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KINEMATICS AND DYNAMICS OF CO-OPERATING MANIPULATORS ON A MOBILE PLATFORM

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Abstract

This paper describes the kinematics and dynamics of multiple robotic arms mounted on a mobile platform and cooperatively handling a common object. The presence of closed kinematic chains along with a mobile platform of comparable mass and inertia with rest of the system makes the whole system dynamic analysis extremely complex. An attempt has been made to derive a complete unified dynamic model and the simulation algorithms for a general type of cooperative robotic system, with maximum emphasis on space robotic systems. All the derivations has been made in relation to space free-flying and free-floating robot manipulators.

1 Introduction

One of the most important and increasingly popular area of robotic systems research is the cooperative robotic systems. This is because of their capability to handle more difficult jobs which can not be easily handled by single manipulator. These jobs include, handling large, heavy or non-rigid objects; mating of mechanical parts; and space robotic applications. The potentiality of these systems can only be achieved through an efficient coordination control strategy. But the control and analysis of coordinated systems are much more complex due to the presence of inherent kinematic and dynamic interactions to maintain any desired cooperation. The dynamics and control of fixed base cooperating manipulators have been studied rigorously in the last decade [1-5].

When the platform is mobile, it makes the system even more complex. A mobile platform with complete motion freedom, where the attitude can rotate about three axes as well as translate along spatial x, y and z axes, can be modelled as a six degree of freedom system such as free-flying or free-floating space robotic systems, or it can be modelled as a partially constrained system with lesser than six degree of freedom. In the case of free-flying space robots both the space craft and manipulator system are controlled simultaneously, whereas once the spacecraft is positioned in a proper working space, to save jet fuel the spacecraft thruster system can be shut off and this is known as a free-floating space robot [25].

The control of such type of mobile base robots is much challenging when its mass and inertia is comparable with that of the rest of the robotic system consisting of all the manipulators and the object it is handling. In this case, the motion of the platform depends on the motion of the rest of the system. The control of such systems must take into account the dynamic coupling between the platform and the rest of the system. This necessitates the proper understanding of the kinematics and dynamics of these systems. Although most of the research on space robotics are concentrated on the analysis and control of single manipulator, but recently multiple manipulator systems are gaining more attention [6,7].

Longman et al. [8] have described kinematics of a satellite mounted robot arm and computed the moments required to drive the reaction wheels, which is only valid for systems with attitude controlled base. Vafa and Dubowsky [9, 10] have used the concept of virtual manipulator to represent kinematics and dynamics of free-floating space manipulators. They have used this concept for path planning of space manipulators which incorporates the minimisation of attitude disturbance [11].

Papadopoulos and Dubowsky [12-14] have used barycentric vector approach to analyse the kinematics and dynamics of single manipulator free-floating robots. Also this method has been extended by Papadopoulos and Moosavian [15, 16] for multiple arm free-floating robotic systems. Moosavian and Papadopoulos [7] have shown that direct path method is more accurate and efficient than barycentric vector approach for modelling multiple manipulator space free-flying robots.

Umetani and Yoshida [17] have presented a generalised Jacobian matrix, which is used for resolved motion rate control of free-flying manipulators. An efficient computational algorithm of generalised Jacobian matrix for multiarm free-flying space robots have been described by Yokohji et al. [18], with the limitation that this method is very slow and is not useful for closed loop robotic systems. Xu and Shum [19] have presented the role of dynamic coupling and their effects on the control aspects of a free-floating single manipulator robotic system. Saha [20] has derived very efficient direct and inverse kinematic algorithms for a space robot manipulator using the concept of primary body, where the end-effector serves as primary body. Also, Saha [21] extended the concepts of primary body along with natural orthogonal complements to derive an efficient recursive inverse dynamics algorithm for a single manipulator free-flying system.

Mukherjee and Nakamura [22] have presented an effective method for the inverse dynamics computation of space robotic systems in the framework of resolved acceleration control. Nakamura and Mukherjee [23] have developed a trajectory planning scheme for six-degree-of-freedom free-flying space robots in the presence of nonholonomic redundancy.

Murphy et al. [24] have described the closed form dynamic equations for a system of two PUMA manipulators installed on a mobile platform where these manipulators cooperatively manipulate a common object. Further, they have considered that the motion variables for both the mobile platform and manipulators are independent of each other. Hu and Vukovich [6] described the kinematics and dynamics of free-floating coordinated robots, and have proposed a position and force control for this free-floating coordinating robotic systems with closed kinematic constraints.

In this paper, an attempt has been made to derive a unified generalised dynamic model for a cooperating robotic system mounted on a mobile platform of comparable mass and inertia, where the later is subject to an external force. Of course, for a fixed base cooperating robotic system whose kinematics and dynamics are much simpler, can be represented as a particular case of this mobile robotic system having zero platform mobility, and infinite platform mass and inertia. Both, inverse dynamics and forward dynamics algorithms have been derived. It has been shown that the simulation of this cooperative mobile system can make use of any simulation algorithms for unconstrained multiple arm mobile system on a mobile platform. Although, a very negligible amount of research has been performed in the area of unconstrained multiple space robotic systems, but this analysis provides the means to incorporate efficient unconstrained multiple manipulator system dynamics methods into the analysis of complex constrained multiple manipulator space robotic systems. It has been shown that the developed model represents the dynamics of both free-flying and free-floating robotic systems. Spatial vector approach has been used through out the entire derivation process. This approach is analogous to the direct path method described by Moosavian and Papadopoulos [7] for the modelling of kinematics of multiple arm free-flying system.

2 System Modelling and Notations

The general model of a coordinated robot manipulator system with m-robots installed on a moving platform , is shown in Fig.1. Each robot consists of n_l links with n_l joints and each joint can be assumed to have d_f degree of freedom. The end-effectors of these robots hold rigidly a common object. When the total mass and inertia of the robotic manipulators and the object is comparable with that of the mass and inertia of the platform, which is the case in space robots where the spacecraft serves as the base, then the motion of the platform depends on the motion of the rest of the system. Because of this dependency of the platform motion on that of the motion of the robotic manipulator and object system, the acceleration of the platform is not known a priori and so the base acceleration remains unknown until the calculation of the acceleration of the rest of the manipulator system. But when the mass and inertia of the platform is much larger than the total mass and inertia of all the manipulators and the object system, then the base acceleration can be represented as an independent motion variable with complete knowledge of the platform motion, which makes the analysis of the entire system easier [24]. In this paper, the mass and inertia of the platform and the rest of the system has been considered to be comparable with each other.

To make the system more generalised, an external spatial force $\mathbf{f}_b$ is applied to the platform of the system. This additional force makes the entire system kinematic and dynamic analysis even more complex. In the absence of any external force, the momentum of the whole system is conserved and the initial momentum of the system can be assumed to be zero. This type of system is called free-floating system in the context of space robotics [25]. In space operation, often to conserve energy the spacecraft thruster is closed once the robotic system acquires the required position, which gives rise to a free-floating system . Now for the free-flying robotic systems the spacecraft thruster force required to control its position and attitude can be equivalently modelled as an external force acting on the platform. Hence, in the case of a free-floating space robotic system this external force can be assumed to be zero.

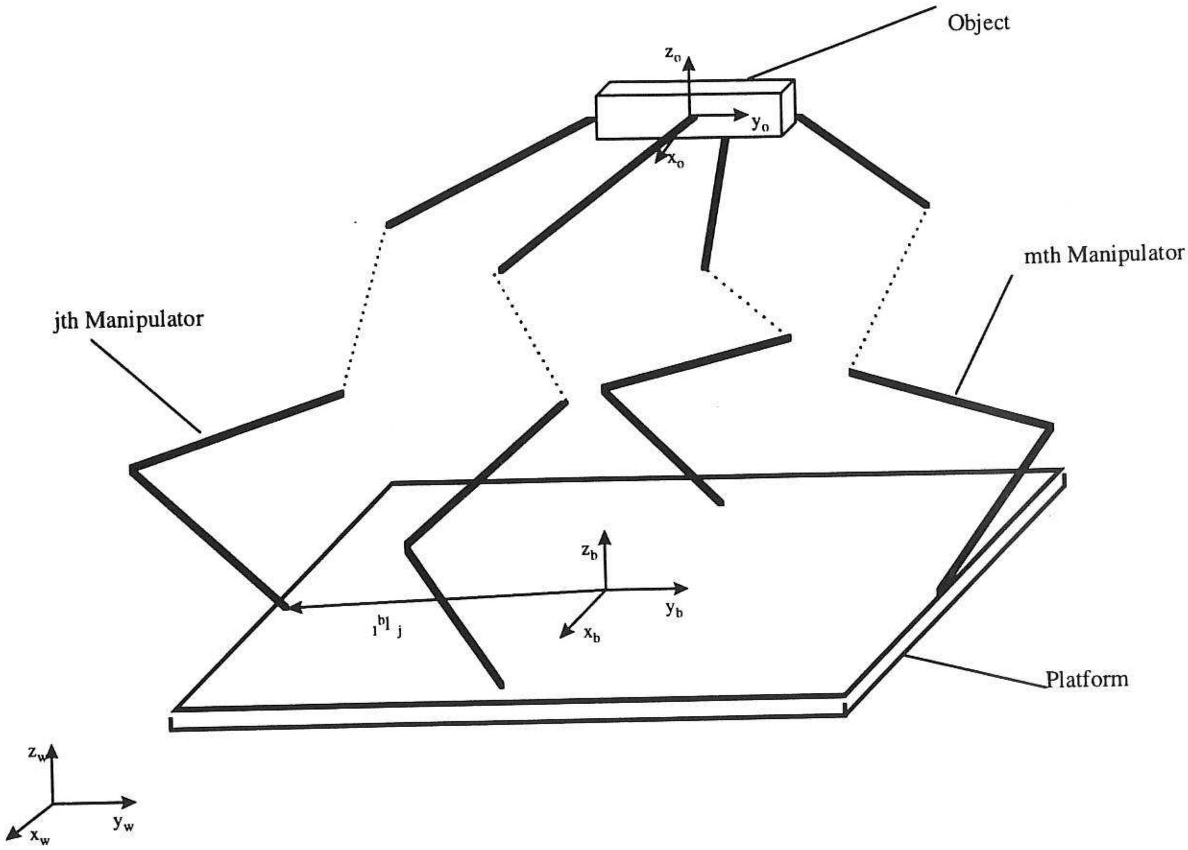

Fig. 1. A Cooperating Robotic System

Here, a co-ordinate free spatial notation has been followed, with many notational conventions from Featherstone [26], Roberson and Schwertassek [27], Brandl et al. [28], Rodriguez [29] and Kreutz-Delgado et al. [30]. The coordinate frames are defined according to a modified form of the Denavit-Hartenberg convention. As per this convention, the coordinate frame of a particular link is attached to that link with frame origin at the near end of the link. The coordinate frames assigned to the inertial reference, platform, end-effectors and object are (x_w, y_w, z_w), (x_b, y_b, z_b), {(x_{ej}, y_{ej}, z_{ej}), j = 1, 2, …., m}, and (x_o, y_o, z_o), respectively.

The spatial velocity, acceleration and force vectors of the ith link of the jth robot resolved in the ith link frame are denoted by 6×1 vector symbols ${}^i\mathbf{v}_j$, ${}^i\dot{\mathbf{v}}_j$ and ${}^i\mathbf{f}_j$, respectively, and are defined as

${}^i\mathbf{v}_j = [{}^i\varpi_j^T \;\; {}^i v_j^T]^T$, where ${}^i\varpi_j$ and ${}^i v_j$ are the 3×1 angular and linear velocity vectors, respectively of the ith link of the jth robot.

${}^i\dot{\mathbf{v}}_j = [{}^i\dot{\varpi}_j^T \;\; {}^i\dot{v}_j^T]^T$, where ${}^i\dot{\varpi}_j$ and ${}^i\dot{v}_j$ are the 3×1 angular and linear acceleration vectors, respectively of the ith link of the jth robot.

${}^{i}\mathbf{f}_j = [{}^{i}f_j^T \;\; {}^{i}\eta_j^T]^T$, where ${}^{i}f_j$ and ${}^{i}\eta_j$ are the 3×1 force and moment vectors, respectively of the ith link of the jth robot at the ith frame origin.

The 6×6 spatial transformation matrix ${}_{i-1}^{\;\;i}\mathbf{X}_j$ transforms a spatial vector from (i-1)th coordinate frame to the ith coordinate frame of the jth robot and is defined as (Featherstone, 1987) [26]

$$ {}_{i-1}^{\;\;i}\mathbf{X}_j = \begin{bmatrix} {}_{i-1}^{\;\;i}\mathbf{R}_j & \mathbf{0} \\ {}_{i-1}^{\;\;i}\mathbf{R}_j \, {}_{i-1}^{\;\;i}\tilde{\mathbf{p}}_j^T & {}_{i-1}^{\;\;i}\mathbf{R}_j \end{bmatrix} $$

where ${}_{i-1}^{\;\;i}\mathbf{R}_j$ is a 3×3 rotation matrix from the (i-1)th link frame to the ith link frame for the jth robot; ${}_{i-1}^{\;\;i}\mathbf{p}_j$ is a 3×1 vector from the origin of the (i-1)th link frame to the origin of the ith link frame for the jth robot; $\tilde{\mathbf{p}}$ for a vector $\mathbf{p} = \begin{bmatrix} p_x & p_y & p_z \end{bmatrix}^T$ is a 3×3 anti-symmetric matrix defined as

$$ \tilde{\mathbf{p}} = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix} $$

The 6×6 spatial inertia matrix of the ith link of the jth robot is denoted by ${}^{i}\mathbf{M}_j$ and is defined as (Featherstone, 1987) [26]

$$ {}^{i}\mathbf{M}_j = \begin{bmatrix} -{}^{i}m_j \, {}_{i}^{c}\tilde{\mathbf{l}}_j & {}^{i}m_j\mathbf{E} \\ {}^{i}\mathbf{I}_j & {}^{i}m_j \, {}_{i}^{c}\tilde{\mathbf{l}}_j \end{bmatrix} $$

where ${}^{i}m_j$ is the mass of the ith link of the jth robot; ${}^{i}\mathbf{I}_j$ is the inertia tensor of the ith link of the jth robot at the ith frame origin; ${}_{i}^{c}\mathbf{l}_j$ is the distance from the ith frame origin to the centre of mass of the ith link of the jth robot; and $\mathbf{E}$ is an identity matrix.

The general joint model used in this paper is described in Roberson and Schwertassek [27], Brandl et al. [28], and Lilly [31]. This general joint model has been defined with the incorporation of the orthogonal vectors ${}^{i}\mathbf{\Phi}_j$ and ${}^{i}\mathbf{\Phi}_j^c$, which represent matrix of free and constrained mode vectors of the ith joint of the jth robot, respectively.

3 Kinematics

3.1 Position and Velocity Relations

The position vector of any arbitrary point p on the link i of the jth robot can be expressed in the inertial frame as

$$ {}^{i}\mathbf{r}_{j}^{p} = \mathbf{r}_{b} + {}_{b}^{1}\mathbf{l}_{j} + \sum_{k=1}^{i-1} {}_{k}^{k+1}\mathbf{l}_{j} + {}_{i}^{p}\mathbf{l}_{j} \tag{1} $$

where $\mathbf{r}_b$ is the position vector of the centre of mass of the base with respect to inertial frame; ${}_{b}^{1}\mathbf{l}_{j}$ is the position vector of the joint 1 of the jth robot with respect to the centre of mass of the platform; ${}_{i}^{i+1}\mathbf{l}_{j}$ is the length of the link connecting frames i and i+1 of the jth robot; ${}_{i}^{p}\mathbf{l}_{j}$ is the position vector of the point p with respect to the ith link frame origin of the jth robot.

The linear velocity of the point p is obtained by differentiating Eq.(1) with respect to time, which can be represented as

$$ {}^{i}v_{j}^{p} = {}^{i}v_{j} + {}_{i}^{p}\tilde{\mathbf{l}}_{j}\ {}^{i}\varpi_{j} \tag{2} $$

where ${}^{i}v_{j}$ is the linear velocity of the ith link frame of the jth robot with respect to the inertial frame.

This equation will be used for the calculation of the total momentum of the system. Now, the spatial velocity of the ith link of the jth robot represented in the ith link frame with rheonomic constraints, i.e., the position constraints are explicit functions of time. Then the spatial velocity can be expressed as [24-27]

$$ {}^{i}\mathbf{V}_{j} = {}_{i-1}^{i}\mathbf{X}_{j}\ {}^{i-1}\mathbf{V}_{j} + {}^{i}\boldsymbol{\Phi}_{j}\ {}^{i}\dot{\mathbf{q}}_{j} + {}^{i}\xi_{j} \tag{3} $$

where ${}^{i}\boldsymbol{\Phi}_{j}$ and ${}^{i}\xi_{j}$ are functions of joint position and time. The matrix ${}^{i}\boldsymbol{\Phi}_{j}$ is a $6\times{}^{i}d_{j}$ matrix of free mode vectors of the joint i of the robot j resolved in the link frame of the jth robot, and has full column rank of ${}^{i}d_{j}$ which is the degree of freedom of the ith joint of the jth robot. It is always possible to construct a 6×6 regular matrix $[{}^{i}\boldsymbol{\Phi}_{j}\ \ {}^{i}\boldsymbol{\Phi}_{j}^{c}]$ [27, 28], where the six columns form a basis of $\Re^{6}$. ${}^{i}\boldsymbol{\Phi}_{j}^{c}$ is a $6\times(6-{}^{i}d_{j})$ matrix of constrained mode vectors of the ith joint of the jth robot.

Now, Eq.(3) can alternatively be represented in terms of the velocity of the base $\mathbf{v}_b$ as

$$ {}^{i}\mathbf{v}_j = {}^{i}_{b}\mathbf{X}_j \ \mathbf{v}_b + \sum_{k=1}^{i} {}^{i}_{k}\mathbf{X}_j \ {}^{k}\boldsymbol{\Phi}_j \ {}^{k}\dot{\mathbf{q}}_j + \sum_{k=1}^{i} {}^{i}_{k}\mathbf{X}_j \ {}^{k}\xi_j $$

$$ = {}^{i}_{b}\mathbf{X}_j \ \mathbf{v}_b + \sum_{k=1}^{i} {}^{i}_{k}\mathbf{X}_j \left({}^{k}\boldsymbol{\Phi}_j \ {}^{k}\dot{\mathbf{q}}_j + {}^{k}\xi_j \right) \tag{4} $$

The spatial velocity of all the links of the jth robot can be expressed concisely as

$$ \mathbf{v}_j = {}_{b}\mathbf{X}_j \ \mathbf{v}_b + \mathbf{X}_j (\boldsymbol{\Phi}_j \ \dot{\mathbf{q}}_j + \xi_j) \tag{5} $$

where $\mathbf{v}_j = [{}^{1}\mathbf{v}_j^T \ {}^{2}\mathbf{v}_j^T \ldots\ldots\ldots {}^{n}\mathbf{v}_j^T]^T$, $\dot{\mathbf{q}}_j = [{}^{1}\dot{\mathbf{q}}_j^T \ {}^{2}\dot{\mathbf{q}}_j^T \ldots\ldots\ldots {}^{n}\dot{\mathbf{q}}_j^T]^T$, ${}_{b}\mathbf{X}_j = [{}^{1}_{b}\mathbf{X}_j^T \ {}^{2}_{b}\mathbf{X}_j^T \ldots\ldots\ldots {}^{n}_{b}\mathbf{X}_j^T]^T$, $\boldsymbol{\Phi}_j = \mathrm{diag}({}^{1}\boldsymbol{\Phi}_j \ {}^{2}\boldsymbol{\Phi}_j \ldots\ldots\ldots {}^{n}\boldsymbol{\Phi}_j)$, $\xi_j = [{}^{1}\xi_j^T \ {}^{2}\xi_j^T \ldots\ldots\ldots {}^{n}\xi_j^T]^T$, and the matrix $\mathbf{X}_j$ is given as

$$ \mathbf{X}_j = \begin{bmatrix} {}^{1}_{1}\mathbf{X}_j & \mathbf{0} & \mathbf{0} & . & . & . & \mathbf{0} \\ {}^{2}_{1}\mathbf{X}_j & {}^{2}_{2}\mathbf{X}_j & \mathbf{0} & . & . & . & \mathbf{0} \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ {}^{n}_{1}\mathbf{X}_j & {}^{n}_{2}\mathbf{X}_j & . & . & . & . & {}^{n}_{n}\mathbf{X}_j \end{bmatrix} $$

For all the m robots the Eq.(5) can be concisely expressed as

$$ \mathbf{v} = \mathbf{X}_b \ \mathbf{v}_b + \mathbf{X}(\boldsymbol{\Phi} \ \dot{\mathbf{q}} + \xi) \tag{6} $$

where $\mathbf{v} = [\mathbf{v}_1^T \ \mathbf{v}_2^T \ldots\ldots\ldots \mathbf{v}_m^T]^T$, $\mathbf{X}_b = [{}_{b}\mathbf{X}_1^T \ {}_{b}\mathbf{X}_2^T \ldots\ldots\ldots {}_{b}\mathbf{X}_m^T]^T$, $\mathbf{X} = \mathrm{diag}(\mathbf{X}_1 \ \mathbf{X}_2 \ldots\ldots\ldots \mathbf{X}_m)$, $\boldsymbol{\Phi} = \mathrm{diag}(\boldsymbol{\Phi}_1 \ \boldsymbol{\Phi}_2 \ldots\ldots\ldots \boldsymbol{\Phi}_m)$, $\dot{\mathbf{q}} = [\dot{\mathbf{q}}_1^T \ \dot{\mathbf{q}}_2^T \ldots\ldots\ldots \dot{\mathbf{q}}_m^T]^T$, and $\xi = [\xi_1^T \ \xi_2^T \ldots\ldots\ldots \xi_m^T]^T$.

The end-effector velocity of the jth robot denoted by $\mathbf{v}_j^e$ and is defined as per Eq.(3) as

$$ \mathbf{v}_j^e = {}^{n+1}\mathbf{v}_j = {}^{n+1}_{n}\mathbf{X}_j \ {}^{n}\mathbf{v}_j $$

Now using Eq.(4), we can get

$$ \mathbf{v}_j^e = {}^{n+1}_{n}\mathbf{X}_j \left[{}^{n}_{b}\mathbf{X}_j \ \mathbf{v}_b + \sum_{k=1}^{n} {}^{n}_{k}\mathbf{X}_j \left({}^{k}\boldsymbol{\Phi}_j \ {}^{k}\dot{\mathbf{q}}_j + {}^{k}\xi_j \right) \right] $$

$$ = {}^{n+1}_{b}\mathbf{X}_j \, \mathbf{v}_b + {}^{n+1}_{n}\mathbf{X}_j \, \mathbf{L}_j (\boldsymbol{\Phi}_j \ \dot{\mathbf{q}}_j + \xi_j) \tag{7} $$

where $\mathbf{L}_j = [{}^{n}_{1}\mathbf{X}_j \, {}^{n}_{2}\mathbf{X}_j \ldots\ldots\ldots {}^{n}_{n}\mathbf{X}_j]^T$.

The end-effector velocity expressed in Eq.(7) is resolved in the end-effector frame, which can be expressed in the base frame:

$$\mathbf{v}^e_j = {}_b\mathbf{R}_j \, {}^{n+1}_{\;\;b}\mathbf{X}_j \, \mathbf{V}_b + {}_b\mathbf{R}_j \, {}^{n+1}_{\;\;n}\mathbf{X}_j \, \mathbf{L}_j(\boldsymbol{\Phi}_j \, \dot{\mathbf{q}}_j + \xi_j)$$

$$= \mathbf{J}_{bj} \, \mathbf{V}_b + \mathbf{J}_{qj} \, \dot{\mathbf{q}}_j + \mathbf{k}_j \, \xi_j \qquad (8a)$$

where ${}_b\mathbf{R}_j = \text{diag}({}^{\;\;b}_{n+1}\mathbf{R}_j \; {}^{\;\;b}_{n+1}\mathbf{R}_j)$ with ${}^{\;\;b}_{n+1}\mathbf{R}_j$ is a rotation matrix from the end-effector frame to the base frame, $\mathbf{k}_j = {}_b\mathbf{R}_j \, {}^{n+1}_{\;\;n}\mathbf{X}_j \, \mathbf{L}_j$, $\mathbf{J}_{qj} = {}_b\mathbf{R}_j \, {}^{n+1}_{\;\;n}\mathbf{X}_j \, \mathbf{L}_j \, \boldsymbol{\Phi}_j$, and $\mathbf{J}_{bj} = {}_b\mathbf{R}_j \, {}^{n+1}_{\;\;b}\mathbf{X}_j$.

For all the m-robots, Eq.(8) can be represented as

$$\mathbf{v}^e = \mathbf{J}_b \, \mathbf{V}_b + \mathbf{J}_q \, \dot{\mathbf{q}} + \mathbf{k} \, \xi \qquad (8b)$$

3.2 Momentum

The total momentum of the entire system with respect to the inertial reference frame is the sum of the individual rigid body momentum due to the base ($\mathbf{P}_{base}$), links of all the manipulators ($\mathbf{P}_{links}$), and object ($\mathbf{P}_{obj}$), each represented in the common inertial reference frame, and can be expressed as

$$\mathbf{P} = \mathbf{P}_{base} + \mathbf{P}_{links} + \mathbf{P}_{obj} \qquad (9)$$

Momentum of the base can be represented as

$$\mathbf{P}_{base} = {}_w\mathbf{M}_b \, \mathbf{V}_b \qquad (10)$$

where ${}_w\mathbf{M}_b$ can be expressed as

$${}_w\mathbf{M}_b = \begin{bmatrix} \mathbf{0} & m_b\mathbf{E} \\ \mathbf{I}_b & m_b\tilde{\mathbf{r}}_b \end{bmatrix},$$ with m_b and $\mathbf{I}_b$ are the mass and moment of inertia of the base.

Now the momentum of the links of all the manipulators can be expressed as sum of momentum of the individual links of all the manipulators:

$$\mathbf{P}_{links} = \sum_{j=1}^{m}\sum_{i=1}^{n} {}^{\;i}_{w}\mathbf{M}_j \, {}^{i}\mathbf{V}^c_j \qquad (11)$$

where ${}^{i}\mathbf{V}_j^c$ is the mass centre velocity of the ith link of the jth robot, and using Eq.(2) this can be expressed in terms of the link spatial velocities as

$${}^{i}\mathbf{V}_j^c = {}^{i}\mathbf{T}_j\,{}^{i}\mathbf{V}_j, \text{ where } {}^{i}\mathbf{T}_j \text{ can be expressed as}$$

$${}^{i}\mathbf{T}_j = \begin{bmatrix} \mathbf{E} & 0 \\ -{}^{c}_{i}\tilde{\mathbf{l}}_j & \mathbf{E} \end{bmatrix} \tag{12}$$

with ${}^{c}_{i}\tilde{\mathbf{l}}_j$ as the position vector of mass centre of the ith link of the jth robot from the ith link frame, and ${}^{i}_{w}\mathbf{M}_j$ can be expressed as

$${}^{i}_{w}\mathbf{M}_j = \begin{bmatrix} \mathbf{0} & {}^{i}m_j\mathbf{E} \\ {}^{i}\mathbf{I}_j & {}^{i}m_j\,{}^{i}\tilde{\mathbf{r}}_j^c \end{bmatrix}$$ with ${}^{i}\mathbf{r}_j^c$ as the position vector of the mass centre of the ith link of the jth robot with respect to the inertial frame.

Now for the jth robot, Eq.(12) can be expressed in the matrix form as

$$\mathbf{V}_j^c = \mathbf{T}_j\,\mathbf{V}_j \tag{13}$$

where $\mathbf{T}_j = \mathrm{diag}({}^{1}\mathbf{T}_j\;{}^{2}\mathbf{T}_j \,\ldots\ldots\ldots\, {}^{n}\mathbf{T}_j)$.

From Eq.(11) the total momentum of all the manipulators can be represented in the vector matrix as

$$\mathbf{P}_{links} = \sum_{j=1}^{m} {}_{w}\mathbf{M}_j\mathbf{V}_j^c \tag{14}$$

where ${}_{w}\mathbf{M}_j = [\,{}^{1}_{w}\mathbf{M}_j\;{}^{2}_{w}\mathbf{M}_j \,\ldots\ldots\ldots\, {}^{n}_{w}\mathbf{M}_j]$, and $\mathbf{V}_j^c = [({}^{1}\mathbf{V}_j^c)^T\;({}^{2}\mathbf{V}_j^c)^T \,\ldots\ldots\ldots\, ({}^{n}\mathbf{V}_j^c)^T]^T$.

Using Eq.(13) and (5) in Eq.(14), we can get

$$\mathbf{P}_{links} = \sum_{j=1}^{m} {}_{w}\mathbf{M}_j\mathbf{T}_j\mathbf{V}_j$$

$$= \sum_{j=1}^{m} {}_{w}\mathbf{M}_j\mathbf{T}_j\,{}_{b}\mathbf{X}_j\,\mathbf{V}_b + {}_{w}\mathbf{M}_j\mathbf{T}_j\,\mathbf{X}_j(\boldsymbol{\Phi}_j\,\dot{\mathbf{q}}_j + \xi_j) \tag{15}$$

Now Eq.(15) can be expressed in the vector matrix form as

$$\mathbf{P}_{links} = \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b\,\mathbf{V}_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}(\boldsymbol{\Phi}\,\dot{\mathbf{q}} + \xi) \tag{16}$$

where $\mathbf{M}_w = [{}_w\mathbf{M}_1 \; {}_w\mathbf{M}_2 \ldots\ldots\ldots {}_w\mathbf{M}_m]$ and $\mathbf{T} = \mathrm{diag}(\mathbf{T}_1 \; \mathbf{T}_2 \ldots\ldots\ldots \mathbf{T}_m)$.

Now the momentum of the object with moment of inertia $\mathbf{I}_o$, mass m_o, velocity $\mathbf{V}_o$ and mass centre position vector in the inertial frame $\mathbf{r}_o$ can be represented as

$$\mathbf{P}_{obj} = {}_w\mathbf{M}_o \mathbf{V}_o \tag{17}$$

where ${}_w\mathbf{M}_o = \begin{bmatrix} \mathbf{0} & m_o\mathbf{E} \\ \mathbf{I}_o & m_o\tilde{\mathbf{r}}_o \end{bmatrix}$.

The object velocity in terms of the end-effector velocity can be represented as

$$\mathbf{V}_o = \mathbf{v}^o = \mathbf{W}^{-T} \mathbf{v}^e \tag{18}$$

where $\mathbf{W}$ is the grasp matrix. This is discussed in detail under the heading of object dynamics.

Now using the value of $\mathbf{v}^e$ from Eq.(8b) into Eq.(18) gives

$$\begin{aligned} \mathbf{V}_o &= \mathbf{W}^{-T}(\mathbf{J}_b \mathbf{V}_b + \mathbf{J}_q \dot{\mathbf{q}} + \mathbf{k}\,\xi) \\ &= \mathbf{J}'_b \mathbf{V}_b + \mathbf{J}'_q \dot{\mathbf{q}} + \mathbf{k}'\,\xi \end{aligned} \tag{19}$$

So from Eq.(17) and (19), the momentum of the object can be expressed as

$$\mathbf{P}_{obj} = {}_w\mathbf{M}_o(\mathbf{J}'_b \mathbf{V}_b + \mathbf{J}'_q \dot{\mathbf{q}} + \mathbf{k}'\,\xi) \tag{20}$$

Hence, the momentum of the entire system using Eqs.(9), (10), (16) and (20), can be expressed as

$$\begin{aligned} \mathbf{P} = &({}_w\mathbf{M}_b + {}_w\mathbf{M}_o \mathbf{J}'_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)\mathbf{V}_b + (\mathbf{M}_w\mathbf{T}\,\mathbf{X}\,\boldsymbol{\Phi} + {}_w\mathbf{M}_o \mathbf{J}'_q)\dot{\mathbf{q}} \\ &+ (\mathbf{M}_w\mathbf{T}\,\mathbf{X} + {}_w\mathbf{M}_o \mathbf{k}')\xi \end{aligned} \tag{21}$$

As discussed earlier, for a free-floating space robotic system the total momentum of the system P will be zero, therefore from Eq.(21) the base velocity can be expressed as

$$\begin{aligned} \mathbf{V}_b &= -({}_w\mathbf{M}_b + {}_w\mathbf{M}_o \mathbf{J}'_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)^{-1}\{(\mathbf{M}_w\mathbf{T}\,\mathbf{X}\,\boldsymbol{\Phi} + {}_w\mathbf{M}_o \mathbf{J}'_q)\dot{\mathbf{q}} \\ &\qquad + (\mathbf{M}_w\mathbf{T}\,\mathbf{X} + {}_w\mathbf{M}_o \mathbf{k}')\xi\} \\ &= \mathbf{J}_r \dot{\mathbf{q}} + \mathbf{k}_1 \xi \end{aligned} \tag{22}$$

where $\mathbf{J}_r = -({}_w\mathbf{M}_b + {}_w\mathbf{M}_o \mathbf{J}'_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)^{-1}(\mathbf{M}_w\mathbf{T}\,\mathbf{X}\,\boldsymbol{\Phi} + {}_w\mathbf{M}_o \mathbf{J}'_q)$ and $\mathbf{k}_1 = -({}_w\mathbf{M}_b + {}_w\mathbf{M}_o \mathbf{J}'_b + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)^{-1}(\mathbf{M}_w\mathbf{T}\,\mathbf{X} + {}_w\mathbf{M}_o \mathbf{k}')$.

The m end-effector velocity vector of a free-floating robotic system can be derived using Eqs.(8b) and (22):

$$\mathbf{v}^e = (\mathbf{J}_b\,\mathbf{J}_r + \mathbf{J}_q)\dot{\mathbf{q}} + (\mathbf{J}_b\,\mathbf{k}_1 + \mathbf{k})\xi = \mathbf{J}\,\dot{\mathbf{q}} + \mathbf{k}_c\,\xi \quad (23)$$

If the manipulator joint constraints are assumed not to exhibit rheonomic constraints, then Eq.(23) can be represented as

$$\mathbf{v}^e = \mathbf{J}\,\dot{\mathbf{q}} \quad (24)$$

Above relationship between $\mathbf{v}^e$ and $\dot{\mathbf{q}}$ will not be valid for free-flying robotic systems as the momentum is no more conserved. Hence the thruster force modelled as an external force acting on the platform plays a crucial role in the analysis of free-flying systems. So, in this case the external spatial force applied to the base can be expressed as the time rate of change of the total momentum $\mathbf{P}$:

$$\mathbf{f}_b = \dot{\mathbf{P}} = ({}_w\mathbf{M}_b + {}_w\mathbf{M}_o\,\mathbf{J}_b' + \mathbf{M}_w\mathbf{T}\,\mathbf{X}_b)\dot{\mathbf{V}}_b + (\mathbf{M}_w\mathbf{T}\,\mathbf{X}\,\boldsymbol{\Phi} + {}_w\mathbf{M}_o\,\mathbf{J}_q')\ddot{\mathbf{q}} + \mathbf{b}_r \quad (25)$$

where $\mathbf{b}_r$ is the net bias force acting on the system, which is a function of position, velocity and time.

The end-effector velocity vector of free-flying space robots without joint rheonomic constraints and using Eq.(8b), can be expressed in the following concise form:

$$\mathbf{v}^e = \mathbf{J}_b\,\mathbf{V}_b + \mathbf{J}_q\,\dot{\mathbf{q}}$$

$$= [\mathbf{J}_b \;\; \mathbf{J}_q]\begin{bmatrix}\mathbf{V}_b \\ \dot{\mathbf{q}}\end{bmatrix} = \mathbf{J}_g\,\mathbf{V}_g \quad (26)$$

where $\mathbf{J}_g$ and $\mathbf{V}_g$ are generalised Jacobian and velocity of the system, respectively.

4 Inverse Dynamics

4.1 Force Relations

It has been shown that the acceleration of the platform $\dot{\mathbf{V}}_b$ is dependent on the motion of the manipulators and object, and so can only be obtained after the computation of the manipulator and object motion variables. The acceleration of the links of the manipulators can be obtained by differentiating Eq.(6) with respect to time, which yields

$$\dot{\mathbf{V}} = \dot{\mathbf{X}}_b \mathbf{V}_b + \mathbf{X}_b \dot{\mathbf{V}}_b + \dot{\mathbf{X}}(\boldsymbol{\Phi}\, \dot{\mathbf{q}} + \boldsymbol{\xi}) + \mathbf{X}(\boldsymbol{\Phi}\, \ddot{\mathbf{q}} + \dot{\boldsymbol{\Phi}}\, \dot{\mathbf{q}} + \dot{\boldsymbol{\xi}})$$

$$= \mathbf{X}\, \boldsymbol{\Phi}\, \ddot{\mathbf{q}} + \dot{\mathbf{X}}_b \mathbf{V}_b + \mathbf{X}_b \dot{\mathbf{V}}_b + \boldsymbol{\zeta} \qquad (27)$$

where $\boldsymbol{\zeta} = \dot{\mathbf{X}}(\boldsymbol{\Phi}\, \dot{\mathbf{q}} + \boldsymbol{\xi}) + \mathbf{X}(\dot{\boldsymbol{\Phi}}\, \dot{\mathbf{q}} + \dot{\boldsymbol{\xi}})$ and is a function of position, velocity and time. In this equation $\dot{\mathbf{V}}_b$ is an unknown variable, whereas $\mathbf{V}_b$ is known.

The force exerted on the ith link of the jth robot is expressed as [24-26]

$${}^{i}\mathbf{f}_j = {}_{i+1}^{\;\;i}\mathbf{X}_j \; {}^{i+1}\mathbf{f}_j + {}^{i}\mathbf{M}_j \, {}^{i}\dot{\mathbf{V}}_j + {}^{i}\mathbf{b}_j \qquad (28)$$

where ${}^{i}\mathbf{b}_j$ is the bias force on the ith link of the jth robot, which is defined as the force applied to the rigid body to produce zero spatial acceleration.

Above Eq.(28) can also be represented in terms of the end-effector force ${}^{n+1}\mathbf{f}_j$:

$${}^{i}\mathbf{f}_j = {}_{n+1}^{\;\;\;i}\mathbf{X}_j \; {}^{n+1}\mathbf{f}_j + \sum_{k=i}^{n} {}_{k}^{i}\mathbf{X}_j \, ({}^{k}\mathbf{M}_j \, {}^{k}\dot{\mathbf{V}}_j + {}^{k}\mathbf{b}_j) \qquad (29)$$

Now the force vector for the jth robot can be described as

$$\mathbf{f}_j = \mathbf{X}_j^T (\mathbf{D}_j \, {}^{n+1}\mathbf{f}_j + \mathbf{M}_j \dot{\mathbf{V}}_j + \mathbf{b}_j) \qquad (30)$$

where $\mathbf{f}_j = [{}^{1}\mathbf{f}_j^T \; {}^{2}\mathbf{f}_j^T \ldots\ldots\ldots {}^{n}\mathbf{f}_j^T]^T$, $\dot{\mathbf{V}}_j = [{}^{1}\dot{\mathbf{V}}_j^T \; {}^{2}\dot{\mathbf{V}}_j^T \ldots\ldots\ldots {}^{n}\dot{\mathbf{V}}_j^T]^T$, $\mathbf{b}_j = [{}^{1}\mathbf{b}_j^T \; {}^{2}\mathbf{b}_j^T \ldots\ldots\ldots {}^{n}\mathbf{b}_j^T]^T$, $\mathbf{D}_j = [0\ldots\ldots\ldots 0 \; {}_{n}^{n+1}\mathbf{X}_j]^T$ and $\mathbf{M}_j = \mathrm{diag}({}^{1}\mathbf{M}_j \; {}^{2}\mathbf{M}_j \ldots\ldots\ldots {}^{n}\mathbf{M}_j)$

for all the m robots this force relationship can be expressed as

$$\mathbf{f} = \mathbf{X}^T (\mathbf{M}_q \dot{\mathbf{V}} + \mathbf{b}) + \mathbf{X}^T \mathbf{D}\, \mathbf{f}_e \qquad (31a)$$

This can be rewritten as

$$\mathbf{f} - \mathbf{X}^T \mathbf{D}\, \mathbf{f}_e = \mathbf{X}^T (\mathbf{M}_q \dot{\mathbf{V}} + \mathbf{b}) \qquad (31b)$$

where $\mathbf{f} = [\mathbf{f}_1^T \, \mathbf{f}_2^T \ldots\ldots\ldots \mathbf{f}_m^T]^T$, $\dot{\mathbf{V}} = [\dot{\mathbf{V}}_1^T \, \dot{\mathbf{V}}_2^T \ldots\ldots\ldots \dot{\mathbf{V}}_m^T]^T$, $\mathbf{b} = [\mathbf{b}_1^T \, \mathbf{b}_2^T \ldots\ldots\ldots \mathbf{b}_m^T]^T$, $\mathbf{D} = \mathrm{diag}(\mathbf{D}_1 \, \mathbf{D}_2 \ldots\ldots\ldots \mathbf{D}_m)$, $\mathbf{M}_q = \mathrm{diag}(\mathbf{M}_1 \, \mathbf{M}_2 \ldots\ldots\ldots \mathbf{M}_m)$, and $\mathbf{f}_e$ is the force exerted by the end-effector on the object, which can be expressed by the relationship $\mathbf{f}_e = [{}^{n+1}\mathbf{f}_1^T \; {}^{n+1}\mathbf{f}_2^T \ldots\ldots\ldots {}^{n+1}\mathbf{f}_m^T]^T$.

From Eq.(29) the force exerted by the base on the first link of the jth robot can be expressed as

$$ {}^{1}\mathbf{f}_j = {}^{1}_{n+1}\mathbf{X}_j\, {}^{n+1}\mathbf{f}_j + \sum_{k=1}^{n} {}^{1}_{k}\mathbf{X}_j\, ({}^{k}\mathbf{M}_j\, {}^{k}\dot{\mathbf{V}}_j + {}^{k}\mathbf{b}_j) $$

This force vector ${}^{1}\mathbf{f}_j$ is nothing but the first equation in the equation set derived in Eq.(30), which can alternatively be represented as

$$ {}^{1}\mathbf{f}_j = [{}^{1}_{1}\mathbf{X}_j\; {}^{1}_{2}\mathbf{X}_j \,\ldots\ldots\ldots\, {}^{1}_{n}\mathbf{X}_j] (\mathbf{D}\, \mathbf{f}_e + \mathbf{M}_q\, \dot{\mathbf{V}} + \mathbf{b}) \tag{32} $$

4.2 Base Dynamics

The force equilibrium equation of the base with an external force acting on it, can be represented as

$$ \mathbf{f}_b = \mathbf{M}_b\, \dot{\mathbf{V}}_b + \mathbf{b}_b + \sum_{j=1}^{m} {}^{b}_{1}\mathbf{X}_j\, {}^{1}\mathbf{f}_j \tag{33} $$

By substituting the value of ${}^{1}\mathbf{f}_j$ from Eq.(32) in Eq.(33), the modified expression for $\mathbf{f}_b$ is given as

$$ \mathbf{f}_b = \mathbf{M}_b\, \dot{\mathbf{V}}_b + \mathbf{b}_b + \sum_{j=1}^{m} {}^{b}\mathbf{X}_j^{T} (\mathbf{D}\, \mathbf{f}_e + \mathbf{M}_q\, \dot{\mathbf{V}} + \mathbf{b}) $$

$$ = \mathbf{M}_b\, \dot{\mathbf{V}}_b + \mathbf{b}_b + \mathbf{X}_b^{T} (\mathbf{D}\, \mathbf{f}_e + \mathbf{M}_q\, \dot{\mathbf{V}} + \mathbf{b}) \tag{34} $$

Now solving Eq.(34) for the base acceleration, which gives

$$ \dot{\mathbf{V}}_b = \mathbf{M}_b^{-1} \{ \mathbf{f}_b - \mathbf{b}_b - \mathbf{X}_b^{T} (\mathbf{D}\, \mathbf{f}_e + \mathbf{M}_q\, \dot{\mathbf{V}} + \mathbf{b}) \} \tag{35} $$

Then substituting the value of $\dot{\mathbf{V}}_b$ in Eq.(27), the acceleration of the manipulators can be expressed as

$$ \dot{\mathbf{V}} = (\mathbf{E} + \mathbf{X}_b\, \mathbf{M}_b^{-1}\, \mathbf{X}_b^{T}\, \mathbf{M}_q)^{-1} \{ \mathbf{X}\, \boldsymbol{\Phi}\, \ddot{\mathbf{q}} + \dot{\mathbf{X}}_b\, \mathbf{V}_b - \mathbf{X}_b\, \mathbf{M}_b^{-1}\, \mathbf{X}_b^{T} (\mathbf{D}\, \mathbf{f}_e + \mathbf{b}) $$

$$ + \mathbf{X}_b\, \mathbf{M}_b^{-1} (\mathbf{f}_b - \mathbf{b}_b) + \zeta \} \tag{36} $$

with known external force and base velocity, $\dot{\mathbf{V}}$ can be computed for the links of all the manipulators.

4.3 Equations of Motion

Now an explicit relation between the force and joint position, velocity and acceleration can be obtained by eliminating $\dot{\mathbf{V}}$ from Eqs.(36) and (31b):

$$\mathbf{f} - \mathbf{X}^T(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1}\mathbf{M}_q^{-1}\mathbf{D}\,\mathbf{f}_e = \mathbf{X}^T(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1}\{\mathbf{X}\,\Phi\,\ddot{\mathbf{q}}+\dot{\mathbf{X}}_b\mathbf{V}_b$$

$$+\mathbf{M}_q^{-1}\mathbf{b}+\mathbf{X}_b\mathbf{M}_b^{-1}(\mathbf{f}_b-\mathbf{b}_b)+\zeta\} \qquad (37)$$

where $(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1} = \mathbf{M}_q(\mathbf{E}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T\mathbf{M}_q)^{-1}$.

The joint torque $\mathbf{T}$ can be obtained by multiplying both the sides of the Eq.(37) with Φ^T, which gives

$$\mathbf{f} - \Phi^T\mathbf{X}^T(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1}\mathbf{M}_q^{-1}\mathbf{D}\,\mathbf{f}_e = \Phi^T\mathbf{X}^T(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1}\{\mathbf{X}\,\Phi\,\ddot{\mathbf{q}}$$

$$+\dot{\mathbf{X}}_b\mathbf{V}_b+\mathbf{M}_q^{-1}\mathbf{b}+\mathbf{X}_b\mathbf{M}_b^{-1}(\mathbf{f}_b-\mathbf{b}_b)+\zeta\}$$

This final expression for the joint torque vector can be concisely represented as

$$\mathbf{T} - \mathbf{J}^T\mathbf{f}_e = \mathbf{M}\,\ddot{\mathbf{q}} + \mathbf{C} \qquad (38)$$

where $\mathbf{M} = \Phi^T\mathbf{X}^T(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1}\mathbf{X}\,\Phi$, $\mathbf{J}^T = \Phi^T\mathbf{X}^T(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1}\mathbf{M}_q^{-1}\mathbf{D}$,

$\mathbf{C} = \Phi^T\mathbf{X}^T(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1}\{\dot{\mathbf{X}}_b\mathbf{V}_b+\mathbf{M}_q^{-1}\mathbf{b}+\mathbf{X}_b\mathbf{M}_b^{-1}(\mathbf{f}_b-\mathbf{b}_b)+\zeta\}$, and $\mathbf{T} = \Phi^T\mathbf{f}$.

The inversion of the matrix in the Eq.(37) can be simplified using the matrix inversion lemma, which gives

$$(\mathbf{M}_q^{-1}+\mathbf{X}_b\mathbf{M}_b^{-1}\mathbf{X}_b^T)^{-1} = \mathbf{M}_q - \mathbf{M}_q\mathbf{X}_b(\mathbf{M}_b + \mathbf{X}_b^T\mathbf{M}_b\mathbf{X}_b)^{-1}\mathbf{X}_b^T\mathbf{M}_q$$

Now in this simplified form the maximum order of the matrix to be inverted has decreased to a 6×6 matrix.

4.4 Object Dynamics

The object is assumed to be held rigidly by m manipulators, then the force at the centre of mass of the object due to all the end-effector forces acting on it, can be represented as [32, 35]

$$\mathbf{f}_o = \mathbf{W}^T\mathbf{f}_e \qquad (39)$$

where $\mathbf{W}$ is known as the grip matrix or grasp matrix and it depends upon the current centre of mass of the object and the end-effector contact points.

Using the principle of virtual work from classical mechanics that establishes the duality between forces and velocities, the end-effector velocity is expressed as

$$\mathbf{v}^e = \mathbf{W}\,\mathbf{v}^o \tag{40}$$

where $\mathbf{v}^o$ is the velocity of the centre of mass of the object.

The end-effector acceleration $\dot{\mathbf{v}}^e$ can be obtained by differentiating Eq.(2) with respect to time:

$$\dot{\mathbf{v}}^e = \mathbf{W}\,\dot{\mathbf{v}}^o + \dot{\mathbf{W}}\,\mathbf{v}^o \tag{41}$$

Still another representation for $\dot{\mathbf{v}}^e$ can be obtained from Eq.(8b):

$$\begin{aligned}\dot{\mathbf{v}}^e &= \mathbf{J}_b\,\dot{\mathbf{V}}_b + \mathbf{J}_q\,\ddot{\mathbf{q}} + \dot{\mathbf{J}}_b\,\mathbf{V}_b + \dot{\mathbf{J}}_q\,\dot{\mathbf{q}} + \mathbf{k}\,\dot{\xi} + \dot{\mathbf{k}}\,\xi \\ &= \mathbf{J}_b\,\dot{\mathbf{V}}_b + \mathbf{J}_q\,\ddot{\mathbf{q}} + \kappa \end{aligned} \tag{42}$$

The force balance equation for this object can be represented as

$$\mathbf{f}_o = \mathbf{M}_o\,\dot{\mathbf{v}}^o + \mathbf{b}_o \tag{43}$$

where $\mathbf{b}_o$ is the bias force on the object.

Now combining Eqs.(39) and (43), the following dynamic equation for the object can be obtained:

$$\mathbf{M}_o\,\dot{\mathbf{v}}^o + \mathbf{b}_o = \mathbf{W}^T\,\mathbf{f}_e \tag{44}$$

5 Forward Dynamics

The forward dynamics analysis of this cooperating manipulators on a mobile platform can be described with reference to Eq.(38), which involves the computation of the joint accelerations $\ddot{\mathbf{q}}$ with the knowledge of the input torques and forces, **T**, current state of the manipulator, $\mathbf{q}$, $\dot{\mathbf{q}}$ and motion of the base.

Simplifying Eq.(25) for $\dot{\mathbf{V}}_b$ gives

$$\dot{V}_b = ({}_wM_b + {}_wM_o\, J_b' + M_w T\, X_b)^{-1}\, \{ f_b - b_r - (M_w T\, X\, \Phi + {}_wM_o\, J_q')\ddot{q} \} \qquad (45)$$

From Eqs.(27) and (45), the value of $\dot{V}$ can be represented as

$$\dot{V} = \{ X\, \Phi - X_b ({}_wM_b + {}_wM_o\, J_b' + M_w T\, X_b)^{-1} (M_w T\, X\, \Phi + {}_wM_o\, J_q') \} \ddot{q} + \dot{X}_b\, V_b$$

$$+ ({}_wM_b + {}_wM_o\, J_b' + M_w T\, X_b)^{-1}\, (f_b - b_r) + \zeta \qquad (46)$$

Now the base acceleration can be calculated using Eqs.(35) and (46), which gives

$$\dot{V}_b = M_b^{-1} [(f_b - b_b) - X_b^T\, D\, f_e - X_b^T\, M_q\, \{ (X\, \Phi - X_b ({}_wM_b + {}_wM_o\, J_b' + M_w T\, X_b)^{-1}$$

$$(M_w T\, X\, \Phi + {}_wM_o\, J_q')) \ddot{q} + \dot{X}_b\, V_b + ({}_wM_b + {}_wM_o\, J_b' + M_w T\, X_b)^{-1} (f_b - b_r)$$

$$+ \zeta \} - X_b^T\, b]$$

$$= M_b^{-1} [(f_b - b_b) - X_b^T\, D\, f_e - X_b^T\, M_q\, \{ (X\, \Phi - X_b\, GH) \ddot{q} + \dot{X}_b\, V_b$$

$$+ G\, (f_b - b_r) + \zeta \} - X_b^T\, b] \qquad (47)$$

where $G = ({}_wM_b + {}_wM_o\, J_b' + M_w T\, X_b)^{-1}$ and $H = (M_w T\, X\, \Phi + {}_wM_o\, J_q')$.

Substituting the values for $\dot{V}_b$ in Eq.(42) gives

$$\dot{v}^e = \{ J_q - J_b\, M_b^{-1}\, X_b^T\, M_q (X\, \Phi - X_b\, GH) \} \ddot{q} + (\dot{J}_b - J_b\, M_b^{-1}\, X_b^T\, M_q\, \dot{X}_b) V_b$$

$$+ J_b\, M_b^{-1} (E - X_b^T\, M_q\, G)\, f_b + J_b\, M_b^{-1} (X_b^T\, M_q\, G b_r - b_b - X_b^T\, b + \zeta)$$

$$+ (\dot{J}_q\, \dot{q} + k\, \dot{\xi} + \dot{k}\, \xi) - J_b\, M_b^{-1}\, X_b^T\, D\, f_e$$

$$= \dot{v}^e_{open} - \dot{v}^e_{constrained} \qquad (48)$$

Hence, the system can be modelled as a superposition of open chain part and a constrained part due to the presence of cooperation.

Now using Eqs.(41) and (48), it is possible to find an explicit relationship between the end-effector force and object acceleration, which gives

$$\dot{v}^e_{constrained} = \dot{v}^e_{open} - W\, \dot{v}^o - \dot{W}\, v^o$$

$$\Rightarrow \quad f_e = (J_b\, M_b^{-1}\, X_b^T\, D)^{-1}\, [\dot{v}^e_{open} - W\, \dot{v}^o - \dot{W}\, v^o] \qquad (49)$$

substituting the value of f_e from Eq.(49) into Eq.(44) gives

$$\dot{v}^o = [M_o + W^T (J_b M_b^{-1} X_b^T D)^{-1} W]^{-1} \{ W^T (J_b M_b^{-1} X_b^T D)^{-1}$$

$$(\dot{v}^e_{open} - \dot{W} v^o) - b_o \} \tag{50}$$

Once the spatial acceleration of the object $\dot{v}^o$ is known, Eq.(49) can give all the end-effector spatial forces. Similarly from Eq.(38) $\ddot{q}$ can be represented as

$$\ddot{q} = M^{-1}(T - C) - J^T f_e$$

$$= \ddot{q}_{open} - \ddot{q}_{constrained} \tag{51}$$

Hence, the joint accelerations of the entire system can be computed with the known open chain joint accelerations and end effector forces. This necessitates the development of efficient algorithms for a relatively easier open chain dynamics.

6 Conclusions

In this paper a unified approach for the kinematics and dynamics of a cooperating robotic system on a mobile platform has been presented. Special emphasis has been given for space robot analysis. In the presence of closed kinematic chain constraints the kinematics and dynamics of space robots become increasingly complex. Both inverse dynamics and forward dynamics of these systems have been addressed. In addition, it has been shown that the simulation of this type of systems can be carried out with the use of any efficient multiarm unconstrained space robotics analysis approaches.

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