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Cooperative Positioning Using Angle of Arrival and Time of Arrival

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***Abstract*—Localization has been one of the most highly researched topics in wireless communications in the past decade. Localization of wireless nodes can be achieved using a variety of techniques, in which range measurement and angle measurement are most commonly used. In the presence of both angle and range measurement, a hybrid model can be developed. In this paper we analyze a hybrid angle of arrival-time of arrival (AoA-ToA) model for localization of wireless nodes, the model is modified to remove the bias from the estimated positions. We also explore the idea of cooperative localization using both angle and range measurements and develop a linear least squares (LLS) scheme. It is shown via simulation that the modified model in unbiased and that the performance of the proposed cooperative LLS is superior to its non-cooperative counterpart.**

***Index Terms*—hybrid localization, angle of arrival, time of arrival, cooperative localization.**

I. Introduction

ONE of the major application area of wireless communications is localization of wireless nodes. Development of low cost and accurate localization system has been a prime research topic in the last few years. Previously localization systems were only used for military purposes e.g. aircraft detection\localization. With the advent of new technologies the applications of localization systems grew significantly. These may include logistics, military and surveillance applications [1]. Different algorithms and techniques are proposed by researchers to improve the accuracy and decrease the complexity of localization systems [2], [3]. Localization can be range based or it can be based on the angle of arrival (AoA) of the impinging signal. Two main techniques to estimate the range between nodes are time of arrival (ToA) that utilizes the delay required for the signal to propagate from one node to another and received signal strength (RSS) that is based on the power dissipation of the signal as it travel through the channel. For both of these techniques the localization problem can be solved using high complexity maximum likelihood techniques [4], [5]. A low complexity linear least squares (LLS) approach has also been proposed for ToA systems [6], its performance is analyzed and enhanced in [7]. The angle of arrival can be estimated using an array of antennas [8] or a rotating beam of radiation [9], and techniques like multiple signal classification (MUSIC) [10] or estimation of signal parameters via rotational invariant techniques (ESPRIT) [11] are used for angle estimation. Localization can also be classed as non-cooperative [12], in which only the anchor nodes (ANs) are used to localize target nodes (TNs) or cooperative [13] in which all the nodes are used for localization. Or it can be classed as centralized in which all the data is transferred and processed at a base station (BN), or distributed, in which the processing is done at node level.

Positioning using AoA, ToA or RSS, each have their own advantages and disadvantages. For example positioning with RSS is the cheapest and least complex form of localization, however the system suffers from performance issues in highly cluttered networks. Positioning via ToA shows good performance, however in order to achieve this accuracy every node in the network must be equipped with a synchronized clock, which adds to the cost and complexity of the system. On the other hand, localization that utilizes the angle of the incoming signal performs well in small networks but the accuracy degrades significantly as the TNs moves away from the ANs. Also in order to measure the angle at which the signal is coming, the nodes must be equipped with an array of antennas or a rotating directional antenna, which adds to the cost and size of the nodes. A new trend is to use all the available information, hence researchers are also working on hybrid systems, that combine two or more of these techniques to make one robust and accurate system.

Another trend is to use cooperation between TNs along with ANs. Cooperative localization shows much better performance when compared to the non-cooperative schemes, however the TNs used in these networks must have substantial processing power. In [13] the author propose a cooperative scheme using metric multidimensional scaling (MDS). A non-metric MDS approach is presented in [14]. Performance of location estimation via MDS is improved in [15]. Obtaining MDS solution for localization of wireless nodes consist of the spectral decomposition of a doubly centered distance matrix which is computationally intensive. To avoid this spectral decomposition the authors in [16] propose an iterative scheme for localization called scaling by majorizing a complicated function (SMACOF). One of the main problems in cooperative localization is noise propagation i.e. noise in one node will add to the noise in other node. In [17] the author proposed an error propagation aware algorithm in which only the nodes with low noise in their communication links are used in a peer-to-peer manner. In [18] a cooperative range free algorithm is proposed which takes into account an approximated distance between two nodes rather than a metric distance. This approximated distance depends upon node density of the network and hence will not work in sparse networks.

In this paper we analyze and modify a hybrid AoA-ToA localization model. The modification is necessary to make the system unbiased. The paper also explores the idea of cooperative hybrid AoA-ToA localization. Via simulation it is shown that the modified model is unbiased, also it is shown that the cooperative location estimation outperforms the non-cooperative model.

The rest of the paper is organized as follows. In section II the hybrid AoA-ToA model is reviewed and the bias of the system is calculated. Furthermore a modified unbiased model is also presented. In section III we develop a linear least squares cooperative localization technique. Section IV presents the simulation results. Future work and the conclusion is presented in section V of the paper.

II. The Hybrid Model

For future use, the following notations are defined. $\mathbb{R}^n$ is the set of n dimensional real numbers. $(.)^T$ represents the transpose operator. $\mathrm{E}(.)$ refers to the expectation operation. $(\mathbf{T})_{ij}$ refers to the element at the i^{th} row and j^{th} column of matrix $\mathbf{T}$. $\mathcal{N}(\mu,\sigma^2)$ denotes the normal distribution with mean μ and variance σ^2.

If both range and angle measurements are available then TN coordinates can be estimated with only one AN. However to improve estimation accuracy more ANs can be introduced to the system. In [19] the author proposes a hybrid location system that utilizes both range and angle of the received signal. However the system produces biased estimates and there is no cooperation between the TNs. In this section a hybrid AoA-ToA technique is developed.

Consider a network of M ANs, where $\left(\begin{array}{cc}\bar{x}_i & \bar{y}_i\end{array}\right)$ are the coordinates of i^{th} AN. Let $\hat{\theta}_i$ and $\hat{d}_i$ be the measured angle and distance between i^{th} AN and the TN, and are given by

$$\hat{d}_i = d_i + n_i,$$

where

$$d_i = \sqrt{(x-\bar{x}_i)^2 + (y-\bar{y}_i)^2},$$

and

$$\hat{\theta}_i = \theta_i + m_i,$$

where

$$\theta_i = \arctan\left[\frac{(y-\bar{y}_i)}{(x-\bar{x}_i)}\right].$$

n_i and m_i represents the zero mean Gaussian noise in the estimate of distance and angle with the i^{th} AN i.e. $n_i \sim \left(\mathcal{N}\left(0,\sigma_{n_i}^2\right)\right)$ and $m_i \sim \left(\mathcal{N}\left(0,\sigma_{m_i}^2\right)\right)$.

Then the x and y coordinate of the target node can be estimated as

$$\hat{x} = \bar{x}_i + \hat{d}_i \cos\hat{\theta}_i \quad \text{for } i = 1,\ldots,M \tag{1}$$

$$\hat{y} = \bar{y}_i + \hat{d}_i \sin\hat{\theta}_i \quad \text{for } i = 1,\ldots,M \tag{2}$$

(1) and (2) can be written in matrix form as

$$\hat{\mathbf{u}}_{\mathrm{H}}\mathbf{A}_{\mathrm{H}} = \hat{\mathbf{p}}_{\mathrm{H}},$$

where

$$\hat{\mathbf{u}}_{\mathrm{H}} = \left[\begin{array}{cc} x & y \end{array}\right]^T,$$

$$\mathbf{A}_{\mathrm{H}} = \mathrm{diag}\left(\mathbf{e},\mathbf{e}\right),$$

where $\mathbf{e}$ is a column matrix of M ones and

$$\hat{\mathbf{p}}_{\mathrm{H}} = \left[\begin{array}{cc} \hat{\mathbf{p}}_x & \hat{\mathbf{p}}_y \end{array}\right]^T,$$

where $\hat{\mathbf{p}}_x = \left[\bar{x}_1 + \hat{d}_1\cos\hat{\theta}_1, \cdots, \bar{x}_M + \hat{d}_M\cos\hat{\theta}_M\right]^T$ and $\hat{\mathbf{p}}_y = \left[\bar{y}_1 + \hat{d}_1\sin\hat{\theta}_1, \cdots, \bar{y}_M + \hat{d}_M\sin\hat{\theta}_M\right]^T$.

The LLS solution for this system is given by

$$\hat{\mathbf{u}}_{\mathrm{H}} = \mathbf{A}_{\mathrm{H}}^{\dagger}\hat{\mathbf{p}}_{\mathrm{H}},$$

where $\mathbf{A}_{\mathrm{H}}^{\dagger}$ is the Moore–Penrose pseudo inverse of matrix $\mathbf{A}_{\mathrm{H}}$ and is given by

$$\mathbf{A}_{\mathrm{H}}^{\dagger} = \left(\mathbf{A}_{\mathrm{H}}^T\mathbf{A}_{\mathrm{H}}\right)^{-1}\mathbf{A}_{\mathrm{H}}^T.$$

A. Bias In the Hybrid Model

The bias of the LLS estimator is given by

$$\mathrm{Bias} = \mathbf{A}^{\dagger}\boldsymbol{\rho}, \tag{3}$$

where $\boldsymbol{\rho} = \mathrm{E}(\hat{\mathbf{p}}) - \mathbf{p}$, for $\mathbf{p}$ representing noise free observation. The i^{th} term of $\boldsymbol{\rho}$ for the hybrid model is given by

$$\rho_i = \epsilon_i\left(\left(\exp\left(-\frac{\sigma_{m_i}^2}{2}\right) - 1\right)\right),$$

where $\epsilon_i = d_i\cos\theta_i$ for the estimation of x coordinate of the TN and $\epsilon_i = d_i\sin\theta_i$ for the estimation of y coordinate of the TN. The hybrid model is modified in the following subsection to remove the bias from the system.

B. Modified Hybrid Model

In order to reduce (3) to zero, (1) and (2) are modified by introducing an unbiasing constant. The modified version of (1) and (2) are as follows

$$\hat{x} = \bar{x}_i + \hat{d}_i\cos\hat{\theta}_i\delta_i \quad \text{for } i = 1,\ldots,M \tag{4}$$

$$\hat{y} = \bar{y}_i + \hat{d}_i\sin\hat{\theta}_i\delta_i \quad \text{for } i = 1,\ldots,M \tag{5}$$

where δ_i is the unbiasing constant used in the i^{th} observation. Its value is given by

$$\delta_i = \exp\left(\frac{\sigma_{m_i}^2}{2}\right).$$

In matrix form (4) and (5) can be written as

$$\tilde{\mathbf{p}}_{\mathrm{H}} = \begin{bmatrix} \tilde{\mathbf{p}}_x & \tilde{\mathbf{p}}_y \end{bmatrix}^T,$$

where $\tilde{\mathbf{p}}_x = \left[\bar{x}_1 + \hat{d}_1 \cos\hat{\theta}_1\delta_1, \cdots, \bar{x}_M + \hat{d}_M \cos\hat{\theta}_M\delta_M\right]^T$ and $\hat{\mathbf{p}}_y = \left[\bar{y}_1 + \hat{d}_1 \sin\hat{\theta}_1\delta_1, \cdots, \bar{y}_M + \hat{d}_M \sin\hat{\theta}_M\delta_M\right]^T$.

The LLS solution for this system is given by

$$\hat{\mathbf{u}}_{\mathrm{H}} = \mathbf{A}_{\mathrm{H}}^{\dagger}\tilde{\mathbf{p}}_{\mathrm{H}}.$$

III. The Cooperative Hybrid Model

In the hybrid model discussed in section 2, there is no communication links between TNs. Now we consider a network in which a TN can communicate with other TNs along with the ANs. Consider a network of M ANs and N TNs. Let $\hat{d}_{ij}$ and $\hat{\theta}_{ij}$ be the measured distance and angle between i^{th} AN and j^{th} TN and are given by (6) and (7) respectively. Here we use the notation x_j and y_j for the x and y coordinate of j^{th} TN.

$$\hat{d}_{ij} = d_{ij} + n_{ij}, \tag{6}$$

where

$$d_{ij} = \sqrt{(\bar{x}_i - x_j)^2 + (\bar{y}_i - y_j)^2},$$

and

$$\hat{\theta}_{ij} = \theta_{ij} + m_{ij}, \tag{7}$$

where

$$\theta_{ij} = \arctan\left[\frac{(y_j - \bar{y}_i)}{(x_j - \bar{x}_i)}\right].$$

On the other hand, let $\hat{D}_{jk}$ and $\hat{\alpha}_{jk}$ be the measured distance and angle between j^{th} and k^{th} TN, which are given by (8) and (9) respectively.

$$\hat{D}_{jk} = D_{jk} + n_{jk}, \tag{8}$$

where

$$D_{jk} = \sqrt{(x_k - x_j)^2 + (y_k - y_j)^2},$$

and

$$\hat{\alpha}_{jk} = \alpha_{jk} + m_{jk}, \tag{9}$$

where

$$\alpha_{jk} = \arctan\left[\frac{(y_k - y_j)}{(x_k - x_j)}\right].$$

n_{ij}, n_{jk}, m_{ij} and m_{jk} represents the zero mean Gaussian noise in the estimate of $\hat{d}_{ij}$, $\hat{D}_{jk}$, $\hat{\theta}_{ij}$ and $\hat{\alpha}_{jk}$ respectively i.e. $n_{ij} \sim \left(\mathcal{N}\left(0, \sigma_{n_{ij}}^2\right)\right)$, $n_{jk} \sim \left(\mathcal{N}\left(0, \sigma_{n_{jk}}^2\right)\right)$, $m_{ij} \sim \left(\mathcal{N}\left(0, \sigma_{m_{ij}}^2\right)\right)$ and $m_{jk} \sim \left(\mathcal{N}\left(0, \sigma_{m_{jk}}^2\right)\right)$.

Then the x coordinate of j^{th} TN can be estimated by the following equations

$$\hat{x}_j = \bar{x}_i + \hat{d}_{ij}\cos\hat{\theta}_{ij}\delta_{ij} \quad \text{for } i = 1, \ldots, M \tag{10}$$

x coordinate of j^{th} TN in cooperation with k^{th} TN is

$$\hat{x}_j = \bar{x}_i + \hat{d}_{ik}\cos\hat{\theta}_{ik}\delta_{ik} - \hat{D}_{jk}\cos\hat{\alpha}_{jk}\delta_{jk} \quad \text{for } i = 1, \ldots, M \tag{11}$$

Similarly the y coordinate can be estimated by

$$\hat{y}_j = \bar{y}_i + \hat{d}_{ij}\sin\hat{\theta}_{ij}\delta_{ij} \quad \text{for } i = 1, \ldots, M \tag{12}$$

and

$$\hat{y}_j = \bar{y}_i + \hat{d}_{ik}\sin\hat{\theta}_{ik}\delta_{ik} - \hat{D}_{jk}\sin\hat{\alpha}_{jk}\delta_{jk} \quad \text{for } i = 1, \ldots, M \tag{13}$$

where δ_{ij} and δ_{jk} are the bias reducing constants whose values are given by

$$\delta_{ij} = \exp\left(\frac{\sigma_{m_{ij}}^2}{2}\right).$$

$$\delta_{jk} = \exp\left(\frac{\sigma_{m_{jk}}^2}{2}\right).$$

(10) and (12) are the same as (4) and (5), which is the contribution from the ANs. On the other hand (11) and (13) is the contribution from the TNs. (10) to (13) can be extended to N TNs and can be written in matrix form as:

$$\mathbf{A}\hat{\mathbf{u}} = \hat{\mathbf{b}}$$

where $\mathbf{A} = \mathrm{diag}[\mathbf{E}, \ldots, \mathbf{E}] \in \mathbb{R}^{2MN^2 \times 2N}$, and $\mathbf{E}$ is column matrix of MN ones.

$$\hat{\mathbf{u}} = [\hat{x}_1, \ldots, \hat{x}_N, \hat{y}_1, \ldots, \hat{y}_N]^T \in \mathbb{R}^{2N \times 1},$$

$$\hat{\mathbf{b}} = \begin{bmatrix} \hat{\mathbf{b}}_x, & \hat{\mathbf{b}}_y \end{bmatrix}^T \in \mathbb{R}^{2MN^2 \times 1},$$

where

$$\hat{\mathbf{b}}_x = \begin{bmatrix} \dot{\mathbf{x}}_1, & \ddot{\mathbf{x}}_1, & \ldots, & \dot{\mathbf{x}}_N, & \ddot{\mathbf{x}}_N \end{bmatrix}^T \in \mathbb{R}^{MN^2 \times 1}.$$

and

$$\dot{\mathbf{x}}_j = \begin{bmatrix} \bar{x}_1 + \hat{d}_{1j}\cos\hat{\theta}_{1j}\delta_{1j} \\ \bar{x}_2 + \hat{d}_{2j}\cos\hat{\theta}_{2j}\delta_{2j} \\ \vdots \\ \bar{x}_M + \hat{d}_{Mj}\cos\hat{\theta}_{Mj}\delta_{Mj} \end{bmatrix} \in \mathbb{R}^{M \times 1}, \ddot{\mathbf{x}}_j = [\bar{\mathbf{X}}_{j1}, \ldots, \bar{\mathbf{X}}_{jN}]^T$$

and

$$\bar{\mathbf{X}}_{jk} = \left[\dot{\mathbf{x}}_k - \hat{D}_{jk}\cos\hat{\alpha}_{jk}\delta_{jk}\mathbf{e}\right] \text{ for } k = 1, \ldots, N \;\; (j \neq k)$$

Similarly vector $\hat{\mathbf{b}}_y = \begin{bmatrix} \dot{\mathbf{y}}_1, & \ddot{\mathbf{y}}_1, & \ldots, & \dot{\mathbf{y}}_N, & \ddot{\mathbf{y}}_N \end{bmatrix}^T \in \mathbb{R}^{MN^2 \times 1}$.

where

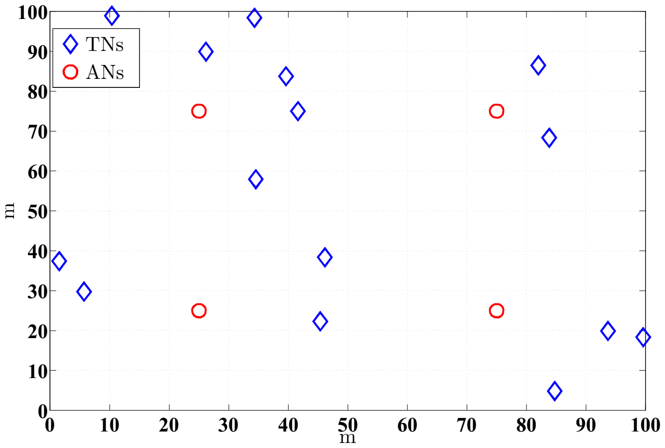

Figure 1. Network deployment

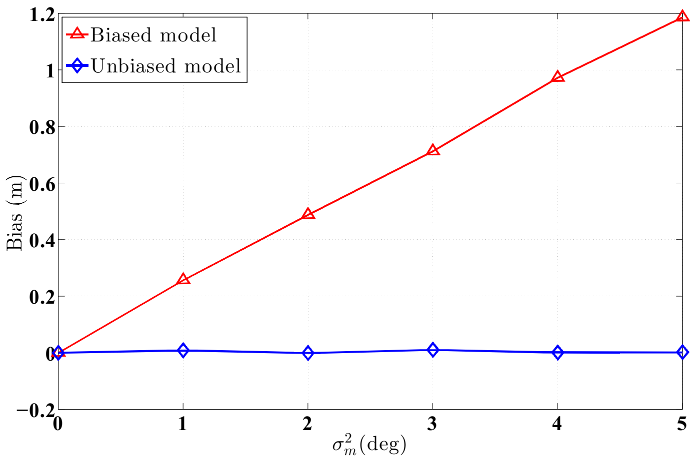

Figure 2. Bias in the biased and unbiased models. ANs=4, TNs=1, $\upsilon = 2500$.

$$\dot{\mathbf{y}}_j = \begin{bmatrix} \bar{y}_1 + \hat{d}_{1j} \sin\hat{\theta}_{1j}\delta_{1j} \\ \bar{y}_2 + \hat{d}_{2j} \sin\hat{\theta}_{2j}\delta_{2j} \\ \vdots \\ \bar{y}_M + \hat{d}_{Mj} \sin\hat{\theta}_{Mj}\delta_{Mj} \end{bmatrix} \in \mathbb{R}^{M\times 1}, \ddot{\mathbf{y}}_j = \left[\hat{\boldsymbol{Y}}_{j1}, \ldots, \hat{\boldsymbol{Y}}_{jN}\right]^T,$$

and

$$\hat{\boldsymbol{Y}}_{jk} = \left[\dot{\mathbf{y}}_k - \hat{D}_{jk} \sin\hat{\alpha}_{jk}\delta_{jk}\mathbf{e}\right] \text{ for } k = 1, 2, 3 \ldots N \, (j \neq k). \tag{14}$$

The LLS solution for the linear system is given by

$$\hat{\mathbf{u}} = \mathbf{A}^{\dagger}\hat{\mathbf{b}}, \tag{15}$$

where $\mathbf{A}^{\dagger}$ is the Moore–Penrose pseudo inverse of $\mathbf{A}$ and is given by $\mathbf{A}^{\dagger} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T$. Matrix $\mathbf{A}^{\dagger}$ can be calculated directly without taking the pseudo inverse if the number of TNs and ANs are known i.e.

$$\mathbf{A}^{\dagger} = \text{diag}\left[\boldsymbol{\eta}, \boldsymbol{\eta}, \cdots, \boldsymbol{\eta}\right] \in \mathbb{R}^{2N\times 2MN^2}, \tag{16}$$

where $\boldsymbol{\eta}$ is a column matrix of MN elements, the value of each element is given by $\frac{1}{MN}$.

IV. Simulation Results

A network of $100\text{m} \times 100\text{m}$ is considered. 4 ANs at $(25, 25), (25, 75), (75, 25)$ and $(75, 75)$ are taken with 15 TN at random. The TNs are taken inside as well as outside the convex hull of the ANs. The network deployment is shown in Fig. 1.

Fig. 2 plots the bias of the system with and without the consideration of the unbiasing constant. The simulation is run independently υ times. The red curve represents the bias when δ_i is ignored. As can be seen the bias increases with the increase in noise variance of angle estimate. On the other hand the blue curve represents the bias when δ_i is considered. Fig. 2 shows that the system is unbiased at all variances if δ_i is taken in the model.

In Fig. 3, the cooperative and non-cooperative hybrid AoA-ToA algorithms are compared. For simplicity, the same noise variance in distance and angle measurements is used for all AN-TN and TN-TN links i.e $\sigma^2_{n_{ij}} = \sigma^2_{n_{jk}} = \sigma^2_n$ and $\sigma^2_{m_{ij}} = \sigma^2_{m_{jk}} = \sigma^2_m$. The performance in terms of the average root mean square error (Avg. RMSE) is compared while the variance in distance and angle estimates is increased. Fig. 3 shows that the cooperative model outperforms the non cooperative counterpart by a large margin.

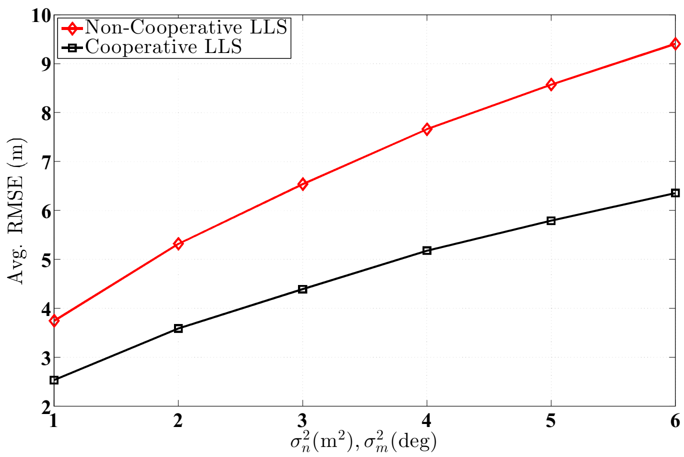

Figure 3. Performance comparison between LLS-Non cooperative and LLS-Cooperative hybrid AoA-ToA localization. ANs=4, TNs=15, $\upsilon = 1500$.

In Fig. 4 the performance of both systems is compared across distance noise variance along x-axis and angle noise variance across y-axis, while z-axis represents the average RMSE. Again as expected the cooperative model shows better performance than the non-cooperative version. From Fig. 4 the effect of both noise variance on system performance can be seen. It can be seen that the performance degrade at a higher rate with the increase in angle noise variance while distance noise variance has less effect on performance of both systems. This is due to the fact that the angle noise variance is distance dependent and thus degrades the performance of the system when distance between nodes is larger.

V. Conclusion and Future Work

In this paper we analyzed hybrid AoA-ToA model for localization. The model was made unbiased by the introduction

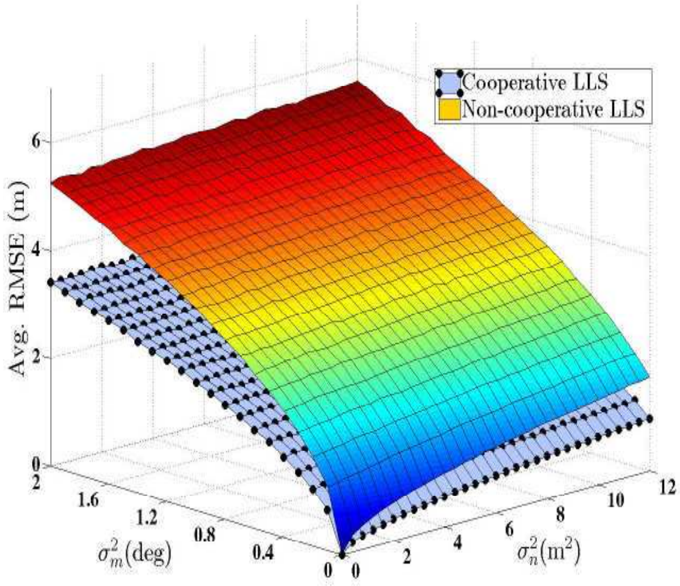

Figure 4. Performance comparison between LLS-Non cooperative and LLS-Cooperative hybrid AoA-ToA localization. ANs=4, TNs=15, x-axis (distance noise variance), y-axis (angle noise variance), z-axis (Avg. RMSE), v = 1500.

of the unbiasing constant. Furthermore a cooperative hybrid AoA-ToA LLS solution was proposed. Through simulation it was shown that by the introduction of unbiasing constant, the bias of the system is reduced to zero. It was also shown through simulation that the cooperative model shows better performance than the non-cooperative model.

The solution proposed in this paper can be extended to partial connectivity between TN-TN. Since the noise in the model is distance dependent, cooperation with nodes at large distance is not convenient as it degrades the overall performance of the system. Thus a new model can be proposed in which cooperation with only the nearby nodes is done. Furthermore the system can be extended to distributed model where the localization of all the TNs is not required.

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