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Maximum entropy model for business cycle synchronization

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Abstract

The global economy is a complex dynamical system, whose cyclical fluctuations can mainly be characterized by simultaneous recessions or expansions of major economies. Thus, the researches on the synchronization phenomenon are key to understanding and controlling the dynamics of the global economy. Based on a pairwise maximum entropy model, we analyze the business cycle synchronization of the G7 economic system. We obtain a pairwise-interaction network, which exhibits certain clustering structure and accounts for 45% of the entire structure of the interactions within the G7 system. We also find that the pairwise interactions become increasingly inadequate in capturing the synchronization as the size of economic system grows. Thus, higher-order interactions must be taken into account when investigating behaviors of large economic systems.

Keywords: Maximum entropy, Business cycle synchronization, Ising model, Interaction network
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1. Introduction

Since the sub-prime mortgage crisis of the United States erupted, all major economies in the world have been inflicted with a severe financial crisis. Indeed, the global economy has experienced the worst recession since the Great Depression of the 1930s. This has in turn prompted an increase of academic interest in global business cycle [1, 2].

Global business cycle can be characterized by simultaneous recessions or expansions of major economies; such dynamical similarity along business cycles is also called business cycle synchronization in the economics literature [3]. And there is quite an extensive literature in this research area. Frankel and Rose presented empirical evidence that higher bilateral trade between two economies is associated with more correlated business cycles [4]. Imbs stressed the linkage between similarity in industrial structure and business cycle synchronization in her paper [5]. Rose and Engel discussed the role of currency unions in business cycle synchronization by empirical analysis [6]. While these researches identified the factors that affect the degree of synchronization between economies, they did not, however, address the synchronization of the overall economic system.

The key to understanding the mechanism of synchronization is to uncover the interaction structure among economies [3]. The most common way of estimating network structure of complex system is to characterize the connection between elements by means of correlation coefficients. However, recent researches have shown that such characterization does not accurately estimate network structure due to significant indirect correlations [7, 8]. We argue that a more effective and informative approach is to derive the network of interaction based on the principle of maximum entropy.

The principle of maximum entropy as an inferential tool was originally introduced in statistical physics by Jaynes [9, 10, 11] and was further developed by other physicists afterwards [12, 13, 14, 15]. Generally, observed signals of any given system are governed by, and therefore are manifestation of, the underlying structure of the system. The principle of maximum entropy provides a simple way by which we can infer the system's least-biased structure capable of generating these signals. Compared with correlation coefficient, the approach succeeds in inferring interactions, from which it reconstructs correlations at all orders, and thus can estimate network structure more accurately [7, 8]. Due to its universality, the approach has been successfully applied to researches in ecology [16, 17, 18, 19], life sciences [20, 21, 22, 23], and neuroscience [7, 24, 25], among other disciplines. In particular, it has been shown that only pairwise interactions are sufficient to describe such complex systems as tropical forests [17], proteins [23], and retinas [7]. In this paper, we apply the principle of maximum entropy, built on pairwise interactions, to the business cycle synchronization of the seven most-developed economies in the world, known as G7.

2. Data

The data in this study are taken from the database OECD.Stat, where quarterly real GDPs of every member of OECD are available. The GDPs are calculated in terms of US dollars, adjusted by fixed PPPs (Purchasing Power Parity). The time period with available date for most countries is from 1960's first quarter to 2009's first quarter (amounting to 197 quarters). The total number of data points of all members is 5,190 observations.

In order to apply a pairwise maximum entropy model, the data need to be converted into a binary representation—recession or expansion, in this case. To this end, we first calculate the average growth rate for each economy. Suppose the available data of GDP for an economy last over N quarters, and the growth rate in the ith quarter is r_i, the average growth rate $\overline{r}$ can be obtained from the following relation:

$$\prod_{i=1}^{N-1}(1+r_i) = (1+\overline{r})^{N-1}. \tag{1}$$

We then define recession and expansion: if growth rate is less than the average growth rate, we define the state as recession and set the value of state variable to 1; otherwise, we define the state as expansion and set the value of state variable to 0.

The size of the system under consideration is limited by the data availability: in order to obtain reliable estimates of the parameters, the number of all possible states of the system should be well below the number of observations, i.e., $2^N < 197$. The G7 economic system is a small, yet meaningful, sub-system of the global economy. Its synchronous behavior can influence the business cycle of the global economy. As such, it is an excellent case study for our approach.

3. Principle of maximum entropy

The first step in the analysis with the principle of maximum entropy is to determine some meaningful constraints that describe the observed signals generated by the system. We then determine the least-structured distribution subject to those constraints. It is possible to prove that the Shannon entropy is the correct measure of the structure whose maximization, under a given set of constraints, would lead to the least-structured distribution [11].

Consider an economic system consisting of N economies. We build a binary representation of the economic state by assigning a binary variable σ_i to economy i: $\sigma_i = 1$ if economy i is in a recession, and $\sigma_i = 0$ if the economy is in an expansion. Then a state for the whole economic system can be denoted by a vector $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_N)$. Our goal is to calculate the probability distribution $p(\boldsymbol{\sigma})$ that maximizes Shannon entropy

$$H = -\sum_{\boldsymbol{\sigma}} p(\boldsymbol{\sigma}) \ln p(\boldsymbol{\sigma}) \tag{2}$$

with the following constrains:

$$\sum_{\boldsymbol{\sigma}} p(\boldsymbol{\sigma}) = 1, \tag{3a}$$

$$\langle \sigma_i \rangle = \sum_{\boldsymbol{\sigma}} p(\boldsymbol{\sigma}) \sigma_i = \frac{1}{T} \sum_{t=1}^{T} \sigma_i^t, \tag{3b}$$

$$\langle \sigma_i \sigma_j \rangle = \sum_{\boldsymbol{\sigma}} p(\boldsymbol{\sigma}) \sigma_i \sigma_j = \frac{1}{T} \sum_{t=1}^{T} \sigma_i^t \sigma_j^t, \tag{3c}$$

where σ_i^t denotes the state of economy i at time t and T the total number of observations. The probability distribution that satisfies the above conditions is in the following form:

$$p(\boldsymbol{\sigma}) = \frac{1}{Z} \exp\left[\frac{1}{2} \sum_{i \neq j} J_{ij} \sigma_i \sigma_j + \sum_i h_i \sigma_i\right], \tag{4}$$

where Z is the partition function or normalization constant, and J_{ij} and h_i are the adjustable parameters to meet the constraints. For a non-interacting system, the probability distribution would factorize into independent single-economy probability distributions. Any deviation from a simple product of independent probability distributions is a measure of the interactions among economies. Thus, J_{ij} can naturally be defined as the interaction strength between economies i and j: a positive J_{ij} favors simultaneous recessions or expansions of economies i and j. Similarly, h_i quantifies an economy's propensity to recession: an economy with a positive h_i is more prone to recession. Eq. (4) is known in the physics literature as Ising model with J_{ij} being interpreted as the coupling between electron spins. **(Chot, why do you say that "a positive J_{ij} favors simultaneous recessions or expansions"? If $\sigma_i = 0$, i.e. expansion, J_{ij} has no influence. In order for J_{ij} to be important, both σ_i and σ_j have to be different from zero. So I'd simply say "a positive J_{ij} favors synchronized recession between economies")**

The maximum entropy model we have introduced above approximates the economic system as a pairwise (undirected) interacting network at stationarity, where the pairwise interactions J_{ij} measure the strength of interaction between economies i and j and they do not depend on time. We call this kind of model a pairwise maximum entropy model. In fact, such a maximum entropy model can easily be extended to incorporate high-order interactions. For example, adding triplet correlations $\langle \sigma_i \sigma_j \sigma_k \rangle$ into the constraints, we can obtain a measure J_{ijk} of triplet interaction. However, the complexity of the algorithm for parameter estimation grows exponentially with the increase in the order. Importantly, pairwise maximum entropy models have been shown to effectively capture much of the underlying structure of a number of other complex systems. Indeed, one of our research questions is whether such a property holds for economic systems.

Here, the model is implemented by means of the algorithm proposed by Dudík and coauthors [26]. Based on the first two moments of the binary representation of the G7 system data, we find h's and J's in Eq. (4). The algorithm incorporates l_1-regularization to avoid the problem of over-fitting. Since system size is sufficiently small in this case, we perform calculations involving all 2^7 possible states of the system (as opposed to Monte Carlo simulations). We terminate the algorithm when the parameter adjustment becomes very small (e.g., in the order of 10^{-5}).

4. Results and discussion

The estimates of J's characterize pairwise interaction network of the G7 system (Fig. 1). The results suggest that the network can be roughly divided

Figure 1: Pairwise interaction network of the G7 system. The red and blue indicate negative and positive strengths of pairwise interactions, respectively. Thick, short lines correspond to strong positive J, whereas thin, long lines correspond to weak or negative J.

Figure 2: Embeddedness and h of each G7 economy.

into three clusters: (i) Continental Europe and Japan, (ii) North America, and (iii) UK. The clustering structure is obviously associated with the region. Indeed, this structure is in general agreement with existing economic studies that employ different analytical methods. For example, *Monfort et al.* [27], by means of Kalman filtering techniques and a dynamic factor model, found that area-specific common factors separate the G7 system into Continental-European and North-American areas, with the UK and Japan being somewhat separate from these areas. Other studies [28, 29] also showed fairly clear evidence of the European and North American cycles. These agreements, to some extent, confirm the validity of applying the Ising model to economic synchronization problems.

While all these countries constitute the G7 economic network, the degrees at which they are embedded or integrated into this network vary. We propose that such "embeddedness" of economy i be measured by $E_i = \sum_{j\neq i} |J_{ij}| / \sum_k \sum_{j\neq i} |J_{kj}|$. The metric measures the magnitude of interaction between a given economy and others—in both synchronous and anti-synchronous ways. Therefore, it offers different, but complementary, information from that in Fig. 1. The results in Fig. 2 show that France is the most embedded/integrated economy, and UK is the least integrated one. Interestingly, the pattern of embeddedness is almost a mirror image of the pattern of h (Fig. 2); recall that h measures how prone to recession an economy is. Together, these patterns indicate that economies with a greater tendency to grow tend to be the same ones as those with greater embeddedness—no attempt on implying any causality is made here. Finally, it is also worth-noting that all h's are negative, i.e., all G7 economies in fact have a tendency to grow.

To examine how effectively the pairwise maximum entropy model reproduces the empirical statistics of the G7 system, we make comparisons be-

Figure 3: The frequencies of different states of G7 system predicted by pairwise maximum entropy model are plotted against the empirical frequencies. The dashed line shows equality.

tween the empirical and predicted frequencies of different states. The results are shown in Fig. 3. We see that the predicted frequencies based on the pairwise maximum entropy model are tightly correlated with the empirical ones. The results indicate that business cycles are significantly correlated with each other within the G7, which is coherent with what economists expected [29, 30], and that the pairwise maximum entropy model captures key characteristics of business cycle synchronization of the G7 system moderately well.

To systematically quantify the model's performance, we adopt the following information-theoretic metric proposed by Schneidman and coauthers [7, 31]. For a system of N economies, we can define the maximum entropy distributions p_K that are consistent with all K^{th}-order constraints for any $K = 1, 2, \ldots, N$. These distributions form a hierarchy, from $K = 1$ where all economies are independent, up to $K = N$, which is exactly the empirical distribution. The entropy difference or multi-information $I_{(N)} = H_1 - H_N$ measures the total amount of interactions in the system. Likewise, $I_{(K)} = H_{K-1} - H_K$ quantifies the amount of the K^{th}-order interactions. Evidently, $I_{(N)} = \sum_{K=2}^{N} I_{(K)}$. Thus, the ratio $I = I_{(2)}/I_{(N)}$ can be used to measure the contribution of pairwise interactions to the overall interactions. We find that $I \simeq 45\%$ for the G7 system. This means that 45% of the entire structure of G7 system can be characterized by pairwise interactions.

The G7 system investigated here is only a subnetwork embedded in a larger global economy network. We wonder whether the estimates of J's are sensitive to incorporating other economies into the network. To investigate this, we include the three next biggest OECD economies, namely Spain, Netherlands and Belgium, and re-estimate J's for the G7 economies. The results are presented in Fig. 4. There are no significant, systematic changes in J's as the system size grows. Thus, we claim that the pairwise maximum entropy model gives reliable estimates of pairwise interactions between the G7 economies.

This issue of network size warrants further investigation. It should be noted that a system with more economies may have richer structure and

Figure 4: The strengths of pairwise interactions from the 10-economy system, which includes G7 as its subsystem, are plotted against those from G7 system. The black line shows equality.

Figure 5: The contribution of pairwise interactions is plotted against the size of economic systems. A box plot shows minimum, lower quartile, median (red line), upper quartile, and maximum, as well as some outliers (red pluses); black stars represent the mean values. The contribution of pairwise interactions declines with the size of economic systems.

therefore possibly larger proportion of high-order interactions. To test this hypothesis, we randomly select N economies from OECD to construct an economic network ($N = 3, \ldots, 10$) and calculate their corresponding I's—the contribution of pairwise interactions to the overall interactions. For each N, we repeat the procedure 150 times. The results are shown in Fig. 5. The average I declines as the system size increases. It is 0.61 in a three-economy system, which indicates that pairwise interaction is the leading factor shaping business cycle synchronization. By contrast, when the system size approaches to 10, it drops to 0.20. At these system sizes, higher-order interactions dominate over pairwise ones, playing more important roles in dictating the behavior of economic system. These results suggest that higher-order interactions are more important in economic systems than in neurons [7] or ecosystems [18], for which pairwise interactions capture most of the structure. This implies that higher-order interactions are necessary to adequately understand economic systems, indicating their greater degree of complexity compared to other, natural systems.

5. Conclusions

In this paper, we investigate business cycle synchronization of the G7 system by means of a pairwise maximum entropy model. We find some clustering structure in the interaction network between the G7 countries, which more or less follows their geographical locations. We also find that France is the most embedded economy, while the UK is the least so. The pairwise interactions account for 45% of the entire structure of the G7 system; this number, although significant, is much lower than its counterparts in other systems like neurons or forests. Indeed, our further analysis shows that the larger system size is, the more important the contribution of higher-order interactions becomes. This has important implications on future studies of interacting economies: if one wants to investigate the behavior of business cycle synchronization of a large economic system, higher-order interactions must be taken into account.

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