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Identification of the Hammerstein Model Using Multiresolution Wavelets

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A new approach is introduced for identifying the Hammerstein model using multiresolution wavelet decompositions. Under some mild assumptions, the linear dynamic part of the Hammerstein model can be identified separately from the static nonlinearity. The static nonlinearity can then be identified and recovered using multiresolution B-spline wavelet decompositions. The main advantages of the new method is that now the static nonlinearity can be an arbitrary function which is ether continuous or discontinuous. Simulation results are included to demonstrate the effectiveness of the new approach.

Keywords: B-spline wavelets; Hammerstein model; identification ; multi-resolution analysis.

1. Introduction

The identification of the Hammerstein model, which consists of a static nonlinearity followed by a linear dynamic system in Figure 1, has been widely studied by several authors. Most of the early work [Narenda and Gallman,1966; Chang and Luus,1971; Thatachar and Ramaswamy, 1973; Gallman,1975,1976; Billings and Fakhouri, 1979; Stoica and Soderstrom, 1982] was mainly concerned with parametric identification methods which assumed that under certain input excitations the unknown static nonlinearity could be approximated using a polynomial with a finite known order. Clearly, if the nonlinearity is not a polynomial and the input is non-Gaussian, many of these algorithms may not converge [Gallman,1975]. Most of the nonparametric algorithms [Greblicki and Pawlak, 1986, 1987, 1989; Hwang and Shyu, 1988; Greblicki, 1989; Krzyzak,1989] are based on kernel regression or infinite series expansions. Although relatively little a priori knowledge about the Hammerstein model is required for these nonparametric approaches, which can generally achieve estimates which converge to the true nonlinear characteristics quite well, the resulting representations are often rather complicated and involve high dimensional parameter estimation problems[Al-Duwaish et al., 1997].

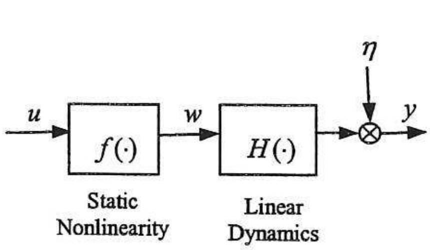

Figure 1 The Hammerstein model

To overcome the drawbacks of the early parametric approaches, some new algorithms for identifying the Hammerstein model have recently been proposed, see, for example, Al-Duwaish et al.[1997], Voros [1997], Li [1999], Zhu [2000], and Giri et al. [2001]. In these algorithms, either the static nonlinearity is assumed to be described using a continuous function, or prior knowledge of the intrinsic properties of the static nonlinearity have to be known beforehand. The application of nonparametric polynomial approaches [Greblicki and Pawlak,1994; Lang, 1997] make the representation of the estimates relatively simple and more applicable, but these methods cannot be used to effectively identify nonlinearities with intrinsic discontinuities. The introduction of multiresolution analysis to nonparametric identification for the Hammerstein model [Pawlak and Hasiewicz1998] provides a more flexible and more accurate representation for describing a static nonlinearity.

In the present study, a new method is introduced for identifying the Hammerstein model based on multiresolution B-spline wavelet decompositions. With some mild restrictions imposed on the static nonlinearity, the identification of the linear dynamic part of the Hammerstein model can be separated from the identification of the static nonlinearity. This enables the identification of the static nonlinearity and the linear dynamics to be independent, but with the advantage that the static nonlinearity can now be an arbitrary function, which is continuous or discontinuous.

2. Multiresolution Wavelet Decompositions

Wavelet decompositions outperform many other approximation schemes and offer a flexible capability for approximating arbitrary functions, even those with sharp discontinuities. Wavelet basis functions have the property of localization in both time and frequency. Due to this inherent property, wavelet approximations provide the foundation for representing arbitrary functions economically using only a small number of basis functions. It can be shown that the intrinsic nonlinear dynamics related to real nonlinear systems can easily be captured by an appropriately fitted wavelet model with a small number of wavelet basis functions.

Under some assumptions, an orthogonal wavelet system can be constructed using *multiresolution analysis* (MRA) [Mallat, 1989; Chui, 1992; Daubechies, 1992]. Assume that the wavelet φ and associated scaling function ϕ constitute an orthogonal wavelet system, then any function $f \in L^2(R)$ can be expressed as a *multiresolution wavelet decomposition*

$$f(x) = \sum_k \alpha_{j_0,k}\phi_{j_0,k}(x) + \sum_{j \geq j_0}\sum_k \beta_{j,k}\varphi_{j,k}(x) \tag{1}$$

where $\varphi_{j,k}(x) = 2^{j/2}\varphi(2^j x - k)$ and $\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k)$, $j,k \in Z$ are the scale and translation parameters, and j_0 is an arbitrary integer representing the lowest resolution or scaling level.

Although many functions can be chosen as scaling and/or wavelet functions, most of these are not suitable in system identification applications, especially in the case of multidimensional and multiresolution expansions. An implementation, which has been tested with very good results, involves B-spline and B-wavelet functions in multiresolution wavelet decompositions [Billings and Coca, 1999; Liu 2000; Coca and Billings, 2001; Wei and Billings, 2002].

B-splines are piece-wise polynomial functions with good local properties, and were originally introduced by Chui and Wang [1992] to define a class of semi-orthogonal wavelets for representing a signal using multiroselution decompositions. The *m*th order B-spline function is defined as

$$N_m(x) = \frac{1}{(m-1)!}\sum_{j=0}^{m} C_j^m (-1)^j (x-j)_+^{m-1}, \; m \geq 2 \tag{2}$$

where $C_k^m = m(m-1)\cdots(m-k+1)/k!$, and $x_+^n = x^n$ for $x \geq 0$ and $x_+^n = 0$ for $x < 0$. The mth order B-spline N_m can be calculated by the following recursive formula:

$$N_m(x) = \frac{x}{m-1} N_{m-1}(x) + \frac{m-x}{m-1} N_{m-1}(x-1),\ m \geq 2 \tag{3}$$

with

$$N_1(x) = \chi_{[0,1)}(x) = \begin{cases} 1 & \text{if } x \in [0,1) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Setting N_m as the scaling function, that is, $\phi(x) = N_m(x)$, then both the scaling function and the associated wavelet can be expressed in terms of the scaling function $N_m(x)$ as follows

$$\phi(x) = \sum_{k=0}^{m} c_k N_m(2x-k) \tag{5}$$

$$\varphi(x) = \sum_{k=0}^{3m-2} d_k N_m(2x-k) \tag{6}$$

with the coefficients given by

$$c_k = \frac{1}{2^{m-1}} C_k^m \tag{7}$$

$$d_k = \frac{(-1)^k}{2^{m-1}} \sum_{j=0}^{m} C_j^m N_{2m}(k-j+1),\ k = 0,1,\cdots,3m-2 \tag{8}$$

Clearly, the support of the m th order B-spline wavelet and the associated scaling function are

$$\begin{cases} \text{supp } \phi = \text{supp} N_m = [0, m] \\ \text{supp } \varphi = [0, 2m-1] \end{cases} \tag{9}$$

Both the B-spline wavelets and the associated scaling functions are symmetric or anti-symmetric within the support. The most commonly used B-splines are those of orders 1 to 4, which can be explicitly expressed as Table 1.

Table 1 The B-splines of order 1 to 4

	$N_1(x)$	$N_2(x)$	$2N_3(x)$	$6N_4(x)$
$0 \leq x < 1$	1	x	x^2	x^3
$1 \leq x < 2$	0	$2-x$	$-2x^2+6x-3$	$-3x^3+12x^2-12x+4$
$2 \leq x < 3$	0	0	$(x-3)^2$	$3x^3-24x^2+60x-44$
$3 \leq x < 4$	0	0	0	$-x^3+12x^2-48x+64$
elsewhere	0	0	0	0

3. Identification of the Hammerstein Model

The new identification approach for the Hammerstein model will be derived using multiresolution B- spline wavelet decompositions to describe the static nonlinearity. The following assumptions on the static nonlinearity will be considered:

The static nonlinear function f in Figure 1 is bounded and saturates outside an interval, so that f can be described as

$$w = f(u) = \begin{cases} \underline{w} & u \le a \\ g(u) & a \le u \le b \\ \overline{w} & b \le u \end{cases} \tag{10}$$

where a, b, $\underline{w} = \lambda_1 a$ and $\overline{w} = \lambda_2 b$ are known constants, $a \le 0 \le b$ and $\underline{w} \le \overline{w}$, $g(\cdot)$ is an unknown nonlinear function which is to be identified and might be ether continuous or discontinuous.

3.1 Separating the linear dynamics from the static nonlinearity

The idea of separating the identification of the linear dynamics and the static nonlinearity is direct and simple under the assumption (10) above. From Figure 1, the output the Hammerstein model be expressed as

$$y(t) = \frac{B(z^{-1})}{A(z^{-1})} w(t) + \frac{1}{A(z^{-1})} \xi(t) = \frac{B(z^{-1})}{A(z^{-1})} f(u(t)) + \frac{1}{A(z^{-1})} \xi(t) \tag{11}$$

where $H(z^{-1}) = \dfrac{B(z^{-1})}{A(z^{-1})}$ is the transfer function of the linear dynamic part, and $\xi(t) = A(z^{-1})\eta(t)$ represents the noise. From the assumption (10), if the input signal $u(t) \le a$ or $u(t) \ge b$, then the internal variable $w(t) = \underline{w}$ or $w(t) = \overline{w}$. Setting $u(t)$ to be a two-step signal with one-step value, say, of $\lambda_0 a$ ($\lambda_0 \ge 1$), and another value, say, of $\lambda_0 b$, then the Hammerstein model becomes

$$y(t) = \lambda \frac{B(z^{-1})}{A(z^{-1})} u(t) + \frac{1}{A(z^{-1})} \xi(t) \tag{12}$$

where $\lambda = \lambda_1 / \lambda_0$ for $u(t) \le a$ and $\lambda = \lambda_2 / \lambda_0$ for $u(t) \ge b$. Using existing linear identification techniques, the linear dynamic model (12) can easily be identified based on the input-output measurements by choosing the input signal $u(t)$ as follows: $u(t)$ is a two-level signal, for example, an offset PRBS, where one amplitude is greater than b, and the other amplitude is less than a.

3.2 Identification of the Static nonlinearity using multiresolution wavelet decompositions

Assume that the linear dynamic model was identified as $H(z^{-1}) = B(z^{-1}) / A(z^{-1})$, where

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_p z^{-p} \tag{13a}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_q z^{-q}, \ p \ge q \tag{13b}$$

Then form wavelet theory [Chui, 1992; Daubechies, 1992], the static nonlinearity $g(\cdot)$ can be represented using a multiresolution wavelet decomposition as

$$g(u)=\sum_{k\in A_0}\alpha_{j_0,k}\phi_{j_0,k}(u)+\sum_{j\geq j_0}\sum_{k\in B_j}\beta_{j,k}\varphi_{j,k}(u)\ ,\ u\in[a,b] \tag{14}$$

where $\phi_{j,k}(x)=2^{j/2}\phi(2^j x-k)$ and $\varphi_{j,k}(x)=2^{j/2}\varphi(2^j x-k)$ are the *m*th order B-spline wavelet and the associated scaling functions, with support [0,*m*] and [0,2*m*-1], respectively. In theory, the decomposition (14) consists of an infinite number of basis functions. In practice, however, the decomposition (14) is usually truncated at an appropriate resolution scale level *J*. The index sets K_0 and K^j, which depend not only on the index j but also the interval [*a*, *b*], can be chosen as $A_0=\{k:2^{j_0}a-m\leq k\leq 2^{j_0}b-1,k\in Z\}$, and $B_j=\{k:2^j a-2m+2\leq k\leq 2^j b-1,k\in Z,j\in Z\}$ for $j_0\leq j\leq J$.

From (10), (11) and (14), for any arbitrary signal $u\in[a,b]$

$$\begin{aligned}
&y(t)+a_1y(t-1)+\cdots+a_py(t-p)\\
&\quad=b_0g(u(t))+b_1g(u(t-1))+\cdots+b_qg(u(t-q))+\xi(t)\\
&\quad=\sum_{k\in A_0}b_0\alpha_{j_0,k}\phi_{j_0,k}(u(t))+\sum_{j\geq j_0}\sum_{k\in B_j}b_0\beta_{j,k}\varphi_{j,k}(u(t))\\
&\quad+\sum_{k\in A_0}b_1\alpha_{j_0,k}\phi_{j_0,k}(u(t-1))+\sum_{j\geq j_0}\sum_{k\in B_j}b_1\beta_{j,k}\varphi_{j,k}(u(t-1))+\cdots\\
&\quad+\sum_{k\in A_0}b_q\alpha_{j_0,k}\phi_{j_0,k}(u(t-q))+\sum_{j\geq j_0}\sum_{k\in B_j}b_q\beta_{j,k}\varphi_{j,k}(u(t-q))+\xi(t)
\end{aligned} \tag{15}$$

Eq. (15) can be rearranged and transferred into a linear-in-the-parameter regression form with respect to the unknown wavelet coefficients $\alpha_{j_0,k}$ and $\beta_{j,k}$. Assume that *M* bases (the dilated and translated mother wavelet and/or scaling functions or their combinations) are involved in the model (15), and for convenience of representation also assume that the *M* wavelet bases are ordered according to a single index *m* to form a wavelet dictionary $D=\{p_m(t)\}_{m=1}^{M}$, then (15) can be expressed as the linear-in-the-parameters form

$$z(t)=\sum_{m=1}^{M}\theta_m p_m(t)+\xi(t) \tag{16}$$

This can be solved using linear regression techniques. Note that for a high resolution scale *J*, the model (16) might involve a great number of model terms or regressors. Experience shows that very often many of the model terms are redundant and therefore are insignificant to the system output and can be removed from the model. An efficient orthogonal least squares (OLS) algorithm and an error reduction ratio (ERR) criterion [Billings et al., 1988, 1989; Korenberg et al., 1988; Chen et al., 1989] can be used to determine which terms should be included in the model.

4. Simulation Studies

Two examples, one with a continuous and the second with a discontinuous static nonlinearity, will be used to illustrate the application of the new identification procedure.

4.1 Example 1

Consider the following Hammerstein model

$$H(z^{-1}) = \frac{0.25z^{-1}}{1-1.8z^{-1}+0.825z^{-2}} \tag{17}$$

$$w(t) = f(u(t)) = \begin{cases} 0 & 0 < |u(t))| < 1 \\ \mathrm{sgn}(u(t)) & 1 \le |u(t))| \le 2 \\ 0.5|u(t)|\,\mathrm{sgn}(u(t)) & 2 \le |u(t))| \le 4 \\ 2\,\mathrm{sgn}(u(t)) & 4 \le |u(t))| \end{cases} \tag{18}$$

where *sgn*(u) is a function whose value is defined to be 1 for u>0, -1 for u<0, and 0 for u=0. The noise $\xi(t)$ was assumed to be a Gaussian white noise with a standard deviation σ_ξ=0.1.

The system was initially simulated using a PRBS input with one amplitude of ± 4. Figure 2 shows the first 200 points of this input. Based on 500 input-output data points and using the forward regression OLS-ERR algorithm[Billings et al., 1988, 1989; Korenberg et al., 1988; Chen et al., 1989], the liner dynamic model was identified to be

$$\hat{H}(z^{-1}) = \frac{0.2504z^{-1}}{1.0201-1.7886z^{-1}+0.8223z^{-2}} \tag{19}$$

This model was then used to identify the static nonlinearity in the next step, where a random input sequence $u(t)$ uniformly distributed in the interval [-5, 5] was used as the system input. Altogether 1000 input-output data points were used to identify the static nonlinearity. The Haar wavelet and scaling functions (the first order B-spline wavelet and scaling functions) were used to describe the static nonlinearity with the initial and the truncated high resolution scale levels j_0=0 and J=3, respectively. The Haar scaling and mother wavelet functions are respectively defined as follows

$$\phi(x) = \begin{cases} 1 & \text{for } 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

$$\varphi(x) = \begin{cases} 1 & 0 \le x < 1/2 \\ -1 & 1/2 \le x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

The static nonlinearity can be described as

$$g(u(t)) = \sum_{k=-4}^{4} \alpha_{0,k}\phi_{0,k}(u(t)) + \sum_{j=0}^{3}\sum_{k=-2^{j+2}}^{2^{j+2}} \beta_{j,k}\varphi_{j,k}(u(t)) \ , \ u(t) \in [-4,4] \tag{22}$$

Finally from Eq. (15), the Hammerstein model can be described as

$$\begin{aligned} z(t) &= 1.0201y(t) - 1.7886y(t-1) + 0.8223y(t-2) \\ &= 0.2504g(u(t-1)) + \xi(t) \\ &= \sum_{k=-4}^{4} 0.2504\alpha_{0,k}\phi_{0,k}(u(t-1)) + \sum_{j=0}^{3}\sum_{k=-2^{i+2}}^{2^{j+2}} 0.2504\beta_{j,k}\varphi_{j,k}(u(t-1)) + \xi(t) \end{aligned} \tag{23}$$

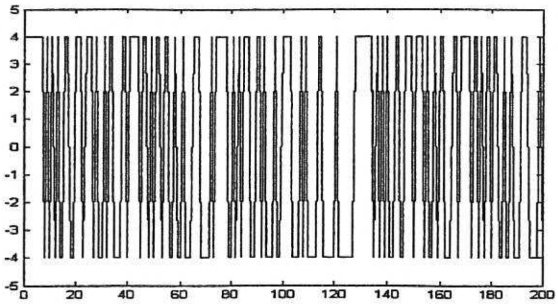

Figure 2 The PRBS used as the system input to identify the linear dynamics for Example 1

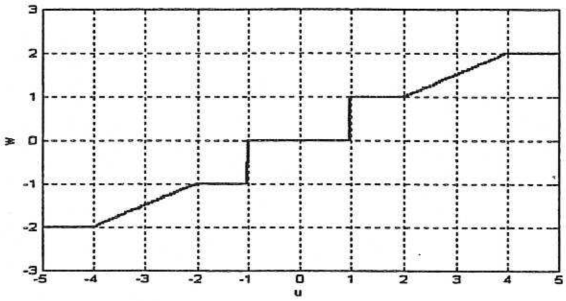

Figure 3 The static nonlinearity recovered from the identified multiresolution Haar wavelet model for Example 1

Note the model (23) contains 133 wavelet basis functions(candidate model terms), and 67 significant model terms (wavelet basis functions) were selected using the OLS-ERR algorithm[Billings et al., 1988, 1989; Korenberg et al., 1988; Chen et al., 1989] with a cutoff value of ρ=0.0005 for algorithm termination. This cutoff value also defines an approximate accuracy for the representation. Ordering the 67 selected significant wavelet basis functions using a single index *m*, the identified model for the static nonlinearity can be expressed as

$$g(u(t)) = \sum_{m=1}^{67} \theta_m B_m(u(t)) \tag{24}$$

where $B_m(\cdot)$ (m=1,2,...,67) indicates the dilated and translated Haar wavelet/scaling basis functions.

By setting the input signal as a ramp function, that is, $u(t)=t$, the static nonlinearity can be recovered from the identified model (24) and this is shown in Figure 3, which clearly indicates that the static nonlinearity was accurately identified.

4.2 Example 2

Consider a Hammerstein system where the linear dynamics were the same as in Example 1, but the static nonlinearity was described by the following continuous function

$$w(t) = f(u(t)) = \begin{cases} \dfrac{1-\exp(-u^3(t))}{1+\exp(-u^3(t))} & -2 \le u(t) \le 2 \\ -\operatorname{sgn}(u(t) & 2 < |u(t))| \end{cases} \tag{25}$$

The linear dynamics were identified in the same way as in Example 1 and the resulted model is the same as model (19). To identify the static nonlinearity, a random sequence $u(t)$ uniformly distributed in the interval [-3,3] was used as the system input. A total of 600 input-output data points were used for identification. The fourth order B-spline wavelet and scaling functions were used to describe the static nonlinearity with the initial and the truncated high resolution scale levels $j_0=0$ and $J=2$, respectively. The static nonlinearity can be described as

$$g(u(t)) = \sum_{k=-5}^{1}\alpha_{0,k}\phi_{0,k}(u(t)) + \sum_{j=0}^{2}\sum_{k=-2^{j+1}-6}^{2^{j+1}-1}\beta_{j,k}\varphi_{j,k}(u(t)) \ , \ u(t)\in[-2,2] \tag{26}$$

From Eq. (15), the Hammerstein model can be described as

$$\begin{aligned} z(t) &= 1.0201y(t) - 1.7886y(t-1) + 0.8223y(t-2) \\ &= 0.2504g(u(t-1)) + \xi(t) \\ &= \sum_{k=-5}^{1}0.2504\alpha_{0,k}\phi_{0,k}(u(t-1)) + \sum_{j=0}^{2}\sum_{k=-2^{j+2}-6}^{2^{j+2}-1}0.2504\beta_{j,k}\varphi_{j,k}(u(t-1)) + \xi(t) \end{aligned} \tag{27}$$

Although altogether 81 wavelet basis functions(candidate model terms) were involved in the initial model (27), only 12 significant model terms (wavelet basis functions) were selected using the OLS-ERR algorithm[Billings et al., 1988, 1989; Korenberg et al., 1988; Chen et al., 1989] with a cutoff value of $\rho=10^{-4}$ for the algorithm termination. This cutoff value also defines an approximate accuracy for the representation. The final identified model for the static nonlinearity was found to be

$$\begin{aligned} g(u) = {} & 1.69771469\,\phi_{0,0}(u) \ +0.18352418\,\phi_{0,-1}(u) \ -0.18724256\,\phi_{0,-3}(u) \\ & -1.69902113\,\phi_{0,-4}(u) \ +1.59785418\,\varphi_{0,0}(u) \ -0.95245890\,\varphi_{0,-1}(u) \\ & +0.28092308\,\varphi_{0,-3}(u) \ -0.27990980\,\varphi_{0,-4}(u) \ +0.96395753\,\varphi_{0,-6}(u) \\ & -1.70618033\,\varphi_{0,-7}(u) \ +0.0320097\,\varphi_{1,0}(u) \ +0.02370121\,\varphi_{1,-6}(u) \end{aligned} \tag{28}$$

where $\varphi_{j,k}(x)=2^{j/2}\phi(2^j x-k)$ and $\phi_{j,k}(x)=2^{j/2}\phi(2^j x-k)$ are the 4th-order B-spline wavelet and scaling functions. The recovered static nonlinearity from the identified model (28) is shown in Figure 4, which clearly indicates that the identified model is excellent.

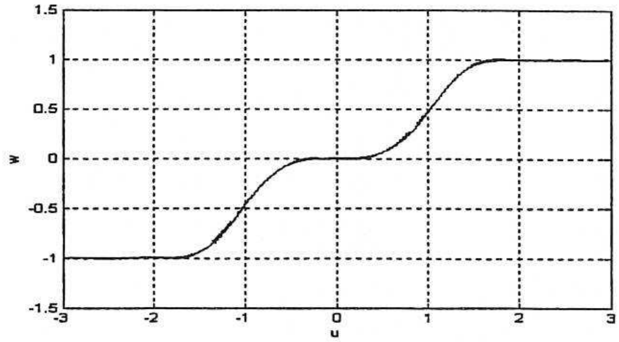

Figure 4 Comparison of the recovered static nonlinearity from the identified multiresolution B-spline wavelet model with the actual static nonlinearity for Example 2. (Solid: the actual; Dashed: the identified).

In the two examples given in sections 4.1 and 4.2, the static nonlinearity function were both anti-symmetric about the origin. The anti-symmetric property of the nonlinear element is not required by the new identification algorithm. A non- symmetric static nonlinearity could also be successfully identified using the new wavelet-based identification approach.

5. Conclusions

A new approach for identifying the Hammerstein model has been proposed using multiresolution wavelet decompositions. The static nonlinearity and the linear dynamic subsystem can be identified separately under some mild constrains, where the static nonlinearity is assumed to be bounded and to saturate outside a known interval. No a priori knowledge is assumed for the static nonlinearity inside the interval. The main advantage of the new approach over existing methods is that, the static nonlinearity can be an arbitrary function, which can contain jumps or discontinuities. It was shown that multiresolution wavelet decompositions can be used to describe any arbitrary static function with a required accuracy. The disadvantage of the new algorithm is that a boundary value for the static nonlinearity is assumed to be known. One aspect of further studies includes removing the boundary constraint imposed on the static nonlinearity.

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