purpose in this paper however is as a convenient link to the natural form of multipass stability criteria.

3. Multipass Processes as 2-D Systems

This section explores the connection between multipass process models and the Roesser model. We begin with an example to motivate the discussion. A general treatment is given in section 3.2.

3.1 An Illustrative Scalar Example

Consider the simplified model of a multipass steel-rolling process

$$y_{k+1}(x) = Ky_k(x) + u_{k+1}(x) \qquad \ldots(16)$$

where x is the distance along the pass and k is the pass index. Assuming proportional control action of the form

$$u_{k+1}(x) = c(r_{k+1}(x) - y_{k+1}(x-X)) \qquad \ldots(17)$$

where X is the sensor delay, c is the controller gain and $r_{k+1}(x)$ is the reference on the $(k+1)^{th}$ pass, yields the closed-loop model of algebraic-delay form

$$y_{k+1}(x) = Ky_k(x) + c(r_{k+1}(x) - y_{k+1}(x-X)) \qquad \ldots(18)$$

Discretizing this system for the purposes of stability analysis by introducing the 'along-the-pass' index i by $x = iX$ and defining the variables $y(k,i) = y_k(iX)$, $r(k,i) = r_k(iX)$ yields the relations

$$y(k+1,i) = Ky(k,i) - cy(k+1,i-1) + cr(k+1,i) \qquad \ldots(19)$$

which can be regarded as a 2-D system with transfer function

$$H(z_1,z_2) = \frac{c}{1 - Kz_2 + cz_1} \qquad \ldots(20)$$

with numerator polynomial $A(z_1,z_2) = c$ and $B(z_1,z_2) = 1-Kz_2+cz_1$. The Huang test for BIBO stability requires that $1+cz_1 \neq 0$ for all $|z_1| \leq 1$ (and hence that $|c| \leq 1$) and that $1-Kz_2+cz_1 \neq 0$ for all $|z_1| = 1$, $|z_2| \leq 1$. This second condition is equivalent to the requirement that the circle $\{z : |z-1| \leq c\}$ should not intersect with the circle $\{z : |z| \leq K\}$ ie Huang's BIBO stability test reduces to the requirement that

$$|c| < 1 - |K| \qquad \ldots(21)$$

This condition is equivalent to that obtained by Edwards[1] using inverse Nyquist frequency domain analysis and also to the condition for stability along the pass derived by Owens[4]. It is clear therefore that there is some equivalence between 2-D BIBO stability tests and the multipass ideas of stability along the pass in this case. This is explored in the next section by considering the multipass equivalent to the 2-D system model due to Roesser.

3.2 Discrete Linear Time-invariant Multipass Processes

Consider a general discrete multipass process obtained by discretizing the general continuous model proposed by Owens[4]. The process is described by the equations

$$x_k(i+1) = Ax_k(i) + B_1y_k(i) + B_2r(i) \quad , \quad x_k(o) = x_o \qquad \ldots(22)$$

$$y_{k+1}(i) = Cx_k(i) + Dy_k(i) \qquad \ldots(23)$$

where $x_k(i) \in R^n$, $y_k(i) \in R^m$, $k \geq 0$ and $0 \leq i \leq I < +\infty$. For the purposes of consistency we have retained the notation of Owens[4], although it does clash somewhat with that of Roesser. Despite these notational differences it is easily seen that the multipass process of equations (22) and (23) takes the form of a 2-D system. More precisely it is a Roesser model with eqn (22) playing the role of equation (8) (and hence with x playing the role of 'horizontally transmitted' information) and equation (23) plays the role of equation (9) (with y representing 'vertically transmitted' information). The multipass process above has no analogue to equation (10), although one could envisage the introduction of other algebraic measurement equations that would play the same role.

Proposition 1: The 2-D stability test of Shanks and Huang in the form of theorem 1 requires that

(a) A is a stability matrix

(b) D is a stability matrix

(c) All eigenvalues of the transfer function matrix

$$G(z_1^{-1}) \triangleq D + C(z_1^{-1}I_n - A)^{-1}B_1 \qquad \ldots(24)$$

with $|z_1| = 1$ lie in the interior of the unit circle in the complex plane.

Consider now the form of the multipass stability criteria. Following Owens[4] the problem can be considered in the context of the Banach space E_I of mappings from the finite integer set $0 \leq i \leq I$ into the vector space C^m of complex m-vectors with norm

$$\| y \|_I \triangleq \max_{o \leq i \leq I} \| y(i) \| \qquad \ldots(25)$$

Writing equations (22), (23) in the equivalent form

$$y_{k+1}(0) = Cx_k(0) + Dy_k(0)$$

$$y_{k+1}(i) = CA^i x_o + Dy_k(i) + \sum_{\ell=0}^{i-1} CA^{i-\ell-1}(B_1 y_k(\ell) + B_2 r(\ell))$$

$$1 \leq i \leq I \qquad \ldots(26)$$

and defining the bounded linear map in E_I by the relations

$$(L_I y)(i) \triangleq \begin{cases} Dy(i) & : i = 0 \\ Dy(i) + \sum_{\ell=0}^{i-1} CA^{i-\ell-1} B_1 y(\ell) & : 1 \leq i \leq I \end{cases} \qquad \ldots(27)$$

and the constant vector $b_I \in E_I$ by

$$b_I(i) \triangleq \begin{cases} Cx_o & : i = 0 \\ Cx_o + \sum_{\ell=0}^{i-1} CA^{i-\ell-1} B_2 r(\ell) & : 1 \leq i \leq I \end{cases} \qquad \ldots(28)$$

then the multipass process takes the standard form[4]

$$y_{k+1} = L_I y_k + b_I \quad , \quad k \geq 0 \qquad \ldots(29)$$

This system is said to be uniformly asymptotically stable[4] if, and only if, given any initial profile $y_o \in E_I$ and known disturbance b_I, the sequence of pass profiles $\{y_k\}$ converges to an equilibrium profile y_∞ satisfying $y_\infty = Ly_\infty + b_I$ and that this property is also possessed by operators L' sufficiently close to L. Necessary and sufficient conditions for uniform asymptotic stability are that[4] the spectral radius $r_\infty(L_I)$ of L_I satisfies

$$r_\infty(L_I) < 1 \qquad \ldots(30)$$

Proposition 2: The multipass process of equations (22)-(23) is uniformly asymptotically stable if, and only if, all eigenvalues of D lie in the open unit disc in the complex plane.

Proof: As E_I is finite dimensional, λ is a spectral value of L_I if, and only if, it is an eigenvalue of L_I. Consider the equation $L_I y = \lambda y$, $y \neq 0$.ie

$$\begin{aligned} x(i+1) &= Ax(i) + B_1 y(i) \quad , \quad x(0) = 0 \\ \lambda y(i) &= Cx(i) + Dy(i) \quad , \quad 0 \leq i \leq I \end{aligned} \qquad \ldots(31)$$

If λ is not an eigenvalue of D then the equations take the form,

$$\begin{aligned} x(i+1) &= (A + B_1(\lambda I - D)^{-1}C)x(i) \quad , \quad x(0) = 0 \\ y(i) &= (\lambda I - D)^{-1}Cx(i) \qquad 0 \leq i \leq I \end{aligned} \qquad \ldots(32)$$

which has the unique solution $y = 0$. It follows that the spectrum of L_I is a subset of the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_m$ of D, that $r_\infty(L_I) \leq \max_i |\lambda_i|$ and hence that the conditions of the result are sufficient. Necessity follows directly by taking $x_o = 0$ and noting that $y_{k+1}(0) = Dy_k(0)$ or $y_k(0) = D^k y_o(0)$. In particular we must have

$$r_\infty(L_I) = \max_{1 \leq i \leq m} |\lambda_i| \qquad \ldots(33)$$

An immediate consequence of this result is that uniform asymptotic stability of the process is not equivalent to BIBO stability of the process regarded as a 2-D system. This is a consequence of the assumption of a finite pass length I. The situation changes if we let $I \to +\infty$ and consider stability along the pass.

The multipass process of equation (29) is said[4] to be stable along the pass if, and only if there exists real numbers $M_\infty > 0$ and $0 < \lambda_\infty < 1$ such that

$$\| L_I^{\,k} \|_I \leq M_\infty \lambda_\infty^{\,k} \quad \forall \; k \geq 0 \;, \; I \geq 0 \qquad \ldots(34)$$

where $\| L_I \|_I$ is the induced operator norm in E_I. Necessary and sufficient conditions for this to hold are that[4]

$$r_\infty \triangleq \sup_{I \geq 0} r_\infty(L_I) < 1 \qquad \ldots(35)$$

and that

$$M \triangleq \sup_{I \geq 0} \; \sup_{|\eta| = \lambda} \; \| (\eta I - L_I)^{-1} \|_I < \infty \qquad \ldots(36)$$

for some real number λ in the range $r_\infty < \lambda < 1$.

Applying these conditions to the discrete multipass process, note from eqn (33) that (35) reduces to the requirement that

$$r_\infty = \max_{1 \leq i \leq m} |\lambda_i| < 1 \qquad \ldots(37)$$

and hence that D must be a stability matrix. Consider now the solution of the equation $\{\eta I - L_I\} y = y_o$ for some arbitrary $y_o \in E_I$ ie

$$\begin{aligned} x(i+1) &= Ax(i) + B_1 y(i) \quad , \qquad x(o) = 0 \\ \eta y(i) &= Cx(i) + Dy(i) + y_o(i) \quad , \quad 0 \leq i \leq I \end{aligned} \qquad \ldots(38)$$

But $|\eta| > r_\infty$ implies that $|\eta I - D| \neq 0$ and hence that y is obtained from the relations,

$$\begin{aligned} x(i+1) &= \{A + B_1(\eta I - D)^{-1} C\} x(i) + B_1(\eta I - D)^{-1} y_c(i) \\ y(i) &= (\eta I - D)^{-1} \{Cx(i) + y_o(i)\} \end{aligned} \qquad \ldots(39)$$

The boundedness requirement of equation (36) is equivalent to the requirement that the system (39) is asymptotically stable for all

$|\eta| = \lambda$ for suitable choice of λ in the range $r_\infty<\lambda<1$. Noting[4] that we can replace (36) by the requirement that

$$M' \triangleq \sup_{I\geq 0} \sup_{|\eta|\geq\lambda} \|(\eta I-L_I)^{-1}\|_I < \infty \qquad \ldots(40)$$

indicates that (39) must be stable at the point $|\eta| = \infty$ and hence we require that A be a stability matrix.

Finally, writing the characteristic polynomial

$$|\mu I_n - A - B_1(\eta I_m-D)^{-1}C|$$

$$\equiv \frac{|\mu I_n - A|}{|\eta I_m - D|} \cdot |\eta I_m - G(\mu)| \qquad \ldots(41)$$

The stability condition hence reduces to

$$|\eta I_m - G(\mu)| \neq 0 \qquad |\eta| = \lambda, \; |\mu| \geq 1 \qquad \ldots(42)$$

which, noting that $\lim_{|\mu|\to+\infty} G(\mu) = D$, and considering the case of $|\mu| = 1$ implies that eigenvalues of $G(\mu)$ with $|\mu| = 1$ lie in the interior of the unit circle in the complex plane. The Shanks/Huang conditions for BIBO stability (as expressed by Proposition 1) are hence necessary for stability along the pass. They are also sufficient as, if D is a stability matrix, then (equation (37)) $r_\infty<1$. Also, if A is a stability matrix and all eigenvalues of $G(\mu)$ with $|\mu| = 1$ lie in the open unit circle in the complex plane, it is possible to choose $r_\infty<\lambda<1$ such that both sides of (41) are non-zero for $|\eta| = \lambda$ and $|\mu| = 1$ and such that all eigenvalues of A have modulus $<\lambda$. Considering the unit contour $\{\mu:|\mu| = 1\}$ traversed in a clockwise manner and applying standard encirclement theorems to equation (41) indicates that all roots of the characteristic polynomial (equation (41)) lie in the interior of the circle $|z|<1$ for all $|\eta| = \lambda$ ie the system (39) is asymptotically stable for all $|\eta| = \lambda$ which verifies equation (36). We have therefore proved the following theorem demonstrating the equivalence of the two-dimensional and multipass stability concepts:

Theorem 2

The discrete multipass process of equations (22)-(23) is stable along the pass if, and only if, when regarded as a 2-D Roesser model, it is BIBO-stable in the sense of Shanks and Huang.

The important conclusion to be drawn from this result is that any of the many tests available[7,14] for checking the BIBO stability of Roesser models can be applied to linear discrete multipass processes. More precisely, following the development of section 2, taking the 2-D z-transform of the multipass equations (22) and (23) provides a 2-D z-transfer function matrix defining process dynamics and, in particular, the characteristic polynomial

$$\rho(z_1,z_2) \triangleq \begin{vmatrix} I_n - z_1 A & -z_1 B \\ -z_2 C & I_m - z_2 D \end{vmatrix} \qquad \ldots(43)$$

It follows that, for example, the following 2-D stability tests are applicable to linear discrete multipass processes:

Corollory 1 (Shanks[8]): The discrete multipass process of equations (22)-(23) is stable along the pass if, and only if, $\rho(z_1,z_2) \neq 0$ for all $|z_1| \leq 1$, $|z_2| \leq 1$.

Corollory 2 (Huang[9]): The discrete multipass process of equations (22)-(23) is stable along the pass if, and only if, $\rho(z_1,0) \neq 0$ for all $|z_1| \leq 1$ and $\rho(z_1,z_2) \neq 0$ for all $|z_1| = 1$ and $|z_2| \leq 1$.

Corollory 3 (Strintzis[10]): The discrete multipass process of equations (22)-(23) is stable along the pass if, and only if, there exists a,b such that $|a| \leq 1$, $|b| = 1$ and (i) $\rho(a,z_2) \neq 0$ when $|z_2| \leq 1$ (ii) $\rho(z_1,b) \neq 0$ when $|z_1| \leq 1$ and (iii) $\rho(z_1,z_2) \neq 0$ when $|z_1| = |z_2| = 1$.

Corollory 4 (De Carlo et al[11,15]): The discrete multipass process of equations (22)-(23) is stable along the pass if, and only if, $\rho(z_1,z_2) \neq 0$ for all $z_1 = z_2 = z$ when $|z| \leq 1$ and $\rho(z_1,z_2) \neq 0$ for $|z_1| = |z_2| = 1$.

Graphical tests similar to the above together with techniques based on diagonal dominance, Schur-Cohn matrices and 2-D co-prime factorization of the transfer function matrix are described elsewhere[7,14].

4. Discussion and Conclusions

Motivated by problems in image enhancement and filtering, the topic of 2-D systems theory has developed extensively in the past decade[16]. In independent work on qualitative stability problems for systems performing repetitive operations Edwards[1-3] has identified an important class of physical processes for which a useful control theory is lacking, although Owens[4,5] has provided a rigorous approach to stability analysis based on abstract functional analytic models.

The present work has identified similarities in the dynamic models of 2-D systems and linear discrete multipass process models. More precisely, it has been shown that the general discrete linear model for multipass processes is identical in general structure to the 2-D system model proposed by Roesser[6]. The result demonstrating the equivalence of the apparently distinct 2-D BIBO stability tests and stability along the pass is particularly important as it releases a large number of 2-D stability tests for application to multipass processes and suggests a strong connection between stability and the denominator polynomial of the 2-D z-transfer function matrix. This in turn suggests that it may be possible to develop 2-D transfer function matrix and consequent block diagram algebra methods as a basis for the development of a rigorous feedback control theory for multipass processes. Moreover the related 2-D concepts of stability, observability, controllability and minimality developed[17] in recent years could be used to form the foundation of a systems theoretical approach to the analysis of multipass process dynamics. These topics are under consideration at the present time.

Finally the above interplay of ideas, concepts and applications should enrich both subject areas whilst retaining their own individuality and differences. For example, it seems unlikely that 2-D system theory will be of direct use in multipass process applications involving continuous integral smoothing operations.

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