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SOME UNIFYING CONCEPTS IN MULTIVARIABLE FEEDBACK DESIGN

BY

D.H. Owens B.Sc., A.R.C.S., Ph.D., A.F.I.M.A., C.Eng., M.I.E.E.

Department of Control Engineering,
Mappin Street,
Sheffield S1 3JD.

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SOME UNIFYING CONCEPTS IN MULTIVARIABLE FEEDBACK DESIGN

D. H. Owens

Department of Control Engineering, University of Sheffield,
Mappin Street, Sheffield S1 3JD, England

Abstract. Frequency domain techniques for computer-aided design of multivariable feedback systems are now well established in the form of several, apparently distinct, design techniques. It is shown that a unified design structure can be developed based on the theoretical concepts of precompensation, eigenvalue approximation and permissible, constant, input/output transformations.

Keywords. Computer-aided system design, multivariable control systems, control system analysis, feedback, linear systems, multivariable systems.

INTRODUCTION

The use of frequency domain ideas in the analysis and design of output feedback configurations for the control of a linear time-invariant plant described by the mxm transfer function matrix (TFM) G(s) is now well established (Owens, 1978; Rosenbrock, 1974). A number of apparently distinct design techniques have been introduced based on reduction of the multivariable problem to a sequence of essentially independent scalar designs. Particular examples of such techniques are the inverse Nyquist array (INA) (Rosenbrock, 1974, Munro, 1979), the method of dyadic expansion (MDE) (Owens 1975, 1978, 1979) and the use of characteristic loci (CL) and approximately commutative control (ACC) (MacFarlane and Kouvaritakis 1977, Postlethwaite 1979, Kouvaritakis 1979). Although the techniques have a fundamentally different conceptual basis it is clear that the design operations have an algebraic and numerical similarity. This paper describes a unified structure encompassing the apparently distinct design techniques based on the ideas of precompensation, eigenvalue approximation and the use of a class of permissible input/output transformations. Not only do these concepts provide a theoretical unity, they can enable the construction of general design programmes of great flexibility containing the above named design techniques simply as special cases.

GENERAL DESIGN PHILOSOPHY

For simplicity, consider the unity negative feedback system of Fig. 1(a). The special case of the diagonal plant $G(s) = \mathrm{diag}\{g_j(s)\}_{1\le j\le m}$ represents m non-interacting systems. Each of these systems can be separately controlled by the diagonal controller $K(s) = \mathrm{diag}\{k_1(s), k_2(s), \ldots, k_m(s)\}$ and

$$|T(s)| = \prod_{j=1}^{m} (1+g_j(s)k_j(s)) \qquad (1)$$

$$H_c(s) = \mathrm{diag}\left\{\frac{g_j(s)k_j(s)}{1+g_j(s)k_j(s)}\right\}_{1\le j\le m} \qquad (2)$$

where $T(s) \triangleq I_m+G(s)K(s)$ is the matrix return-difference and $H_c(s)$ is the closed-loop TFM (CLTFM). In effect the closed-loop system is noninteracting and its stability and transient performance is only 'as good as' the responses of the scalar systems

$$h_j(s) \triangleq \frac{g_j(s)k_j(s)}{1+g_j(s)k_j(s)}, \quad 1\le j\le m \qquad (3)$$

Although trivial, this problem does underline the observation that the presence of off-diagonal/interaction terms in the plant are a major source of design problems. It is therefore assumed that design procedures will, in practice, attempt to reduce or eliminate (in some sense) the need to consider interaction effects in the design process and hence reduce the design to a sequence of independent scalar designs. The theoretical tools developed will, of course, reflect this general design philosophy.

TRANSFORMATION OPERATIONS

The natural approach to the diagonalization of a square matrix is the use of eigenvector methods as in the CL design method. In Fig. 1(a) let the forward path TFM $Q \triangleq GK$ have eigenvector matrix W(s) with inverse V(s) and corresponding eigenvalues $q_j(s)$,

$1 \leq j \leq m$ ie $Q(s) = W(s)\ \mathrm{diag}\{q_j(s)\}_{1\leq j\leq m} V(s)$ or

$$|T(s)| = \prod_{j=1}^{m} (1+q_j(s)) \tag{4}$$

$$H_c(s) = W(s)\ \mathrm{diag}\left\{ \frac{q_j(s)}{1+q(s)} \right\}_{1\leq j\leq m} V(s) \tag{5}$$

In effect eigenvector transformations formally reduce the design problem to the analysis and design of the 'characteristic transfer functions' $q_j(s)$, $1\leq j\leq m$, (subjected to unity negative feedback) and the eigenvector matrix $W(s)$.

The pseudo-classical nature of this analysis is deceptive in its simplicity as it provides few explicit design algorithms. It does however establish transformation techniques as an important component in the general design philosophy.

For conceptual and numerical simplicity, the transformations suggested here (Owens 1979) (a) introduce no extra dynamics into the system and (b) have a meaningful physical interpretation. The transformations take the form $G(s) \mapsto \tilde{G}(s)$ defined by

$$G(s) = P_1\ \tilde{G}(s)\ P_2 \tag{6}$$

where P_1, P_2 are square, nonsingular matrices. Such transformations certainly introduce no new dynamics and the TFM $\tilde{G}(s)$ can be interpreted as the TFM between $\hat{u}(s) = P_2 u(s)$ and $\hat{y}(s) = P_1^{-1} y(s)$.

It is a natural first step to demand that the transformation pair (P_1, P_2) be real.

In such a case the operations are a generalization of many previously defined design operations:

Constant precompensation: The map $G(s) \to G(s)K$ (where K is a constant, nonsingular controller) can be regarded as the map $G \to \tilde{G} = GK$ induced by the transformation pair (I_m, K^{-1}) and describes such operations as alignment of high frequency eigenvectors in the CL and ACC method and pseudodiagonalization in the direct Nyquist array (Rosenbrock 1974).

Constant pre- and post-compensation: Writing equation (6) in the form

$$\hat{\tilde{G}}(s) = P_2\ \hat{G}(s)\ P_1 \tag{7}$$

(where $\hat{L}(s)$ denotes the inverse of the TFM $L(s)$) it is seen that the pair (P_1, P_2) can be given an interpretation in terms of constant post- and pre-compensation operations as used, for example, in the INA.

There are sound theoretical and physical reasons (Owens 1979) for permitting the transformation matrices to have complex elements in a restricted form, just as it is necessary to allow complex conjugate pair eigenvectors in spectral analysis of oscillatory linear systems. This device also allows many basic operations of the MDE (Owens 1979) to be described by transformations of the form described. The class of transformations permitted are as follows:

Definition (Owens (1979)

The mxm nonsingular complex matrices

$$P_1 = [\alpha_1, \alpha_2, \ldots, \alpha_m]\ ,\ P_2 = \begin{bmatrix} \beta_1^T \\ \beta_2^T \\ \vdots \\ \beta_m^T \end{bmatrix} \tag{8}$$

are said to be permissible if the columns (resp. rows) of P_1 (resp. P_2) are real or exist in complex conjugate pairs and if $\beta_j = \beta_{\ell(j)}$ whenever $\alpha_j = \bar{\alpha}_{\ell(j)}$, $1\leq j\leq m$.

(Note: the map $j \mapsto \ell(j)$ is a one-to-one map of the integer set $(1,2,3,\ldots,m)$ onto itself by the structure and nonsingularity of P_1 and $\ell(\ell(j)) = j$, $1\leq j\leq m$). The potential benefit of introducing complex transformations can be illustrated by examples (Owens 1979) and the basic theorem of the MDE.

Theorem 1 (Owens 1979)

If $|G(i\omega_1)| \neq 0$ and $G(-i\omega_1)G^{-1}(i\omega_1)$ has a complete set of eigenvectors then there exists a permissible transformation $(P_1(\omega_1), P_2(\omega_1))$ such that $\tilde{G}(s)$ is diagonal at the point $s = i\omega_1$ and diagonally dominant in an open frequency interval containing that point.

Equivalently, under mild (generic!) conditions, permissible transformations can be used to transform a given system into a system possessing no interaction at a specified frequency point. Moreover it follows that the class of permissible transformations is the smallest class that can achieve this objective.

We now describe some properties of permissible transformations (the proofs are straightforward and omitted here):

Theorem 2

If the transformation pair (P_1, P_2) is permissible, then (a) $P_o \triangleq P_1 P_2$ is real and nonsingular and (b) $(P_1, \bar{P}_1^{-1})$, (c) $(\bar{P}_2^{-1}, P_2)$ and (d) (P_2^{-1}, P_1^{-1}) are permissible.

TRANSFORMATION OF THE CONTROL PROBLEM

The previous section has shown that constant plant and controller operations can be regarded as permissible transformations and that this is true of all three design techniques considered. Another major application of the ideas is in complete transformation of the design problem. More precisely (Owens 1979), the analysis and design of the configuration of Fig. 1(a) can be replaced by that shown in Fig. 1(b), provided that

$$K(s) = P_2^{-1}\tilde{K}(s)P_1^{-1} \tag{9}$$

ie if the forward path control systems are related by the permissible (Theorem 2(d)) transformation (P_2^{-1}, P_1^{-1}). In particular, the relations

$$|I_m + G(s)K(s)| \equiv |I_m + \tilde{G}(s)\tilde{K}(s)| \tag{10}$$

$$(I_m + GK)^{-1}GK \equiv P_1(I_m + \tilde{G}\tilde{K})^{-1}\tilde{G}\tilde{K}P_1^{-1} \tag{11}$$

indicate that the closed-loop stability of the two systems is identical and that the CLTFM's are related by the permissible (similarity) transformation (P_1, P_1^{-1}) (Theorem 2(b)).

The unifying influence of this transformation can be illustrated as follows:

Constant precompensation. The design of the factored controller $K(s) = K_pK_c(s)$ (where K_p is a real, nonsingular precompensator) for the plant $G(s)$ can be regarded as the design of the controller $\tilde{K}(s) = K_c(s)$ for the transformed plant $\tilde{G}(s) = G(s)K_p$. The permissible transformation in this case is (I_m, K_p^{-1}).

Constant pre- and post-compensation. Consider Fig. 1(c) with real, nonsingular constant postcompensator L and forward path controller $K_1(s) = K_pK_c(s)$. The design process can be regarded as the design of the controller $\tilde{K}(s) = K_c(s)$ for the transformed plant $\tilde{G}(s) = LG(s)K_p$ generated by the permissible transformation (L^{-1}, K_p^{-1}). The transformation of equation (9) then generates the configuration of Fig. 1(a) with forward path controller $K(s) = K_pK_c(s)L = K_1(s)L$ derived from Fig. 1(c) by 'moving L around the loop'.

Approximately commutative control. An approximately commutative controller takes the form $K(s) = W_1 \mathrm{diag}\{k_j(s)\}_{1\le j\le m} V_1$ where W_1 (resp. V_1) is a real approximation to the eigenvector (resp. inverse eigenvector) matrix of $G(s)$ at some desired frequency point $s = i\omega_1$. Comparing this with equation (10) it is seen that $K(s)$ can be regarded as generated from $\tilde{K}(s) = \mathrm{diag}\{k_j(s)\}_{1\le j\le m}$ and induced by the permissible transformation $(P_1, P_2) = (V_1^{-1}, W_1^{-1})$. The corresponding transformed plant is $\tilde{G}(s) = V_1G(s)W_1$.

Method of dyadic expansion. This is the only method (at this stage) that explicitly makes use of complex permissible transformations. The pair (P_1, P_2) are computed (Owens 1978) to generate (Theorem 1) a transformed plant $\tilde{G}$ that is diagonal at a desired frequency point $s = i\omega_1$. The controller $\tilde{K}(s)$ is set equal to a diagonal matrix $\mathrm{diag}\{k_j(s)\}_{1\le j\le m}$ and the controller $K(s)$ generated by equation (10).

REALIZABILITY CONDITIONS

In practice, both G and K have rational polynomial elements. For physical realizability these polynomials must have real coefficients. These conditions are satisfied (for G say) iff

$$\overline{G(s)} \equiv G(\bar{s}) \tag{12}$$

Theorem 3

If (P_1, P_2) is permissible, then eqn. (12) holds iff

$$\overline{\tilde{G}_{jk}(s)} \equiv \tilde{G}_{\ell(j)\ell(k)}(\bar{s}) \ \forall\ j,k \tag{13}$$

Proof Substitute eqns (6) and (8) into (12)

$$\sum_{j,k} \overline{\tilde{G}_{jk}(s)\alpha_{\ell(j)}\beta_{\ell(k)}^T} \equiv \sum_{j,k} \tilde{G}_{jk}(\bar{s})\alpha_j\beta_k^T \tag{14}$$

and equate coefficients of $\alpha_j\beta_j^T \ \forall\ j,k$.

It is important to note that this theorem can also be applied to construct suitable constraints on $\tilde{K}(s)$ to ensure the physical realizability of $K(s)$. More precisely, as (P_2^{-1}, P_1^{-1}) is permissible (Theorem 2(d)), then:

Corollary

$$\overline{K(s)} \equiv K(\bar{s}) \text{ iff } \overline{\tilde{K}_{jk}(s)} \equiv \tilde{K}_{\ell(j)\ell(k)}(\bar{s}) \quad \forall\ j,k$$

The following result is also important:

Theorem 4

The set of TFM's $\tilde{G}$ satisfying equation (13) is closed under multiplication and inversion.

Proof Suppose that $\tilde{G}$ and $\tilde{K}$ satisfy (13) and let $\tilde{Q} = \tilde{G}\tilde{K}$ then

$$\overline{\tilde{Q}_{jk}(s)} \equiv \sum_{i=1}^{m} \overline{\tilde{G}_{ji}(s)}\ \overline{\tilde{K}_{ik}(s)}$$

$$\equiv \sum_{i=1}^{m} \tilde{G}_{\ell(j)\ell(i)}(\bar{s})\tilde{K}_{\ell(i)\ell(k)}(\bar{s})$$

$$\equiv \tilde{Q}_{\ell(j)\ell(k)}(\bar{s}) \qquad (15)$$

If $\tilde{G}$ satisfies (13) with inverse $\tilde{K}$ then

$$\delta_{jk} = \sum_{i=1}^{m} \overline{\tilde{G}_{ji}(s)}\ \overline{\tilde{K}_{ik}(s)}$$

$$= \sum_{i=1}^{m} \tilde{G}_{\ell(j)\ell(i)}(\bar{s})\ \overline{\tilde{K}_{ik}(s)} \qquad (16)$$

$$\delta_{\ell(j)\ell(k)} = \sum_{i=1}^{m} \tilde{G}_{\ell(j)\ell(i)}(\bar{s})\tilde{K}_{\ell(i)\ell(k)}(\bar{s}) \qquad (17)$$

The result follows from the uniqueness of the inverse and $\delta_{jk} = \delta_{\ell(j)\ell(k)}\ \forall\ j,k$.

DIAGONAL DOMINANCE

The choice of (P_1,P_2) can be based on physical grounds, trial and error or theoretical synthesis procedures. In all cases a well-defined objective is required. Our attitude is that permissible transformation techniques can only be of value if the transformed plant $\tilde{G}$ is 'easier to control' than G. Within our general design philosophy, design problems reduce as interaction effects are reduced. Therefore the attitude taken here is that permissible transformations will normally be constructed to reduce interaction effects in $\tilde{G}$ over a desired frequency band.

The natural (but non-unique) measure of interaction effects in $\tilde{G}$ in a given frequency interval is its degree of diagonal dominance (Rosenbrock 1974, Owens 1978, 1979). In practical terms, if we can choose permissible (P_1,P_2) such that $\tilde{G}$ (or its inverse) is diagonally dominant (d.d.) on the Nyquist D-contour the basic theorems of the INA method can be used to design a diagonal $\tilde{K}(s)$. If attention is restricted to real (P_1,P_2) then the alignment and pseudodiagonalization algorithms (say) can be applied, but success is not guaranteed even over a finite frequency interval. The potential benefit of accepting complex permissible transformations is illustrated by the basic theorem (Theorem 1) of the MDE ie diagonal dominance over a frequency interval containing a specified point is (almost) always possible. In the case of dyadic transfer function matrices (DTFM's) (Owens 1978, 1979), when G is diagonal, the design approach is particularly simple.

PRECOMPENSATION

Constant permissible transformation of G does not guarantee the d.d. of $\tilde{G}$ over a frequency interval large enough to have practical value. In such cases we can follow the INA by writing $\tilde{K}(s) = \tilde{K}_p(s)\tilde{K}_c(s)$ where $\tilde{K}_p(s)$ is a dynamic precompensator introduced to ensure that $\tilde{G}\tilde{K}_p$ (or its inverse) is d.d. over a desired frequency interval and $\tilde{K}_c$ is a compensator introduced to inject phase and gain compensation. We will assume that the compensator $\tilde{K}_c$ is diagonal, which is the natural choice if $\tilde{G}\tilde{K}_p$ is approximately diagonal in the frequency interval of interest.

The only constraint on the choice of $\tilde{K}_p$ and $\tilde{K}_c$ is the physical realizability of K. This is ensured (theorems 3 and 4) if $\tilde{K}_p$ and $\tilde{K}_c$ (or their inverses) satisfy constraints of the form of equation (13). The choice of $\tilde{K}_p$ can proceed on a trial and error basis of row or column operations or algorithmic bases as in pseudodiagonalization. The precise method is not relevant here. If, however, $\tilde{K}_p$ is generated as a product of trial precompensators, theorem 4 indicates that each factor must satisfy constraints of the form of equation (13). The resulting controller K can be implemented directly or in the form of cascaded blocks $K = K_pK_c$ where (Theorems 2,3) $K_p(s) \triangleq P_2^{-1}\tilde{K}_p(s)P_1^{-1}$ and $K_c(s) \triangleq P_1\tilde{K}_c(s)P_1^{-1}$ are physically realizable.

A GENERAL DESIGN ALGORITHM

The basic building blocks of permissible transformation, diagonal dominance checks and precompensation have been shown to provide a unified description of most of the basic operations in the INA, CL and ACC and MDE design techniques. We now show that they do, in fact, generate these algorithms and can generate new algorithms by (a) new combinations of these building blocks and (b) introducing new algorithms for constructing (P_1,P_2), $\tilde{K}_p$ and $\tilde{K}_c$.

A common feature in the three design techniques is manipulation and compensation of plant behaviour in an open, connected subset Ω of the Nyquist D-contour. A general design algorithm for this purpose is proposed in Fig. 2. The INA, CL and ACC, and the MDE are simply special cases of this algorithm generated by specific choices of (P_1,P_2), $\tilde{K}_p$ and $\tilde{K}_c$:

INA: Take $\Omega = D$ and the design algorithm in terms of inverse systems. Choose real pre- and post-compensators P_2^{-1} and P_1^{-1} respectively to achieve row dominance of $\hat{\tilde{G}} = P_2GP_1$ on Ω. Next choose an inverse dynamic precompensator $\hat{K}_p$ to improve the dominance of $\hat{K}_p\hat{G}$ if necessary. Next choose a diagonal compensator $\hat{K}_c$ to compensate the diagonal terms and satisfy the d.d. conditions of the INA stability theorem. Finally, construct $K = P_2^{-1}K_pK_cP_1^{-1}$ by moving the postcompensator P_1^{-1} around the loop.

ACC: Consider the representative case of high frequency alignment and intermediate frequency compensation. Set $\Omega = \{s = i\omega: \omega \geq \omega_m\}$ where ω_m is an intermediate frequency above which compensation is required. Choose $(P_1,P_2) = (V_m^{-1},W_m^{-1}K_h^{-1})$ where K_h is a real nonsingular matrix constructed to align the characteristic directions of GK_h at high frequencies and W_m (resp. V_m) are real, nonsingular approximations to the eigenvector (resp. inverse eigenvector) matrix of $G(i\omega_m)K_h$. As originally described (MacFarlane and Kouvaritakis, 1977) the technique can be regarded as choosing $\tilde{K}_p(s) \equiv I_m$ and $\tilde{K}_c(s) = \text{diag}\{k_j(s)\}_{1\leq j\leq m}$ on the assumption that $\tilde{G} = P_1^{-1}GP_2^{-1}$ is diagonal with diagonal terms equal to the CL of GK_h. The general design algorithm enables the testing of this assumption by examination of the degree of d.d. of $\tilde{G}$ in Ω and also suggests that the inclusion of a precompensator $\tilde{K}_p$ may improve the success of the technique.

MDE: Consider, for simplicity, the situation of an intermediate frequency point $s = i\omega_m$ around which compensation of system CL is desired. Choose (P_1,P_2) such that $\tilde{G}$ is diagonal at $s = i\omega_m$ and hence d.d. in a vicinity Ω of this point. Set $\tilde{K}_p(s) \equiv I_m$ and then $\tilde{K}_c(s) = \text{diag}\{k_j(s)\}_{1\leq j\leq m}$, choosing the $\{k_j(s)\}$ to produce the desired compensation to the diagonal terms of $\tilde{G}$ (regarded as rational approximations to the CL). If successful the controller $K = P_2^{-1}\tilde{K}_cP_1^{-1}$ is constructed. As with the ACC the general design algorithm suggests that the MDE can be extended by inclusion of a precompensator K_p to increase the interval Ω of diagonal dominance and hence extend the frequency interval over which accurate CL compensation can be achieved.

(Note: the ACC and MDE algorithms are, in practice, applied at high-intermediate and low frequencies and the controllers combined in a standard manner (Owens 1978,1979)).

DISCUSSION AND CONCLUSIONS

The apparently diverse nature of several frequency domain multivariable design techniques has been unified in the theoretical framework of permissible transformations, diagonal dominance checks and precompensation. A general design algorithm for system compensation in a subset Ω of the Nyquist D-contour has been constructed from these three modules and it has been seen that the INA, CL and ACC and the MDE fit into this framework simply as special cases.

A major step is the introduction of complex permissible transformations and, hence, TFM's with rational polynomial elements with complex coefficients. This departure from normal engineering practice is meaningful and, moreover, vital if further progress in design methods is to be made at a theoretical level. Examples (Owens 1979a) certainly suggest that the benefits can outweigh any apparent difficulties and the basic theorem of the MDE (Theorem 1) also implies that, if diagonality/diagonal dominance is taken as an important part of our design philosophy, then the acceptance of complex permissible transformations is necessary if design algorithms are to have any (generic) guarantee of success!

The structure of the general design algorithm suggests that any division of the three design techniques (in design programme terms) is purely artificial. In particular, the general design algorithm could be realized as a general design package of great flexibility, containing these techniques simply as alternative routes through the package obtained by specialized choices of transformation and the use (or otherwise) of precompensators. Such a programme would need a reconsideration of data handling to allow complex elements but the potential benefits obtained by (a) breaking down the barriers between established techniques and (b) the generation of new algorithms by, for example, the introduction of precompensation and diagonal dominance checks into the ACC and MDE, will, in the authors opinion, far outweigh the initial interpretive difficulties.

Finally, the emergence of a root-locus theory for multivariable systems (Owens 1978,1979b, Postlethwaite 1979) has focussed attention on the configuration of Fig. 1(d). Minor loop feedback H plays an important role in compensation studies for multivariable root-loci (Owens 1979b) and can be a great help in achieving diagonal dominance as is easily seen by examining the inverse forward path system $\hat{K}_c\{\hat{K}_p\hat{G}+H\}$. The introduction of permissible transformations into this picture is achieved by performing the transformations $G = P_1\tilde{G}P_2$, $K_p = P_2^{-1}\tilde{K}_pP_1^{-1}$, $K_c = P_1\tilde{K}_cP_1^{-1}$, $H = P_1\tilde{H}P_1^{-1}$ and considering the configuration with G,K_p,K_c and H replaced by $\tilde{G},\tilde{K}_p,\tilde{K}_c$ and $\tilde{H}$ respectively. The identities (c.f. eqns (10),(11))

$$|I_m+(I_m+GK_pH)^{-1}GK_pK_c| \equiv |I_m+(I_m+\tilde{G}\tilde{K}_p\tilde{H})^{-1}\tilde{G}\tilde{K}_p\tilde{K}_c| \quad (18)$$

$$(I_m+(I_m+GK_pH)^{-1}GK_pK_c)^{-1}(I_m+GK_pH)^{-1}GK_pK_c$$
$$\equiv P_1(I_m+(I_m+\tilde{G}\tilde{K}_p\tilde{H})^{-1}\tilde{G}\tilde{K}_p\tilde{K}_c)^{-1}(I_m+\tilde{G}\tilde{K}_p\tilde{H})^{-1}\tilde{G}\tilde{K}_p\tilde{K}_cP_1^{-1} \quad (19)$$

indicate that the two configurations have identical stability characteristics with CLTFM's related by the permissible transformation (P_1,P_1^{-1}).

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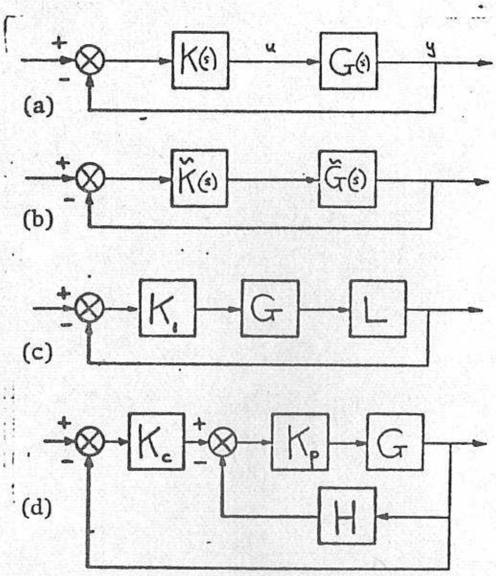

Fig.1 Multivariable Feedback Systems

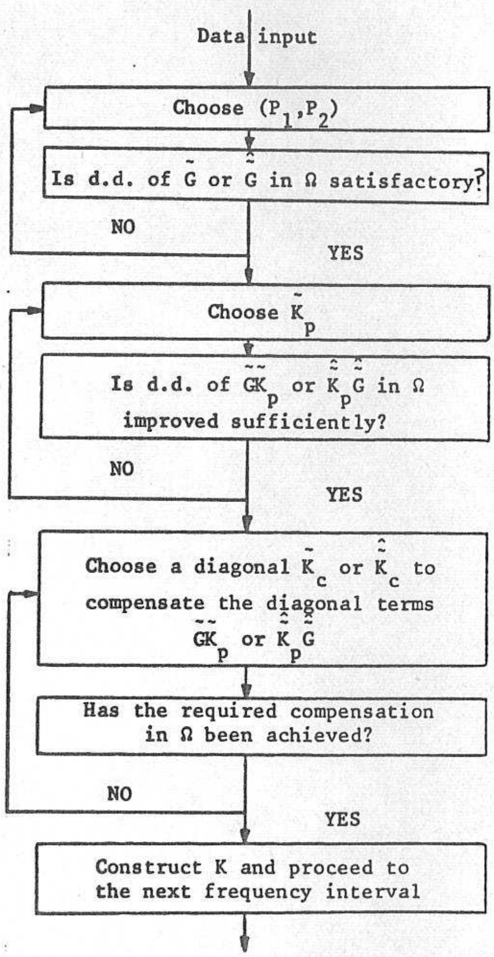

Fig. 2. General Design Algorithm