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Prediction of bubble departure in forced convection boiling: A mechanistic model

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Abstract

In the context of computational fluid dynamic simulations of boiling flows using time-averaged Eulerian multi-phase approaches, the many sub-models required to describe such a complex phenomena are of particular importance. Of interest here, wall boiling requires calculation of the contribution of evaporation to global heat transfer, which in turn relies on determination of the active nucleation site density, bubble departure diameter and frequency of bubble departure. In this paper, an improved mechanistic model for the bubble departure diameter during flow boiling is developed. The model is based on the balance of forces acting on a bubble at a single nucleation site, with a new equation governing bubble growth proposed. The formulation accounts for evaporation of the micro-layer under the bubble, heat transfer from superheated liquid around the bubble surface, and condensation on the bubble cap due to the presence of sub-cooled liquid. Validation of the growth equation is provided through comparison against experiments in both pool boiling and flow boiling conditions. Introduction of condensation on the bubble cap allows reproduction of the growth of the bubble for different sub-cooling temperatures of the surrounding liquid. In addition, a sensitivity study guarantees dependency of the bubble departure diameter on relevant physical quantities such as mass flow rate, heat flux, liquid sub-cooling and pressure, with any inclination of the channel walls correctly accounted for. Predictions of bubble departure diameter and bubble lift-off are validated against three different databases on sub-cooled flow boiling with water and an additional database on saturated boiling with refrigerant R113. The whole data set guarantees validation is performed over a range of parameters and operating conditions as broad as possible. Satisfactory predictive accuracy is obtained in all conditions. The present formulation provides an appropriate starting point for prediction of the behaviour of vapour bubbles under more general conditions which include lift-off after sliding, the frequency of bubble departure, bubble merging and bubble shrinking and collapse due to condensation.

KEYWORDS: Forced convection boiling; bubble departure diameter; mechanistic model; multi-phase flow; bubble growth equation; sub-cooled boiling.

1. Introduction

Nucleate boiling and two-phase flow are complex processes involving mass, momentum and energy transfer at the liquid-vapour interface, and frequently involve close interaction with solid walls. As a consequence, research in these areas is ongoing within many engineering disciplines, and in relation to thermal hydraulics in particular, despite them having been studied for decades. The ability to predict two-phase boiling flow is also of significant interest in many industrial fields, including the chemical and process industries, refrigeration and air conditioning among many others. In the nuclear energy sector, it is essential for the safe operation of boiling water reactors (BWRs) and the design of new passive nuclear reactor systems operating under natural circulation.

The development of computational fluid dynamic (CFD) approaches for predicting such flows has proved promising and of value in engineering design, in particular through the Eulerian time-averaged models generally used in practice. In such models the phases are treated as interpenetrating continua, and all the information on the interface structure is lost due to the averaging process [1]. Consequently, models are needed for the inter-phase exchanges of mass, momentum and energy to close the system of equations. In particular, a specific model is needed to describe nucleate boiling at the wall. Heat flux partitioning models, such as that of Kurul and Podowski [2], have been adopted in most CFD models of boiling flows to date. Heat flux from the wall is portioned into contributions due to single-phase convection, transient conduction and evaporation. This evaluates the amount of vapour generated from several parameters, such as the active nucleation site density, the bubble departure diameter and the bubble departure frequency. A review of heat flux partitioning models can be found in [3] and [4].

In this type of model, the proper evaluation of bubble growth is particularly important. In the initial stages of the growing transient, growth of the bubble is controlled by the inertia of the surrounding liquid, whereas it is later limited by the amount of heat that can be transferred from the surroundings [5]. Numerous mechanisms occur in heat transfer from the wall [6]. During bubble growth, a thin liquid micro-layer is trapped under the bubble which then evaporates as heat flows from the superheated wall. Diffusion of heat from the superheated layer surrounding the bubble cap also takes place. Partial dry-out of the micro-layer due to evaporation can form a dry patch on the wall surface and a three-phase contact line. Evaporation at the latter contact line supplies heat to the bubble that in turn contributes to bubble growth. In addition, growth of the bubble can perturb the flow field around the bubble itself, resulting in additional energy transfer by micro-convection. Further complexity is added by condensation at the top of the bubble in the case of sub-cooled boiling. The dominant heat transfer mechanisms have been debated over many years, and a number of different bubble growth models have been proposed, although no general agreement has been reached as yet. Recently, Kim [6] stated that experiments suggest that a bubble gains the great majority of the energy from the bubble cap rather than from processes at the wall. In contrast, Gerardi et al. [7] observed during pool boiling of water that a bubble gains a significant amount of the heat required for its growth through direct heat transfer from the wall. Therefore micro-layer evaporation is considered the dominant mechanism. In addition, various authors have suggested a dependency on fluid properties, based, for example, on observations in [6] related to refrigerants.

Forster and Zuber [8], and Plesset and Zwick [9], modelled bubble growth in a uniform superheated liquid. In their models, which only differ in a numerical constant, after an initial period when hydrodynamic forces are dominant, bubble growth is governed by heat diffusion from a thin superheated boundary layer around the bubble. Zuber [10] extended this model to non-uniform temperature fields, while Mikic et al. [11], and Prosperetti and Plesset [12], derived dimensionless relations valid throughout both inertia-controlled and heat diffusion-controlled growth. Cooper and Loyd [13], and Cooper [14], identified the evaporation of a thin liquid micro-layer trapped under the bubble as the major heat source sustaining bubble growth and modelled it accordingly. The same concept was later adopted by Unal [15] to derive correlations for bubble growth rate and maximum bubble diameter in a sub-cooled boiling flow of water. Van Stralen et al. [16] proposed a model based on the mutually dependent contributions of evaporation of the micro-layer under the bubble and heat diffusion from a relaxation micro-layer around the bubble surface.

Despite efforts to derive a more mechanistic description of bubble growth, the nucleation site density, bubble departure diameter and bubble departure frequency are most frequently predicted through empirical correlations. A thorough review of available correlations can be found in Cheung et al. [17]. For bubble departure diameter, in particular, such correlations are normally implemented in commercial CFD packages. Among the most frequently used are the Tolubinsky and Kostanchuk [18] and the Kocamustafaogullari [19] correlations, both of which were developed from pool boiling experiments. Tolubinsky and Kostanchuk [18] developed a correlation that evaluates the bubble departure diameter from a reference value as a function of sub-cooling. On the other hand, in [19] bubble departure diameter is considered a function of the system pressure and fluid conditions.

More recently, mechanistic models have been developed, focused on flow boiling in particular, to account for all the complex phenomena involved. Klausner et al. [20] developed a model based on the balance of the forces acting on the bubble during its growth phase and leading to bubble departure. These authors obtained a satisfactory predictive accuracy against their own data on the saturated flow boiling of refrigerant R113. Zeng et al. [21, 22] subsequently extended the original model to both pool and flow boiling conditions, with satisfactory agreement found for different experimental data on pool boiling and a relative deviation of 19 % obtained against the authors' experiments on bubble lift-off in the saturated flow boiling of refrigerant R113. Over the years, slightly modified versions of the Klausner et al. [20] model have been used by many authors to predict their own experimental data. Situ et al. [23] validated their model against experimental data for bubble lift-off in the vertical sub-cooled flow boiling of water, claiming an average relative error of ± 35.2 %. Wu et al. [24] studied experimentally a sub-cooled horizontal flow of boiling refrigerant R134a, demonstrating an average relative error between predictions and data of 5.4 % for bubble departure and 4.0 % for bubble lift-off. Yun et al. [25] implemented their bubble departure diameter model in the STAR-CD 4.12 software to simulate the DEBORA [26] sub-cooled boiling data of refrigerant R-12, showing a satisfactory predictive accuracy. Cheung et al. [27] included their model for bubble departure in a global heat flux partitioning model, obtaining good predictions against data for the vertical sub-cooled flow boiling of water. Klausner et al. [20] used the Mikic et al. [11] model to simulate bubble growth, whereas all other authors employed the Zuber [10] formulation for bubbles growing in a uniformly superheated liquid, with the exception of Yun et al. [25]. The latter authors used the Zuber [10] model for a non-uniform temperature field coupled with the Ranz and Marshall correlation [28] to evaluate condensation on the bubble cap.

In most cases, the introduction of different improvements based on validation against a specific database resulted in a large number of relatively accurate, but not extensively validated, models. As an example, Zuber [10] included in his formulation a parameter b to account for bubble sphericity. This parameter has been used by many other authors to fit their models with experiment data, resulting in a wide range of suggested values. In their original papers, Zeng et al. [20, 21] found the best results with b = 1.0 for flow boiling and used values of b between 0.24 and 24.24 to fit different pool boiling data sets. Situ et al. [23] adopted b = 1.73, whereas b = 1.2 was used by Wu et al. [24], b = 1.56 by Yun et al. [25] and b = 0.21 by Cheung et al. [27]. In more recent work, Sugrue and Buongiorno [29] proposed a modified version of the model, showing improved accuracy with respect to both Klausner et al. [20] and Yun et al. [25]. The former authors provide an extensive validation of their model against numerous data sets, showing in general good agreement. Bubble growth is simulated through the Zuber [10] model, using b = 1.56, with the largest errors observed against the lift-off diameter data of Situ et al. [23].

Model development has always been reliant on the availability of experimental data required for validation. On bubble growth and bubble departure diameter, an enormous amount of experimental works have been published and are available to modellers. Unal [15] studied the sub-cooled boiling of water and bubble departure diameter at high pressure in a steam generator pipe. Klausner et al. [20] and Zeng et al. [22] measured bubble departure and lift-off diameters during saturated flow boiling of refrigerant R113 in a horizontal square test section. Bibeau and Salcudean [30] studied bubble growth, detachment and condensation during sub-cooled boiling of water in a vertical annulus. Throncroft et al. [31] measured bubble departure and lift-off diameter, and waiting time between consecutive bubbles, in the vertical up-flow and down-flow boiling of refrigerant FC-87. Prodanovic et al. [32] studied bubble behaviour from inception to collapse for sub-cooled boiling of water in a vertical annulus. Situ et al. [23] measured bubble lift-off for forced convective sub-cooled boiling of water in a BWR scaled vertical channel. Chen et al. [33] measured boiling heat transfer, active nucleation sites, bubble departure diameter and bubble departure frequency for sub-cooled flow boiling of refrigerant R-407C in a horizontal annular duct. Sugrue [34] measured bubble departure diameter during sub-cooled flow boiling of water in a square channel for different orientation angles of the channel. More recently, the evolution of experimental techniques has allowed very detailed measurements of the bubble growth phase, including wall temperature distribution and the dry area under the bubble. This new data allows detailed validation and improved modelling of the different mechanisms sustaining bubble growth. Kim et al. [35] obtained very detailed measurements of bubble growth for pool boiling of refrigerant R113. A micro-scale heater array allowed accurate measurements of the heat flow rate under the bubble with high spatial and temporal resolutions. Gerardi et al.

[7] used infrared thermometry and high-speed video to study bubble nucleation and heat transfer during pool boiling of water. Bubble departure diameter and frequency, active nucleation site density, growth and wait times were measured, together with temperature distribution underneath the growing bubble.

In this paper, attention is focused on bubble departure diameter during flow boiling and a mechanistic model is developed starting from the work of Klausner et al. [20]. Modifications are introduced in the surface tension force following more recent findings and a new equation governing bubble growth is proposed. The new formulation accounts for evaporation of the micro-layer under the bubble and heat transfer from superheated and sub-cooled liquid around the bubble surface. To the authors' knowledge, no attempt has yet been made to include all the possible heat transfer contributions in a mechanistic model for bubble departure diameter. Providing that no single heat transfer mechanism can be considered dominant in all conditions, inclusion of all contributions aims to improve the model's general validity. In addition, evaluation of condensation on the bubble cap is crucial in view of the relevancy of sub-cooled boiling in many engineering applications. An extensive validation of the model is also provided. In view of its relevance, the equation governing bubble growth is independently validated against experimental measurements in both pool and flow boiling conditions. Departure diameter predictions are then tested to verify that the dependency on relevant parameters is correctly reproduced. Finally, a quantitative comparison is carried out against three data sets for the sub-cooled flow boiling of water (Prodanovic et al. [32], Situ et al. [23] and Sugrue [34]) and the data of Klausner et al. [20] for the saturated flow boiling of refrigerant R113. The whole data base allows validation over a wide range of experimental parameters and operating conditions. In addition, the model's ability to predict bubble lift-off as well as bubble departure diameter is evaluated using the data of Situ et al. [23]. The experimental data used and the new mechanistic model are presented in Sections 2 and 3, respectively. Section 4 contains a validation of the equation governing bubble growth. A sensitivity study on the influence of different parameters on bubble diameter at departure is provided in Section 5, and Section 6 is focused on global comparisons with experiments, followed by conclusions in Section 7.

2. Experimental databases

Three databases of sub-cooled flow boiling of water are used for model validation, i.e. those of Prodanovic et al. [32], Situ et al. [23] and Sugrue [34]. These combined databases allow an extensive validation over a wide range of mass flux, heat flux and liquid sub-cooling (250 kg m^{-2} s^{-1} < G < 900 kg m^{-2} s^{-1}, 50 kW m^{-2} < q'' < 1200 kW m^{-2}, 1.5 °C < ΔT_{sub} < 60 °C). The system pressures considered are in the range 1-5 bar. An additional comparison is made with the data of Klausner et al. [20], related to the saturated boiling of refrigerant R113. The details of each data base are summarised in Table 1. Experimental uncertainties for each measurement of bubble departure are provided in Sugrue [34], and are included between the lower limit of resolution ± 0.019 mm and a maximum of ± 0.113 mm. In Situ et al. [23], the measurement error of bubble diameter is estimated as the pixel distance, equal to 0.016 mm, whereas it is estimated equal to ± 0.03 mm in Klausner et al. [20]. No information on uncertainty of the bubble diameter measurements is provided in Prodanovic et al. [32].

As mentioned in the introduction, the framework of the present work is the improvement of available Eulerian multiphase CFD models, which requires bubble departure diameter as an input from a dedicated sub-model. In the past, correlations for bubble departure diameter were used in the vast majority of CFD calculations and are normally implemented in commercial CFD codes [36-38], although they started to be replaced by more mechanistic models in recent years [25, 27, 39]. To verify their accuracy, two of the most frequently used correlations (those of Tolubinsky and Kostanchuk [18] and Kocamustafaogullari [19]) have been compared against experiments.

Table 1 Databases used for validation of the bubble departure diameter model.

	Sugrue [34]	**Situ et al. [23]**	**Prodanovic et al. [32]**	**Klausner et al. [20]**
Fluid	Water	Water	Water	R113
Orientation	0°, 30°, 45°, 60°, 90°, 180°	Vertical	Vertical	Horizontal
Channel	Rect.; D_h =16.7 mm	Ann.; D_h=19.1 mm	Ann.; D_h=9.3 mm	Rect.; D_h=25 mm
G / kgm^{-2}s^{-1}	250 - 400	466 - 900	76.6 - 766	112 - 287
q'' / kWm^{-2}	50, 100	54 - 206	200 - 1000	11 - 26
ΔT_{sub} / °C	10, 20	2 - 20	10, 20, 30	Saturated
p / bar	1.01, 2.02, 5.05	1.01	1.05 - 3.00	1.01

These correlations are compared in Figure 1 with the data of Sugrue [34], whilst in Figure 2 the data of Klausner et al. [20] are compared with the correlation of Kocamustafaogullari [19]. Both correlations were developed for pool boiling and adopt a simple formulation, including the effect of sub-cooling and/or system pressure. As might be anticipated, large discrepancies are found between these correlations and the flow boiling experimental data. Significant physical effects, which are evident from the flow boiling experiments, such as the influence of mass flow rate and thermal flux, are neglected in their formulations. In particular, for the mass flow rate, neglecting its influence on bubble departure can be considered as one of the major sources of error when formulae derived for pool boiling are applied to flow boiling conditions. Results from this brief comparison aim to further demonstrate the larger errors that can be found when using pool boiling correlations to predict flow boiling experiments, and support the need to switch to more detailed mechanistic models for bubble departure in the context of the CFD simulation of boiling flows.

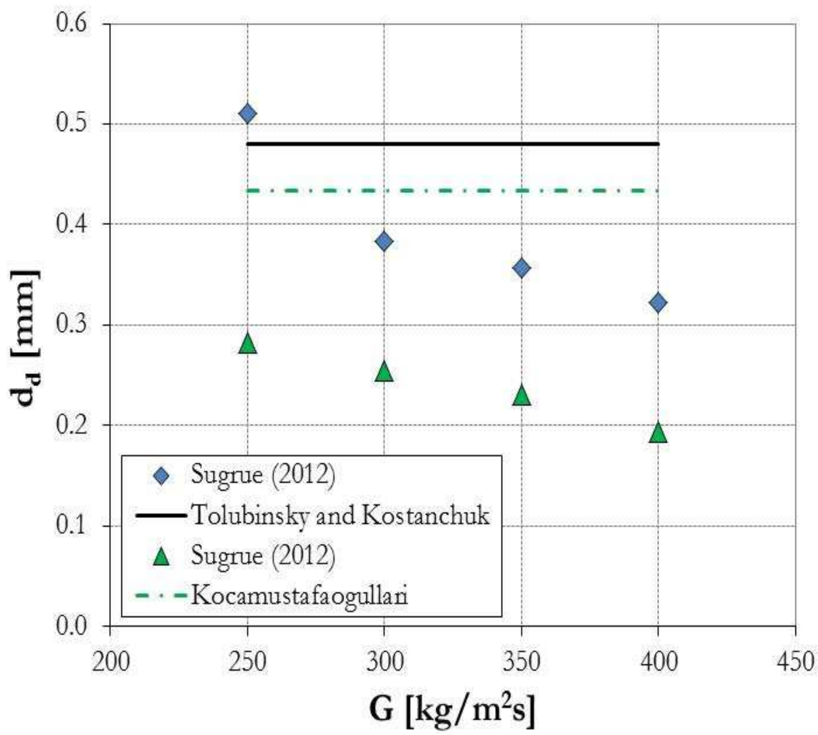

Figure 1. Comparison between the experimental data of Sugrue [34] and the correlations of Tolubinsky and Kostanchuk [18] and Kocamustafaogullari [19]. (◊ , –) : p = 1.01 bar, ΔT_{sub} = 20 °C, q'' = 50 kW m^{-2}; (Δ , -·-) : p = 5.05 bar, ΔT_{sub} = 10 °C, q'' = 100 kW m^{-2}.

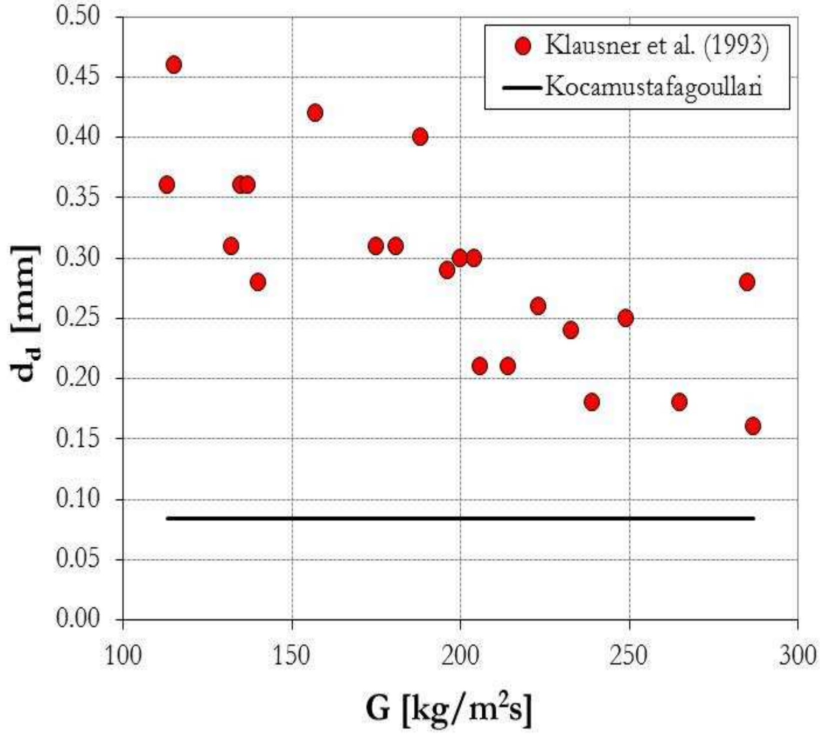

Figure 2. Comparison between the experimental data of Klausner et al. [20] and the correlation of Kocamustafaogullari [19].

3. Mechanistic model

The model has been developed based on the work of Klausner et al. [20] and it is based on the balance of forces acting on a single bubble at its nucleation site. In the direction parallel (*x*-direction) and perpendicular (*y*-direction) to the heating surface, the forces acting on the growing bubble are:

$$\sum F_x = F_{stx} + F_{qsd} + F_b \sin\theta + F_{udx} = 0 \tag{1}$$

$$\sum F_y = F_{sty} + F_{sl} + F_b \cos\theta + F_{udy} + F_p + F_{cp} = 0 \tag{2}$$

In these equations, F_{st} is the surface tension force, F_{qsd} the quasi-steady drag force, F_b the buoyancy force, F_{sl} the shear lift force, F_{ud} the unsteady drag force due to asymmetrical bubble growth, F_p the force due to hydrodynamic pressure and F_{cp} the contact pressure force. Subscripts *x* and *y* refer to forces acting in the *x* and *y* directions, respectively. A graphical representation of all the acting forces is provided in Figure 3. Whilst the sum of the forces in both directions equals zero, the bubble grows from the nucleation site without detaching. Detachment occurs when Eq. (1) or Eq. (2) is violated and detaching forces overcome those forces that keep the bubble attached to the surface. If the balance in the *x*-direction is violated first, the bubble departs from the nucleation site and starts sliding along the heated surface until it lifts off from the wall towards the liquid stream. The point at which the sum of F_x becomes greater than 0 is then used as the condition for bubble departure. Conversely, the bubble lifts off from the wall without sliding at the point at which the sum of F_y becomes greater than zero, and this can again be used as the condition for bubble departure. In the present formulation, modifications have been introduced in the surface tension force and in the equation governing bubble growth, which is directly included in the unsteady drag force but which also influences most of the other terms in the balances. Quasi-steady drag, shear lift, unsteady drag, hydrodynamic and contact pressure forces are taken from [20].

Expressions for the surface tension force [20] are:

$$F_{stx} = -1.25 d_w \sigma \frac{\pi(\alpha-\beta)}{\pi^2-(\alpha-\beta)^2}(\sin\alpha - \sin\beta) \tag{3}$$

$$F_{sty} = -d_w \sigma \frac{\pi}{(\alpha-\beta)}(\cos\beta - \cos\alpha) \tag{4}$$

In these equations, α is the advancing contact angle, β the receding contact angle and d_w the contact diameter between the bubble and the heated surface. Measurements and reliable models for determining these parameters are rather scarce in the literature, therefore they are one of the major sources of uncertainty in the present model, which has been proved to be quite sensitive to their values [29]. Klausner et al. [20] recommended $\alpha = \pi/4$ and $\beta = \pi/5$ from their measurements in R113. For the contact diameter d_w, a value of 0.09 mm was given. Instead, a constant ratio with bubble diameter $d_w = d_B/15$ was used by Yun et al. [25]. Some measurements of contact angles have been provided by Sugrue [34] for water, namely 90.63° for the advancing and 8.03° for the receding contact angle. In addition, a much lower contact diameter to bubble diameter ratio was reported to give fairly good agreement with data [29]. In this work, the suggestions from Klausner et al. [20] for contact angles and the Yun et al. [25] formula for contact diameter are used. By virtue of the good agreement reported, values from Sugrue and Buongiorno [29] are employed to predict their data.

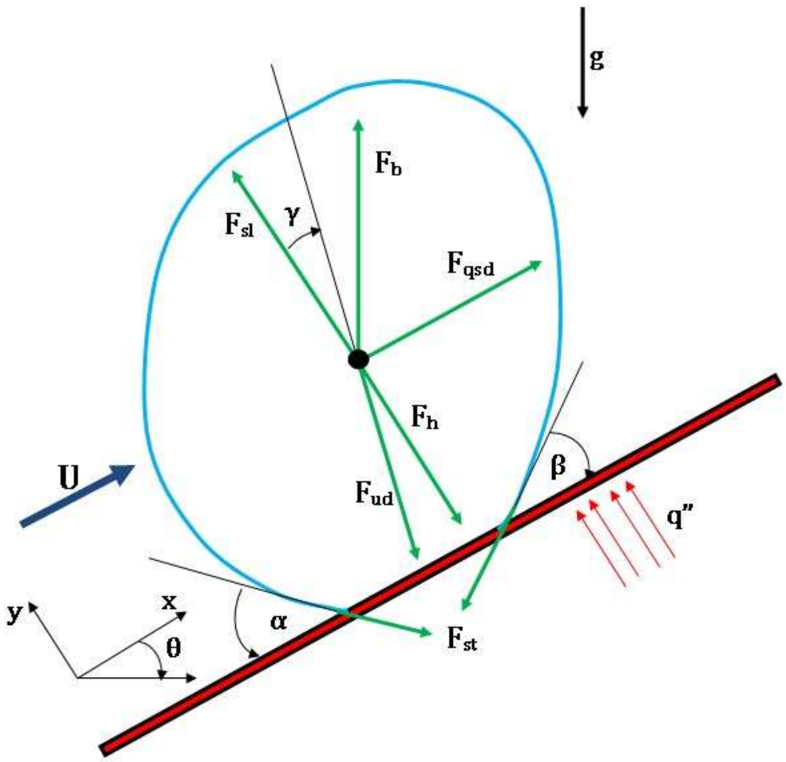

Figure 3. Schematic of the forces acting on a bubble growing at a nucleation site.

Prior to departure, bubble diameter behaviour is governed by a growth equation. Bubbles are assumed hemispherical during growth. Additionally, only the asymptotic phase of the bubble growth is considered, which is characterised by the diffusion of heat from the surrounding liquid. Conversely, inertia controlled bubble growth has been neglected. This hypothesis is justified by the very short duration of the inertia controlled growth phase [15]. Differently from the majority of other available models [20, 23, 29], the present approach includes the evaporation of the thin liquid micro-layer formed under the bubble during its growth. As reported in Gerardi et al. [7], this micro-layer can be considered as the dominant, or at least a significant, energy source during bubble growth. Micro-layer evaporation is evaluated accordingly to Cooper and Lloyd [13]:

$$\frac{dR(t)}{dt} = \frac{1}{C_2} Pr^{-0.5} Ja \left(\frac{k_l}{\rho_l C_{p,l}} \right)^{0.5} t^{-0.5} \tag{5}$$

Here, the constant C_2 is related to the initial width of the micro-layer. Optimisation against experimental data returned a value of 1.78 which is higher than the value estimated by Copper and Lloyd [13] of between 0.8 and 1.2. A higher value, which reduces the contribution of the micro-layer, is to be expected since the value from Copper and Lloyd [13] was calculated considering micro-layer evaporation as the only heat source sustaining bubble growth. In contrast, in the present model the heat provided by the superheated boundary layer around the bubble surface is also considered. In his review paper, Kim [6] ascribes the major contribution to bubble growth to heat diffusion from the superheated boundary layer. Therefore, it seems reasonable to account for both contributions in a generalised model. The superheating contribution is based on the model developed by Plesset and Zwick [9]:

$$\frac{dR(t)}{dt} = \sqrt{\frac{3}{\pi}} k_l (T_l - T_{sat}) \left(\frac{k_l}{\rho_l C_{p,l}} \right)^{0.5} t^{-0.5} \tag{6}$$

Finally, condensation on the bubble cap as it comes in contact with the sub-cooled liquid is also accounted for since sub-cooled boiling is relevant in many engineering applications. It is therefore important to account for the dependency of bubble diameter on sub-cooling. The condensation heat transfer coefficient is evaluated from the Ranz and Marshall [28] correlation:

$$h_c = \frac{k_l}{d_B} (2 + 0.6 Re^{0.5} Pr^{0.3}) \tag{7}$$

Superimposing the contribution of the micro-layer, superheated liquid and sub-cooling, the complete bubble growth equation then reads:

$$\begin{aligned} \frac{dR(t)}{dt} = \frac{1}{C_2} Pr^{-0.5} Ja \left(\frac{k_l}{\rho_l C_{p,l}} \right)^{0.5} t^{-0.5} + \sqrt{\frac{3}{\pi}} k_l (T - T_{sat}) \left(\frac{k_l}{\rho_l C_{p,l}} \right)^{0.5} (1-b) t^{-0.5} \\ - \frac{h_c}{\rho_v h_{lv}} (T_{sat} - T_{sub}) b \end{aligned} \tag{8}$$

Combining the superheating and sub-cooling contributions gives an expression similar to those of Yun et al. [25] and previously Zuber [10], except for some numerical parameters. The parameter b determines the portion of the bubble surface in contact with the sub-cooled liquid. It is calculated determining the location of the saturation temperature in the boundary layer from a temperature profile scaled on the single-phase wall function from Kader [40]:

$$\theta^{+} = Pry^{+}e^{-\Gamma} + \left\{2.12\ln\left[(1+y^{+})\frac{2.5\left(2-{}^{y}/_{\delta}\right)}{1+4(y-\delta)^{2}}\right] + \beta(Pr)\right\}e^{-1/\Gamma} \tag{9}$$

where:

$$\beta(Pr) = \left(3.85Pr^{1/3} - 1.3\right)^{2} + 2.12\ln Pr \tag{10}$$

and:

$$\Gamma = \frac{0.01(Pry^{+})^{4}}{1+5Pr^{3}y^{+}} \tag{11}$$

Scaling requires knowledge of the actual wall temperature. Since wall temperature is not available for all the data, it has been calculated using the heat flux provided in the experiments and the heat transfer coefficient from the correlation of Chen [41]. The correlation of Chen [41] obtains the heat transfer coefficient as the superposition of a convective contribution and a nucleate boiling contribution:

$$h_{tp} = h_{conv} + h_{nb} \tag{12}$$

The convective component is represented as a Dittus-Boelter type equation:

$$h_{conv} = 0.023\left[\frac{G(1-x)D_{h}}{\mu_{l}}\right]^{0.8}\left(\frac{\mu_{l}C_{p,l}}{k_{l}}\right)^{0.4}\left(\frac{k_{l}}{D_{h}}\right)F \tag{13}$$

whilst the nucleate boiling component uses a slight modification of the analysis by Forster and Zuber [42]:

$$h_{nb} = 0.00122\left[\frac{k_{l}^{0.79}C_{p,l}^{0.45}\rho_{l}^{0.49}}{\sigma^{0.5}\mu_{l}^{0.29}i_{lv}^{0.24}\rho_{v}^{0.24}}\right]\Delta T_{sat}^{0.24}\Delta p_{sat}^{0.75}S \tag{14}$$

Knowledge of the temperature distribution allows calculation of the value of b and the average temperatures for the superheated and sub-cooled regions. A higher limit is imposed on the value of b since, according to [15], the presence of other bubbles on the heated surface prevents contact between the sub-cooled liquid stream and the bottom half of the bubble.

The unsteady drag force due to asymmetrical growth of the bubble is written as:

$$F_{dux} = -\rho_l \pi R^2 \left(\frac{3}{2}\dot{R}^2 - R\ddot{R}^2\right) \sin\gamma \tag{15}$$

$$F_{duy} = -\rho_l \pi R^2 \left(\frac{3}{2}\dot{R}^2 - R\ddot{R}^2\right) \cos\gamma \tag{16}$$

The angle γ is the inclination angle measured in the y-direction. Based on [20], it has been fixed to $\pi/18$.

The quasy-steady drag force has been derived by Mei and Klausner [43] for an unbounded uniform flow over a spherical bubble:

$$F_{qsd} = 6\pi\rho_l \nu U R \left\{\frac{2}{3} + \left[\left(\frac{12}{Re}\right)^{0.65} + 0.862\right]^{-1.54}\right\} \tag{17}$$

The shear lift force was given in Klausner et al. [20] as an interpolation between the Mei and Klausner [44] expression for the shear lift force on a spherical bubble in an unbounded flow field at low Reynolds number, and the result of Auton [45] for a bubble in an inviscid flow with a low shear rate:

$$F_{sl} = \frac{1}{2}\pi\rho_l U^2 R^2 \{3.877 G_s^{0.5}[Re^{-2} + (0.344 G_s^{0.5})^4]^{0.25}\} \tag{18}$$

where G_s is the dimensionless shear rate of the oncoming flow:

$$G_s = \left|\frac{dU}{dy}\right| \frac{R}{U} \tag{19}$$

The buoyancy force is given by:

$$F_b = \frac{4}{3}\pi R^3 (\rho_l - \rho_v) g \tag{20}$$

Finally, the hydrodynamic pressure and contact pressure terms are written as:

$$F_p = \frac{9}{8}\rho_l U^2 \frac{\pi d_w^2}{4} \tag{21}$$

$$F_{cp} = \frac{\sigma}{R}\frac{\pi d_w^2}{4} \tag{22}$$

The balance of forces (Eq. (1) and Eq. (2)) are advanced in time using Eq. (8) to calculate the diameter of the growing bubble. When Eq. (1) is violated first, the current diameter is taken as the value at departure and the calculation is carried on neglecting the contact diameter between the bubble and the heated surface. Bubble lift-off is then obtained when Eq. (2) is also violated. If Eq. (2) is violated first, the bubble directly lifts off from the nucleation site and the bubble departure and lift-off diameter coincide.

4. Validation of bubble growth model

Before comparing model results with experimental data for bubble diameter and lift-off, the accuracy of the bubble growth equation (Eq.(8)) has been evaluated against experiments. First, some pool boiling experiments are considered. Figure 4 shows comparisons with the experimental work of Gerardi et al. [7] for saturated pool boiling of water, with an experiment at a thermal flux of 50 kW m^{-2} selected. A good prediction of the growing transient is obtained, with most of the heat sustaining bubble growth coming from micro-layer evaporation, in agreement with experimental findings [7].

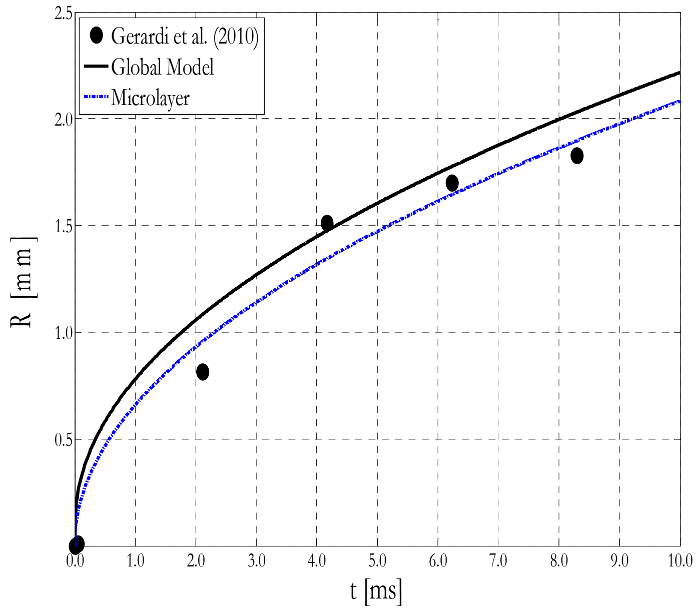

Figure 4. Bubble radius prediction compared against pool boiling experiment at $q'' = 50$ kW m^{-2} from Gerardi et al. [7].

Further comparison has been made with the pool boiling experiment of Kim et al. [35]. While saturated boiling conditions were tested in [7], the Kim et al. [35] experiment targeted the influence of pool temperature on the boiling of refrigerant R113. Therefore, it makes possible an evaluation of the effectiveness of including sub-cooling in Eq.(8).

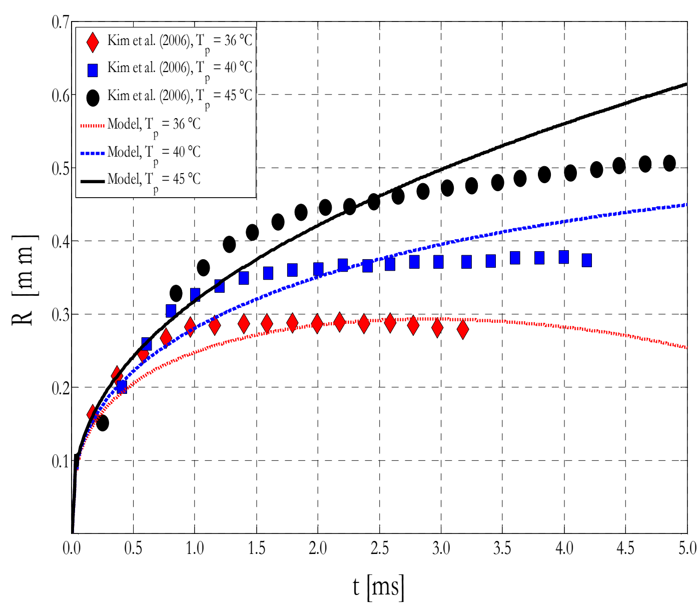

Figure 5. Bubble radius prediction compared against pool boiling experiments at different pool temperatures from Kim et al. [35].

Figure 5 compares model predictions against three experimental data sets at different levels of sub-cooling. Overall, the decrease of bubble radius with sub-cooling is well reproduced, although discrepancies are observed in some parts of the transients. In particular, bubble radius is slightly underestimated at initial times, probably as a consequence of neglecting the inertia-controlled growth region. However, this seems not to affect prediction of the following part of the transient. Growth rate is subsequently overestimated in the final part of the transient, except for the highest sub-cooling case. This suggests that the accurate quantitative estimation of the influence of sub-cooling is difficult to obtain using the Ranz and Marshall [28] correlation. Although frequently used for inter-phase heat transfer [25, 38], this correlation was derived for the evaporation of drops in a gas stream and may not be entirely suited for condensation in the cap of a growing bubble attached to a wall. However, a model derived for the specific situation of interest is not available at the present time. Another drawback of this correlation has been identified in the excessive condensation rate predicted with water at the beginning of the growth phase. Since the correlation includes the bubble radius, its initial low value can turn into too high a condensation rate causing shrinkage of the bubble. The inability of the model in its current form to accurately simulate the contribution of the reduction in bubble volume makes these conditions difficult to predict. A similar argument has also been put forward by Sugrue and Buongiorno [29] who note that the Ranz and Marshall [28] correlation was derived in the bulk fluid and not for small radius growing bubbles. In addition, the common definition of bubble Reynolds number, which includes the relative velocity between the water and vapour, may be inappropriate for a bubble growing while attached to a wall. Therefore, it has been defined using flow parameters in the remainder of this work. All things considered, however, the effect of sub-cooling on bubble radius can be considered to be satisfactorily predicted (Figure 5), although condensation on the bubble cap has been identified as a major area for future improvement.

Following the validation against pool boiling data, Eq. (8) is compared with the flow boiling experiments of Prodanovic et al. [32] in Figure 6 and Figure 7. As well as the bubble diameter, the separate contributions of the micro-layer, superheating and sub-cooling are shown. To facilitate the comprehension, the absolute value of the negative sub-cooling contribution is shown in these plots. A major contribution is provided by the micro-layer evaporation. Again, major discrepancies are found in the latter parts of the transient, after the maximum bubble diameter is reached in the experiments. As already noted, the present formulation of the model is not able to correctly accommodate bubble shrinkage due to condensation. However, such bubble shrinkage is expected in only some of the data of Prodanovic et al. [32], for which the entire bubble lifetime before and after departure from the wall has been recorded. Therefore, in the majority of cases bubble shrinkage is expected to happen after bubble departure, and to have a negligible effect on the bubble departure diameter. In both Figure 6 and Figure 7 the effect of sub-cooling is seen to be rather small. Even if the sub-cooling is significant, both data sets are characterised by a high heat flux. For higher levels of sub-cooling or lower heat fluxes, the contribution of sub-cooling becomes more relevant, as shown in Figure 8 for a set of conditions considered by Sugrue [34]. Experimental data for the bubble growth are not available for this data set. In some limited situations, sub-cooling was also observed to overcome the contribution from the superheated liquid layer around the bubble surface.

In view of the validation presented in this section, the formulation proposed in Eq. (8) can be considered to describe with a satisfactory accuracy the growth of vapour bubbles in the conditions of interest.

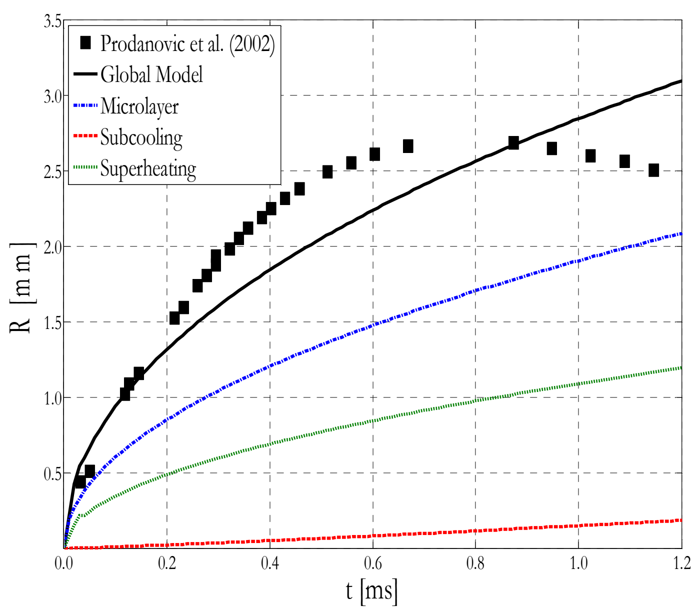

Figure 6. Comparison between the bubble growth model and a bubble growing transient from Prodanovic et al. [32]. p = 1.05 bar, G = 410.4 kg m^{-2} s^{-1}, q'' = 600 kW m^{-2}, ΔT_{sub} = 30°C.

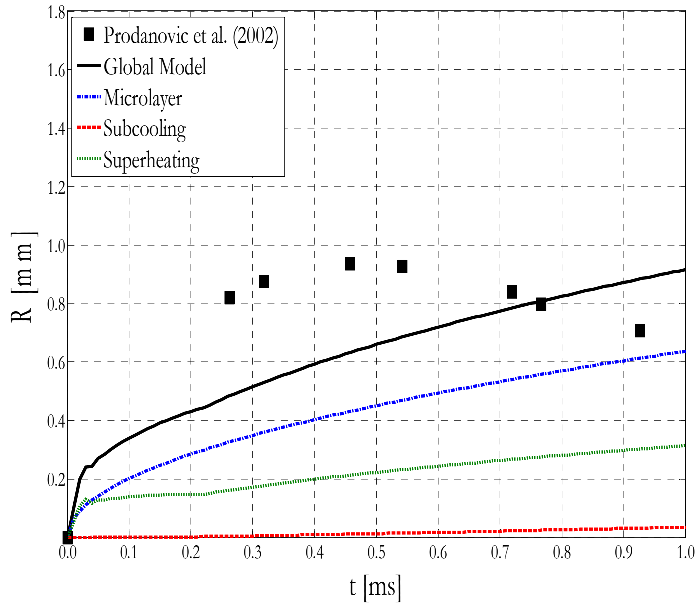

Figure 7. Comparison between the bubble growth model and a bubble growing transient from Prodanovic et al. [32]. p = 3 bar, G = 391.7 kg m^{-2} s^{-1}, q'' = 600 kW m^{-2}, ΔT_{sub} = 29.4°C.

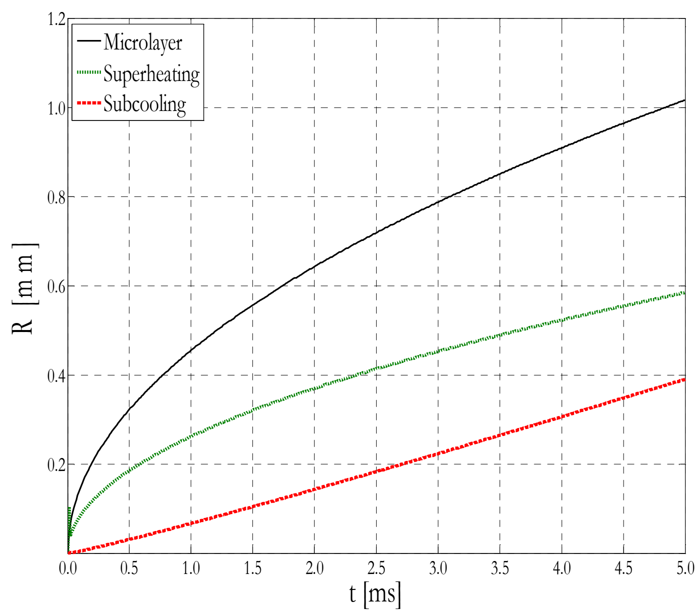

Figure 8. Simulated growth transient for the experimental conditions of Sugrue [34]. p =1.01 bar, G = 350 kg m^{-2} s^{-1}, q'' = 100 kW m^{-2}, ΔT_{sub} = 20°C.

5. Model sensitivity to system parameters

As far as the determination of the bubble departure diameter is concerned, numerous system and flow parameters influence the dynamics of bubble growth and the point at which the bubble departs from the wall. As noted in Section 2, neglecting some of these physical effects can result in significant deviations from experiments (Figure 1 and Figure 2). Therefore, it is very important to ensure that the model reproduces the physical dependencies of bubble departure diameter on parameters such as mass flow rate, thermal flux, pressure and fluid sub-cooling. In this section, comparison is made against a selected group of data to investigate the sensitivity of the model to these parameters.

An increase in the mass flow rate usually causes a decrease in the bubble diameter at departure [20]. Among the various forces acting, the quasi-steady drag (Eq. (17)) and shear lift (Eq. (18)) forces are strongly dependent on the liquid velocity of the flow and can be thought of as the tendency of the liquid to pull the bubble from its nucleation site. Therefore, increasing the liquid velocity leads to an increase of both these forces, which act to promote bubble departure and bubble lift-off, respectively. In Figure 9, model predictions are compared against three sets of data from Sugrue [34] obtained under different experimental conditions. For all the data, the mass flux is in the range 250 kg m^{-2} s^{-1} < G < 400 kg m^{-2} s^{-1}. It is evident from these results that the bubble departure diameter is reduced with increasing mass flux. This decreasing trend is correctly reproduced by the model, which returns accurate predictions for all the data considered.

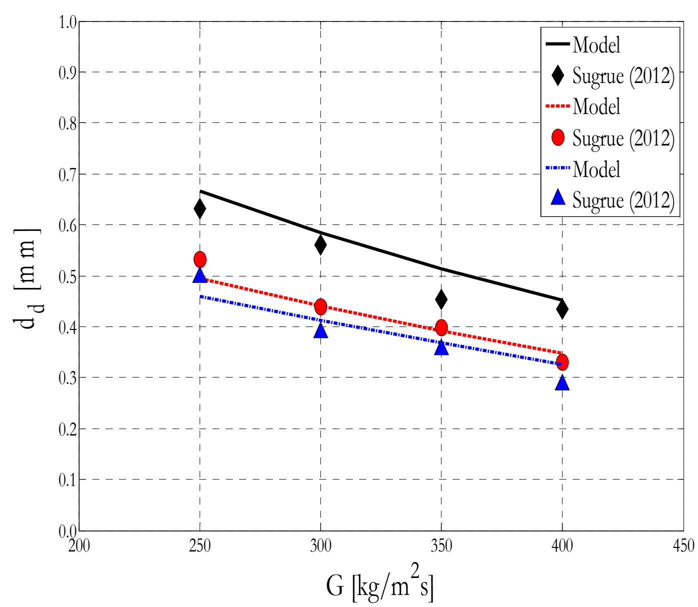

Figure 9. Predicted bubble departure diameter compared against data from Sugrue [34] at different mass flux. (◊ , –) : p =1.01 bar, q'' = 100 kW m^{-2}, ΔT_{sub} = 20°C, θ = 0°; (o , --) : p = 1.01 bar, q'' = 100 kW m^{-2}, ΔT_{sub} = 20°C, θ = 45°; (Δ , -·-) : p =1.01 bar, q'' = 100 kW m^{-2}, ΔT_{sub} = 20°C, θ = 60°.

The thermal flux from the wall provides the heat supporting the growth of the bubble and a higher heat flux increases its growth ratio. This leads to an increase in the force due to bubble growth which opposes bubble departure (Eq. (15) and Eq. (16)), this being a consequence of the increased pressure distribution around the bubble surface caused by the inertia of the surrounding liquid. In addition, during flow boiling the bubble is distorted and inclined in the flow direction at an angle γ by the liquid drag. As a consequence of this asymmetry, pressure is reduced in the direction facing the flow, resulting in a component parallel to the heated surface in the direction opposite to the mean flow which delays bubble departure. Therefore, higher heat fluxes cause a larger bubble diameter at departure or lift-off [20]. Among all the parameters considered in this section, the dependence on heat flux is the most uncertain. Increases in the bubble departure diameter with heat flux have been reported by Situ et al. [23] and Sugrue [34], although these authors state that the effect is slight and not very significant. In contrast, Prodanovic et al. [32] reported a decrease in bubble diameter with increasing heat flux. Their experiments were made with generally higher levels of sub-cooling and include cases of bubble collapse before lift-off or bubble merging, potentially leading to more complex interactions between the relevant parameters. Figure 10 shows bubble lift-off diameters at different heat fluxes from Situ et al. [23], with all others system parameters constant. The bubble lift-off diameter is clearly increased as a consequence of an increase of the thermal flux, although some scatter is observed in the data. The sensitivity of the bubble lift-off diameter to increases in heat flux is reproduced correctly by the model, which gives predictions close to average values for each group of data.

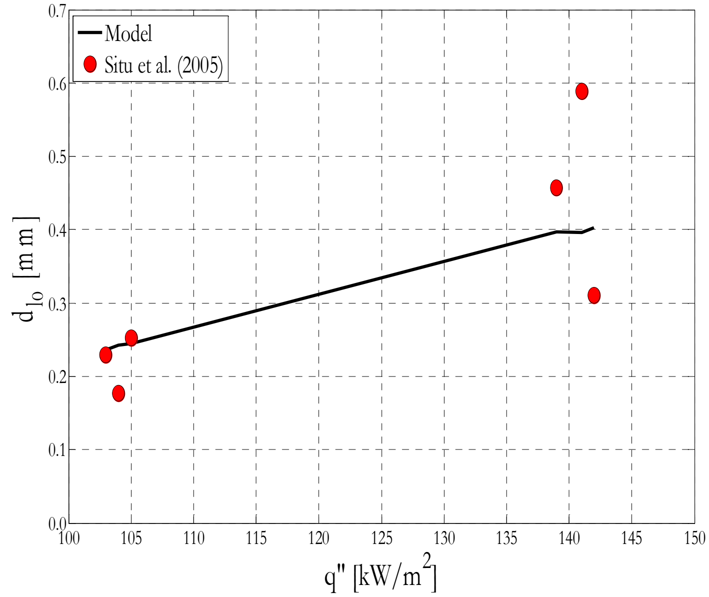

Figure 10. Predicted bubble departure diameter compared against data from Situ et al. [23] for different heat fluxes. p ~ 1 bar, G ~ 720 kg m^{-2} s^{-1}, ΔT_{sub} ~ 8°C.

An increased value of the liquid sub-cooling tends to reduce bubble diameters at departure from the surface. Higher sub-cooling causes higher temperature gradients in the liquid and a higher rate of condensation on the bubble cap, thus reducing bubble volume. In Figure 11 bubble departure diameter data from Prodanovic et al. [32] are shown for an increasing value of the inlet sub-cooling, all other system parameters being equal. Despite a slight underestimation of the bubble diameter, the effect of the fluid sub-cooling is correctly reproduced by the predictive approach, which provides further evidence in favour of including sub-cooling in the equation governing bubble growth (Eq. (8)). In terms of the underestimation, it is difficult to identify why this occurs given the limited data considered for this comparison. It may be attributable to the superposition of different factors, including experimental uncertainty. In addition, the model contains some coefficients that have been optimized over a range of parameters, and correlations, whose accuracy may also play a role. Additionally, the underestimation observed is not reflected elsewhere in relation to any general identified trends, e.g. as an underestimation of data which increases with the degree of sub-cooling. Therefore at this point, the underestimation must be attributed to a combination of different factors.

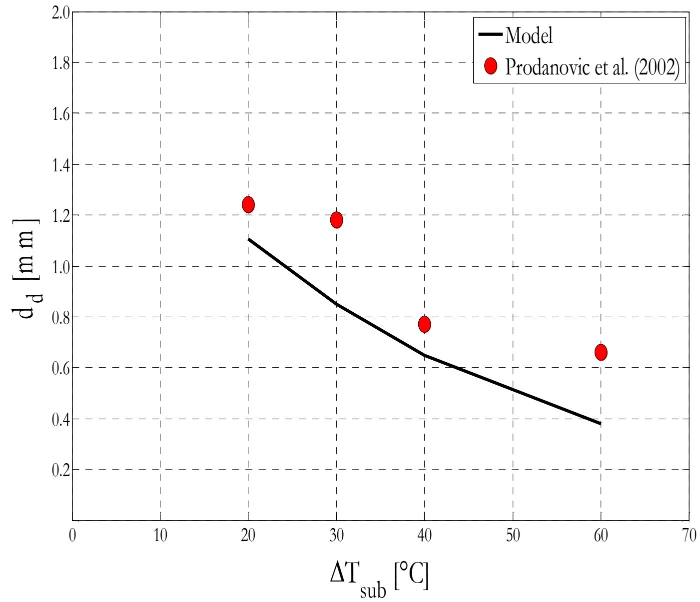

Figure 11. Predicted bubble departure diameter compared against data from Prodanovic et al. [32] for different fluid sub-coolings. p ~ 1 bar, G ~ 800 kg m^{-2} s^{-1}, q'' ~ 400 kW m^{-2}.

A change in pressure significantly modifies the fluid properties, which in turn has a large influence on the bubble diameter. In particular, vapour to liquid density ratio is increased following the increase of vapour density with pressure. Therefore, bubble volume is reduced for the same amount of heat following an increase in the system pressure, with a similar effect on the bubble departure diameter. Some data from Sugrue [34] at different system pressures are shown in Figure 12 and the decrease in bubble departure diameter with increasing pressure is evident. Despite showing an unremarkable level of accuracy, the model does reproduce the effect of system pressure quite satisfactorily. It must be pointed out that the predictions given in Figure 12 (at q'' = 100 kW m^{-2} and ΔT_{sub} = 10°C) were characterized by the highest error amongst the entire data base of [34]. The Ranz and Marshall correlation [28], being already identified as one of the major sources of error for the whole model, seems unable to account properly for the effect of sub-cooling in these particular conditions (sub-cooling is at its minimum and heat flux is at its maximum). Generally, lower errors were found for all other conditions in the database, with no significant error trends a function of any parameter in particular.

Sugrue [34] also performed a very detailed study on the influence of channel inclination on bubble departure diameter. The largest bubble diameters were observed in the horizontal (downward facing) configuration, whilst the smallest were seen in the vertical case. The buoyancy force contributes to bubble departure in a vertical channel, but to bubble lift-off in the horizontal case. Therefore, bubble departure should be promoted in moving from a horizontal to a vertical surface, at least when departure followed by sliding and lift-off is observed. Groups of data from Sugrue [34] at different channel inclinations are shown in Figure 13, where increases in bubble departure diameter in moving towards a horizontal inclination are observed. An accurate sensitivity to inclination is demonstrated by the model and the global quantitative accuracy is also good, with significant overestimation observed only for the vertical case. In addition, it can again be observed that the mass flow rate influence is correctly predicted, as in Figure 9. Globally then, the model is able to reproduce correctly the effect of mass flow rate, heat flux, liquid sub-cooling, pressure and channel orientation on bubble departure diameter.

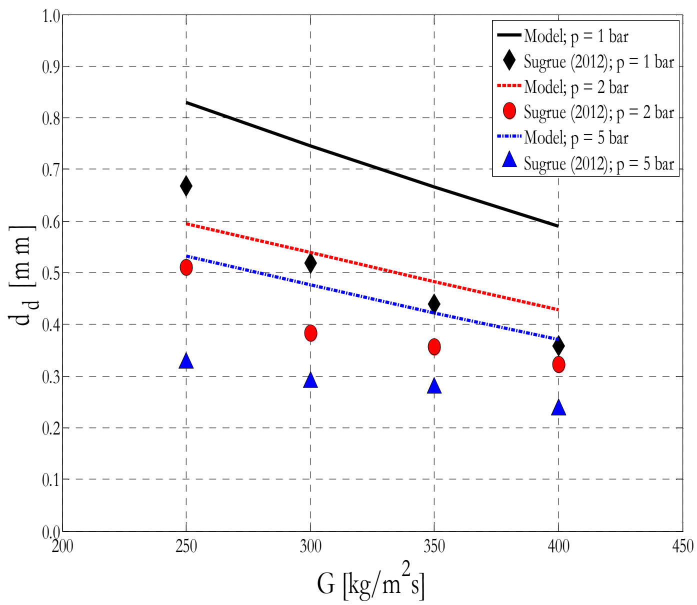

Figure 12. Predicted bubble departure diameter compared against data from Sugrue [34] at different system pressures. q'' = 100 kW m^{-2}, ΔT_{sub} = 10°C.

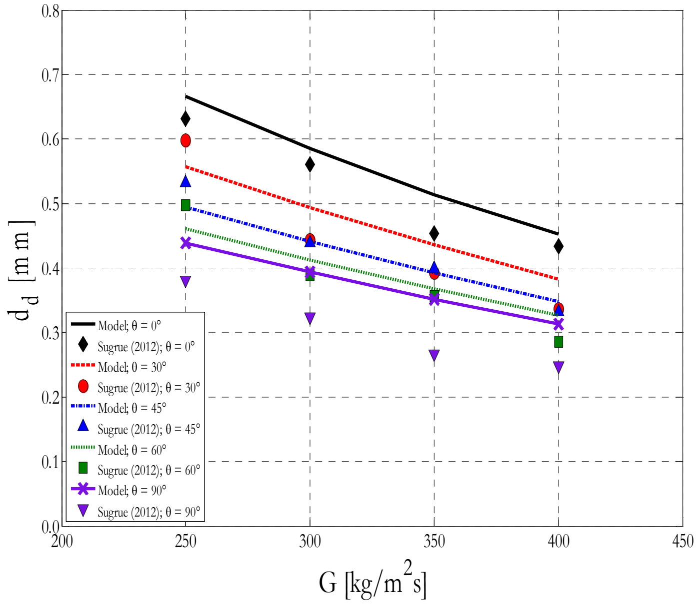

Figure 13. Predicted bubble departure diameter compared against data from Sugrue [34] for different inclinations of the channel.

6. Comparison with experiments

This section presents global comparisons with experimental data. The model derived has been compared against the three databases of Prodanovic et al. [32], Situ et al. [23] and Sugrue [34] for sub-cooled boiling at approximately atmospheric pressure. The comparisons are shown in Figure 14 to Figure 16. The new model shows reasonable accuracy with a combined average relative error of 27.8% with respect to these data. Therefore, the ability to give satisfactory estimations in a wide range of conditions is demonstrated. Good accuracy is shown for the Sugrue [34] data with the smallest average relative error of 20.6 %. A slightly higher average relative error is found for the Situ et al. [23] data, with a satisfactory 24.4% error shown. From Figure 15, it is seen that the bubble lift-off diameter tends to be underestimated. The highest errors are found for the database of Prodanovic et al. [32] (Figure 14), with an average relative error over 40%. Given that a comparable error is also reported by Sugrue and Buongiorno [29], again the highest among all the data considered by these authors, the database of Prodanovic et al. [32] seems to be the most difficult to predict. In addition, comparing Figure 14 with results from Figure 15 to Figure 17, the Prodanovic et al. [32] data show the highest scatter amongst the whole database considered.

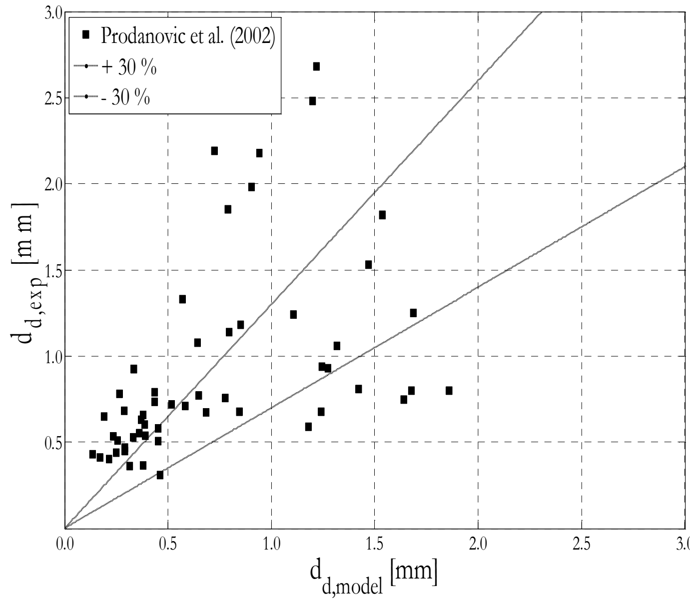

Figure 14. Comparison between model predictions and experimental data of Prodanovic et al. [32].

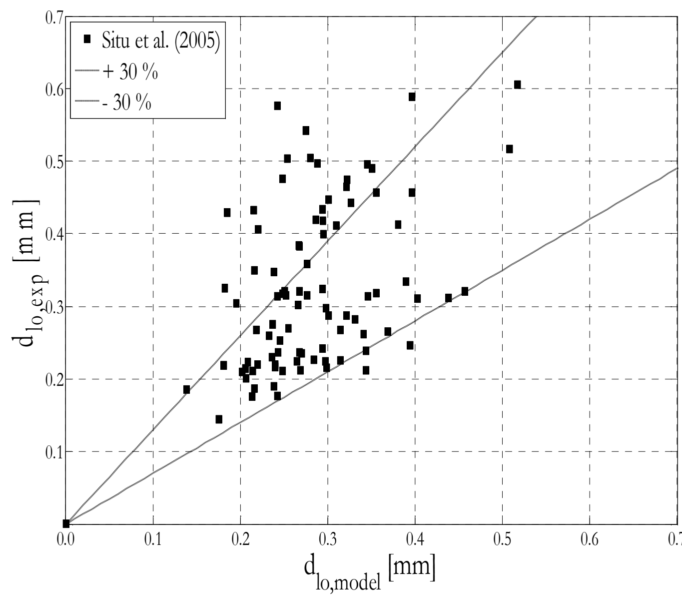

Figure 15. Comparison between model predictions and experimental data of Situ et al. [23].

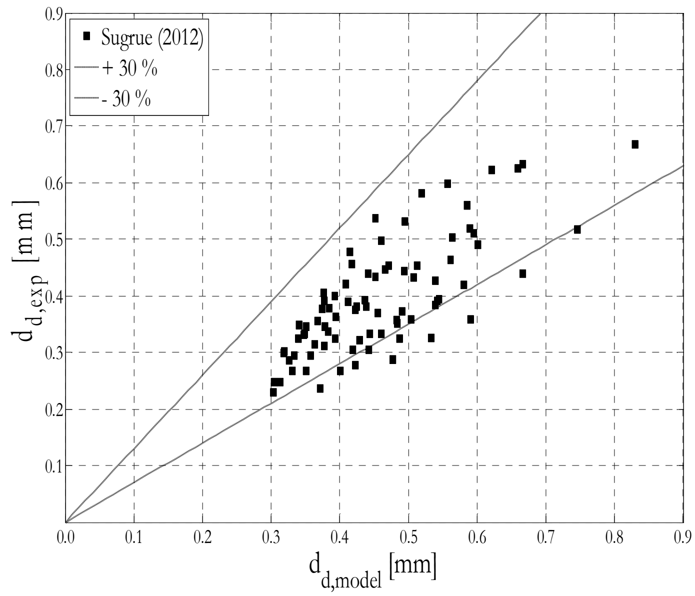

Figure 16. Comparison between model predictions and experimental data of Sugrue [34].

It is useful to focus this discussion on the comparisons with the Situ et al. [23] data, where the bubble lift-off was measured. Predictions of bubble lift-off are obtained when the force balance in the *y*-direction is violated. When the balance in the *x*-direction is violated first, the calculation is continued, neglecting the contact diameter between the bubble and the heated surface, and hence the surface tension force. The bubble lift-off diameter is then obtained when the balance in the *y*-direction is also violated. In [23], for most of the cases considered, bubbles were observed to depart by first sliding on the wall and then lifting off. Therefore, the present model seems able to predict not only the diameter of the bubble at departure, and whether it slides on the heated surface or directly lifts-off from the surface, but also to some extent the lift-off diameter after bubble sliding.

The data from Klausner et al. [20] for saturated flow boiling of refrigerant R113 allowed further extension of the comparisons, as shown in Figure 17. With an average relative error of 18.9%, the model remains accurate despite changing the fluid and thermal hydraulic conditions, further demonstrating its wide range of applicability. Considering the whole database [20, 23, 32, 34], the average relative error is satisfactory at 26.8%. A summary of the average relative error for the different data sets and for the database as a whole is provided in Table 2.

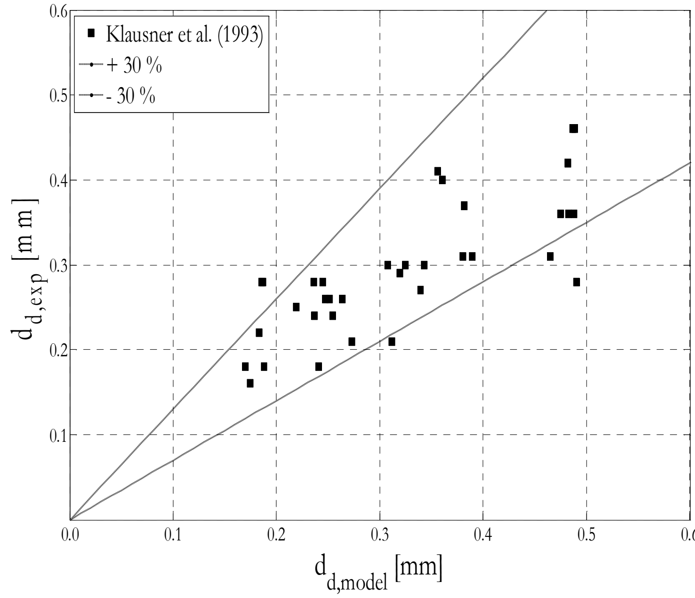

Figure 17. Comparison between model predictions and experimental data of Klausner et al. [20].

Table 2. Summary of the model accuracy for the single data sets and for the whole database.

Data	**Average Relative Error**
Prodanovic et al. [32]	44.8%
Situ et al. [23]	24.4%
Sugrue [34]	20.6%
Klausner et al. [20]	18.9%
Subcooled boiling	27.8%
Whole database	26.8%

7. Conclusions

An improved mechanistic model of vapour bubble departure in forced convection boiling has been developed. Starting from the model proposed by Klausner et al. [20], modifications have been included in the surface tension force prescription and a new equation governing bubble growth has been developed. In the present formulation, the model accounts for the evaporation of the micro-layer under the bubble and the heat transfer around the bubble surface, including both conduction of heat from the superheated liquid and condensation on the bubble cap. The equation governing bubble growth has been successfully validated against experimental data. The first comparison was made with pool boiling data, and the validation was later extended to the sub-cooled flow boiling of water. The influence of liquid sub-cooling on the growth of vapour bubbles has been successfully predicted, although a major area for future research has been identified in the development of an improved model for condensation on bubble cap.

In a sensitivity study, the model reproduced correctly the dependency of bubble departure diameter on relevant parameters such as mass flow rate, heat flux, liquid sub-cooling, pressure and channel inclination. Quantitative predictions of bubble departure diameter and bubble lift-off were finally validated against three data sets for the sub-cooled flow boiling of water and an additional database of saturated flow boiling of refrigerant R113. The whole validation database guaranteed assessment of the model over a wide range of experimental conditions. Satisfactory accuracy has been reported, with a global average relative error of 26.8%. Higher errors were found only for the data set of Prodanovic et al. [32] (44.8 %), which proved to be difficult to predict as in studies by other authors. In addition to comparisons with data for bubble departure, the database for bubble diameter at lift-off from Situ et al. [23] was also well predicted. Therefore, the present formulation seems an appropriate starting point to predict lift-off after bubble departure from a nucleation site and sliding along a heated wall, although further development would be useful. In this regard, it seems feasible to extend the model to account for the behaviour of vapour bubbles in general, including bubble shrinking and collapse due to condensation, the frequency of bubble departure and bubble merging and interaction with neighbouring bubbles. Uncertainties still remain on the surface tension force, which has a significant influence on the results in some conditions. The development of more accurate and general models for the evaluation of bubble contact diameter with a heated wall and contact angles would therefore be useful in improving the overall model accuracy.

Acknowledgements

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Nomenclature

b parameter

C_2 constant

C_p specific heat

D_h	hydraulic diameter
d	diameter
d_w	bubble-heated wall contact diameter
F	force
G	mass flux
G_s	dimensionless shear rate
g	gravitational acceleration
h	heat transfer coefficient
i	enthalpy
Ja	Jakob number [$\rho_l\ C_{p,l}\ (T_l\text{-}T_{sat})\ /\ \rho_v\ i_{lv}$]
k	thermal conductivity
Pr	Prandtl number [$\mu_l\, C_{p,l}\ /\ k_l$]
p	pressure
Δp	pressure difference
q''	thermal flux
R	bubble radius
Re	Reynolds number [$\rho_l\, U_l\, R\ /\ \mu_l$]
Re_B	bubble Reynolds number [$\rho_l\, (U_v\text{-}U_l)\ d_B\ /\ \mu_l$]
S	suppression factor
T	temperature
ΔT	temperature difference
t	time
U	velocity
x	quality
y	wall distance
y^+	non-dimensional wall distance

Greek symbols

α	advancing contact angle
β	receding contact angle
γ	bubble inclination angle

δ boundary layer thickness

θ channel inclination angle

θ^+ non-dimensional temperature

μ viscosity

ν kinematic viscosity

ρ density

σ surface tension

Subscripts

B bubble

c condensation

conv convection

d departure

exp experimental

l liquid

lo lift-off

nb nucleate boiling

p pool

sat saturation

sub sub-cooling

tp two-phase

v vapour

x *x*-direction

y *y*-direction

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