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STABILITY ANALYSIS OF A MULTI-PHASE CAR-FOLLOWING MODEL

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Abstract

This paper presents a numerical stability analysis of a multi-phase car-following model under mild to severe disturbances. The results show that local stability was always conformed. An asymptotically unstable region was found for traffic in congested states. One of the previously calibrated boundary conditions for close-following situations was found to be in conflict with the stable condition required by the car-following model, which had attributed to speed oscillations during transition of the traffic from a non-congested to a congested state. Suggestions were made to the choice of model parameter values to meet the stability conditions and ways to improve the model.

1. Introduction

Traffic flow has attracted multidisciplinary interests in recent years due to the increasing traffic congestion problem on highways and the complexity of the traffic flow system [1-7]. Traffic behaviour has been studied by microscopic and macroscopic models, and by a variety of approaches ranging from car-following models [2,3], cellular automation models [4,5], to gas kinetic and hydrodynamic models [6,7].

Car-following model is a microscopic description of the behaviour of vehicles following one another in a single stream of traffic [1-3], and is one of the fundamental building blocks of microscopic representation of traffic flow. There are many formulations of car-following models and some attempt to generalise them [1]. Some of the earliest car-following models were developed by General Motors (GM) based on vehicle-following data collected on their test-tracks [2]. The GM models represent the response of a following vehicle in terms of its acceleration and deceleration to the stimulus it received from the vehicle ahead and its driver's sensitivity. The stimulus is usually represented as a function of the relative velocity and spacing between the two vehicles (e.g. [2, 3]). This type of models is often referred to as *psycho-physical car-following models* in reference to their combined representation of drivers' reactions with the laws of physics on the dynamic equation of vehicle motion.

Another type of models, so called *safety-distance models*, is based on the simple idea that, whatever the following vehicles do, they want to keep a safe distance behind so that they do not collide with the vehicle(s) in front. One of the most well-known safety-distance based car-following models is the Gipps model [8], which is widely adopted in traffic micro-simulation software [9,10].

Most of these models represent a single state of traffic, i.e. there is a single, fixed rule for the car-following behaviour throughout. Moreover, many of the models including the GM models, are based on empirical investigations carried out at relatively low speeds (mostly in the region

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of 30-60 km/hr), which may not reflect more general car-following behaviour of traffic on high-speed road networks.

In recent years, the technical progress and the instrumentation of many of our highway networks have enabled collection and analysis of large sets of empirical traffic flow data and across a wide range of different traffic conditions [11-12]. Studies of such highway traffic flows have revealed the existence of different traffic phases ranging from free-flow, synchronized traffic with lower velocity but still high flows, to complete flow breakdown and traffic jams [1, 13]. This has led the development of *multi-regime car-following models* which apply different car-following rules to represent drivers' adapting to different driving behaviour under different traffic conditions [e.g. 14, 15].

Two phenomena of highway traffic have received particular attention recently; namely *close-following* behaviour and *traffic hysteresis*. Considerably closer following than often found in urban traffic is observed when highway traffic is near capacity, but before the breakdown [16, 17]. This is characterised by vehicles driving at very high speed but keeping very small gaps, sometimes with a time gap as low as 0.8 second [18]. It is believed that close-following is one of the main causes of traffic instability and therefore traffic jams [11, 19, 20]. Traffic hysteresis is a phenomenon characterized by a loop structure from empirical observed flow-occupancy plots, where the capacity of a traffic flow recovering from a flow breakdown does not reach the capacity before the breakdown [21-24].

Wang et al [25] proposed a multi-phase car-following model which explicitly included a close-following phase and introduced the concept that drivers' reaction times are different during different phases. The model was shown to be capable of reproducing the full spectrum of traffic states, and in particular close-following and traffic hysteresis. The key model parameters were calibrated using aggregated traffic detector data [26]. The main contribution of the current paper is to study the stability properties of this model using numerical simulation method.

The paper is organised as follows: Section 2 of the paper describes briefly the model concerned. Section 3 introduces the concept of local and asymptotic stabilities and presents results showing the performance of the model in the stability tests. An unstable, oscillatory behaviour of the model was found and questions were raised on the model parameter values used. Section 4 discusses the implications and suggests ways the model can be improved. Finally conclusions are drawn in Section 5.

2. A Multi-Phase Car-Following Model

The model by Wang, Liu and Montgomery [25], hereafter WLM model, was designed to represent traffic flow on high-speed freeway networks, in particular to capture some of the key characteristics of freeway traffic flow such as the close-following behaviour and traffic hysteresis phenomenon.

For the description and the formulation of the WLM model, we consider a simple car-following situation as illustrated in Fig. 1 where vehicle *n* follows vehicle *n-1* in a single stream of traffic. The variable $x_n(t)$ denotes the position vehicle *n* as measured from an arbitrary starting point, whilst $v_n(t)$ its velocity at time *t*. L_{n-1} is the "effective" size of vehicle *n-1*, which includes the physical length of the vehicle plus a safe margin.

The WLM model was built on the concept that drivers in different traffic conditions (or states) behave differently. This concept was represented in the model by drivers applying different accelerations and reaction times in different states. The states considered were: traffic build-up from free-flow towards congestion, close-following, flow break-down and recovery.

The model assumed that, as the traffic was getting congested, drivers' behaviour would change and they would become more alert to their surroundings. This change was characterised in the model by a critical driving speed, v_c *at 50 km/hr* [17]. Above this speed threshold (i.e. when traffic moves more freely), drivers were considered to be in a *non-alert state* with longer reaction times and lower acceleration and deceleration. Below this critical speed (when traffic becomes congested), drivers were considered to be in an *alert state* with shorter reaction times and higher acceleration and braking power. After a highly alert state during congestion and flow breakdown and during the recovery state, the drivers were assumed to want to relax a little bit and return back to the non-alert state with longer reaction time and lower acceleration/deceleration.

Situated in between the non-alert and alert states during traffic build-up is a close-following state. The exact definition of the close-following state is described later.

In its mathematical formulation, the WLM model combines the idea of the safe-distance model of Gipps [8] for alert and non-alert states, with the model of Leutzbach and Wiedemann [14] for close-following state. The WLM model then covers the full spectrum of traffic states and allows vehicles to move from one state to another in a continuous space-time domain.

Gipps model gives the speed of each vehicle *n* at time $t + \tau$ in terms of its speed at the earlier time *t* as:

$$v_n^G(t+\tau) = \min\{v_n(t) + 2.5A_n\tau[1 - v_n(t)/V_n]\sqrt{0.025 + v_n(t)/V_n},\ B_n\tau + \sqrt{B_n^2\tau^2 - 2B_n[x_{n-1}(t) - x_n(t) - L_{n-1}] + B_n\tau v_n(t) + B_n v_{n-1}^2(t)/B'}\} \quad (1)$$

where V_n is the desired speed of the driver drawn randomly from a distribution with the free-flow speed v_f as its mean. Here $A_n > 0$ *and* $B_n < 0$ are the acceleration and deceleration of vehicle *n* respectively. The Gipps model applies the same reaction time (τ) and the perceived deceleration (B') to all vehicles.

The WLF model applies the Gipps' car-following rule in both the alert and non-alert states, but with different accelerations and reaction times. We use symbols (a_1, τ_1) to represent the acceleration and reaction time when drivers are in the alert state, and (a_2, τ_2) for when they are in non-alert state.

As first proposed in [14], traffic is said to be in a *close-following* state if it falls within a region of small relative speed (between ΔV_a and ΔV_b) and relative space headway (between d_{min} and d_{max}) to the vehicle ahead. The spatial boundary conditions for close-following was further defined in [9] as:

$$d_{min} = L_n + C_1\sqrt{v_n(t)} \text{ and } d_{max} = L_n + C_1\sqrt{C_2 v_n(t)} \quad (2)$$

where *C1* and *C2* are constants.

In the WLM model, the close-following state is further confined to a situation where none of the vehicles downstream in the platoon appears to be braking and the following vehicle will then to need to break very hard either, even when keeping a close headway. The decelerations of the two front vehicles in the platoon are used as indicators: if they are not decelerating at a rate noticeable to the following vehicle (i.e. the model parameter D_C), the following vehicle is regarded as in a close-following state.

The trajectories of the vehicle in a close-following state can be illustrated in Fig. 2 as a bounded circle in a relative speed and space headway domain. The model applies either a constant accelerate or deceleration to a vehicle depends on its space headway to the vehicle in front. The speed of the following vehicle is simply updated according to Newtonian equation of motion:

$$v_n^C(t+\tau) = \begin{cases} v_n(t) + a_3\tau_3 & \textit{for acceleration when } \Delta X_n \geq (d_{min} + d_{max})/2 \\ v_n(t) - a_3\tau_3 & \textit{for deceleration when } \Delta X_n < (d_{min} + d_{max})/2 \end{cases} \quad (3)$$

where a_3 and τ_3 are the acceleration and reaction time respectively applied by drivers in close-following state.

Therefore, the speed of a following vehicle *n* at time $t + \tau$ is determined by different functions (eq. (4)) according to the state it is in. The main model parameters which distinguish these different states are the vehicle's acceleration (*a)* and the driver's reaction time (τ). Table 1 summarises the conditions of these different traffic states.

$$v_n(t+\tau) = \begin{cases} v_n^G(a_1, \tau_1) & \textit{for alert traffic state, i.e. traffic condition during a flow breakdown} \\ v_n^G(a_2, \tau_2) & \textit{for non-alert states: free-flow condition or recover from a breakdown} \\ v_n^C(a_3, \tau_3) & \textit{for close-following state} \end{cases} \quad (4)$$

The values of acceleration and reaction time for alert and non-alert traffic of the WLM model were calibrated using aggregated detector data [26]. Table 2 lists the default values of the model parameters applied in this study.

A macroscopic interpretation of the model in terms of the steady-state traffic flow (*q*) and density (ρ) relationships of the different driving states was derived [31] and the results are reproduced here in equation (5) and illustrated in Fig. 3.

$$q = v_f \rho \quad \text{for the non-alert state, line OA in Fig. 3} \quad (5a)$$

$$q = \frac{2}{3\tau_2}(1 - \frac{\rho}{\rho_j}) \quad \text{for the non-alert state, line ABJ} \quad (5b)$$

$$q = \frac{2}{3\tau_1}(1 - \frac{\rho}{\rho_j}) \quad \text{for the alert state, CJ} \quad (5c)$$

$$q = \frac{4}{C_1^2(1+\sqrt{C_2})^2}(\frac{1}{\rho} - \frac{1}{2\rho_j} + \frac{\rho}{\rho_j^2}) \quad \text{for the close-following state, curve DE} \quad (5d)$$

where ρ_j is the jam density, v_f and v_C the free-flow and the critical speed respectively.

It was clear from the simplified illustration of the model in Fig. 3 that the reaction times of the drivers affect the traffic flow levels significantly and the bigger the difference between τ_1 and τ_2, the more clearly a traffic hysteresis would occur. Treiber and Helbing [27] proposed a similar concept in which different adaptation or reaction times being applied to different types of congested traffic to explain the observed inverse-λ shaped and the wide scattering of flow-density data.

The ability of the model to reproduce flow breakdown, shock waves and the correct gap distributions was further demonstrated in [25]. In this paper, we present an investigation of the local and asymptotic stability of the model.

3. Stability Analysis of the WLF Model

Car-following models are generally assessed by their properties to agree satisfactorily with experimental evidence, to be physically and psychologically plausible, and to possess local and asymptotic stability.

Stability of a car-following model is concerned with the growth of a small disturbance in speed and spacing when traffic is in a regime close to a steady state and how the perturbation propagates over time and down a line of vehicles [28]. Two types of stabilities are of general concern: local and asymptotic stability. *Local stability* is concerned with the response of a following vehicle to the fluctuation of the vehicle directly ahead and examines its distance-headway and speed over time. *Asymptotic stability* is concerned with the propagation of the fluctuation of the lead vehicle through a platoon of vehicles [29].

Linear stability analysis develops criteria which characterize the types of motions allowed by the model and determines, with a given range of model parameter values, if a disturbance would be damped, bounded or amplified [29]. The analysis is often mathematically complex and applied mostly to simple, linear car-following models such as the earlier GM models and under uniform flow conditions [30, 31].

Herman et al [28] derived criteria for both local and asymptotic stability for the first GM model proposed in [2] by means of Laplace transform of the equations of motion of a line of identical vehicles. Wilson [31] derived a linear stability formulation of the Gipps model under uniform conditions. Numerical simulation method provides a flexible means to examine the properties of a model. Ez-Zahraouy et al [32, 33] studied the stability of optimal velocity models under the effect of reaction time and its fluctuation.

In this section, we present a numerical simulation analysis of the WLM car-following model. No mathematical derivations of the stability conditions were attempted. The analysis was based on the examination of the simulated individual vehicles' speeds and trajectories. A platoon of seven vehicles was simulated with different initial conditions for the lead and the following vehicles. We present below the test scenarios and the results for the stabilities examined.

3.1 Local Stability

Four scenarios with different trajectories of the lead vehicle and different initial distance headways of the following vehicle were tested. In each scenario, three sub-scenarios (cases) were further considered. The speed-profiles of the lead vehicle tested are shown in Fig. 4. Scenario I was concerned with the influence of different initial distance headway to the local stability. Three cases with different initial distance headways were set up; they were at 30, 40 and 50m respectively for case 1, 2 and 3 of Scenario I. Their equivalent densities are 33.3, 25 and 20 veh/km for case 1, 2 and 3 respectively. The other three scenarios were set up to identify the impacts of different speed perturbations from the lead vehicle to the following vehicle. The same initial distance headway, at 40m (or equivalent density of 25 veh/km), was used in these scenarios.

The simulation results in terms of the changes over time of the distance headway and speed of the following vehicle are shown in Fig. 5, and from which, we can draw the following observations:

1. In all scenarios, a local stable condition was reached, as the speeds and distance headways have all converged to a stable condition; however
2. There were initial speed oscillations of the following vehicle, with different amplitudes;
3. These oscillations were found to occur in the speed range of 10m/s to 14m/s; and finally
4. The final distance headway and speed were only related to the final speed of leading vehicle.

The last observation above suggests that the initial speed of leading vehicle and initial distance headway do not affect the final travel pattern of following vehicle.

It is noted that the upper boundary of 14m/s (~50 km/hr) where the speed oscillations occurred coincides with the critical speed (v_c) which defines the distinction between the non-alert situation and the alert situations (see Fig. 3). We postulate therefore that these speed oscillations are caused by the modelled transition between these two situations. Further discussion on this phenomenon is presented in Section 4.

3.2 Asymptotic Stability

A platoon of seven vehicles was simulated to study the behaviour of asymptotic stability of the model. Four scenarios with different speed profiles of the lead vehicle (Fig. 6) were simulated.

The vehicles were initially put at a distance-headway of 40m (or an equivalent density of 25 veh/km) for Scenario I and II, and 35m (or density 28.6 veh/km) for the other two scenarios.

The four scenarios were designed to represent most of the manoeuvre of a lead vehicle [34]. In Scenario I, a mild disturbance which involved a liner deceleration and acceleration of the lead vehicle was set to test the reactions of following vehicles. In Scenario II, the lead vehicle's speed was varied according to a sinusoidal function and was used to represent a severe disturbance. Scenarios III and IV represented a simple acceleration and deceleration of the lead vehicle respectively.

The simulation results are shown in Fig. 7, from which we observed the following:

1. In scenarios I, II and III, asymptotic stability was reached, as the variations in distance headway and speed were gradually damped to a stable condition;
2. In scenario IV, asymptotic stability was not reached, the disturbances in speed and distance headway were amplified along the platoon;
3. In scenario IV, the final speed was lower compared to the other three scenarios and was also much lower than the critical speed; and
4. As in the local stability, some initial speed oscillations were found in all scenarios and they occurred within the range of 10 m/s to 14 m/s, suggesting again a non-smooth transition between the non-alert and alert conditions.

In scenarios I, II and III, the speeds are converged to 15 m/s which is above the critical speed v_c at 50 km/hr (or 13.9 m/s), whilst in scenario IV the final speed of 5m/s is lower than v_c. In the WLF model, travel speeds above v_c represents the non-congested state, whilst those below v_c represent the congested states. The above observation points 1 - 3 suggest therefore that asymtotic stability is reached in non-congested states, and not reached in congested state.

4. Analysis and Discussion

4.1 Speed Oscillations

One common feature revealed by the results in both local and asymptotic stability analysis is speed oscilattions. The results seemed to suggest that frequent transitions between the non-alert and alert states led to the observed speed oscillations in the range of 10m/s (36 km/hr) and 14m/s (~50km/hr) in the modelled traffic. We explain the conditions upon which this could have happened, show that one of the conditions for close-following was never met with the parameter values used, and suggest how the WLM model could be improved to overcome this problem.

To do so, we refer back to the different states of the model as depicted in Fig. 3. Let's assume that a vehicle is travelling at a speed above the critical speed v_c and it is therefore considered to be in a non-alert, non-congested state (in the region marked by O-A-B in Fig. 3). In the model, this vehicle would be modelled with a reaction time $\tau = \tau_2 = 1.4s.$ We further assume that a vehicle's desired headway equals to its current speed and its reaction time, as $h^{des} = v\tau.$

It follows, therefore, that if the current headway of a vehicle is less than its desired one**,** the vehicle would ***decelerate*** to increase the headway. If, when, the speed of the vehicle is reduced to below v_c, the vehicle would move into the alert state and would then immediately adopt a smaller reaction time $\tau = \tau_1 = 0.4s.$ The driver would now have a smaller desired headway ($h^{des} = v\tau 1 < v\tau_2$) *and* would then want to ***accelerate*** to reach its new, smaller desired headway. As a result, the vehicle could be in a constant acceleration or deceleration mode and traversing between the non-alert and the alert states, which results in speed oscillation.

We illustrate the above interpretation with a schematic drawing of the possible trajectories of the vehicle in a speed and distance-headway plane, shown in Fig. 8. On this plane, there is an angular area defined by two lines originated at the origin and with slopes defined by τ_1 and τ_2. They represent the corresponding desired headways for the alert and the non-alert state, with $h^{des} = 0.4v$ and $h^{des} = 1.4v$ respectively. A verticle line at $v = v_c = 14\ m/s$ marks the transition between the alert and the non-alert state.

We demonstrate here that within this angular area, a vehicles's speed would oscillating. Let's look at a vehicle which starts at an arbitury point A in the speed-headway plane of Fig. 8. As its state are: $v < v_c$ and $h > h^{des} = 0.4v$, the vehicle would want to accelerate in order to reduce its headway. So it follows a path with increasing speed and decreasing headway (from point A to B in Fig. 8). When its speed increases so that $v > v_c$, i.e. it reaches the non-alert state, its desired headway changes to $h^{des} = 1.4v$. Now, with a headway less than the h^{des}, and the vehicle would now have to decelerate in order to increase its headway.

This process of acceleration and deceleration repeats itself and results in an oscillatory (or zig-zag) pattern of the vehicle's speeds and headways, until it reaches a stable condition indicated by point B_1 where the vehicle's headway matches its desired headway when it is in the alert state.

For completion, we describe below what happens when a vehicle's speed-headway lies outside the above "oscillation" region.

In the area above the slope of $h^{des} = 1.4v$, the vehicle's headway is aways higher than its desired headway. For a vehicle at point C in Fig. 8 (i.e. in the alert state with $v < v_c$), its desired headway would be $h^{des} = 0.4v$. Whilst its desired headway would be $h^{des} = 1.4v$ if it were at point D (i.e. in the non-alaert state). Therefore, the vehicle would want to accelerate to reduce its headway to its desired headway. Such acceleration could lead them to reach a stable condition as shown by the solid arrows originated from points C and D in Fig. 8. It is also possible that the acceleration may lead the vehicle into the "oscillation" region, shown by the dashed arrow from point C, from there the above oscillatory behaviour would follow.

An opposite effect happens to vehicles in the area below the slope of $h^{des} = 0.4v$. Here, the vehicle's headway is aways lower than its desired headway. The vehicle would therefore want to decelerate to increase its headway, the result of which may lead them to a stable state as shown by the solid arrow originated from points E and F in Fig. 8, or into the "oscillation" region.

An example trajectory of a vehicle, in the speed oscillation region, is illustrated in Fig. 9. In this example, the vehicle is following a lead vehicle which is travelling at a constant speed. The following vehicle decelerates (or accelerates) at a constant rate when its headway is less (or greater) than its desired headway. Its desired headway is the product of its speed and reaction time, and the later swapps between 0.4s and 1.4s dependent on its speed. So the whole process is a circular one. This is illustrated in Fig. 9 that, in the speed oscillation region, the following vehicle's speed-headway distribution shows a spiral structure, whilst its spatio-temporal distribution shows a periodical structure in the oscillation region.

The above analysis illustrate that, whilst the multi-state WLM model employing different reaction times to represent the different driver-alertness in different traffic conditions which had elegantly explained the traffic hysterisis phenomenon, it had made vehicles in certain driving conditions constantly changing their desired headways which leads to speed oscillations.

While fixging the reaction times, Ez-Zahraouy et al [32] also found discontinuous velocity oscillation between two velocity values, for low vehicle extraction rate probability, in optimal velocity traffic flow models.

4.2 Close-Following Conditions

The above analysis led us to exam the boundary conditions of the different states in the WLM model. In the model formulations, the traffic from the non-congested, non-alert state (OA) is expected to transit firstly into the close-following state (area marked by ADEC in Fig. 3) before moving into the congested alert state of CJ. Since the close-following of vehicles at high speeds is a common feature of freeway traffic, we would not expect large amount of direct transitions between the non-congested state and congested alert states.

However, examining in detail the above simulation experiements, we found none of the vehicles simulated went into the close-following state at any time. Instead, they all moved directly from the non-congested non-alter state to the congested alert state. Detailed examinations revealed that using the calibrated parameter values [18, 26], the close-following conditions in the model could never be met.

The WLM model requires that, for a vehicle to be in the close-following state, all four of the following conditions need to be met:

(a) *Moving in non-alert traffic build-up state:* $v_n(t) \geq v_c$

(b) *The front vehicles are not braking apparently:* $b_{n-1}(t-\tau) > D_c$ *and* $b_{n-2}(t-\tau) > D_c$

(c) *A minimum and a maximum desired following distance:* $d_{min} < [x_n(t)-x_{n-1}(t)] < d_{max}$

(d) *A threshold for recornising a small negative (closing-in) and a small positive (moving-away) relative speed:* $\Delta V_a \leq [v_n(t)-v_{n-1}(t)] \leq \Delta V_b$

The first two conditions were adoped from the stability condition [35]. The last two were first suggested in [14] and their boundary conditions (d_{min}, d_{max}, ΔV_a, ΔV_b) were calibrated in [18].

We found that the conditions (a), (b) and (d) were met in the simulation. We illustrate below that condition (c) could not be met with the parameter values listed in Table 2. For condition (a) to be met, vehicles would be in the non-alert state and their car-following movements are described by eq. (1). Re-arranging eq. (1), we derive the desired distance headway as:

$$\Delta x_n = x_{n-1}(t) - x_n(t) = \frac{[v_n(t+\tau) - B_n\tau]^2 - (B_n\tau)^2}{-2B_n} + \frac{v_n(t)\tau}{2} + \frac{v_{n-1}^2(t)}{2B'} + L_{n-1} \tag{6}$$

We assume a steady-state traffic and simplify $v_n(t) = v_{n-1}(t) = v$, $L_{n-1} = L_n = L$, and adopt the calibrated parameter values for the non-alert state: $\tau = \tau_2 = 1.4$s, $a_2 = 1.7 m/s^2$, $b_2 = -2a_2 = -3.4 m/s^2$, $B' = -3 m/s^2$. The distance headway can be calculated from eq. (6), and the boundaries of condition (c) from eq. (3). The results are presented in Fig. 10(a). It can be seen that, the actual distance headway for speed $v > v_c$ (condition (a)) with reaction time $\tau = 1.4$s is always higher than the maximum desired headway d_{max}.

The distance headway with reaction time values 1.0s - 0.6s were also calculated and displayed in Fig. 10(b). The results show that, in order to meet the close-following condition (c), a smaller reaction times (in the region of 0.7s – 1.0s) would be required.

5. Conclusion

We presented in this paper a numerical analysis of the stability conditions of a car-following model developed for freeway traffic. Different scenarios were set-up, with different speed profiles of the lead vehicle and different initial conditions of the following vehicles, to represent a range of possible driving conditions. The movements of a platoon of seven vehicles were simulated. Local and asymptotic stabilities of the model were analysed from the following vehicles' speeds and distance headways distributions over time.

In all the scenarios simulated, local stability was observed. The critical speed (v_c) played an important role in determining asymptotic stability in that the stability can be achieved where the converged speeds were above v_c, not where they are well below v_c. In the model, traffic speeds above v_c represent the non-congested state, whilst those below v_c represent the congested states (the alert state during flow breakdow and non-alert state during recovery). This suggests that disturbances in a congested traffic conditions would be amplified, a sensible result.

Speed oscillations were found in speeds range between 10 – 14m/s; the higher end of this range conincides with the critical speed. Investigations into the simulated states of the individual vehicles suggest that the oscillations were caused by vehicles jumping between these two states, whilst none of them transit through the intermediate, close-following state of the model designed to represent the high-speed and close-keeping (short headway) behaviour of freeway traffic.

Detailed examination revealed that the close-following conditions of the model could not be met with the calibrated parameter values used in the study. With a reaction time of 1.4s calibrated for the non-congested state [26], the desired distance headway requirement of the close-following conditions [18] could not be met. We examined the effect of different reaction times. The results suggested that, with a smaller reaction time (in the region of 0.7 – 1.0s) for the non-congested state, the above close-following condition could be met. Further calibration of model parameter values and empirical observations of driver behaviour, especially during close-following conditions, are required to improve our modelling of the traffic conditions on high-speed freeway networks.

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