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COMPENSATION OF MULTIVARIABLE FIRST-ORDER TYPE SYSTEMS

by

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Abstract

Recent results on the unity feedback control analysis of multivariable first-order type systems are extended to provide an analytic solution for a unity feedback controller producing different response speeds from each channel.

In a recent paper[1] the concept of a multivariable first-order type system has been introduced, and closed-form solutions derived for proportional and proportional plus integral unity feedback controllers capable of producing a high performance feedback system with fast response speeds and small interaction effects. For the purpose of this letter a multivariable first-order system is described by an mxm transfer function matrix of the form

$$G(s) = \sum_{j=1}^{m} (s+b_j)^{-1}\alpha_j\beta_j^+ \quad , \qquad |G(s)| \neq 0 \qquad \ldots(1)$$

where $\{\alpha_j\beta_j^+\}_{1\leq j\leq m}$ is a set of dyads such that $\bar{b}_j = b_\ell$ implies $\overline{\alpha_j\beta_j^+} = \alpha_\ell\beta_\ell^+$. This implies[1] that $\{\alpha_j\}_{1\leq j\leq m}$ and $\{\beta_j\}_{1\leq j\leq m}$ are sets of linearly independent vectors and

$$G_\infty = \lim_{s\to\infty} sG(s) \qquad \ldots(2)$$

exists and is non-singular. Note that this definition extends the previous[1] to include the possibility of open-loop plant integrators and open-loop unstable systems.

The suggested proportional controller[1] for such a class of system takes the form

$$K(s) = kG_\infty^{-1} - G^{-1}(s)\big|_{s=o} \qquad \ldots(3)$$

which produces a closed-loop transfer function matrix

$$\{I_m + G(s)K(s)\}^{-1}G(s)K(s) = \frac{k}{s+k} M(k) \qquad \ldots(4)$$

where M(k) is a frequency independent matrix satisfying the relation

$$\lim_{k\to\infty} M(k) = I_m \qquad \ldots(5)$$

As k increases, the speed of response increases and closed-loop interaction effects and steady state errors become arbitrarily small.

An important observation in the above analysis is that the proposed controller (eqn (3)) produces identical response speeds from each loop (eqn (4)) whereas, in practical applications, a large spread in system time constants $\{b_j^{-1}\}$ and the use of the controller of eqn (3) may require the use of unrealistically high gains. Alternatively, a low interaction feedback system may be required and different response speeds from each loop. The following analysis indicates that this objective cannot be achieved, in general, by a dyadic controller[1,2].

Consider the dyadic proportional controller,

$$K(s) = G_\infty^{-1} \sum_{j=1}^{m} (k_j - b_j)\alpha_j\gamma_j^+ \qquad \ldots(6)$$

where[1] $\gamma_j^+\alpha_k = \delta_{jk}$. This controller reduces to previous results if[1] $k_j = k$, $1 \leqslant j \leqslant m$. Also

$$G(s)K(s) = \sum_{j=1}^{m} \frac{(k_j - b_j)}{(s+b_j)} \alpha_j\gamma_j^+ \qquad \ldots(7)$$

and

$$\{I_m + G(s)K(s)\}^{-1}G(s)K(s) = \sum_{j=1}^{m} \frac{(k_j - b_j)}{(s+k_j)} \alpha_j\gamma_j^+ \qquad \ldots(8)$$

Taking, for example, the case of m = 2 and $k_1 >> k_2$, then the response to a unit step demand in channel one can be approximated at t = 0+ by

$$y(t) = \frac{k_1 - b_1}{k_1} \{1 - e^{-k_1 t}\}\alpha_1\{\gamma_1^+ e_1\} \qquad \ldots(9)$$

so that a time constant of k_1^{-1} in channel one with small interaction effects can only be obtained if $\alpha_1 \simeq e_1$. Hence, in general, a dyadic controller of the form of equation (6) cannot achieve the objectives of different response speeds in each channel and low interaction effects. The following analysis provides a solution to this problem.

If K(s) is a general mxm forward path controller, then the step response of the closed-loop system is represented by the mxm matrix Y(s) where $Y_{ij}(s)$ represents the response in loop i to a unit step demand in output j. Also,

$$\{I_m + G(s)K(s)\}Y(s) = G(s)K(s)I_m \frac{1}{s} \qquad \ldots(10)$$

Setting $K(s) = G_\infty^{-1}K_1(s)$, noting[1] that $G^{-1}(s) = sG_\infty^{-1} + G^{-1}(s)|_{s=o}$ and defining

$$A = G_\infty G^{-1}(s)|_{s=o} \qquad \ldots(11)$$

then

$$\{sI_m + A + K_1(s)\}Y(s) = \frac{1}{s}K_1(s) \qquad \ldots(12)$$

Defining

$$K_1(s) = \mathrm{diag}\{k_j(s)\}_{1\leq j\leq m} - [A_{ij}(1-\delta_{ij})] \qquad \ldots(13)$$

where $\{k_j(s)\}_{1\leq j\leq m}$ are scalar controller transfer functions, then

$$sY(s) = \mathrm{diag}\left\{\frac{k_j(s)}{s+A_{jj}+k_j(s)}\right\} - \mathrm{diag}\left\{\frac{1}{s+A_{jj}+k_j(s)}\right\}[A_{ij}(1-\delta_{ij})] \qquad \ldots(14)$$

To illustrate that the desired objective has been achieved, consider the case of proportional control with $k_j(s) = p_j - A_{jj}$, $1\leq j\leq m$, then

$$sY(s) = \mathrm{diag}\left\{\frac{p_j - A_{jj}}{s+p_j}\right\} - \mathrm{diag}\left\{\frac{1}{s+p_j}\right\}[A_{ij}(1-\delta_{ij})] \qquad \ldots(15)$$

That is, the response to a step demand in output j, is expressed as

$$y(t) = \frac{(p_j - A_{jj})}{p_j}\{1 - e^{-p_j t}\}e_j + \varepsilon_j(t) \quad \ldots(16)$$

and if $p = \min\{p_j\}$,

$$\lim_{p\to\infty} \sup_{t\geq 0} \|\varepsilon_j(t)\| = 0 \quad \ldots(17)$$

Examination of equations (16),(17) indicates that, provided gains are high, the proposed controller produces a low interaction system with different response speeds from each channel. Choice of any particular pole set $\{-p_j\}$ does not guarantee a low interaction system unless p is large. However, the above analysis does indicate that it is possible to choose relative response speeds and obtain a low interaction feedback system.

To illustrate the result, consider the multivariable first order system

$$G(s) = \frac{1}{15(s+1)(s+10)}\begin{bmatrix} 15s+159 & -36 \\ 36 & 15s+6 \end{bmatrix} \quad \ldots(18)$$

Note that $G_\infty = I_2$ so that $K(s) = K_1(s)$ and (equation (13)),

$$K(s) = \frac{1}{15}\begin{bmatrix} 15p_1-6 & -36 \\ 36 & 15p_2-159 \end{bmatrix} \quad \ldots(19)$$

and the interaction term in equation (15) takes the form

$$\frac{1}{15}\begin{bmatrix} \frac{1}{s+p_1} & 0 \\ 0 & \frac{1}{s+p_2} \end{bmatrix}\begin{bmatrix} 0 & -36 \\ 36 & 0 \end{bmatrix} \quad \ldots(20)$$

Here, for example, if $p = \min(p_1,p_2)>12$, interaction effects will be less than 20% and an arbitrary relative loop response speed p_1/p_2 can be obtained. Interaction effects can be reduced by increasing p and introducing integral action into the controller (see equation (14)).

References

1. D. H. Owens: 'First and second-order type structures in linear multivariable control systems design' Proc.IEE, 122( ), 1975, pp.

2. D. H. Owens: 'Dyadic approximation method for multivariable control systems design with a nuclear reactor application' Proc.IEE, 120(7), 1973, pp.801-809.