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ERROR CONVERGENCE IN AN ADAPTIVE ITERATIVE LEARNING CONTROLLER

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Abstract

Stability theorems for an adaptive iterative learning control (ILC) system are motivated and described in compact terms that relate the properties of the schemes to systems structure and other important aspects of systems dynamics. The use of high gain feedback is reviewed and a full proof of the convergence of a 'Universal' adaptive scheme based on adaptive gain concepts is given. The results indicate clearly that successful ILC can be achieved in the presence of substantial uncertainty in the detailed knowledge of plant parameters and order. They also suggest that the form and success of the controller will depend crucially on the plant's relative degree and also on its minimum-phase properties.

Key words - learning control, iterative learning control, adaptive control

1. Introduction

One aspect of learning that has grown out of the area of robotics is that of iterative learning control (ILC) [1], [2] where the problem for the control algorithm is to construct (or learn) inputs that generate required outputs from a dynamical system. The mechanism of learning is that of repeated trials and the updating of control inputs from trial to trial based on observed performance. That is, given a dynamical system (assumed linear in this paper)

$$\dot{x}(t) = Ax(t) + Bu(t) \in R^n, \quad x(0) = x_0 \in R^n, \quad y(t) = Cx(t) \in R^m, \quad u(t) \in R^l \qquad (1)$$

and a desired output signal $r(t)$ on a fixed (finite) time interval $0 \le t \le T$, construct an iterative experimental procedure that automatically generates a sequence of input signals $(u_k(t))_{k\ge0}$ and a corresssponding sequence of outputs $(y_k(t))_{k\ge0}$ with the properties that:

(a) *CAUSALITY:* The input $u_{k+1}(t)$ at time t on trial/experiment/iteration $k+1$ is generated from only known trial data and the previous trial input $u_k(t)$.

(b) *CONVERGENT LEARNING:* The experiment has the exact convergence property

$$\lim_{k \to \infty} (r(\cdot) - y_k(\cdot)) = 0 \tag{2}$$

the limit being taken with respect to a chosen topology in a suitable linear vector space of output signals.

(c) *CONSISTENCY:* The input sequence converges to an acceptable limit that generates the required output r.

(d) *REALIZABILITY:* Any parameters associated with adaptive or other time varying elements converge in the limit as the iteration index $k \to \infty$.

The ILC problem can be approached by a wide variety of techniques, some described in [1], [2] and others elsewhere including frequency domain techniques [3], [4], optimal control-based methods [1], [6], [7] and, less frequently seen, adaptive schemes [1], [2], and [5]. In general, the $2D$ nature of the problem [8], [9] makes analysis more complex than in the non-ILC case. In particular, the recursive/iterative/$2D$ nature of the ILC problem combined with the inevitably nonlinear nature of any adaptive schemes suggests that, even in the special case of a linear plant, a mathematical analysis of the problem is likely to be very challenging. At this point in time, only a limited number of results are available and the relationship between systems structure and the choice of adaptation mechanism is poorly understood. What is known is that, in the non-adaptive case, systems structure is a vital contributor to convergence and algorithm performance. This issue can only become more important when uncertainty is introduced into the problem.

In the following paper, section 2 presents the background to the problem and states the main result and its relationship to high gain stabilization ideas [10], [11],13]. The results are seen to be an extension of previous work in the area by the authors. Section 3 is devoted to a proof of this result using a recursive description of the algorithm and a "comparison system" argument. This section contains most of the technical detail of the paper. The paper concludes in section 4 with a brief discussion of the practical need for minimum-phase properties of the plant.

The stability analysis is global (in the state and parameter product space $R^n \times R$) and is based on high gain concepts. It also includes the possibility that the open loop system is unstable. The consequence is that the analysis, in the form presented, does not allow the possibility of control saturation.

2. Statement of the Problem and The Main Stability Result

The paper aims to address issues of adaptive iterative learning control ($AILC$) through a special case that sets the scene for future, more general, algorithms. The special case has three aspects described as properties of the class of systems Σ_1:

Definition: The system class Σ_1 is defined by the statements

1. The system to be controlled is linear and time-invaria t.

2. The system is single-input-single output i.e. $m = l = 1$

3. The first Markov parameter $CB \neq 0$.

Definition: The system class $\Sigma_1^{(-)}$ is defined to be the sub-class of Σ_1 of systems that are also minimum-phase.

Note: The relevance of the minimum-phase assumption is discussed in section 4.

The adaptive controller is in the spirit of Universal Adaptive Stabilization (UAS). More precisely, for any system in $\Sigma_1^{(-)}$, it is known that the adaptive (UAS) control law [10]:

$$u(t) = -sgn(CB)K(t)y(t), \qquad \dot{K}(t) = cy^2(t), \quad c > 0, \quad K(0) = K_0 \tag{3}$$

has the properties that, for all values of x_0 and for all choices of K_0,

$$\lim_{k \to \infty} y(t) = 0. \quad \& \quad \lim_{t \to \infty} K(t) = K_\infty(x_0, K_0) < \infty \tag{4}$$

That is, the nonlinear control algorithm (consisting of a proportional controller with time-varying adaptation on the gain) is capable of stabilizing any system with the desired structure from any initial conditions $x(0) = x_0$ and $K(0) = K_0$ of the plant state and control "gain" $K(t)$. This stabilization of both the system output and controller parameters is achieved without more than a scrap of knowledge of the detailed plant dynamics and without any attempt to identify plant dynamics or embed additional information into the control structure or parameters. It is natural to conclude that the convergence of the adaptive rule is a consequence of the compatibility of the control law with the structure of the plant. An underpinning intuition of this paper is that similar observations can be derived and stated about $AILC$.

It will be shown that the above UAS result can be transferred into an $AILC$ context by the introduction of algorithms of the type described below. In what follows, no attempt is made to derive "the best" or "the most general" adaptive rules. Instead, attention is focussed on a simple case to indicate the form of result that can be obtained and the theoretical procedure underpinning the analysis. The issues of performance and optimisation of the available parameters is left for further research.

The first decision is the choice of data to be used in the update algorithm. An important aspect of the proposed algorithm is the inclusion of current error feedback i.e. the use of the signal e_{k+1} in the construction of the update rule for u_{k+1} on trial $k+1$. Without the use of such feedback, any high gain analysis can be expected to fail. It can also be argued that the inclusion of current error feedback is beneficial in practice for several reasons including:

- the most recent error data more closely reflects current performance of the system.
- the use of traditional feedback could enable the stabilization of unstable plants during each trial and

- it may offer the opportunity to reduce the effects of noise and modelling errors on algorithm performance.

Although based on non-adaptive reasoning, there is no reason to expect that these observations will not be valid for adaptive ILC.

PROPOSED *AILC* ALGORITHM

1. The control input on the $(k+1)^{th}$ trial is given by the update rule

$$u_{k+1}(t) = u_k(t) + sgn(CB)[(K_{k+1}e_{k+1})(t) + (F_{k+1}e_k)(t)], \qquad 0 \leq t \leq T \qquad (5)$$

   where K_{k+1} is a causal *feedback* "learning" operator feeding back the current trial error signal $e_{k+1} = r - y_{k+1}$ and F_{k+1} is a *feedforward* learning operator feeding forward the previous trial error data $e_k = r - y_k$.

2. The feedback control operation is simply a varying gain (sequence) with variation generated by the (nonlinear in data) update law

$$K_{k+1} = K_k + c||e_k||^2, \qquad c > 0 \qquad (6)$$

   which bases the gain choice on trial $k+1$ on its previous value and a measure of the magnitude of the recorded tracking error on trial k. The process is initiated by the choice of a gain K_0 for $k = 0$. In theoretical terms, this choice can be arbitrary.

3. The feedforward control operation is taken simply to be a constant gain F.

Note on Notation: In the above and in what follows the notation $||f|| = (\int_0^T f^2(t)dt)^{\frac{1}{2}}$ denotes the $L_2(0,T)$ norm of its argument $f \in L_2(0,T)$. The notation $< f_1, f_2 >= \int_0^T f_1(t)f_2(t)dt$ denotes the associated inner product.

The similarity between the above *AILC* algorithm and the *UAS* algorithm lies essentially in the use of quadratic update rules for adaptive system's gains. The details differ as the *AILC* problem has a $2D$ structure but it will be seen that the effects on closed-loop systems dynamics are essentially the same. This similarity is formally expressed through the following theorem that forms the main result of this paper:

Theorem 1:

Suppose that the plant is in Σ_1 and the *AILC* algorithm described above is applied with an arbitrary choice of input $u_0 \in L_2(0,T)$ generating an initial error $e_0 \in L_2(0,T)$. Suppose also that the reference signal r can be generated exactly by an input $u_\infty \in L_2(0,T)$ (i.e. $r \in R(G)$). Under these conditions, the resultant closed-loop, nonlinear iterative system has the following properties:

1. The tracking error converges to zero in $L_2(0,T)$ in the sense that

$$\sum_{k=0}^{\infty} ||e_k(\cdot)||^2_{L_2(0,T)} < \infty \qquad (7)$$

   and hence

- The error (norm) sequence $\{||e_k||\}_{k\geq 0}$ is ' ounded in $L_2(0,T)$.
- Conve:gence of the error to zero is guaranteed in the norm topology in $L_2(0,T)$

2. The monotonically increasing adaptive feedback gain parameter sequence $\{K_k\}_{k\geq 0}$ converges in the sense that a limit gain K_∞ (dependent on K_0 and x_0) exists i.e.

$$\lim_{k\to\infty} K_{k+1} = K_\infty < \infty \tag{8}$$

3. There exists a gain K^* with the property that, whenever $K_k > K^*$, the error norm sequence $\{||e_k||\}_{k\geq k^*}$ is strictly monotonically decreasing.

4. If the plant is in $\Sigma_1^{(-)}$, then it is possible to choose K^* to be independent of trial length T.

Note: The first two statements are formal descriptions of the convergence and realizability of the algorithm. The third statement provide somes information on the form of convergence under high gain conditions whilst the fourth suggest the effects of minimum-phase characteristics.

The proof of the theorem is given in the next section. It is worth noting that the basis of the proof is a proof of the convergence of the formal series

$$K_\infty = K_0 + c\sum_{k=0}^{\infty} ||e_k||^2 \tag{9}$$

To conclude this section, note that the authors have shown in a previous paper that the gain update law

$$K_{k+1} = K_k + c||e_k - e_{k-1}||^2_{L_2(0,T)}, \qquad c > 0 \tag{10}$$

based on the error difference $e_k - e_{k-1}$ leads to a convergent adaptive gain but it could only be proved that convergence of the error is in the *weak* topology in $L_2(0,T)$. The proof required additional assumptions such as boundedness of the error sequence in $L_2(0,T)$ or that the plant is positive-real. The previous paper also did not include the feedforward term in the control update rule. The main contribution of this paper is hence a substantial generalisation of previous work and hence a clear demonstration of the theoretical potential of *AILC*. The paper does not propose that the algorithm discussed is ideal for practice. Rather, it demonstrates the potential for achieving convergence of *ILC* under conditions of extreme uncertainty. With this in mind, the community can confidently address the issue of improved general purpose algorithms based on more complete systems information (possibly derived from identification procedures).

3. Proof of the Main Theorem

The proof is approached using two stages. Firstly, the iterative properties of the algorithm in the case where the gain sequence diverges are derived. This result (Theorem 2) provides vital properties of the error difference sequence in this situation of "instability". A parallel "comparison" process is then used to demonstrate that convergence of an appropriate error difference algorithm implies the convergence of the algorithm described above in this paper.

Note: To simplify the presentation, the following assumptions and notation will be assumed:

- It is assumed without loss of generality that $CB > 0$ and hence that $sgn(CB) = 1$.
- Note [6], [7] that, with no loss in generality, it is possible to assume that $x(0) = 0$.
- It is also possible to write $y = Gu$ where the convolution operator G maps $L_2(0,T)$ into itself. This operator notation has been shown to be useful in deriving the basic relationships governing the ILC evolution.

With the above notation, a little algebra and the update rule $u_{k+1} = u_k + K_{k+1}e_{k+1} + Fe_k$ leads to the following relationship between errors:

$$e_{k+1} = \hat{S}_{k+1}e_k, \qquad \hat{S}_{k+1} = S_{k+1}(I - GF), \qquad S_{k+1} = (I + GK_{k+1})^{-1} \tag{11}$$

Note the presence of the well-known sensitivity function $S = (I + GK)^{-1}$ in the update rule. It is useful to introduce the complementary sensitivity and a number of related identities

$$L_{k+1} = I - S_{k+1} = (I + GK_{k+1})^{-1}GK_{k+1} = S_{k+1}GK_{k+1} \tag{12}$$

and, in a parallel manner, whenever $K_{k+1} \neq 0$,

$$\hat{L}_{k+1} := I - \hat{S}_{k+1} = L_{k+1}(1 + \frac{F}{K_{k+1}}) = S_{k+1}G(K_{k+1} + F) = \hat{S}_{k+1}(K_{k+1} + F)(I - GF)^{-1}G \tag{13}$$

Note that L_k has a state space realization of the form $S(A - BK_kC, BK_k, C)$ and hence that $\hat{L}_k$ has a state space realization of the form $S(A - BK_kC, B(K_k + F), C)$. Note also that the signal

$$v_{k+1} := -(e_{k+1} - e_k) = \hat{L}_{k+1}e_k \tag{14}$$

Theorem 2 - The Case of Unbounded Gain Sequences

Suppose that the following conditions hold:

1. the plant is in Σ_1,
2. the control law takes the form $u(t) = -sgn(CB)(K_{k+1}e_{k+1}(t) + Fe_k(t))$
3. and the gain sequence $\{K_k\}_{k\geq 0}$ is monotonically increasing and unbounded.

Then the error difference sequence satisfies the condition

$$\sum_{k=1}^{\infty} ||e_k - e_{k-1}||^2_{L_2(0,T)} < \infty \tag{15}$$

Proof: Without loss of generality, assume that $CB > 0$ and hence that $sgn(CB) = 1$. The proof is similar in structure to that given in previous work and goes as follows: note that, in $L_2(0,T)$,

$$||e_{k+1}||^2 = ||e_k||^2 - 2 < v_{k+1}, e_k > + ||v_{k+1}||^2 \tag{16}$$

In a similar manner to previous results [10], [13] the relationship $v = \hat{L}e$ has a representation of the form (trial indices dropped for notational convenience)

$$x(t) = \begin{bmatrix} v(t) \\ z(t) \end{bmatrix}, \quad A - BKC = \begin{bmatrix} A_{11} - CBK & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B(K+F) = \begin{bmatrix} CB(K+F) \\ 0 \end{bmatrix} \tag{17}$$

(Note: With this description, the eigenvalues of A_{22} are the zeros of the original system G and hence the system is in $\Sigma_1^{(-)}$ if, and only if, the matrix A_{22} is asymptotically stable.)

The relationship $v_{k+1} = \hat{L}_{k+1}e_k$ can now be written in the form

$$\dot{v}_{k+1}(t) = (A_{11} - CBK_{k+1})v_{k+1}(t) + \hat{v}_{k+1}(t) + CB(K_{k+1} + F)e_k(t) \tag{18}$$

where

$$\dot{z}_{k+1}(t) = A_{22}z_{k+1}(t) + A_{21}v_{k+1}(t), \quad \hat{v}_{k+1}(t) = A_{12}z_{k+1}(t) \tag{19}$$

where the initial conditions on both differential equations are zero.

Note that the map $v_{k+1} - \hat{v}_{k+1}$ maps $L_2(0,T)$ into itself. The fact that T is finite ensures that the map is norm bounded. More precisely, there exists $M_T > 0$ (independent of v_{k+1}) such that $||\hat{v}_{k+1}|| \leq M_T||v_{k+1}||$.

A more detailed analysis also yields:

- In general M_T depends on T.
- For minimum-phase systems (i.e. systems in $\Sigma_1^{(-)}$), M_T can be chosen to be independent of T.

It is easily seen that

$$\begin{aligned} \frac{d}{dt}\left(\tfrac{1}{2}v_{k+1}^2(t)\right) &= v_{k+1}(t)\dot{v}_{k+1}(t) \\ &= (A_{11} - CBK_{k+1})v_{k+1}^2(t) + v_{k+1}(t)\hat{v}_{k+1}(t) + v_{k+1}(t)CB(K_{k+1} + F)e_k(t) \end{aligned} \tag{20}$$

Integrating on $[0,T]$ then gives

$$\begin{aligned} 0 &\leq \tfrac{1}{2}v_{k+1}^2(T) \\ &= \int_0^T[(A_{11} - CBK_{k+1})v_{k+1}^2(t) + v_{k+1}(t)\hat{v}_{k+1}(t) + v_{k+1}(t)CB(K_{k+1} + F)e_k(t)]dt \\ &= (A_{11} - CBK_{k+1})||v_{k+1}^2|| + < v_{k+1}, \hat{v}_{k+1} > + CB(K_{k+1} + F) < v_{k+1}, e_k > \end{aligned} \tag{21}$$

or, writing $< v_{k+1}, \hat{v}_{k+1} > \leq ||v_{k+1}|| \cdot ||\hat{v}_{k+1}|| \leq M||v_{k+1}||^2$, it follows that, for all sufficiently high values of trial index k, $K_{k+1} + F > 0$ and hence

$$-2 < v_{k+1}, e_k > \leq 2\frac{(M + A_{11} - CBK_{k+1})}{CB(K_{k+1} + F)}||v_{k+1}||^2 = 2\left[\frac{(M + A_{11}) + CBF}{CB(K_{k+1} + F)} - 1\right]||v_{k+1}||^2 \tag{22}$$

As a consequence,

$$0 \leq ||e_{k+1}||^2 \leq ||e_k||^2 + \left[\frac{2(M + A_{11} + CBF)}{CB(K_{k+1} + F)} - 1\right]||v_{k+1}||^2 \tag{23}$$

The Theorem now follows from an induction argument and noting that $\lambda_{k+1} := \left[\frac{2(M+A_{11}+CBF)}{CB(K_{k+1}+CBF)} - 1\right] < 0$ for all large enough values of k. More precisely, let k^* be any integer such that $\lambda_{k+1} < -\frac{1}{2}$. An inductive argument then yields the observation that

$$||e_{k^*}||^2 \geq ||e_{k^*+N}||^2 + \frac{1}{2}\sum_{j=k^*+1}^{k^*+N} ||v_j||^2 \geq 0 \tag{24}$$

the result follows by letting $N \to \infty$. □

The following corollary plays an important role in the development.

Corollary (Unbounded Gain and the Error Sequence)

Under the conditions of Theorem 2, the following inequalities hold true whenever the reference signal $r \in R(G)$ $(\subset L_2(0,T))$

$$\sum_{k=0}^{\infty} ||e_k||^2(K_k + F)^2 < \infty \tag{25}$$

In particular,

$$\sum_{k=0}^{\infty} ||e_k||^2 < \infty \tag{26}$$

(Note: The first inequality implies the second)

Proof: If $r \in R(G)$, then $e_0 = r - y_0 = r - Gu_0 \in R(G)$. Note that $R(G) = R((I - GF)^{-1}G)$ and write $e_0 = (I - GF)^{-1}G\hat{e}_0$ with $\hat{e}_0 \in L_2(0,T)$. Now note that the operators $\{\hat{S}_k\}_{k\geq 1}$ are a commuting set and consider the iteration $\hat{e}_{k+1} = \hat{S}_{k+1}\hat{e}_k$, $k \geq 0$, to obtain the formula

$$\begin{aligned} \hat{e}_k - \hat{e}_{k-1} &= \hat{S}_k....\hat{S}_1\hat{e}_0 - \hat{S}_{k-1}.....\hat{S}_1\hat{e}_0 \\ &= \hat{S}_{k-1}...\hat{S}_1(\hat{S}_k - I)\hat{e}_0 \\ &= -\hat{S}_{k-1}...\hat{S}_1\hat{L}_k\hat{e}_0 \\ &= -\hat{S}_{k-1}...\hat{S}_1\hat{S}_k(K_k + F)(I - GF)^{-1}G\hat{e}_0 \\ &= -(K_k + F)\hat{S}_k...\hat{S}_1 e_0 \end{aligned} \tag{27}$$

Note that Theorem 1 applies for any initial error and hence applies to the modified iteration with initial error $\hat{e}_0$. Using Theorem 1 then gives

$$\infty > \sum_{k=1}^{\infty} ||\hat{e}_k - \hat{e}_{k-1}||^2_{L_2(0,T)} = \sum_{k=1}^{\infty} ||-\hat{S}_k...\hat{S}_1(K_k + F)e_0||^2 = \sum_{k=1}^{\infty} ||e_k||^2_{L_2(0,T)}(K_k + F)^2 \tag{28}$$

where $e_{k+1} = \hat{S}_{k+1}e_k$ for $k \geq 0$. The result follows as this is just the required recursive formula for the original error sequence $\{e_k\}_{k\geq 0}$. □

Turning now to the proof of Theorem 1, it is important to note that the gain adaptation algorithm leads to the formula

$$K_k = K_0 + c\sum_{j=0}^{k-1} ||e_j||^2 \tag{29}$$

and hence to the observation that the convergence of the gain sequence is closely linked to the convergence of the infinite series

$$\sigma = c^{-1}(K_\infty - K_0) = \sum_{j=0}^{\infty} ||e_j||^2 \tag{30}$$

Proof of Theorem 1:

Parts 1 and 2: As the gain sequence is monotonically increasing, it either converges to a finite limit K_∞ or it diverges to $+\infty$. If it converges, then the infinite series σ converges and the conclusions of the theorem are valid for this case. If the gain diverges, then the corollary to theorem 2 indicates that σ is finite and hence K_∞ is finite which is a contradiction. Divergence of the gain sequence is hence not possible and parts 1 and 2 of the theorem are proved.

Part 3: Using the notation of the proof of theorem 2, note that the inequality (23) is valid for any sequence $\{K_k\}$ and does not depend on its monotonicity or divergence. However, if the sequence is monotonic and ultimately exceeds a value K^* derived by ensuring that $\lambda^* < \frac{1}{2}$, then the monotonicity of $\{||e_k||\}_{k\geq 0}$ is guaranteed for all large enough trial indices k.

Part 4: The "sufficiently large" gain required for Part 3 is dependent on T only because M_T can depend on T. In the case of minimum-phase systems this is not the case. □

4. A Note on the Minimum-phase Assumption

A review of the statement and proof of Theorems 1 and 2 suggests that, at the formal level, the minimum-phase condition is not necessary for stabilization and realizability of the AILC algorithm. The minimum-phase property was reflected only in the observation that the bound M_T on the map $v_{k+1} \to \hat{v}_{k+1}$ was then independent of trial length T. This suggested that the asymptotic properties of the algorithm (in the special case of sufficiently high values of K_∞ are beneficial in the sense that error convergence can be both monotonic and independent of the trial length T. This says nothing about any possible problems for non-minimum-phase systems. The reality is that one of the implications of using the algorithm on a non-minimum-phase system include the possibility that, for large values of limit gain K_∞, the algorithm will be using a destabilising feedback gain! The assumption of minimum-phase is therefore required for practical reasons as it guarantees that:

- The feedback gain is ultimately stabilising (proved by the analysis at least for the situation when K_∞ is sufficiently large) and
- the error convergence (equation (23)) is ultimately monotonic in $L_2(0, T)$ norm for all trial lengths T whenever the limit gain is sufficiently large.

5. Discussion and Conclusions

The paper has formulated, clarified and extended previous work by the first author in an area of adaptive iterative learning control ($AILC$). For a well defined class of systems, it has been demonstrated that $L_2(0,T)$ error norm convergence is possible under conditions of extreme uncertainty of plant parameters and order. In particular, it is seen that convergence is possible without the use of plant identification algorithms. All that is required is a knowledge of plant structure as defined by the set Σ_1 and an adaptive scheme that uses a fixed feedforward term (arbitrarily chosen) and an adaptive proportional feedback gain updated between trials.

The effects of minimum-phase characteristics have been included in the analysis with the conclusion that, although convergence does not depend on this system property, it should be anticipated that convergence rates and the acceptability of the limiting gains do make it a practical requirement.

The possibility of convergent learning in the presence of extreme uncertainty underlines the inherent robustness of the ILC process. The study also underpins the possibility of high quality adaptive control in the ILC context. The proposed algorithm is unlikely to be "optimal" in its present form but its analysis provides a bedrock of theoretical support to initiate the development of more easily implementable adaptive schemes.

Finally, for brevity, the paper has concentrated on one of the simplest special cases. With more technical effort,

1. Using techniques similar to those seen in references [12], [13] the class of systems can be extended to include multi-input/multi-output systems with symmetric, positive definite Markov parameter matrix CB.

2. The adaptation algorithm can be extended to include adaptation laws of the form

$$K_{k+1} = K_k + p(||e_k||) \tag{31}$$

and

$$K_{k+1} = K_k 1 + (K_k + F)^2 p(||e_k||) \tag{32}$$

where $p(\cdot)$ is any polynomial (or more generally any entire function) with the property that $p(a) \geq \epsilon a^2$ for some $\epsilon > 0$ and any $a \geq 0$.

Much of this work relies on similar principles but requires much more technical computation in the proofs. These proofs will be reported separately.

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