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Prediction of traveller information and route choice based on real-time estimated traffic state

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Accurate depiction of existing traffic states is essential to devise effective real-time traffic management strategies using Intelligent Transportation Systems (ITS). Existing applications of Dynamic Traffic Assignment (DTA) methods are mainly based on either the prediction from macroscopic traffic flow models or measurements from the sensors and do not take advantage of the traffic state estimation techniques, which produce estimate of the traffic states which has less uncertainty than the prediction or measurement alone. On the other hand, research studies which highlight estimation of real-time traffic state are focused only on traffic state estimation and have not utilized the estimated traffic state for DTA applications. In this paper we propose a framework which utilizes real-time traffic state estimate to optimize network performance during an incident through traveller information system. The estimate of real-time traffic states is obtained by combining the prediction of traffic density using Cell Transmission Model (CTM) and the measurements from the traffic sensors in Extended Kalman Filter (EKF) recursive algorithm. The estimated traffic state is used for predicting travel times on alternative routes in a small traffic network and the predicted travel times are communicated to the commuters by a variable message sign (VMS). In numerical experiments on a two-route network, the proposed estimation and information method is seen to significantly improve travel times and network performance during a traffic incident.

Keywords: cell transmission model; extended Kalman filter; traffic estimation; dynamic traffic assignment; route choice; traveller information; incident management.

1. Introduction:

Real-time traffic state estimation has been an active field of research for many years. With the development of new technologies and the improvement in existing techniques for acquiring real-time traffic data, more emphasis is being given to proper utilization of such data, to obtain a more accurate and widespread picture of the state of a network. However, there are limitations in the data directly obtained from traffic sensors. Firstly, such data does not include all the required parameters for devising traffic management strategies in real-time and does not portray a complete picture of the traffic state across a network. Another problem with obtaining real-time traffic data is that it requires a good communication infrastructure. On the other hand, the prediction of traffic state using only traffic flow models based on long-term historic information might contain significant error in prediction, especially when actual traffic conditions depart from their historical trend due to external factors. To obtain complete traffic data for the whole network, traffic flow models along with measurements from sensors are used for better estimation with less uncertainty in the final estimate of the existing traffic state compared to the prediction or measurement alone. Thus, 'real-time traffic state estimation' refers to estimation of traffic flow variables (traffic flow, density) for a segment of road or network, with an adequate time and space resolution based on limited available measurements from traffic sensors (Wang, Papageorgiou and Messmer 2008).

Recently, many research studies have focused on traffic state estimation problem (Wang et al. 2011; Munoz et al. 2003 ; Munoz et al. 2006; Tampere and Immers 2007; Sun, Munoz and Horowitz 2004, Ngoduy 2008, Ngoduy 2011a). Of particular relevance to the present paper is the work of Wang and Papageorgiou (2005). They presented a methodology for estimating traffic states by combining real-time traffic data from sensors with predictions from a second-order traffic flow model. In this approach, they utilized the Extended Kalman Filter (EKF) variation on the approach originally proposed by Kalman (1960) to combine predictions and measurements by minimizing the sum of squared of errors between the measurement and prediction. Wang and Papageorgiou (2005) also proposed a method for online estimation of the model parameters by converting these parameters into stochastic variables. The proposed model was designed and applied for a stretch of freeway with on-ramps and off-ramps. Wang et al. (2011) applied this methodology for traffic state estimation in a freeway network of 100 km in Italy. Ngoduy (2008) proposed a framework that utilizes a particle filtering algorithm with a second-order traffic flow model to estimate traffic for a section of freeway; and in Ngoduy (2008; 2011a) utilized an unscented Kalman filter algorithm with a macroscopic traffic flow model for freeway traffic state estimation. Park and Lee (2004) used a Bayesian technique to estimate travel speed for a link of an urban arterial using data from a dual loop detector. Gang, Jiang, and Cai (2007) presented a traffic state estimation scheme based on the Cell Transmission Model (CTM) and Kalman filter for a single urban arterial street under signal control. Liu et al. (2012) proposed a travel time estimation approach for a long corridor with signalized intersections based on probe vehicle data. Long et al. (2008) developed a model based on the CTM for congestion propagation and bottleneck identification in an urban traffic network. They also estimated average journey velocity for vehicles in the network. Long et al. (2011) applied CTM for simulating traffic jams caused due to an incident in an urban network. They assumed that both traffic flow parameters and the duration of the incident were known during the incident and used a CTM alone for traffic prediction. Zhang, Nai and Qian (2013) compared travel-time computed using three different traffic flow models that could be used for predicting network traffic namely the point queue model, the spatial queue model and the CTM, and concluded that the CTM is better than the other two models for predicting travel times especially when queue spillback prevails. Sumalee et al. (2011) and Zhong et al. (2011) proposed a Stochastic CTM for network traffic flow prediction, the stochasticity intended to address uncertainties in both traffic demand and capacity supplied by the network.

Alongside the problem is that of control, by which some network controller may attempt to influence the system in some desirable way, by adjusting signal timings or speed limits, by providing information through variable message signs or in-vehicle navigation systems, or by charging tolls at some points in the network. In this case the controller may influence both the dynamic flow of traffic and the time-dependant route choices of travellers; the mutual interaction of these phenomena is the focus of Dynamic Traffic Assignment

(DTA). Within this field, Kachroo and Ozbay (1998) highlighted the problem of short-term non-recurrent congestion which might be caused due to some incident, addressing this issue by assigning time-dependent split parameters at some diversion points. They used a feedback linearization method to obtain optimum split rate, so as to optimize network performance. In their method, they assumed availability of data from measurement sensors and only utilized these measurements, without using any kind of traffic flow model. Lo (2001) proposed a method for determining dynamic signal control timing plans based on system optimal principle using CTM based network model, which optimize network performance by keeping the density at an optimum level so as to ensure maximum flow on all links approaching a signalized intersection. The results indicated that green progression could reduce delays on the network. Smith and Mounce (2011) presented an idealized splitting rate model when travellers seek to change their route either day-to-day or within a day. This model uses split rates at nodes to change exit flows in such a way that Wardrop equilibrium is obtained. This approach also incorporates dynamic signal green-time reallocation to reduce delays. The model is an extension of formulation proposed by Smith (1984), which suggests that for each pair of routes joining the same O-D pair, traffic flow swaps from a more costly route to a less costly route at a rate which is proportional to the product of the flow on the more expensive route and the difference in cost between the two routes. Many other studies (e.g. Chow 2009; Wu and Huang 2010; Carey and Watling 2012) presented DTA-based solution for improving traffic congestion without considering utilization of traffic state estimate. In contrast, Ziliaskopoulos (2000) developed a CTM-based approach to compute the dynamical system optimal assignment for a network with single origin and destination, formulating the DTA problem as a linear program. In conclusion, then, DTA-based research studies into the control/optimization of networks have typically not considered the availability and reliability of real-time estimates of the traffic states. Such studies have generally assumed that all the data for the scenario is known, and there is no data available on underling changes in the traffic or road environment conditions during the time period under study.

The main contribution of this research work is to integrate model based traffic state estimation with DTA for real-time applications to improve travel times and network performance. Traffic state estimation can be considered equivalent to traffic flow prediction or traffic state reconstruction using observations from measurement sensors, as all these techniques aim to determine the state of the network (traffic flow, density, speed, travel times, etc.). Existing literature contains many studies combining traffic flow models and DTA, for example, Lo (1999), Ziliaskopoulos (2000), Lo (2001), Gomes and Horowitz (2006), Liu, Lai and Gang (2006), Chiu et al. (2007). Similarly, measurements from traffic sensors have also been used for DTA and traffic management in real-time, e.g., Kachroo and Ozbay (1998), Mirchandani and Head (2001), Dotoli, Fanti, and Meloni (2006). In a similar manner, this research combines model-based traffic state estimation and DTA as the estimated traffic state is considered more reliable than the prediction of traffic state using a traffic flow model or observations from traffic sensors alone.

The present paper, therefore, will focus on developing methods that combine real-time traffic state estimation with a DTA-based model of driver's route choice, with an aim to produce accurate and effective traffic management strategies. Therefore, the novelty of the proposed framework is the combination of real-time traffic state estimation with DTA, as the existing literature in DTA only utilizes prediction from traffic flow models or measurements from the sensors and the literature focusing traffic state estimation problem has not utilized traffic estimation techniques for traffic management using DTA. A framework is proposed in this paper in which predictive traveller information is estimated based on real-time traffic state estimates for a traffic network affected by a traffic incident. The objective is that travel times predicted on alternative routes based on real-time traffic state estimates can make the traveller information more reliable and travel decision more accurate during a traffic incident, when its impact and duration are uncertain. The predictive traveller information is derived through a real-time traffic estimation model for the traffic network with online parameter estimation (In our case, this involving online estimation of the parameters of the CTM).

The proposed framework is applied to a simulated scenario involving an affected route, an alternative route, and a spillback link under a hypothetical traffic incident. In numerical experiment, it is demonstrated that the real-time parameter estimation method enables both identification of the incident and an improvement in actual traffic state estimate for the network. This real-time traffic state estimate is utilized in predicting travel times on alternate routes of the network and then displayed to commuters using VMS. Based on traveller information transmitted via VMS, commuters' en route choice behaviour is simulated using a logit model based on predicted travel times, from which split-rates at the diverging intersection are determined.

This paper consists of six sections. Section 2 describes the notation used for all the variables and parameters in formulating the proposed framework. Section 3 explains the overall research framework adapted in this study. Section 4 describes in the framework for traffic state estimation and extracting predictive traveller information for en route choice during the incident. Section 5 describes the test network for application of the proposed framework and presents the results of the simulation experiments. Finally, section 6 draws the conclusions of the research study and identifies direction for future research.

2. Notation

Table 1 describes the notation used for all the variables, parameters and vectors in development of the proposed framework in this research paper.

Table 1. Variables and parameters for proposed model

Variable/ Parameters	**Description**	**Unit**
i	Number of cells in the network	
j	Number of links in the network	
k	Number of simulation time-steps	
a	Number of cells with measurement sensor	
t	Duration of one time-step	hours
l_j	Length of a cell in link j	km
u_j	Free-flow speed for link j	km/hr
w	Backward wave speed	km/hr
$\rho_i^J(k)$	Jam density for cell i	veh/km
$\rho_i^c(k)$	Critical density for cell i	veh/km
c_i (k)	Maximum flow capacity for cell i	veh/hr
ρ_i (k)	Traffic density in cell i	veh/km
$\hat{\rho}_i(k)$	Estimated traffic density for cell i	veh/km
q_i (k)	Traffic inflow to cell i	veh/hr
β_j (k)	Split-rate for link j	
$m_i^\rho(k)$	Measurement of traffic density for cell i	veh/km
τ_j (k)	Predicted travel time for link j	
$\mathbf{z}$	Array containing traffic density for all cells	
$\mathbf{d}$	Array containing parameters for estimation	
$\boldsymbol{\varepsilon}$	Array containing all noise in variable predictions	
$\mathbf{x}$	All variables and parameter for estimation	
$\boldsymbol{y}$	Measurement from the sensor	
$\hat{\boldsymbol{x}}$	Estimated traffic state variables	
$\mathbf{K}$	Kalman Gain matrix	
$\mathbf{P}$	State covariance matrix	

3. Research methodology

In this study, we use cell transmission model (CTM) to predict traffic densities for a traffic network. The CTM proposed by Daganzo (1994) is an approximation of first-order LWR traffic flow model, named after its proposers of Lighthill and Whitham (1955) and Richards (1956). In comparison to other higher order traffic flow models, CTM has lesser number of output variables and input parameters which qualifies CTM as a suitable model for real-time applications. CTM has been used for traffic state estimation (Munoz et al. 2003; Munoz et al. 2006; Gang, Jiang, and Cai 2007; Tampere and Immers 2007, Long et al. 2008; Long et al. 2011) as well as for DTA applications of traffic network optimization (Lo 1999, Ziliaskopoulos 2000, Lo 2001, Gomes and Horowitz 2006, Liu, Lai and Gang 2006, Chiu et al. 2007).

This research proposes utilization of real-time traffic state estimation in management of traffic network and improving travel times using DTA. In this paper, traffic state estimation represents estimation of traffic densities for all the cells in the network and estimation of parameter of fundamental traffic flow diagram (critical density) for the cells with measurement sensors. In this numerical study, we assume that the free flow speed is fixed due to the link speed limit while the initial value for jam density is pre-determined from the vehicle dimensions. So, we only consider the critical density as unknown model parameter to be estimated. The other parameters of the cells with a measurement sensor, such as traffic flow capacity and jam density are determined based on the estimated value of critical traffic density. In traffic density prediction method, we will use the CTM based on Daganzo (1995), Munoz et al. (2003), and Szeto et al. (2009). In our approach, a module 'CTM-network' predicts traffic densities for all the links in the network for one time-step ahead, based on estimated traffic densities at the previous time-step. The prediction from CTM-network and the real-time measurements from the sensors are then combined in an Extended Kalman Filter (EKF), following Wang and Papageorgiou (2005), with the aim to obtain an estimate of the current traffic density which has less uncertainty than the prediction or measurement alone. This real-time traffic density estimate is then assigned to another CTM-based prediction module that predict the travel times users would experience on the alternative routes available to them, based on the forecasted evolution of traffic flow. The predicted travel times are communicated to travellers using a Variable Message Sign (VMS). The route choice behaviour of travellers is modelled using a logit model, based on the predicted travel times. For illustrative purpose, the frequency of measurements from the sensors, prediction of traffic density from CTM, estimation of the cell parameters and updating of VMS is kept same. However, the proposed framework is general and can be applied to the systems with different frequencies of sensor measurements, CTM predictions and VMS updating.

Overall our objective is to integrate real-time traffic estimation methodology proposed by Wang and Papageorgiou (2005) with the application of Dynamic Traffic Assignment (DTA) methods, in order to improve network performance due to unexpected changes in network capacity. Figure 1 explains the conceptual framework of the research problem. In figure 1, $\hat{x}(k|k-1)$ represents the estimated traffic state; x(k+n) is predicted traffic state based on estimated traffic state; y(k) is the measurement obtained from traffic sensor and $\tau_j(k)$ is predicted travel time for link j.

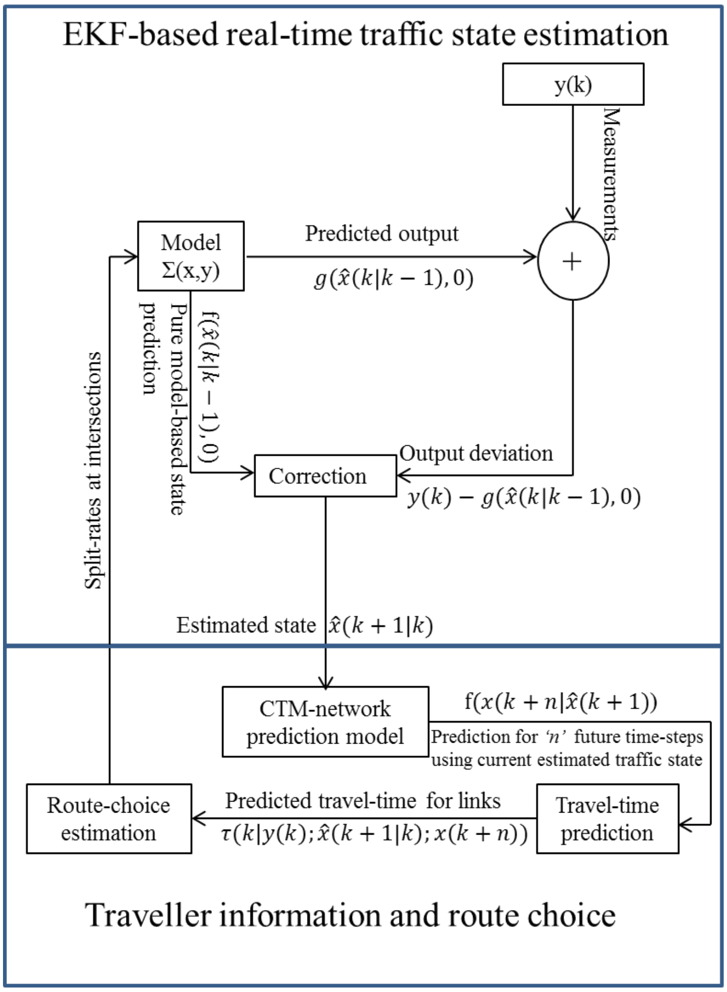

Figure 1.Process flow diagram for traveller information and route choice based on real-time estimated traffic state

The state-space model for the prediction of traffic densities and for mapping the prediction to the measurement, $\sum(x, y)$, predicts the traffic density for a future time-step, based on the estimated traffic density which was obtained using CTM-EKF model at the previous time-step. The traffic density prediction function based on CTM, $f(\hat{x}(k|k-1, 0))$, predicts traffic density for time step k, based on all available measurements until time-step k-1. A differentiable function $g(\hat{x}(k|k-1,0))$ transforms the predicted output for time-step k into the variable measured by the traffic sensor $y(k)$. The measurement of traffic density at time-step k is compared with the predicted output from CTM and a correction factor called the Kalman Gain **K** is estimated using the variance in the prediction and the measurement of the traffic density. This correction is then added to the pure model-based output to obtain a final estimate of traffic density for time-step k+1. The estimated traffic density $\hat{x}(k+1|k)$ for current time-step is then forwarded to another similar CTM-network prediction model $f(x(k+n|\hat{x}(k+1))$ which predicts traffic conditions for a further n time-steps, until all the vehicles on the network are cleared. The predicted time taken to traverse the link j for a vehicle that entered at time-step k, $\tau_j(k|\boldsymbol{y}(k);\hat{\boldsymbol{x}}(k+1);\boldsymbol{x}(k+n))$, thus depends on all measurements until the current time-step, the estimated traffic density for one time-step ahead and the predicted traffic density for the future n time-steps. The predicted travel times are then communicated to travellers using a VMS, and travellers chose their route based on the available information about predicted journey times. The route-choice behaviour of travellers is modelled using a logit model and split-rate at route decision points are provided to the CTM-based model to predict traffic flow for another time-step.

4. A feedback dynamic traffic flow model

4.1 Sensor measurements

We assume that there are traffic measurement sensors installed along various links in the network, which measure traffic density and communicate it to the controller in real-time. In reality, the observations obtained from the sensors include sensor occupancy, flow rate, vehicle classification and speed. Most of the sensor technologies such as magnetic loop detector, inductive loops, video cameras, passive infrared, microwave radar and passive acoustic sensor collect measurements for traffic occupancy (Klein, Mills and Gibson 2006). Traffic density can be determined based on measurements of traffic occupancy obtained from the sensors. However, the proposed model can be used for any other type of real observations obtained from measurement sensors such as traffic flow or speed.

In this research, we generate synthesized measurements by simulating reality using a CTM model. The CTM model to generate traffic density measurements is based on actual network capacity and provided with the values of traffic flow parameters which reflect duration, location and impact of the incident on cell/route capacity. Furthermore, a white Gaussian noise is also added in the simulated reality to depict stochasticity in the measurements obtained in real-world. Let $m_a^{\rho}(k)$ denotes the measurement of traffic density during time period [(k-1)t, kt] and $\varnothing_a^{\rho}(k)$ is Gaussian white noise in measurement of traffic density. The frequency for acquiring sensor measurements is assumed equal to the CTM prediction frequency (30 seconds), but this assumption does not limit the application of proposed framework to the systems with different measurement and prediction frequencies. The measurements obtained from a traffic sensor for a given time-step is related to predicted traffic density based on the following equation:

$$m_a^{\rho}(k) = \rho_a(k) + \varnothing_a^{\rho}(k) \tag{1}$$

4.2 Cell-based state-space model:

Traffic flow models for a single link may be translated into models for a network in two main ways, based either on simple node models (Zhang, Nie and Qian 2013; Daganzo 1995) or link-node model (Zhong et al. 2013; Szeto et al. 2009). In this study we adopt the link-node model proposed by Szeto et al. (2009) for the CTM, which mainly comprises the methodology proposed by Daganzo (1995) to model uncontrolled merging and diverging intersections, with that proposed for signalized intersections by Lo (1999, 2001). The concept of variable cell lengths introduced by Munoz et al. (2003, 2006) is adapted which allows changing the length of cells within the network.

In the CTM representation for network traffic, the network is divided into homogeneous cells and each upstream cell is connected to a downstream cell by a connector.

The traffic outflows from upstream to downstream cells and the traffic inflows to downstream from upstream cells are dictated by properties of the connector. The properties of the connectors are defined based on their location in the link and the network. To incorporate the effects of different geometries of intersections and traffic control, eight different types of connectors are defined. An ‘ordinary connector’ connects the upstream cell of a link to the downstream cell of the same link. A ‘signalized simple connection’ connects a cell of an upstream link to a downstream link controlled by a traffic signal. An ‘origin connection’ connects an origin dummy cell to the first cell of a link. A ‘destination connection’ connects the last cell of a link to the destination dummy cell. An 'unsignalized merge connection’ is used to model unsignalized merging intersections, and a ‘signalized merge connection’ to model a signalized merging intersection. The modelling of unsignalized diverging intersection is carried out by ‘unsignalized diverge connection’ and for signalized diverging intersection, ‘signalized diverge connection’ is used. Szeto et al. (2009) can be referred for a detailed description of the connectors defined above.

We suppose that the network is divided into j links such that j=1,2,3, ..., and i homogeneous segments labelled i=1, 2, 3 ..., with the duration of each simulation time-step t, measured in hours. The free-flow speed on link j is u_j and length of a cell in link j is chosen such that a vehicle can traverse the cell in one time-step if the cell is in a free flow condition, thus the length of each cell in link j is $l_j=u_j$ t km. The cells in the network equipped with a measurement sensor are denoted with a subscript a, such that a= 1, 2, 3, …, .The simulation horizon is divided into k time-steps labelled $k=1, 2, 3, \ldots,$. We assume that each cell in the network has a maximum flow capacity of $c_i(k)$ veh/hr, a corresponding critical density of $\rho_i^c(k)$ veh/km and a jam-density represented by $\rho_i^J(k)$ veh/km based on a triangular fundamental traffic flow diagram as shown in figure 2. In general, any shape of the fundamental traffic flow diagram can be used in place of figure 2, as the EKF is applicable for any nonlinear traffic models (Wang and Papageorgiou 2005). The CTM-as a special discretisation version of the LWR- with a general shape of the fundamental diagram can be embedded in the EKF framework.

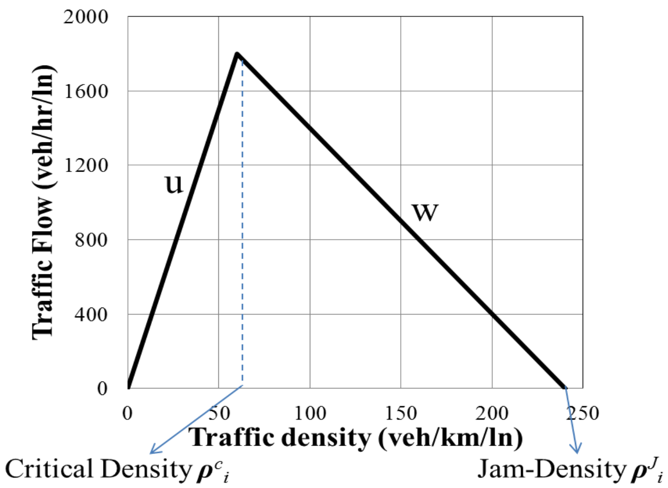

Figure 2. Fundamental traffic flow diagram

Inflow to cell i, $q_i(k)$, and outflow from cell i, $q_{i+1}(k)$, are therefore determined based on one of the connector types, as described above, following Szeto et al. (2009). For an ordinary cell, the inflow $q_i(k)$ is the minimum of traffic demand from upstream cell and the available capacity from the target cell.

$$q_i(k) = min \{\rho_{i-1}(k)\, u_j(k),\, c_{i-1}(k),\, c_i(k),\, w(\rho_i^J(k) - \rho_i(k)\} \quad (2)$$

After finding the inflows and outflows for all the connectors in the network for the current time step k, the traffic state for a future time-step k+1 can be predicted using conservation of traffic flow for all the cells in the network based on Munoz et al. (2003, 2006):

$$\rho_i(k+1)=\rho_i(k)+\frac{t}{l_j}\{q_i(k)-q_{i+1}(k)\} \quad (3)$$

4.2.1 Prediction of traffic flow parameters

The accuracy of traffic density predicted using equations (3) is highly dependent on the accuracy of the cell parameters such as critical density, flow capacity and jam density which govern the relationship between $q_{i+1}(k)$ and $q_i(k)$. In reality, the values of these parameters may be affected by several factors, such as weather conditions, change in the traffic mix, and traffic incidents. The present research specifically focuses on real-time estimation during a traffic incident, when there is no external information about the occurrence and duration of the incident, and when accurate estimation of the changing parameters plays a significant role. This research utilizes a similar approach based on previous research studies (Wang and Papageorgiou 2005; Wang, Papageorgiou and Messmer 2008; Ngoduy 2011b) by converting parameters of fundamental traffic flow diagram into stochastic variables using a random walk equation and utilizing real-time measurements from the sensors.

In general, all the traffic flow parameters can be estimated in real-time but for illustration purpose, we only estimate critical density in real-time and by assuming that free-flow speed and backward wave speed do not change over time. Other two parameters, traffic flow capacity and jam density are calculated using the estimated value of critical density. The parameters are estimated for all the cells with measurement sensors at each simulation time-step and the estimated parameter values are assigned to all the downstream cells until a new estimate for parameters is obtained at the downstream measurement sensor. The estimation of only critical density for the cells with measurement sensors keeps the number of variables to a minimum level and makes the model feasible for real-time applications. The traffic flow capacity and jam-density are calculated for each time-step based on fundamental traffic flow diagram shown in figure (2) using following relations:

$$c_a(k)=\hat{\rho}_a^c(k)\,u_j \tag{4}$$

$$\rho_a^J(k)=\hat{\rho}_a^c(k)(1+u_j/w) \tag{5}$$

where $c_a(k)$ is traffic flow capacity for cell with measurement sensor, $\hat{\rho}_a^c(k)$ is estimated critical density for time-step k, u_j is free flow speed, $\rho_a^J(k)$ is jam density, and w is backward wave speed. The index $a = 1, 2, 3, \ldots$ represents the cells equipped with a measurement sensor. Critical traffic density is transformed into a stochastic variable by adding a white Gaussian noise with standard deviation $\varepsilon_a^c(k)$.

$$\rho_a^c(k+1) = \rho_a^c(k) + \varepsilon_a^c(k) \tag{6}$$

The above equation is used in the estimation algorithm with real-time measurements of traffic density which allows tracking and estimating any unexpected change in this parameter.

4.2.2 State-space model

To simplify the presentation of variables and parameters to be estimated using CTM-EKF model, the proposed model is transformed into state-space form.

Traffic density prediction: $\boldsymbol{z} = [\,\rho_1 \quad \rho_2 \quad \rho_3 \quad \ldots \quad \rho_N]$ (7)

where vector **z** contains traffic densities for all the cells in the network, predicted based on equation (3) using the CTM. The CTM prediction for traffic densities can be described using a function f_1, as follows:

$$\boldsymbol{z}(k+1) = f_1(\boldsymbol{z}, \boldsymbol{\varepsilon}^{\rho}) \tag{8}$$

where $\boldsymbol{\varepsilon}^{\rho}$ is the noise in prediction of traffic density using CTM. The parameters of fundamental traffic flow diagram are also included in the estimation framework. The critical density of cells with measurement sensors is estimated based on equation (6) and measurements from the traffic sensors. The critical density estimated at each time-step for cells with the measurement sensors can be represented by a vector **d** and a function f_2 based on equation (6).

Traffic flow parameter prediction: $\boldsymbol{d} = [\rho_1^c \quad \rho_2^c \quad \rho_3^c \quad \ldots\,]$ (9)

$$\boldsymbol{d}(k+1) = f_2(\boldsymbol{d}, \boldsymbol{\varepsilon}^{c}) \tag{10}$$

where $\boldsymbol{\varepsilon}^{c}$ represents uncertainty in prediction of critical density. Since both vectors **z** and **d**, are estimated in real-time, combining them to one augmented matrix and function we get:

$$\boldsymbol{x}=[\boldsymbol{z}^T \quad \boldsymbol{d}^T] \tag{11}$$

$$\boldsymbol{x}(k+1)=f\,[\boldsymbol{x}(k),\boldsymbol{\varepsilon}(k)] \tag{12}$$

where f is the augmented function, representing functions f_1 and f_2. Similarly, the measurements obtained from the sensors can also be combined using a vector **y**(k) and written as a linear differentiable function g of the traffic state at time-step k. The function g, which relates the measurements obtained from the sensor to the predicted traffic state, is based on equation (1).

$$y=[m_1^\rho \quad m_2^\rho(k) \quad m_3^\rho(k) \;\; \ldots\,] \tag{13}$$

$$\boldsymbol{y}(k)=g[\boldsymbol{x}(k),\boldsymbol{\varphi}(k)] \tag{14}$$

In equation (14) $\boldsymbol{\varphi}(k)$ is the noise in transforming the predicted output to the direct measurement obtained from the sensor.

4.3 Kalman filter based estimation framework

4.3.1 Extended Kalman Filter for traffic state estimation:

CTM has been used in recent research studies for estimation of real-time traffic states using techniques based on the Kalman Filter (KF) (Munoz et al. 2003, 2006). The KF method is based on minimizing the square of error between the predicted and measured values for the traffic state. The EKF is an extension of KF for non-linear systems and obtaining optimal solution using EKF is not always guaranteed. CTM is a non-linear model and therefore is more suited to estimation using the EKF rather than KF. Munoz et al. (2003, 2006) transformed CTM into a linear model by introducing the Switch Mode Model (SMM). CTM-based SMM was derived based on five different traffic modes to avoid the non-linearity caused due to the minimum condition in the CTM. At any given time-step, one of the five modes is selected for the whole link to estimate traffic density, based on the measurement of densities at the upstream and downstream cells of the link. CTM-SMM assumes existence of a maximum of one wave front in the whole link. This means that the whole link is considered to have the same traffic flow condition and road segments with more than one wave-front are impossible to model using CTM-SMM. Furthermore, the selection of mode for the segment requires direct measurements of traffic density at the upstream and downstream of the segment, which might not be available for every urban link or freeway stretch. Sun, Munoz and Horowitz (2004) used mixture Kalman filter with CTM-SMM for appropriate selection of switch mode. Tampere and Immers (2007) adapted the estimation model proposed by Wang and Papageorgiou (2005) and applied linear Kalman Filer for CTM.

For traffic density estimation of a traffic network as shown in figure 3, we adapt the framework described in Wang and Papageorgiou (2005) for the CTM. The EKF is considered as a de facto estimation algorithm for estimating state of a non-linear dynamic system and traffic flow models including CTM are nonlinear in nature. EKF has been consistently used for estimation of traffic state (Wang et al. 2011; Wang, Papageorgiou, and Messmer 2008, Wang and Papageorgiou 2005; Meier and Wehlan 2001; Cremer 1991). Other estimation algorithms for nonlinear systems such as particle filters and unscented Kalman filter are computationally expansive algorithms when compared with EKF. The EKF is more efficient in computation and can be applied in large scale networks, which is our ongoing work. The objective of EKF at each time-step k is to find a state estimate which minimizes the covariance of the estimation error using all available measurements until time-step k, i.e. it minimizes:

$$E\left\{\left[\boldsymbol{x}(k+1)-\hat{\boldsymbol{x}}(k+1|k)\right]^T.\left[\boldsymbol{x}(k+1)-\hat{\boldsymbol{x}}(k+1|k)\right]\right\} \tag{15}$$

The recursive algorithm of EKF estimates a new state for each time-step using the following equation:

$$\hat{\boldsymbol{x}}(k+1|k)=f[\hat{\boldsymbol{x}}(k|k-1),0)]+\boldsymbol{K}(k)[\boldsymbol{y}(k)-g(\hat{\boldsymbol{x}}(k|k-1),0)] \tag{16}$$

where **K** is the Kalman gain matrix and it is estimated at each time-step using the following relation:

$$\mathbf{K}(k)=[\mathbf{A}(k)\mathbf{P}(k|k-1)\mathbf{B}^{T}(k)+\mathbf{\Delta}(k)\mathbf{M}(k)\mathbf{\Pi}^{T}(k)].[\mathbf{B}(k)\mathbf{P}(k|k-1)\mathbf{B}^{T}(k)+\mathbf{\Pi}(k)\mathbf{R}(k)\mathbf{\Pi}^{T}(k)]^{-1} \quad (17)$$

A(k) represents first-order partial derivative of prediction function f with respect to the **x**, which contains all output variables. This is also known as Jacobian matrix. Jacobian matrix **B**(k) represents first-order partial derivative of function g with respect to vector **x**. Jacobian matrix $\boldsymbol{\Delta}$(k) is first-order partial derivative of prediction function f with respect to prediction error $\boldsymbol{\varepsilon}$(k) and Jacobian matrix $\boldsymbol{\Pi}$(k) is first-order partial derivative of function g with respect to measurement error $\boldsymbol{\varphi}$. **P**(k|k-1) is covariance matrix and **R**(k) represents variance of noise in measurement. Appendix A can be referred for further explanation of the EKF algorithm and description of various matrices mentioned in equation (17). The first part of figure (1) illustrates conceptual framework of traffic state estimation in EKF using prediction from CTM and measurements from traffic sensors, which is based on the framework proposed by Wang and Papageorgiou (2005).

4.3.2 Prediction of travel time and traveller information system:

Advanced Traveller Information Systems (ATIS) have been widely used to inform commuters about real-time changes in road capacity, traffic congestion and delays. ATIS helps commuters to take informed decision about their route choice and improves network performance. Many research studies have been conducted to highlight the significance of ATIS in facilitating travellers for better route choice decision and improved network performance (Zhang and Levinson 2008; Kusakabe et al. 2012; Wang and Khattak 2013; Zhou, Chen and Bekhor 2012; Khattak, Schofer and Koppelman1995; Abdel-Aty, Kitamura and Jovanis 1997; Al-Deek and Kanafani 1993).

In this research, it is assumed that variable message signs are installed before the decision points on alternative routes in a network. These decision points are usually diverging intersections, which can be either signalized or unsignalized. A VMS displays travel-times for alternative routes, which are updated at each time-step. The travel-time to the destination following each of the downstream links is predicted based on the estimated traffic in the links connecting a decision point. The estimated traffic state using CTM-EKF model provides an update of the traffic state at each time-step, and this estimate is based on measurements from sensors, which are located so as to detect any unexpected changes in road capacity or traffic demand. The estimated traffic state at time-step k, [$\hat{\boldsymbol{x}}(k+1|k)$], is provided to the CTM prediction model, which then predicts the future propagation of traffic flows and predicts the travel time that will be experienced by a vehicle entering the link at time-step k. In our case, we consider a simple network in which the links emanating from a decision point constitute the complete route to the destination, and so predicting the link travel times means predicting the complete travel time to the destination. The CTM model for predicting travel-times for vehicles that enter a link at time-step k is simulated up to time-step k+n, where n is the maximum number of time-steps required to traverse the link by the last vehicle entering the link at time-step k. This predicted travel time is communicated to travellers using a VMS, placed before the decision point at time-step k+1.

A discrete choice, multinomial logit model is applied to model the behaviour of drivers in adapting their route choice, when the information about predicted travel-times on alternative routes is available. Logistic regression measures the relationship between a categorical dependent variable and one or more independent variables, which are usually continuous, by using probability scores as the predicted values of the dependent variable. If there are j number of exit links/routes emerging from a diverging intersection such that j= 1, 2, 3… , the number of travellers choosing exit link/route j is given by:

$$\beta_j(k+1) = e^{-\theta\tau_j(k|\,\boldsymbol{y}(k);\,\hat{\boldsymbol{x}}(k+1|k);\,\boldsymbol{x}(k+n))} \Big/ \sum_{j=1}^{J} e^{-\theta\tau_j(k|\,\boldsymbol{y}(k);\,\hat{\boldsymbol{x}}(k+1|k);\,\boldsymbol{x}(k+n))} \tag{18}$$

where $\tau_j(k|\,\boldsymbol{y}(k); \hat{\boldsymbol{x}}(k+1); \boldsymbol{x}(k+n))$ is the predicted travel time for link/route j at time-step k which depends on all measurements available until current time-step, $[\boldsymbol{y}(k)]$, one step ahead estimate of traffic state using CTM-EKF model, $[\hat{\boldsymbol{x}}(k+1|k)]$, and prediction of traffic state based on estimated traffic state using CTM, $[\boldsymbol{x}(k+n)]$, that predicts travel time by simulating traffic flow for another n time-steps by using traffic that entered link/route j at time-step k such that $n=1, 2, 3, \ldots \tau_{max}$. θ is the logit coefficient which is specified so as to represent commuters' variation in perception of expected travel time. In practice, this coefficient could also be estimated in real-time, or by using offline data by performing logit regression with known split-rates obtained from measurements and traveller information resulting in measured split-rates. This study does not aim to estimate or evaluate the value of the logit model coefficient. However, in modelling route choice behaviour using logit model, split-rates highly depend on the value of the coefficient, and in reality determination of an accurate value of the coefficient is significant for the accurate estimation of traffic states.

5. Simulation setup and results

5.1 Simulation scenario

A hypothetical diverging network is considered to demonstrate the application of the proposed traffic estimation and en route choice model. Figure 3 describes the network used for this experiment. The network consists of three links, each of length 4.5 km and divided into 10 cells of equal lengths. The first cell (cell-1) of link-1 is a dummy cell which generates traffic demand, with the last cells (cell-20 and cell-30) of link-2 and link-3 also dummies absorbing traffic arriving at the destination. There are two measurement sensors installed, one in cell-15 along link-2 and other in cell-25 along link-3, which measure traffic density in real-time and communicate to the controller.

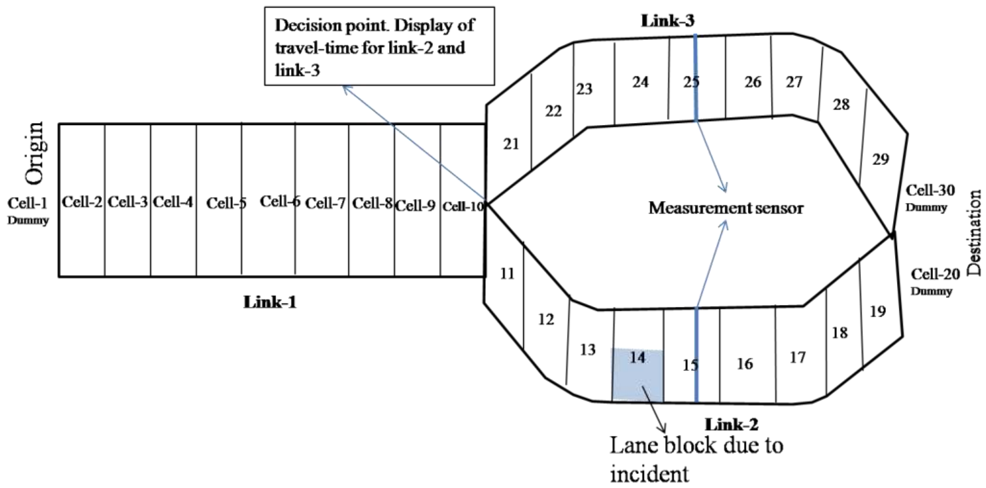

Figure 3. A Simple network for real-time traffic estimation and en route choice modelling

All the links in the network are three lane roads. The traffic demand for the network is show in figure 4. There is a diverging intersection at downstream of cell-10, and traffic is diverging at this intersection on link-2 and link-3, each of these links leading traffic to the same destination. For illustrative purpose, the lengths, traffic flow capacities and speed limits of the alternative routes are considered equal, as it allows an easy base scenario for comparison of the incident scenario with the normal traffic conditions. In dynamic user equilibrium for this symmetric network, for any given departure time at the origin, the traffic will be equally divided between the routes when there is no incident. However, the methodology is general and can be applied to a network with any lengths and capacities of the alternative routes. A variable message sign is installed at link-1, at an appropriate distance before the intersection which displays the predicted travel time on the alternative routes.

This experiment was simulated for 700 time-steps of 30 seconds each, with a traffic incident occurring at time-step k=120 in cell-14 of link-2. This incident blocks two lanes of link-2 until time-step k=360, i.e. a duration of two hours. All the cells in the network have the same initial parameter values with traffic flow capacity of 5400 veh/hr, critical density of 90 veh/km and jam-density of 360 veh/km. The free flow speed of all the cells remains constant at 60 km/hr and backward wave speed is 20 km/hr.

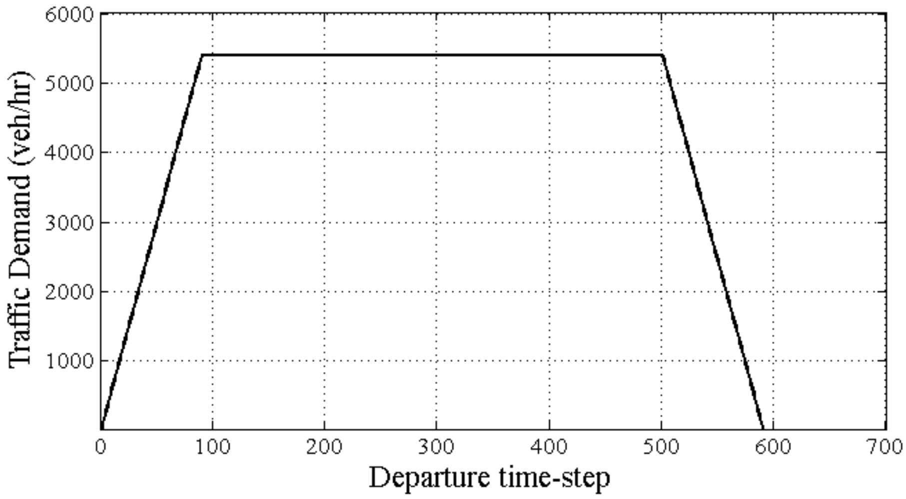

Figure 4.Traffic demand for the traffic network

5.2 Simulation results

The algorithm proposed for estimation of traffic flow parameters was tested with different traffic volumes and different conditions of traffic incident along the link. The proposed algorithm was able to correctly track the drop in capacity due to the incident and also able to bring parameter values back to their normal values once the incident is cleared. Figure 5 shows that reduction in traffic flow capacity due to the incident (which occurred during time-steps 120-360) was accurately identified and estimated by CTM-EKF model. The incident occurred in cell-14 of link-2, but it can only be identified and capacity can be estimated at the downstream measurement location. When there are several sensors installed along the road, any change in traffic flow parameters can be tracked and estimated at the downstream sensor. The estimated capacity, which dropped due to the traffic incident, was subsequently brought back to its actual value by the estimation method, once the measurement of traffic flow becomes high and the link acquires its capacity flow as can be seen in figure 5.

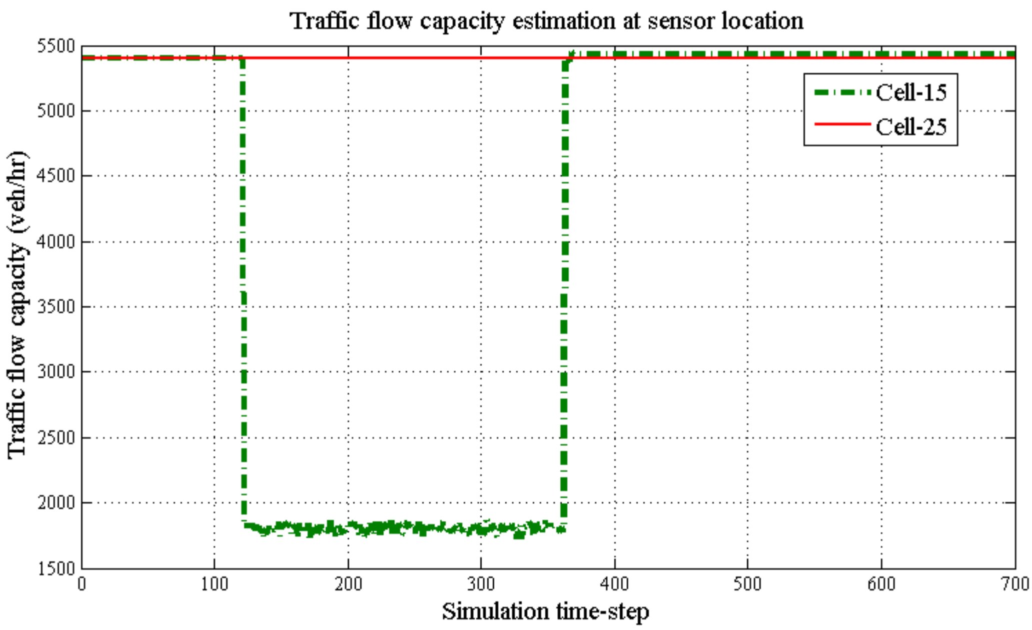

Figure 5. Estimation of traffic flow capacity at cell-15 and cell-25

The performance of the proposed model, which influences the dynamic route choice of commuters through the provision of predicted travel times conveyed to the commuters through a VMS, is compared with the case of real-time traffic state estimation model without any traveller information. In the no information scenario, the split rate is static since the commuters are unaware of the incident and prevailing/predicted travel times on the alternative routes.

For the scenario, in which predicted travel times are provided to drivers, figure 6 shows the dynamic split-rate obtained through traveller information. It can be seen that, as anticipated, information provision helps alleviate congestion on the affected link by diverting traffic to the alternative route with lesser travel time. This also improves underutilization of existing network capacity and network travel time for all commuters.

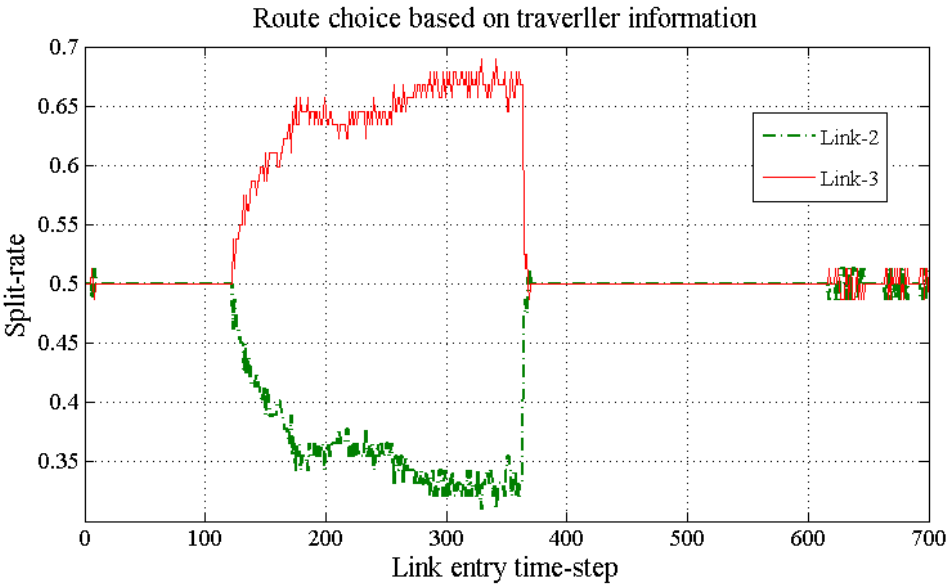

Figure 6. Dynamic split-rate obtained through traveller information system

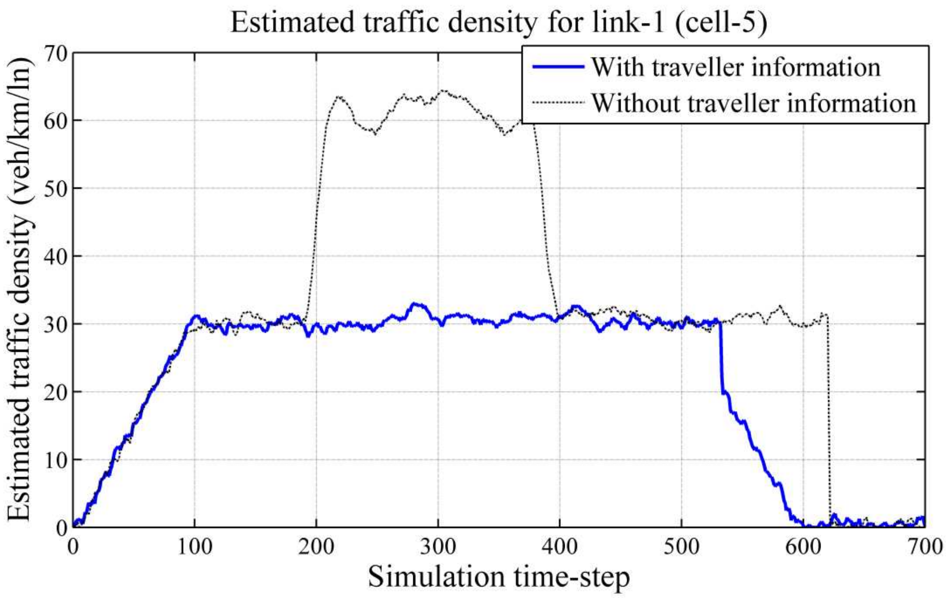

Figure 7. Comparison of traffic states for link-1(cell-5) with and without traveller information

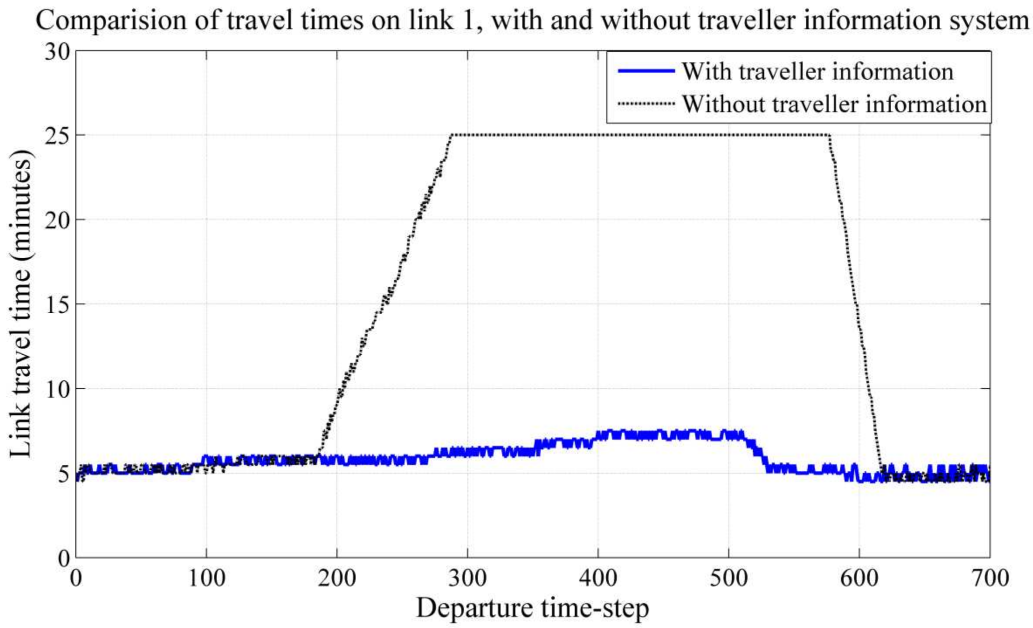

Figure 8. Comparison of travel times for link-1 with and without traveller information

Figure 7 compares the estimated traffic density for cells 5 of link-1 with traveller information and dynamic split rate with the scenario of no information. Only one cell is selected to compare the state of traffic in the link, as all other cells of link-1 show a similar traffic density profile. Figure 7 shows that the traffic flow in link-1 with traveller information is most of the time either at capacity flow or is free-flowing, but for a short period of time it also exceeds capacity flow. Whereas, in the case of no information, it can be observed that after few time-steps of the incident occurrence, congestion starts to build on link-1. This is because travellers are unaware of the incident ahead on link-2, and still the same proportion of traffic selects link-2 as a route as in the normal traffic conditions. Since the travellers trying to take link-2 are not able to propagate, this causes a blockage for vehicles directed towards link-3. Thus, congestion spills back to affect all the upstream cells of link-1. The comparison of estimated traffic density for link-1 with and without traveller information system reveals that with the traveller information system, traffic flow in link-1 was in a good condition when compared with the no information scenario. All the cells of link-1 throughout simulation horizon were almost in free-flow condition with a dynamic split-rate, whereas with a constant split-rate the cells of link-1 become congested during the incident interval. Similarly, a significant improvement in travel times for link-1 can be observed from figure 8. The maximum experienced travel time on link-1 with traveller information was 7.5 minutes which lasted for a short interval of time, whereas without traveller information the maximum travel time on link-1 increased to 25 minutes and it remained for a longer interval of time.

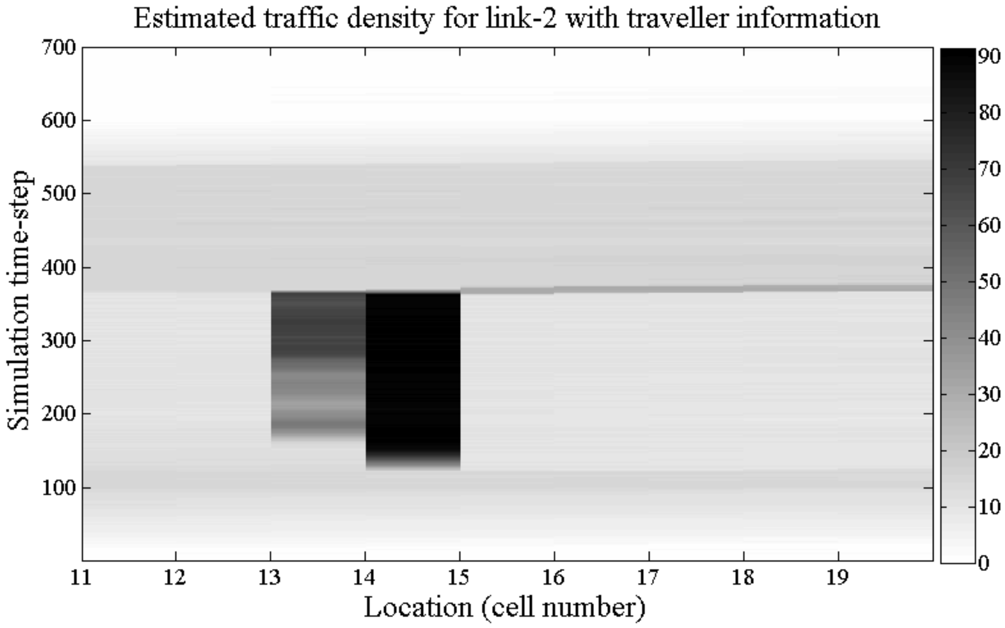

Figure 9. Estimated traffic density (veh/km/ln) for link-2 with traveller information

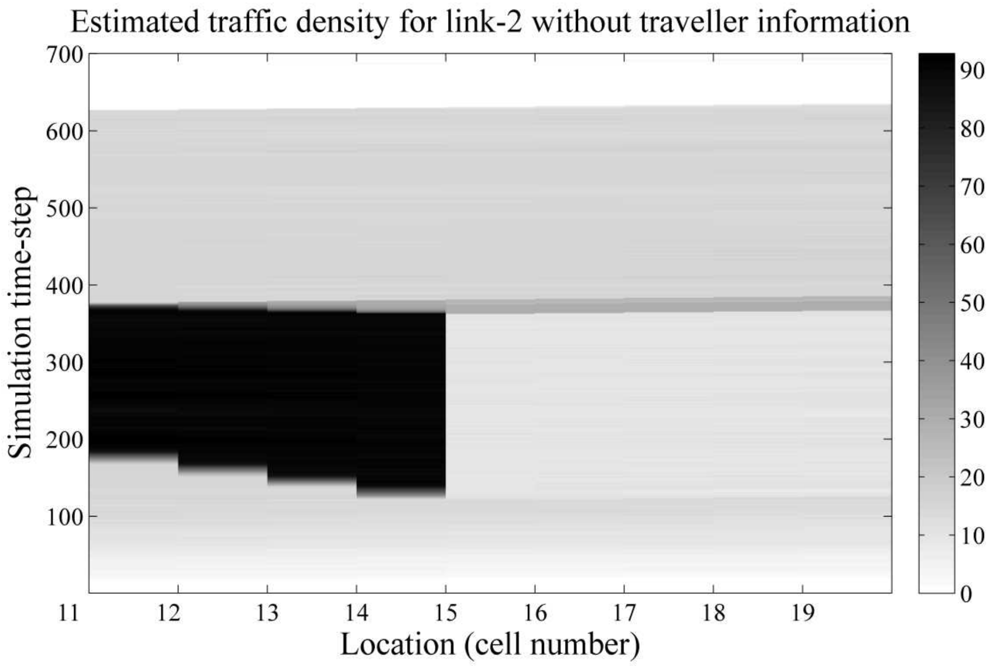

Figure 10. Estimated traffic density (veh/km/ln) for link-2 without traveller information

Figure 9 shows estimated traffic density for link-2 with traveller information and dynamic split rate and figure 10 shows estimated traffic density for link-2 without traveller information. Before occurrence of traffic incident, link-2 was in a free flow condition. After the incident, congestion starts building up in the cells upstream of the incident location in link-2. The comparison of figure 9 and 10 reveals that the traffic state in link-2 shows a significant improvement with a dynamic split-rate when compared with the scenario of no information. Only cells 13 and 14 were partly congested during the traffic incident with a dynamic split-rate, whereas without traveller information all the cells upstream of the sensor location are in a congested state for a comparatively longer interval of time. This fact is further supported by the comparison of travel times for link-2 in figure 11. The maximum value of travel time with a dynamic split-rate for link-2 was 12.5 minutes whereas without traveller information and dynamic split-rate, the maximum travel time increased to 22 minutes.

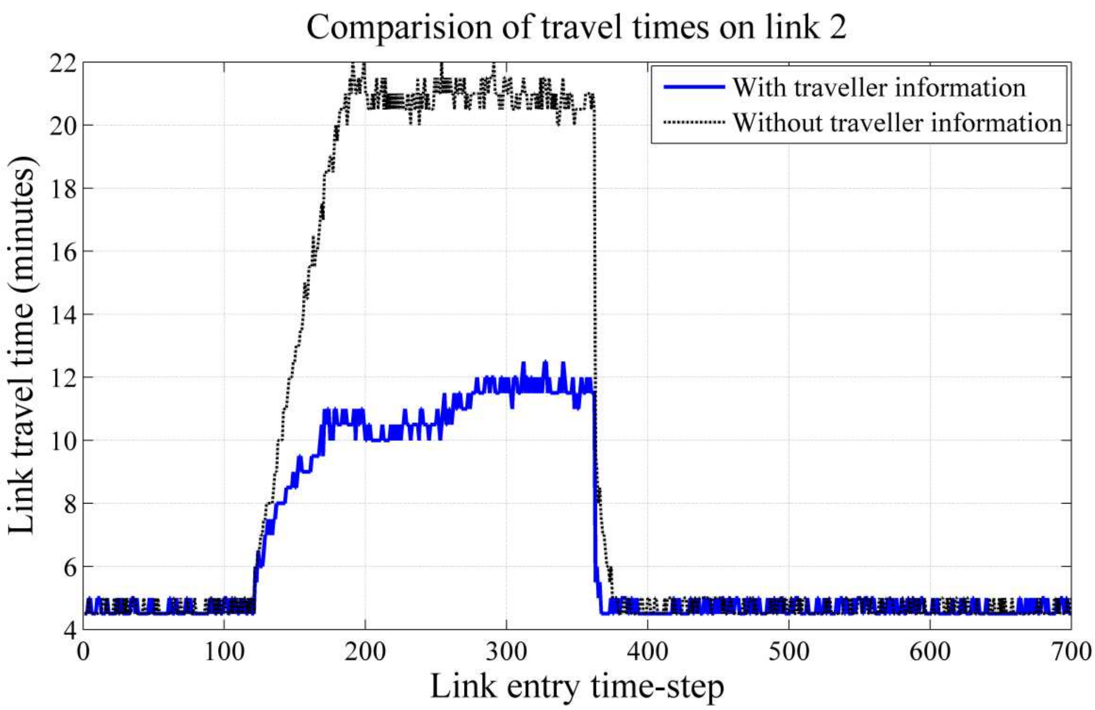

Figure 11. Comparison of travel times for link-2 with and without traveller information

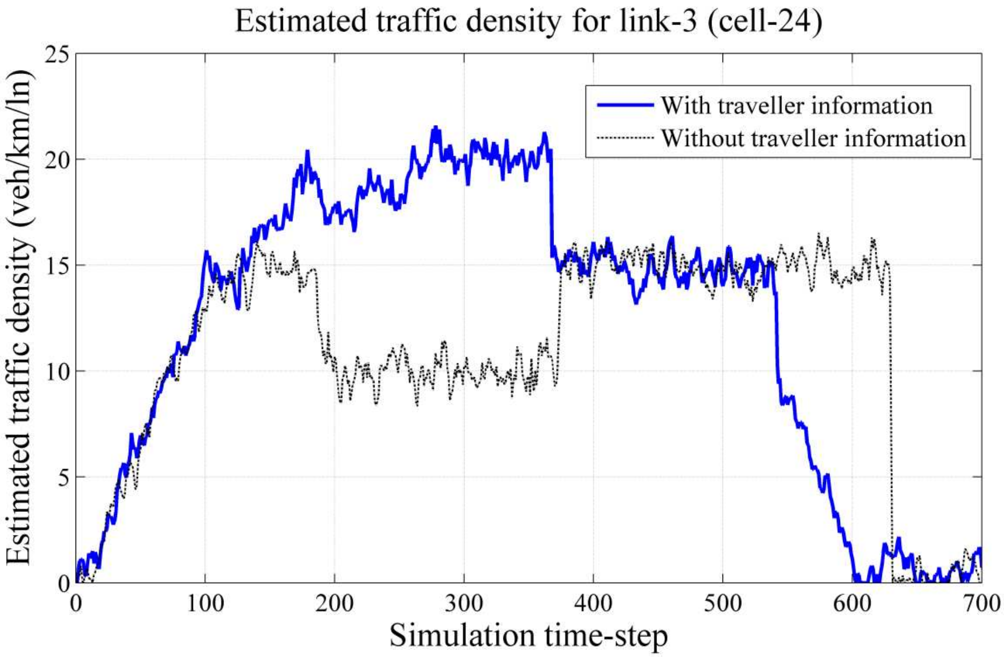

Figure 12. Comparison of estimated traffic density for link-3 (cell-24) with and without traveller information

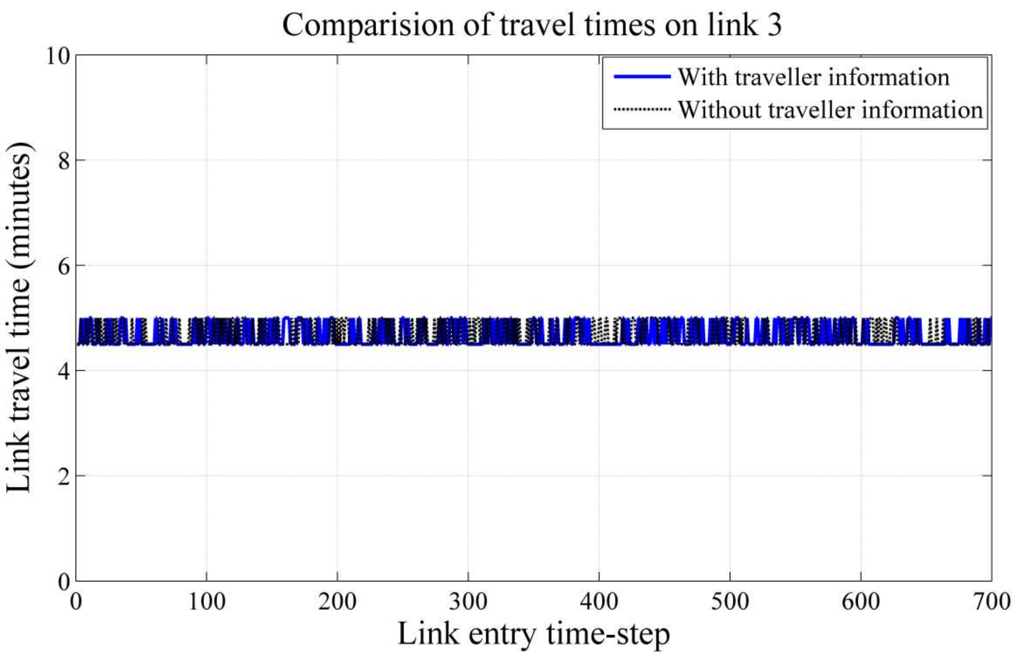

Figure 13. Comparison of travel times for link-3 with and without traveller information

Figure 12 compares the estimated traffic density for cell-24 on link-3 with and without traveller information. All the other cells in link-3 exhibit a similar pattern; therefore, only one cell is selected for the comparison of traffic state in link-3. It can be observed from the figure-

12 that during the interval of the incident, for no-information scenario, while other links are in a congested traffic state the available capacity on link-3 is underutilized. This is due to the fact that vehicles trying to take link-3 are blocked because traffic directed towards link-2 is unable to propagate. The comparison of the estimated traffic density with and without traveller information further confirms that the available capacity of link-3 was better utilized with the dynamic split-rate acquired through the traveller information system. However, traffic density in link-3 does not exceed critical density (90 veh/km) during the simulation horizon in either of the scenarios. The comparison of link travel times for link-3, with and without traveller information is shown in figure 13. It can be observed from figure 13 that link-3 remains in a free flow state throughout the simulation horizon, as the inflow to link-3 is not exceeding the available capacity of the link in either of the scenarios. The fluctuation around free-flow travel time in figure-13 is due to the stochastic nature of traffic density estimation, which is due to noise in measurement from traffic sensors and random error in prediction of traffic densities.

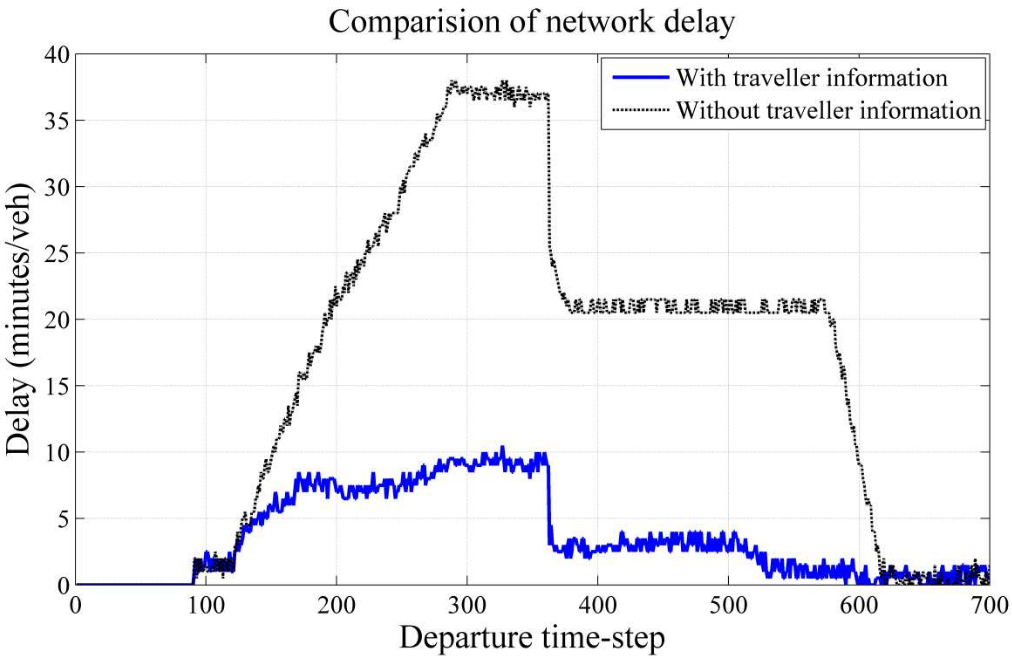

Figure 14. Comparison of network travel delay with and without traveller information

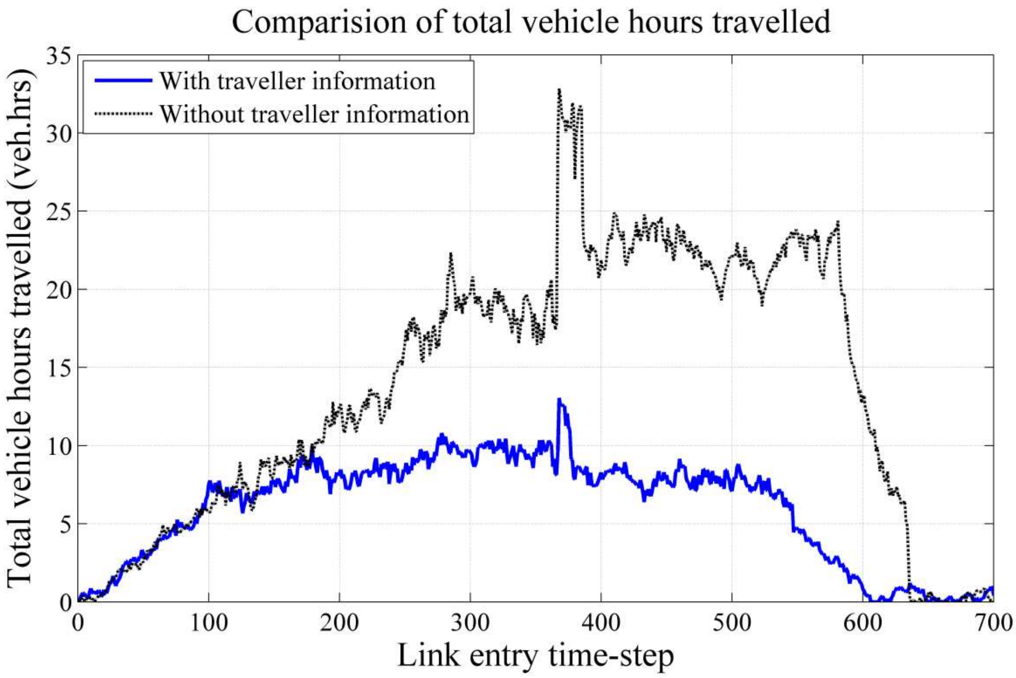

Figure 15. Comparison of total network travel time with and without traveller information

The overall improvement in the network performance by applying the proposed framework for integrating real-time traffic state estimation and DTA for this application is shown in figures 14-15. Figure 14 compares total delay that each vehicle had to encounter to arrive at the destination for both the scenarios. The traffic is in free-flow condition throughout the network for the beginning of the simulation period, therefore no delay is observed till departure time-step 90. A small amount of delay can be observed from figure 14 between time-steps 90 to 120, which is same for both the scenarios. The delay in arriving at the destination increases gradually after the incident. The total delay in the case of no-information is significantly higher than the delay in the scenario with traveller information. The delay in the no-information scenario increased to 37 minutes per vehicle, whereas with the information the maximum value of delay per vehicle was recorded as 10 minutes per vehicle. Figure 15 shows total vehicle hours travelled (VHT) for the vehicles entering the links at each time-step during the simulation horizon. The total VHT is equal in both the scenarios till the occurrence of the incident. However, after the incident the VHT becomes significantly higher in the no-information scenario when compared with the scenario of traveller information.Table-2 provides a link-wise breakdown of total vehicle hours travelled for each link. An overall improvement of 5337.4 vehicle-hours (55.9%) in total VHT is obtained by the implementation of the proposed framework, which is highest for link-1 with 74.3% improvement. A higher value of VHT is observed for link-3 with the provision of traveller information when compared with no-information scenario, as number of vehicles selecting link-3 has increased with the provision of traveller information while travel time on link-3 is similar in both the scenarios.

Table 2. Comparison of total travel time for traffic network

	Vehicles hours travelled without ATIS (veh.hrs)	Vehicles hours travelled with ATIS (veh.hrs)	Improvement in vehicle hours travelled (veh.hrs)
Link-1	6871.1	1766.1	5105 (74.3%)
Link-2	1774.8	1114.3	660.5 (37.2%)
Link-3	899.8	1327.9	-428.1 (-47.5%)
Total	9545.7	4208.3	5337.4 (55.9%)

6. Conclusion and Recommendations

In this research paper, a CTM-based real-time traffic estimation and management model has been proposed for a traffic network. It has been demonstrated in our numerical experiment that real-time traffic states estimated based on measurements from the sensor using EKF can improve the reliability of the estimate. Online estimation of traffic flow parameters enables the model to track any unexpected changes in capacity of the network. The main contribution of this research work is to combine traffic state estimation with the applications of DTA models, whereby the real-time estimated traffic state is utilized for influencing route choice through the provision of predicted travel time information and thus improved travel times and network performance during a traffic incident.

The proposed method has been applied to a simple, hypothetical, two-route network with one of its links affected by an incident. In our numerical experiment, the proposed model was seen to accurately identify and estimate the drop in capacity due to the incident. Predicted travel times communicated to travellers were seen to reduce demand for the affected link and helped traveller to utilize existing capacity on the alternative route. The proposed traffic management model reduced the total vehicle hours travelled by 54% during the simulation horizon.

The research framework proposed in this paper can be extended to model day-to-day dynamics of network traffic flows and route choices with ATIS under uncertain traffic demand or network capacity. The parameter estimation in real-time provides an opportunity to model and estimate behaviour parameter of commuter's route choice such as logit parameter for perception variation. The estimation of such behaviour parameters based on real-time observations can significantly improve the modelling accuracy and it will enable to analytically determine the impact of traveller information, traffic control measures and other factors on route choice behaviour. The proposed framework for combining real-time estimated traffic state and DTA can be extended to optimize signal traffic control and obtain system optimal solutions for urban traffic networks.

Furthermore, the proposed framework in this paper can be extended to model large traffic systems. However, while extending the proposed methodology for larger networks, computational time might be a limiting factor as the traveller information or traffic state is updated with a high temporal and spatial resolution in the proposed model. The spatial and temporal resolution of traffic state estimation and frequency of updating traveller information can be reduced to make it feasible for larger networks. A more aggregate macroscopic traffic flow model, such as Two-regime Transmission Model (TTM) by Balijepalli, Ngoduy and Watling (2013) can be used for traffic state prediction to improve computation and modelling demand. Modelling of split-rates at intersections with multiple origin-destinations can be improved by pre-defining a subset of available routes that travellers can follow for each destination at any node. For a traffic network with multiple O-D flows, the CTM for multiple O-D flows can be used which distinguish traffic occupancy and flows based on origin and destination of traffic departed at each time-step by following Ukkusuri, Han and Doan (2012) or Carey et al. (2014) .

Another practical issue in implementing the proposed framework other than modelling large network using CTM is related to the information provided to the commuters, which traffic management authorities can encounter while implementing the proposed model. For example, the definition of destinations to which the travel time on a particular road is communicated to users could be a practical problem. There could be various routes leading to a destination from the location of a VMS and the consideration of communicated number of routes leading to the destination could be another implementation problem. The design of a VMS regarding the information provided is also an important consideration and various designs of VMS can be considered while implanting the ATIS. In this paper we have not addressed this issue explicitly. However, the details of implementation in real-world will depend on the nature of the problem, so a general solution is difficult to suggest and not covered in the scope of this paper.

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Appendix A: EKF for estimation of traffic state

This appendix describes mathematical formulation of traffic state estimation using EKF and elaborates section-6 of this paper. For traffic density estimation of a traffic network, the framework described in Wang and Papageorgiou (2005) is adapted for CTM. The objective of EKF at each time-step k is to find a state estimate which minimizes covariance of estimation error using all available measurements till time-step k.

$$E\left\{\left[\boldsymbol{x}(k+1)-\hat{\boldsymbol{x}}(k+1|k)\right]^T.\left[\boldsymbol{x}(k+1)-\hat{\boldsymbol{x}}(k+1|k)\right]\right\} \quad \text{(i)}$$

For any estimation problem using EKF, the following three conditions must be satisfied.

i) Noises in measurement $\boldsymbol{\varphi}(k)$ and in prediction process $\boldsymbol{\varepsilon}(k)$ are zero-mean Gaussian white random processes. For any $k>0$ and $l>0$:

$$E[\boldsymbol{\varepsilon}(k)]=0 \quad \text{(ii)}$$

$$E[\boldsymbol{\varphi}(k)]=0 \quad \text{(iii)}$$

$$E[\boldsymbol{\varepsilon}(k)\boldsymbol{\varepsilon}^T(l)]=\begin{cases}\boldsymbol{Q} \text{ if } k=l,\\ 0 \text{ otherwise}\end{cases} \quad \text{(iv)}$$

$$E[\boldsymbol{\varphi}(k)\boldsymbol{\varphi}^T(l)]=\begin{cases}\boldsymbol{R} \text{ if } k=l,\\ 0 \text{ otherwise}\end{cases} \quad \text{(v)}$$

$$E[\boldsymbol{\varepsilon}(k)\boldsymbol{\varphi}^T(l)]=\begin{cases}M \text{ if } k=l,\\ 0 \text{ otherwise}\end{cases} \quad \text{(vi)}$$

where, **Q** and **R** are known symmetric matrices representing variance of noise in prediction of model and noise in measurements, respectively.

ii) Initial state **x**(0) is a Gaussian random with known mean and covariance matrix.

$$\hat{\boldsymbol{x}}_0=E[x(0)] \quad \text{(vii)}$$

$$P_0=E\left\{\left[\boldsymbol{x}(0)-\hat{\boldsymbol{x}}_0\right].\left[\boldsymbol{x}(0)-\hat{\boldsymbol{x}}_0\right]\right\} \quad \text{(viii)}$$

iii) Initial state **x**(0) is not correlated with model prediction or measurement noise at any time instant.

The recursive equation of EKF that recursively estimates the current traffic state based on prediction of traffic state from CTM and observation of traffic state is given by:

$$\hat{\mathbf{x}}(k+1|k)=f[\hat{\mathbf{x}}(k|k-1,0),0] + \boldsymbol{K}(k)\ [\boldsymbol{y}(k)-g(\hat{\mathbf{x}}(k|k-1,0))] \tag{ix}$$

where K is Kalman Gain Matrix and which is estimated at each time-step:

$$\boldsymbol{K}(k)=[\boldsymbol{A}(k)\boldsymbol{P}(k|k-1)\boldsymbol{B}^T(k)+\boldsymbol{\Delta}(k)\boldsymbol{M}(k)\boldsymbol{\Pi}^T(k)].[\boldsymbol{B}(k)\boldsymbol{P}(k|k-1)\boldsymbol{B}^T(k)+\boldsymbol{\Pi}(k)\boldsymbol{R}(k)\boldsymbol{\Pi}^T(k)]^{-1} \tag{x}$$

The covariance for next time-step is also predicted and given by:

$$\boldsymbol{P}(k+1|k)=[\boldsymbol{A}(k)-\boldsymbol{K}(k)\boldsymbol{B}(k)].\boldsymbol{P}(k|k-1)\boldsymbol{A}^T(k)+\boldsymbol{\Delta}(k)\boldsymbol{Q}(k)\boldsymbol{\Delta}^T(k) - \boldsymbol{K}(k)\ \boldsymbol{\Pi}(k)\boldsymbol{M}^T(k)\boldsymbol{\Delta}^{\boldsymbol{T}}(k) \tag{xi}$$

In equation (3.79) and (3.80), **A**(k) represents first-order partial derivative of prediction function f with respect to the **x**, which contains all output variables. This is also known as Jacobian matrix. Jacobian matrix **B**(k) represents first-order partial derivative of function g with respect to vector **x**. Jacobian matrix $\boldsymbol{\Delta}$(k) is first-order partial derivative of prediction function f with respect to prediction error **ε**(k) and Jacobian matrix $\boldsymbol{\Pi}$(k) is first-order partial derivative of function g with respect to measurement error $\boldsymbol{\varphi}$.

$$\boldsymbol{A}(k)=\frac{\partial f}{\partial x}(\hat{\boldsymbol{x}}(k|k-1),0) \tag{xii}$$

$$\boldsymbol{B}(k)=\frac{\partial g}{\partial x}(\hat{\boldsymbol{x}}(k|k-1),0) \tag{xiii}$$

$$\boldsymbol{\Delta}(k)=\frac{\partial f}{\partial \varepsilon}(\hat{\boldsymbol{x}}(k|k-1),0) \tag{xiv}$$

$$\boldsymbol{\Pi}(k)=\frac{\partial g}{\partial \varphi}(\hat{\boldsymbol{x}}(k|k-1),0) \tag{xv}$$