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Karrari, M and Nicholson, H. (1990) A Simplified Nonlinear Model for a Combined Heat and Power (CHP) System. Research Report. ACSE Research Report 416 . Department of Automatic Control and Systems Engineering

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**A SIMPLIFIED NONLINEAR MODEL FOR  
A COMBINED HEAT AND POWER(CHP) SYSTEM**

**by:**

*M. Karrari and H. Nicholson*

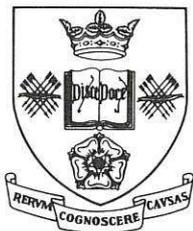
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Research Report No. 416  
November 1990

**Abstract:**

*Steady state errors, hunting and high interactions have been reported in the operation of combined heat and power (CHP) systems working with steam turbines. The design of currently employed regulators in industry is usually based on a linear model of the system. An improved system regulation can only be achieved if a more accurate mathematical model is available. In this paper, a simplified nonlinear model is derived which can be used to investigate the present problems and evaluate new controller performances.*

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### LIST OF SYMBOLS

- $A_{th}$  : throat area  
 $A$  : free surface area of water in drum  
 $D$  : rotor damping coefficient  
 $H$  : total energy stored in boiler  
 $i_d, i_q$  : d- and q- axis armature current  
 $i_{fd}$  : field current  
 $K_A$  : AVR gain  
 $K_c$  : constant gain  
 $m_s$  : rate of steam evaporation  
 $p$  : drum steam pressure  
 $P_D$  : power demand  
 $P_i$  : input power  
 $P_M$  : mechanical load  
 $P_o$  : output power  
 $P_{v_1}, P_{v_2}$  : inlet and pass-out valve position  
 $Q_D$  : heat demand  
 $r_a$  : armature resistance  
 $r_{fd}$  : field resistance  
 $s$  : rotor speed  
 $T$  : absolute temperature  
 $T_e$  : exciter time constant  
 $T_s$  : fall in mass of water in boiler,  
per unit increase in evaporation rate  
 $T_p$  : pipe time constants  
 $T_{v_1}, T_{v_2}$  : inlet and pass-out valve time constants  
 $u_1$  : fuel flow  
 $u_2$  : inlet-valve input signal  
 $u_3$  : feedwater flow  
 $u_4$  : pass-in valve input signal  
 $u_5$  : voltage reference value  
 $v$  : specific volume  
 $v_d, v_q$  : d- and q- axis terminal voltage

- $V_f$ : specific volume of saturated water
- $v_{fd}$ : field voltage
- $V_{ref}$ : voltage reference value
- $V_t$ : terminal voltage
- $W$ : mass flow
- $W_1, W_2$ : mass flows in high and low pressure turbines
- $W_e$ : extracted or pass-out steam flow
- $x_1$ : drum steam pressure
- $x_2$ : drum water level
- $x_3$ : inlet valve position
- $x_4$ : pass-out steam pressure
- $x_5$ : pass-in valve position
- $x_6$ : rotor angle
- $x_7$ : rotor speed deviation
- $x_8$ : flux linkage
- $x_9$ : exciter voltage
- $X_d, X_q$ : d- and q- axis synchronous reactances
- $X_{ad}$ : stator-rotor mutual reactance
- $X_e$ : transmission line reactance
- $X_{fd}$ : field reactance
- $y$ : drum water level
- $\pi_1, \pi_2$ : inlet and pass-out steam pressures
- $\phi_d, \phi_q$ : d- and q- axis stator flux linkages
- $\phi_{fd}$ : field flux linkages
- $\omega$ : angular frequency of rotor
- $\omega_o$ : infinite busbar angular frequency
- $\delta$ : rotor angle

## 1: Introduction

Cogeneration or combined heat and power(CHP) generation is a means of simultaneous generation of both electrical and thermal energy, using a single primary heat source. A typical power system, having an efficiency of 35% , wastes 15% of its input energy in boiler operation and 48% in the condensor, while a typical CHP system, with back-presure turbines, avoids most of the condensor losses, and can have an overall efficiency of more than 80%. This improved efficiency, which offers huge fuel saving, is the main motive behind cogeneration.

As the environment and energy conservation are very important issues concerning international bodies, cogeneration is becoming more and more a focus of attention. It not only helps to conserve energy sources, but also reduces CO<sub>2</sub> emissions substantially, when considered in a nationwide dimension.

CHP generation is a relatively old technology, which was practised even in the 19th century, but previously it has not been economically feasible except for its application in some large industries. Today, with a huge escalation of energy prices, the availability of advanced and cheap microprocessors capable of supporting decision-making on a real-time basis and new legislation to conserve energy sources are among the important factors which have given cogeneration a new life. The American Federal Energy Regulatory Comissions (FERC)'s report [1] shows a steady increase in the number of facilities for cogeneration after 1979 in the USA, from 28 facilities in 1980 to 419 in 1983 and 719 in 1985.

In a conventional power system, the main disturbance affecting system stability is that of electrical power demand. In combined heat and power (CHP) systems, two kinds of disturbances, electrical and thermal, may occur. This phenomena makes the systems more difficult to control. The objective of a controller in such a system is to control the electrical output at a constant voltage and frequency, while maintaining the extracted heat at desired characteristics.

Although numerous papers have been published concerning economical issues of CHP systems, little attempt has been made the overcome the problems of steady state errors, hunting or high interactions reported in CHP systems working with steam turbines. The design of an effective controller for such a highly interactive system requires an accurate mathematical model. Most of the published turbine models for CHP systems are linear [2],[3], which clearly are inadequate for

the study of global stability and regulation. In the present study, a simplified nonlinear model is developed for such CHP systems.

## 2: Modelling of a CHP system

A model of a CHP system as illustrated schematically in fig. 1 can be divided into three sub-systems: boiler, back-pressure turbine and synchronous generator:

### 2.1: Boiler model

Astrom, K.J. et al [4] have derived a simplified nonlinear model of a drum boiler using a combination of data analysis and physical arguments, with the boiler considered essentially as a reservoir of energy. Energy was fed to the reservoir by the fuel and the feedwater and the boiler delivered energy in the form of active power. The energy stored in the metal, water and steam masses was given by:

$$\frac{dH}{dt} = P_i - P_o \quad (1)$$

To obtain a simple model, the distribution of energy stored in the metal and water masses was considered to be constant during transients, and the energy stored in the boiler was approximated by:

$$H = a.p + b \quad (2)$$

where  $a$  and  $b$  are constants. Also the enthalpy difference between the feedwater and saturated state in the drum was assumed to be constant. The input power was thus given by:

$$P_i = k_1 u_1 - k_2 u_3 \quad (3)$$

where  $k_1$  and  $k_2$  are constants. The output power was modelled mainly by data analysis obtained by experiments on the boiler. The output power was given by:

$$P_o = k_3 ( u_2 p^{5/8} - k_4 ) \quad (4)$$

where  $k_3$  and  $k_4$  are constants. Using equations 4, 3 and 2 in the equation 1 gives:

$$\frac{dp}{dt} = -a_1 (u_2 p^{5/8} - a_5) + a_2 u_1 - a_3 u_3 \quad (5)$$

where  $a_i$  are constants.

The simulation results in [4] illustrated a relatively high accuracy of the boiler model. However, studies here showed that using this model for other boiler systems requires major modifications. The output power model obtained using data analysis based on the data obtained from a particular boiler system was inadequate for the present application. To obtain a more applicable simple boiler model, the same physical arguments and simplifications for the stored energy and input power were adopted, but the output power is also modelled by physical argument to avoid the particular problem.

In the literature (such as [5]) a complicated relation between pressure and mass flow is derived. However, for a given pressure ratio across the section, the relation can be simplified to:

$$\frac{W}{A_{th}} = K_1 \frac{p}{\sqrt{T}} \quad (6)$$

or:

$$\frac{W}{A_{th}} = K_2 \sqrt{\frac{p}{v}} \quad (7)$$

where  $K_1$  and  $K_2$  are constants.

These relations suggest that mass flow can be considered proportional to either the pressure or its square root, depending on the assumption of temperature or specific volume remaining constant over the operating range.

The relation of equation 7 is more accurate than that of equation 6 which assumes the characteristics of steam to be the same as a perfect gas. However equation 6 is of greater use because temperatures are more readily available than specific volumes and the relation is quite accurate over a relatively wide range [5].

The mass flow depends also on the valve position. Assuming a linear dependence of the mass flow and the valve position, and similar to the assumption made for the input power model, the output power is considered proportional to the mass flow, then the output power is given by:

$$P_o = \alpha_4 (u_2 p) \quad (8)$$

Incorporating these modifications in the boiler model gives:

$$\frac{dp}{dt} = -\alpha_1 (u_2 p) + \alpha_2 u_1 - \alpha_3 u_3 \quad (9)$$

where  $\alpha_i$  are revised constants .

The above boiler model describes only the gross behaviour of drum steam pressure of a boiler in a typical CHP system, and a simple water level model can be added to the boiler model. The mass flow of water in a boiler drum may increase either from excess of feedwater flow over evaporation rate, or by displacement of water from the tubes into the drum [6]. A simple water level model then takes the form:

$$\frac{dy}{dt} = \frac{V_f}{A} \cdot (u_3 - m_s + T_s \cdot \frac{dm_s}{dt}) \quad (10)$$

It is often good enough to regard the steam space as being of relatively negligible volume, which means that the steam flow and the rate of evaporation can be considered equal. This leads to a simpler water level model of the form:

$$\frac{dy}{dt} = \frac{V_f}{A} \cdot (u_3 - m_e + T_s \cdot \frac{dm_e}{dt}) \quad (11)$$

where  $m_e$  is the steam flow which is proportional to the product of the drum steam pressure and the inlet valve position, i.e:

$$m_e = G \cdot u_2 \cdot p \quad (12)$$

where  $G$  is a constant.

## 2.2: Back-pressure Turbine:

The back-pressure turbine considered here is a two stage, low and high pressure turbine. The inlet valve controls the steam mass flow to the high pressure turbine and the pass-out valve at the end of the high pressure turbine divides the steam flow between the low pressure turbine and the

heat supply pipes. The same physical principles used in boiler modelling are used here to model the relations between steam pressure, valve positions, mass flow and power. i.e:

$$W_1 = \beta_1 \pi_1 P_{v_1} \quad (13)$$

$$W_2 = \beta_2 \pi_2 P_{v_2} \quad (14)$$

$$W_e = W_1 - W_2 \quad (15)$$

where  $\beta_1$  and  $\beta_2$  are constants.

Valve dynamics are represented traditionally by a first order linear system with limits on the output of the system. The approach is adopted here for the inlet and pass-out valve dynamics.

Thus:

$$T_{v_1} \frac{dP_{v_1}}{dt} = u_1 - P_{v_1} \quad 0.0 \leq P_{v_1} \leq 1.0 \quad (16)$$

$$T_{v_2} \frac{dP_{v_2}}{dt} = u_2 - P_{v_2} \quad 0.0 \leq P_{v_2} \leq 1.0 \quad (17)$$

A first order system is also considered for the pipes transferring heat to the consumer [7].

i.e.:

$$T_p \frac{d\pi_2}{dt} = K_c (W_e - Q_D) - \beta_t \pi_2 \quad (18)$$

or:

$$T_p \frac{d\pi_2}{dt} = K_c (\beta_1 P_{v_1} \pi_1 - \beta_2 P_{v_2} \pi_2 - Q_D) - \beta_t \pi_2 \quad (19)$$

Fig. 2 shows steam flow-output power characteristics for a typical CHP system. As the mass flow and power are proportional for the whole range of operations, the mechanical output power of the turbine is a linear combination of the steam flows in the high and low pressure turbines. i.e.:

$$P_M = h_1 \beta_1 P_{v_1} \pi_1 + h_2 \beta_2 P_{v_2} \pi_2$$

where  $h_1$  and  $h_2$  are constants and the rotor speed variations are modelled by:

$$M \cdot \frac{ds}{dt} = h_1 \beta_1 P_{v_1} \pi_1 + h_2 \beta_2 P_{v_2} \pi_2 - P_D + D \cdot s \quad (20)$$

Equations 16, 17, 19 and 20 describe a fourth order turbine model developed for the CHP system.

### 2.3:Synchronous Generator:

The synchronous generator considered here is a single machine connected to an infinite busbar by a double circuit transmission line. The two axis theory [8], by which the phase coils of the synchronous generator are replaced by two fictitious ( $d,q$ ) axis coils, is adopted. Neglecting the damper windings, the armature resistance, the time derivatives of the stator flux linkages and the voltage variations caused by the rotor speed, the machine equations can be simplified to:

$$v_d = \Psi_q \quad (21)$$

$$v_q = -\Psi_d \quad (22)$$

$$v_{fd} = \frac{1}{\omega_o} \cdot \dot{\Psi}_{fd} + r_{fd} i_{fd} \quad (23)$$

where:

$$\Psi_{fd} = \omega_o \Phi_{fd} = X_{fd} i_{fd} + X_{ad} i_d \quad (24)$$

$$\Psi_d = \omega_o \Phi_d = X_{ad} i_{fd} + X_d i_d \quad (25)$$

$$\Psi_q = \omega_o \Phi_q = X_q i_q \quad (26)$$

For the transmission line:

$$v_q = e \cos \delta + X_e i_d \quad (27)$$

$$v_d = e \sin \delta - X_e i_q \quad (28)$$

The above equations can then be manipulated to give:

$$\dot{\Psi}_{fd} = K_1 \cdot U_1 + K_2 \cdot \Psi_{fd} + K_3 \cdot \cos \delta \quad (29)$$

where:

$$K_1 = \frac{\omega_o r_{fd}}{x_{ad}} \quad K_2 = \frac{-r_{fd} (X_d + X_e) \omega_o}{X_{fd} (X'_d + X_e)} \quad K_3 = \frac{X_{ad} \omega_o r_{fd} e}{X_{fd} (X'_d + X_e)}$$

$$U_1 = \frac{X_{ad}}{r_{fd}} v_{fd} \quad X'_d = X_d - \frac{X_{ad}^2}{X_{fd}}$$

As the machine is connected to an infinite busbar, the position of the reference axis is the same as that of the infinite busbar.i.e.:

$$\theta_r = \omega_o t \quad (30)$$

The rotor position is then given by:

$$\theta = \omega_o t - \delta \quad (31)$$

Differentiating gives:

$$\omega = \omega_o - \dot{\delta} \quad (32)$$

Now considering the rotor motion equations:

$$s = \dot{\delta} \quad (33)$$

$$\dot{s} = \frac{1}{M} \cdot (P_M - P_e - Ds) \quad (34)$$

where  $P_M$  is the shaft mechanical power, and  $P_e$  the electrical output power is given by:

$$P_e = v_d i_d - v_q i_q \quad (35)$$

Substituting voltage and current values from above equations gives:

$$\dot{s} = \frac{1}{M} P_M - \frac{D}{M} s - K_4 \psi_{fd} \sin \delta - K_5 \sin \delta \cos \delta \quad (36)$$

where:

$$K_4 = \frac{e X_{ad}}{M (X'_d + X_e) X_{fd}} \quad K_5 = \frac{(X'_d - X_q) e^2}{M (X'_d + X_e) (X_q + X_e)}$$

The exciter is considered to be a fast thyristor excitation system and together with the automatic voltage regulator (AVR) is represented by a first order, high gain, short time constant

linear model. i.e.:

$$T_e \dot{v}_{fd} = K_A (V_{ref} - V_t) - v_{fd} \quad (37)$$

Saturation in the magnetic field is considered by limiting the excitation field voltage between  $\pm 5$  p.u..

Equations 33, 36, 37 and 29 provide a fourth order synchronous generator model.

#### 2.4: Combined CHP model:

Equations 9,11,16,17,19,29,33,36 and 37 describe the behavior of a CHP system, and provide an improved form of representation. The state variables and control variables of the system are:

$$\dot{x}_1 = -\alpha_1 (x_2 x_1 - \alpha_5) + \alpha_2 u_1 - \alpha_3 u_3 \quad \text{--- Boiler model}$$

$$\dot{x}_2 = u_3 - G x_2 x_1 + T_s \cdot (G x_2 \dot{x}_1 + G x_1 \dot{x}_2)$$

$$\dot{x}_3 = (1/T_{v1}) u_2 - (1/T_{v1}) x_3$$

$$\dot{x}_4 = (K_c/T_p) \cdot (\beta_1 x_1 x_3 - \beta_2 x_4 x_5 - Q_D) - (\beta_f/T_p) x_4$$

$$\dot{x}_5 = (1/T_{v2}) u_4 - (1/T_{v2}) x_5$$

$$\dot{x}_6 = x_7$$

$$\dot{x}_7 = (h_1 \beta_1/M) x_1 x_3 + (h_2 \beta_2/M) x_4 x_5 - (D/M) x_7 - K_4 \sin x_6 \cdot x_8 - K_5 \sin 2 x_6$$

$$\dot{x}_8 = \omega_c x_9 - K_2 x_8 + K_3 \cos x_6$$

$$\dot{x}_9 = (K_e/T_e) \cdot (u_5 - V_t) - (1/T_e) x_9$$

The model can then be expressed in the state-space form:

$$\dot{X} = A X + B U + N_x$$

where  $A$  and  $B$  are system coefficient matrices of orders  $9 \times 9$  and  $9 \times 5$  respectively.  $X$  is a state-variable vector of order  $9 \times 1$ ,  $U$  is the input vector of order  $5 \times 1$  and  $N_x$  is a vector of order  $9 \times 1$  which contains the non-linearities.

The non-zero elements of the  $A$ ,  $B$  and  $N_x$  matrices are given in Appendix A and the parameters values of a typical CHP system are given in Appendix B.

### 3: Simulation Results

The fourth order Runge-Kutta integration method with an integration interval of 1 ms was used for all the simulations. Each subsystem is simulated separately, then the overall CHP system is considered.

#### 3.1: Boiler Model

The boiler model was obtained using the model derived in [4]. To investigate the validation of the new model, the same experiments as in [4] were carried out on the new model by simulation. It was found that there is little difference between the error of this model compared with the real data obtained by experiments, and that of the previous model.

Fig. 3 shows the value of output power against pressure in the two models. It is shown that for a wide range of operation (e.g.  $p > 0.6$  p.u.) the characteristics are very similar. Considering the fact that the previous model was not exactly consistent with the experimental results, the revised model can be justified for the CHP system.

Fig. 4 shows the response of the original simulated model with a variation of fuel flow. Feedwater flow was changed similar to the experiment as in [4] to keep drum-level within acceptable limits. Fig. 5 shows the simulation results from the same experiment with the new model. Comparing the pressure variation in the two models confirms the validation of the new model.

In summary, the original model was accurate over approximately the whole range of the boiler operations, but only for that particular boiler from which the experimental data had been obtained. The new model, however, is valid for shorter operating conditions, but more applicable for other boiler systems since it is based on purely physical arguments.

### 3.2: Turbine Model

Granelli, G.P. et al [9] report on a CHP system which consists of two back-pressure turbines working in parallel. For the smaller turbine, two proportional regulators are used, while for the larger turbine, a proportional regulator for rotor speed and a PI regulator for the pass-out steam-pressure are employed. The block-diagram of the regulator for the larger turbine is shown in fig. 6.

To demonstrate the response of the back-pressure turbine, with such a regulator, the regulator parameters were chosen to be:

$$KPS = 0.05$$

$$KPP = 0.075$$

$$KIP = 0.083$$

$$\alpha = 1.41$$

Fig. 7 shows the response of the back-pressure turbine with such a regulator. Fig. 7a shows the response after a 0.01 p.u. step change to the mechanical power demand, which indicates the interaction on the pass-out steam pressure and some steady state rotor speed error. Fig. 7b shows the response after a 0.023 p.u. step change to the thermal power demand, which indicates a large interaction on the rotor speed and again some steady state rotor speed error. Unacceptable steady rotor speed state error and high interaction between the loops are the weaknesses of such a regulator.

In [9], attempts have been made to tackle these problems by regulation of rotor speed by a PI rather than a P regulator and decoupling of the system loops. Fig. 8 shows the arrangement of such a regulator. To demonstrate the response of the back-pressure turbine, with such a regulator the parameters of the controller were chosen to be:

$$K11=1.0$$

$$K12=1.19$$

$$K21=1.0$$

$$K22=0.38$$

$$KIS=0.05$$

$$KPS=0.11$$

$$KIP=0.559$$

$$KPP=0.376$$

Fig. 9 shows the response of the turbine with the new regulator. The step change to mechanical power demand (9a), and thermal power demand (9b), are the same size as that of fig. 7. With

this new regulator, the rotor speed steady state error has been diminished but the system is more oscillatory . Although the effect of thermal output power and pass-out steam pressure changes on the rotor speed has been reduced, the effect of the mechanical output power and rotor speed changes on the pass-out steam pressure has been increased. The more the operating conditions deviate from the normal operating point for which the regulators have been designed, the more oscillations and interactions occur.

### **3.3: Synchronous Generator Model**

Fig. 10 shows the performance of the synchronous generator following a three-phase short circuit. The system is brought back to steady state values by the high-gain AVR and the fast excitation system. Nevertheless it shows a considerable amount of oscillation. The voltage is recovered very quickly, but the rotor angle, rotor speed and the output power settle after 4 seconds.

### **3.4: Combined CHP System Model**

When the combined heat and power system delivers power to a local load, the response of the system is not much different from the simulation results illustrated earlier when each subsystem was considered separately. The system is more oscillatory, but the increased oscillations can be reduced by changing the regulator parameters. However, when the CHP system is connected to an infinite bus-bar, the electrical frequency and rotor speed are imposed by the utility and subsequently any rotor speed steady state error is converted to output power steady state error. Fig. 11 shows the response of the turbine with the regulators of fig. 6, when the synchronous generator is connected to an infinite bus-bar.

## **4: Conclusion**

To avoid the shortcomings of previously published linear models, and to be able to investigate and overcome the current regulator problems, a simplified nonlinear model of a CHP model has been derived.

In the boiler-turbine, except limits which were considered fully in the model, the main non-linearity is the relation between mass flow and valve position. In the linear models reported in the literature, they are considered to be proportional which is widely invalid for global system regulation. Here, mass flow was considered to be proportional to the product of valve position and steam pressure. Although this relation is also based on some simplifications, it describes the experimental results more accurately. Consideration of the full relation would result in different problems of model complication and regulator design difficulties.

The other simplification in the boiler-turbine modelling is the assumed proportional relation between the output power and the mass flow. Considering experimental results, this assumption except for very low load conditions where the power loss in the system compared with the output power is not negligible, is quite reasonable.

A 9th order nonlinear model has been obtained for the overall CHP system by combining the three subsystem models. This model has been used to simulate and investigate the problems occurring in linear regulators. The simulation results show the consistency of the model with a typical CHP system and also the inadequacy of the regulators currently employed in industry. This model has been used to evaluate a typical new regulator performance [11].

## APPENDIX A

The non-zero elements of the  $A$ ,  $B$  and  $N_x$  matrices in the state space equations are:

$$\begin{aligned}
 A(3,3) &= -1.0/T_{v_1} & A(4,4) &= -\beta_1/T_p \\
 A(5,5) &= -1.0/T_{v_2} & A(6,7) &= 1.0 \\
 A(7,7) &= -D/M & A(8,8) &= -K_2 \\
 A(8,9) &= -\omega_o & A(9,9) &= -1.0/T_e \\
 B(1,1) &= \alpha_2 & B(1,3) &= -\alpha_3 \\
 B(2,3) &= -1.0 & B(3,2) &= 1.0/T_{v_1} \\
 B(5,4) &= 1.0/T_{v_2} & B(9,5) &= 1.0/T_e \\
 N_x(1) &= -\alpha_1 x_2 x_1 \\
 N_x(2) &= -G x_2 x_1 + T_s \cdot (G x_2 \dot{x}_1 + G x_1 \dot{x}_2) \\
 N_x(4) &= -(\beta_1/T_p) x_2 x_1 - (\beta_2/T_p) x_4 x_3 \\
 N_x(7) &= -(h_1 \beta_1/M) x_1 x_2 + (h_2 \beta_2/M) x_3 x_4 - A_2 \sin x_5 x_7 - B_2/2 \sin 2x_5 \\
 N_x(8) &= -C_2 \cos x_5 \\
 N_x(9) &= -V_f/T_e
 \end{aligned}$$

## APPENDIX B

Typical parameters for a CHP system in p.u. are [10]:

$$\begin{aligned}
 \alpha_1 &= 3.44e-3 & h_1 &= 0.24 & K_A &= 100.0 \\
 \alpha_2 &= 4.60e-3 & h_2 &= 0.76 & T_e &= 0.01 \\
 \alpha_3 &= 1.28e-3 & \beta_1 &= 2.33 & X_d &= 1.25 \\
 \alpha_4 &= 1.0 & \beta_2 &= 2.16 & X_q &= 0.7 \\
 G &= 1.0 & \beta_t &= 1.0 & X_e &= 0.2 \\
 V_f/A &= 1.0 & K_c &= 5.0 & M &= 0.0185 \\
 T_s &= 25.0 & T_p &= 6.0s & D &= 0.005 \\
 T_{v_1} &= 0.24s & T_{v_2} &= 0.33s & X'_d &= 0.3
 \end{aligned}$$

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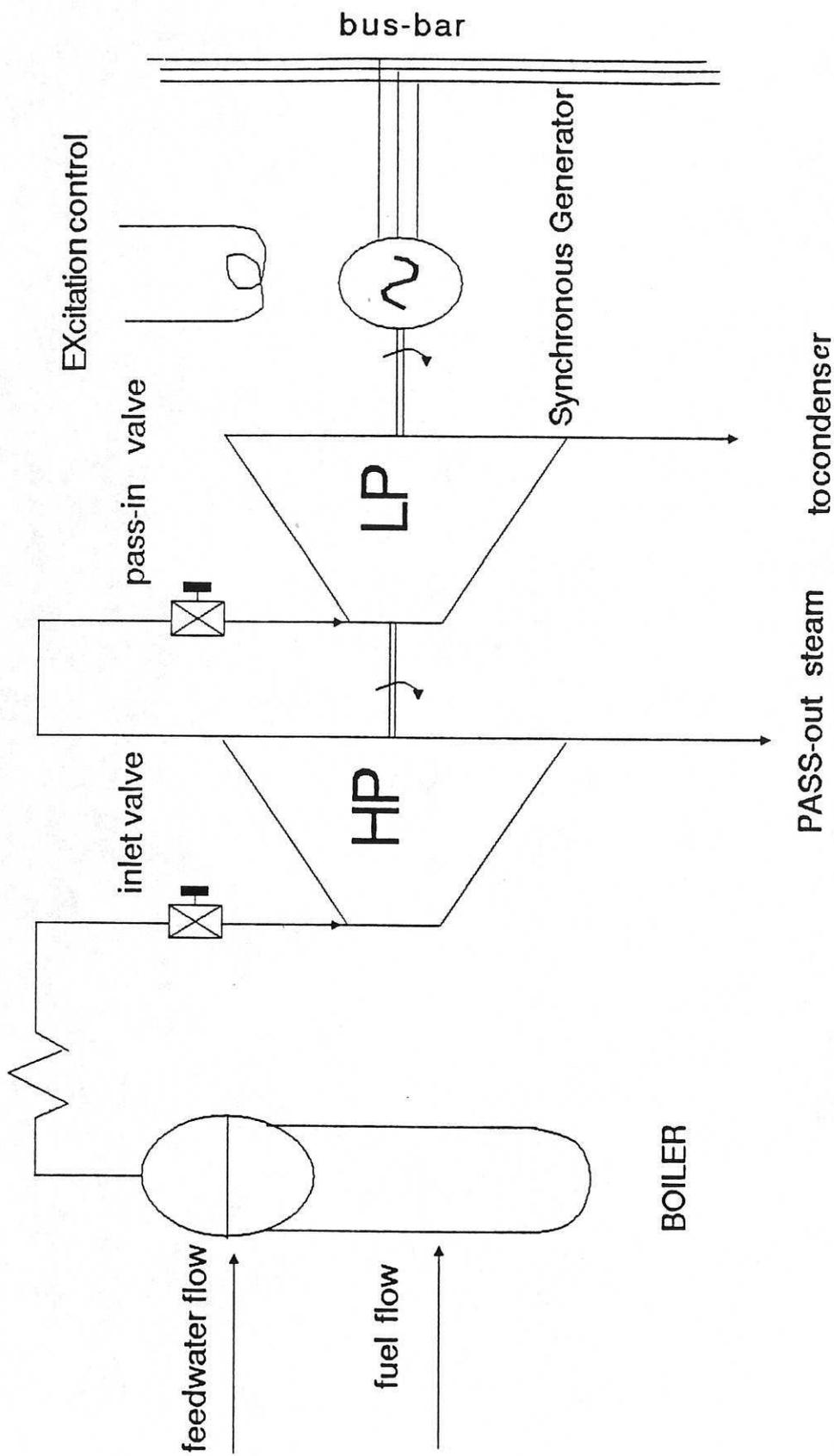
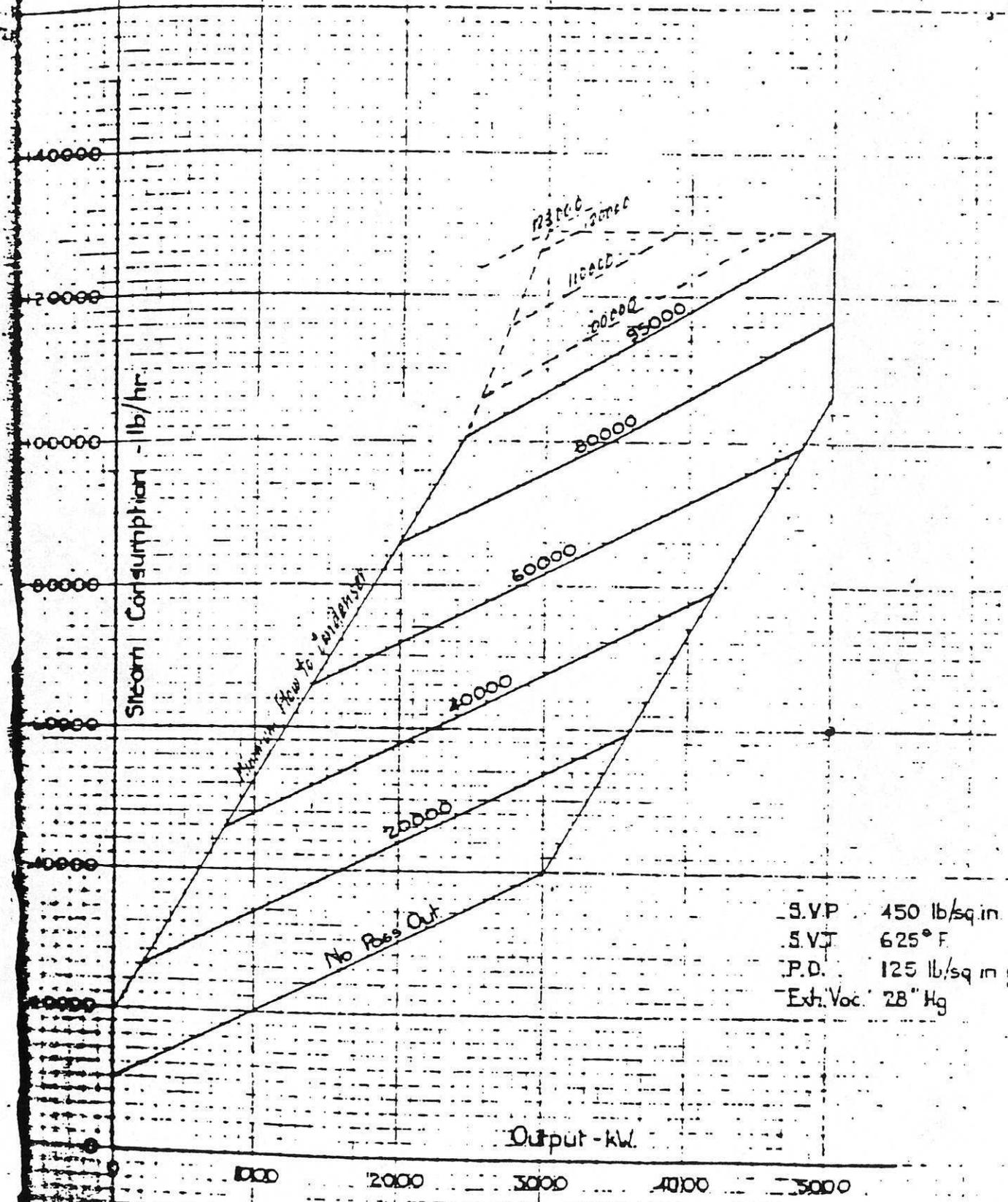


Fig. 1: Schematic diagram of a typical CHP system

WOODALL DUCKHAM FOR NATIONAL COAL BOARD  
 AVENUE PLANT



S.V.P. 450 lb/sq in g  
 S.V.T. 625° F  
 P.D. 125 lb/sq in g  
 Exh. Vac. 28" Hg

W. H. Allen, Sons & Co. Ltd  
 Bedford

C.S. 2465

Fig. 2: Steam flow - output power characteristics for a typical CHP system

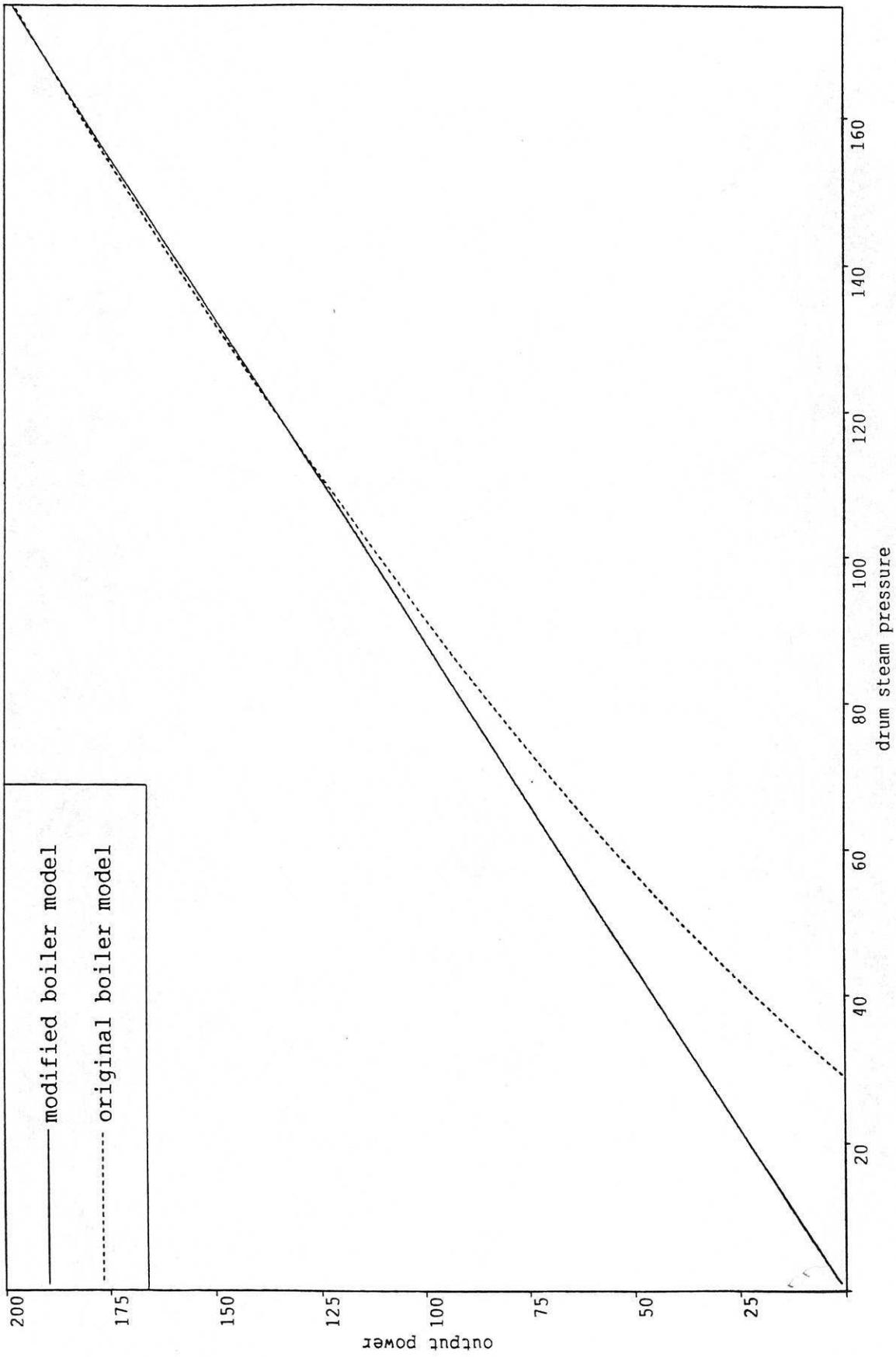


Fig. 3 : boiler model power-pressure characteristics:

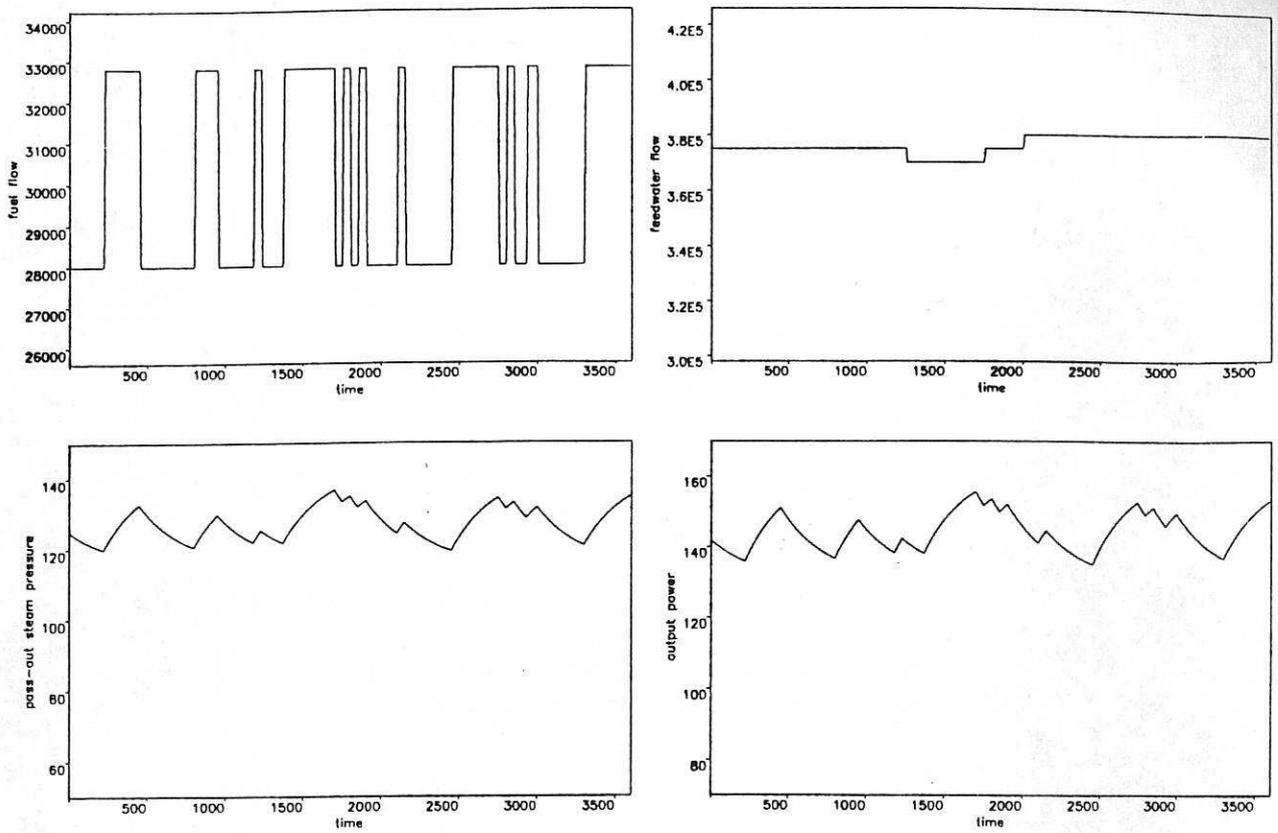


fig. 4 : Simulation of original drum-boiler model

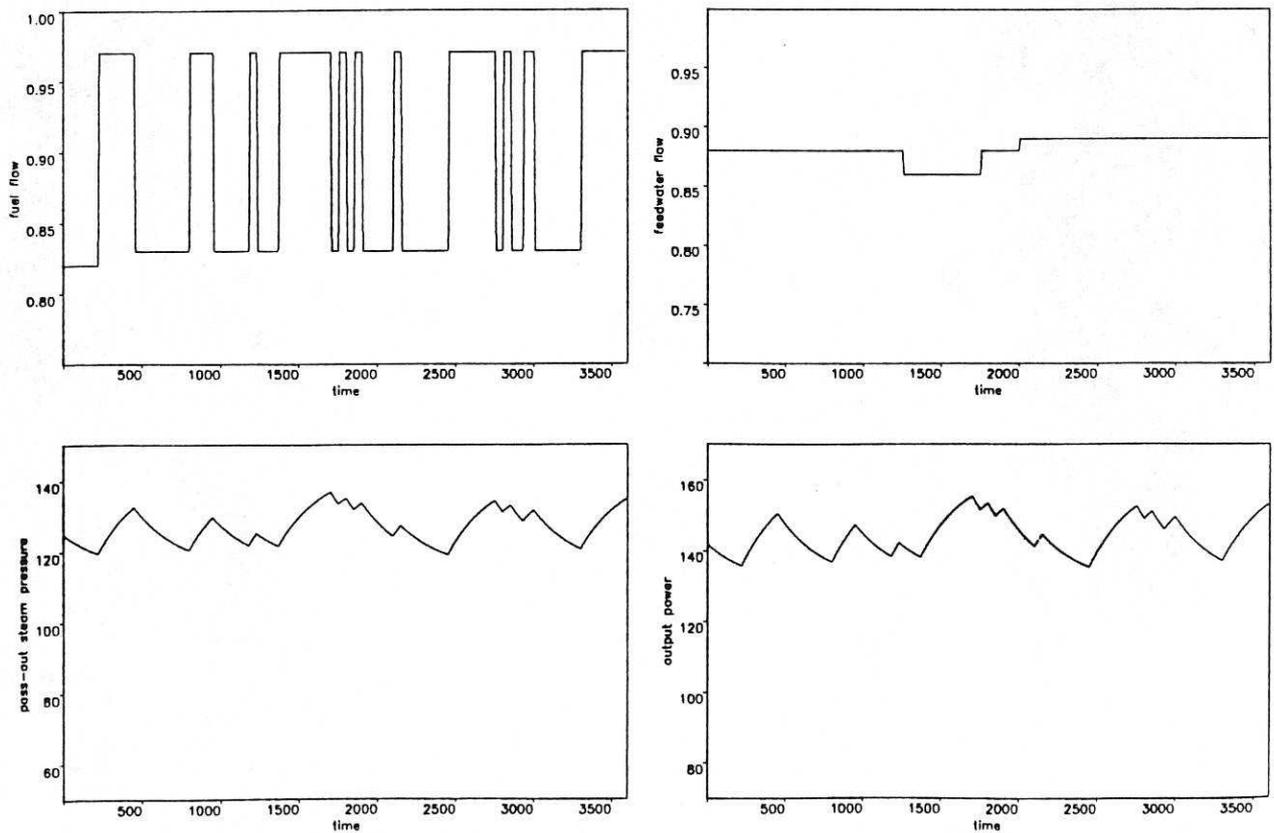


fig. 5 : Simulation of new drum-boiler model

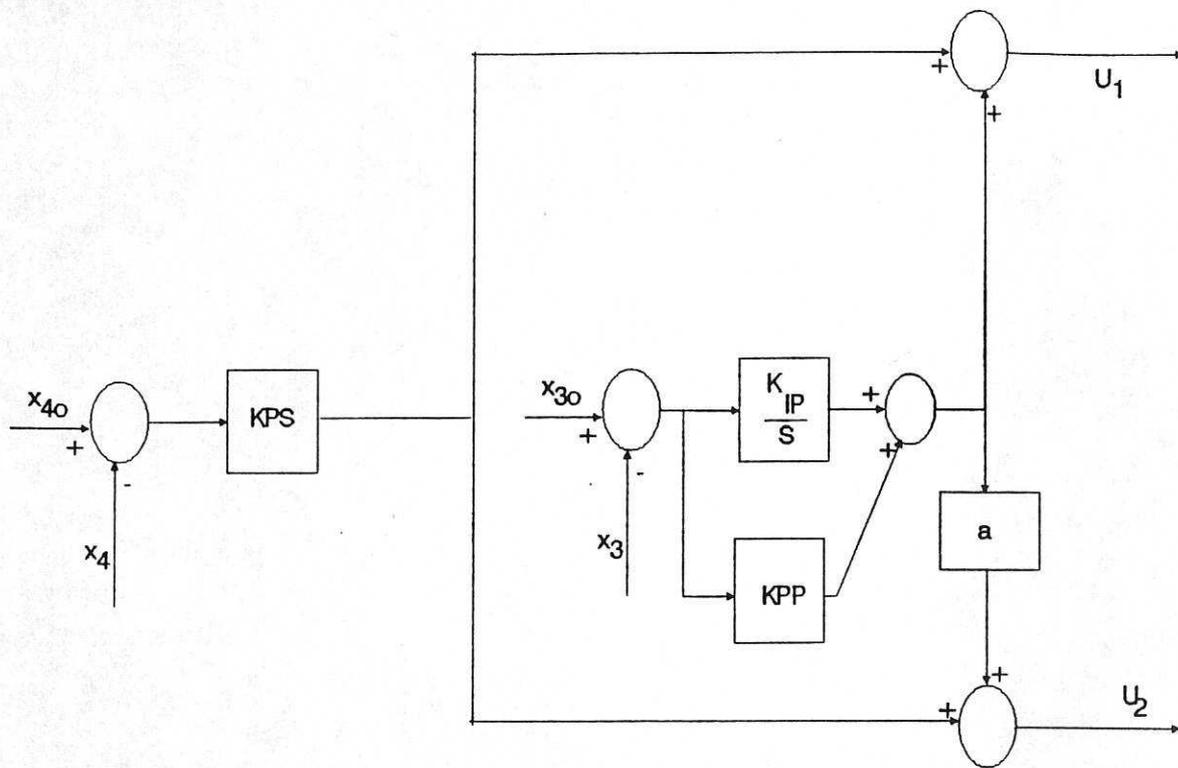
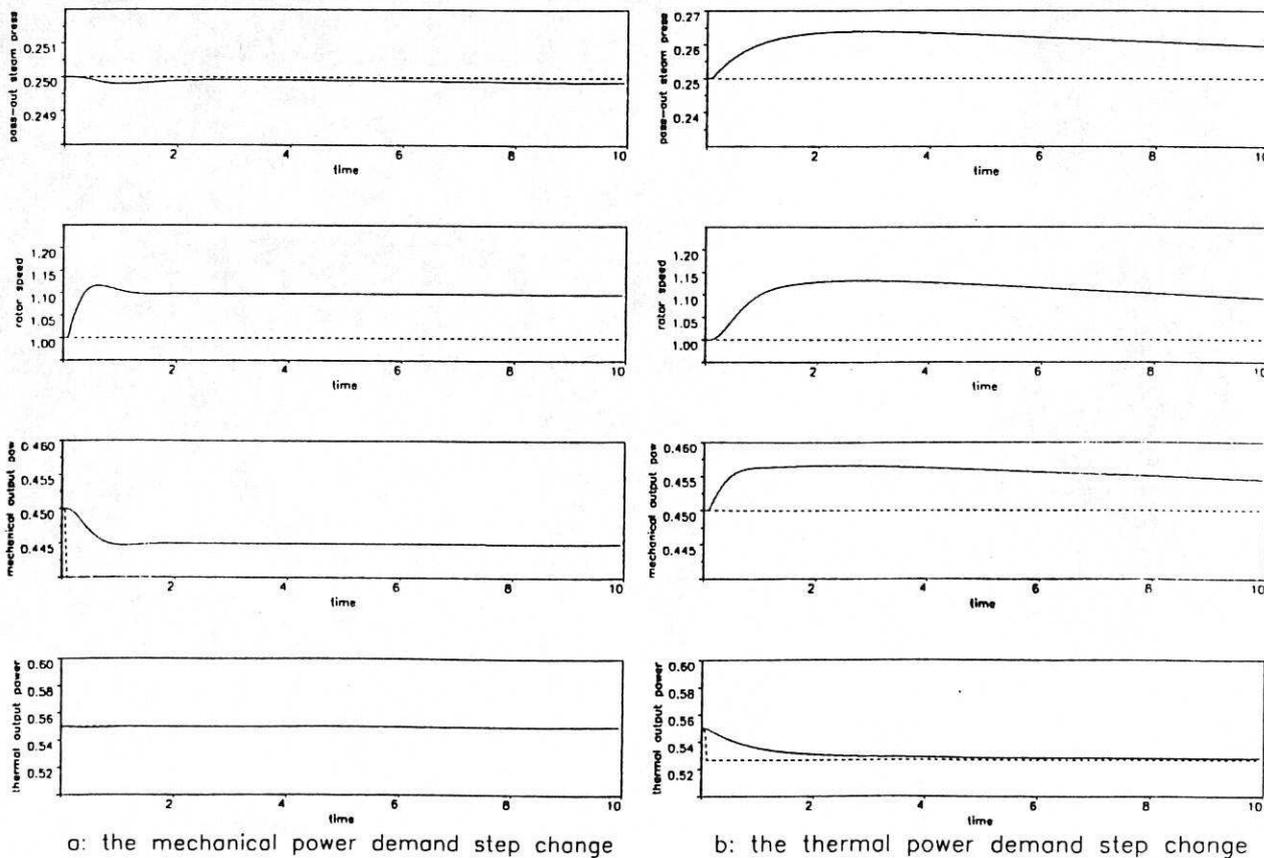


Fig. 6: The regulator with a PI for paa-out steam pressure and a P for rotor speed



a: the mechanical power demand step change

b: the thermal power demand step change

Fig. 7 : The turbine response with a PI regulator for pass-out steam pressure and a P regulator for rotor speed

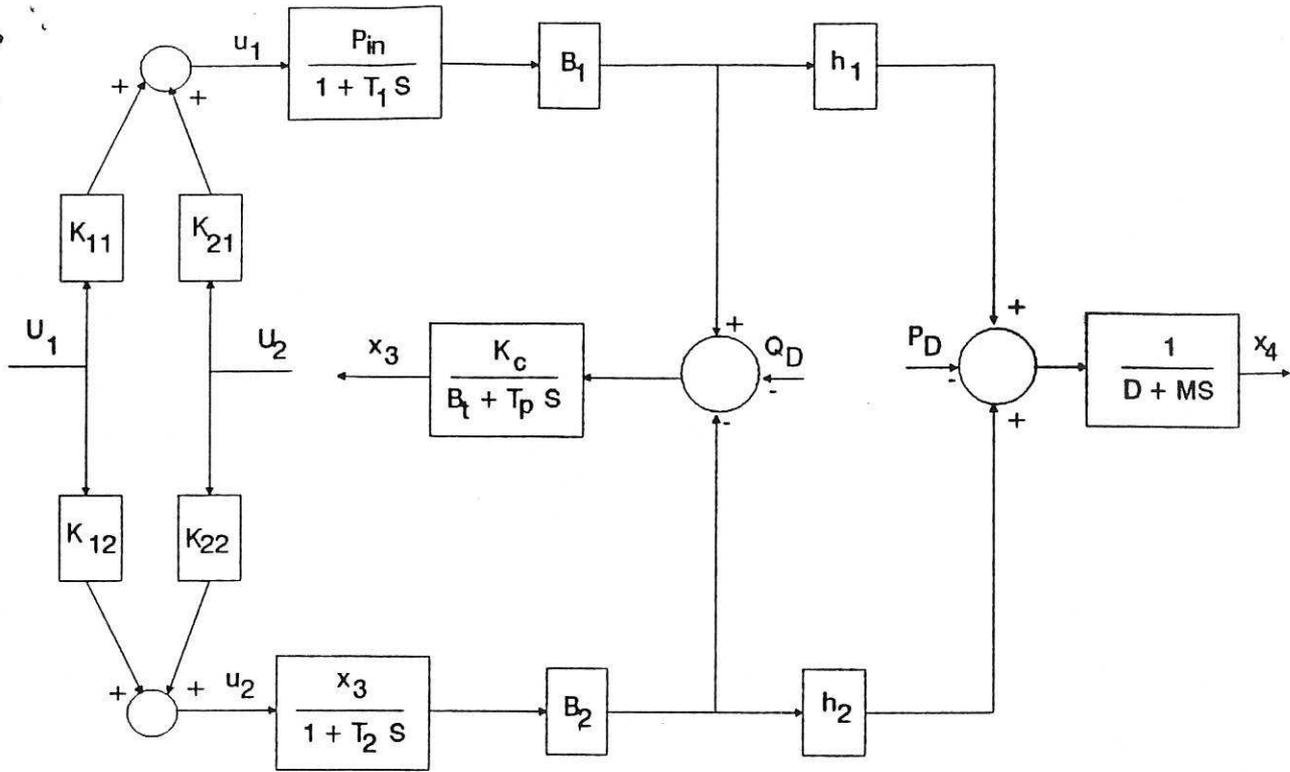


Fig. 8 : The time-varying linear system with precompensators

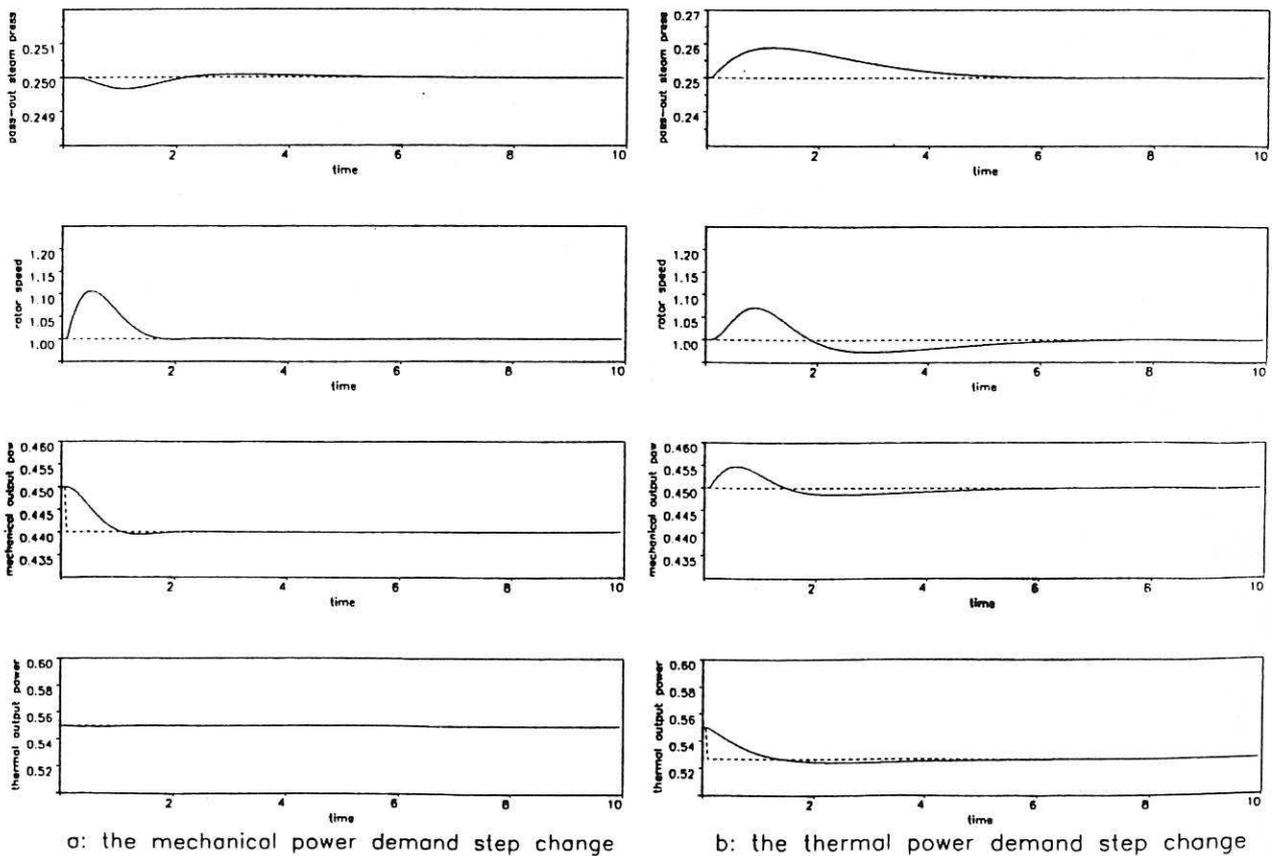


Fig. 9 : The turbine response with two PI regulators for pass-out steam pressure and rotor speed