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Measuring vulnerability of road network considering the extent of serviceability of critical road links in urban areas

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Abstract

Road networks are vulnerable to natural disasters such as floods, earthquakes and forest fires which can adversely affect the travel on the network that remains intact after an event. However, not all road links equally affect the travel conditions in a given network; typically some links are more critical to the network functioning than the others. It is noted that the majority of the existing indices designed to measure vulnerability offer a good measure of network-wide accessibility in sparse regional networks, but they rarely consider the extent of serviceability of critical links in dense urban road networks. This paper describes a number of vulnerability indices from the literature, applies them to the case of urban network of York and discusses the results. It proposes a new vulnerability index considering the serviceability of road links and illustrates its computation. Finally, this paper uses the results of the new vulnerability index and outlines a traffic diversion plan in the event of flooding in York using traffic network modelling techniques combined with Geographic Information Systems (GIS) application.

Key words: Vulnerability, degraded network, serviceability, emergency response plan, traffic diversion plan, traffic network modelling, GIS

1. Introduction

Road networks are vulnerable to natural disasters such as floods, earthquakes and forest fires which can cause immense damage to the infrastructure resulting in adversely affected travel on the degraded network. Typically after the occurrence of an event some places become less accessible e.g. following a bridge collapse due to an earthquake in Kobe (Chang and Nojima, 2001), structural damage after an earthquake in Haiti in 2010 (Bono and Gutierrez 2011) or after a heavy snow fall or even during/after a forest fire. Studying and analysing vulnerability of road networks will help in prioritising the planning, budgeting and maintenance of roads and also will be useful in preparing emergency response plans.

It is noted, however, that not all road links of a network are equally critical to its functioning, that is, to say, some links have a greater impact on network flows than the rest. Therefore it is very important to analyse the vulnerability of a network considering the importance of the roads within. The importance of analysing vulnerability caught the attention of researchers especially in the past 6-10 years due to an evident increase in the frequency of natural disasters over the past decade or so (Taylor et al 2006). There is a good volume of literature

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available on the vulnerability of road networks with some valuable research methods described. Some of the methods developed were based on accessibility measures (Taylor et al 2006) which analysed the vulnerability in sparse regional networks. Some others have been based on network topological measures e.g. Latora and Marchiori (2001). Then there are methods which consider the importance of the links and analyse the impact of complete disconnection (Jenelius et al 2006) and the consequences of geographical disparities (Jenelius 2009). Whatever the approach, all methods focus on accessibility in sparse networks and rely on considering complete link failures to arrive at an indication of the vulnerability of the network. In contrast, we are interested in studying the vulnerability of dense urban road networks which pose new challenges as accessibility alone may not be the central issue anymore (due to a large number of alternative routes being available), but it is the importance of the road link to the functioning of the network that will need greater attention. Moreover, in real life situations, after an event, roads may not always become completely unusable, but they may still be partially available for use. This then raises several questions – (i) whether the indices developed in the past are indeed suitable to the context of denser urban networks; (ii) if so do they consider the importance of the road links within; and, finally (iii) do they allow for analysing partially available roads as against completely damaged ones?

In this paper, firstly, we set out to investigate the indices known from the literature and aim to answer the questions raised. Secondly, we introduce a new measure to analyse the vulnerability of a road network considering the importance of the roads. Road networks usually follow a scheme of hierarchy e.g. Motorways, A-roads, B-roads and C-roads in England (DfT 2012a) based on the significance of their functioning and in this paper we aim to capture the importance of the roads based on their hierarchy. We also aim to make the new measure flexible enough to consider both partially and fully damaged roads. This allows the new vulnerability measure applicable in practical real life situations and it also allows for many possible 'what-if' scenarios to be built and tested.

Studying and analysing vulnerability will help in developing the mitigating measures. The analysis of vulnerability has far wider applications such as in planning and maintaining road networks, prioritising and budgeting, preparing for emergency response (Walker et al 2004). In this paper, as an example, we develop an outline plan to divert traffic in case of an urban network under flooding. The diversion plan prepared uses traffic assignment modelling technique together with Geographic Information System (GIS) and is aimed at maintaining connectivity as well as minimising the travel time on the degraded network.

This paper is set out in six sections including this one. Section 2 reviews the literature identifying the relevant approaches to analysing vulnerability of road networks. Section 3 introduces the new measure of vulnerability. Section 4 describes the case of York introducing the geography and flood prone roads in York. Section 4 also describes the steps involved in computing the vulnerability indices. Section 5 illustrates the indices and analyses the consequences of road failures. Section 5 also outlines a traffic diversion plan aimed at maintaining connectivity during floods as well as minimising the travel time on the degraded network. Section 6 concludes the research work reported in this paper.

2. Literature review

This section defines the term vulnerability and then presents an overview of the network vulnerability indices before specifying them with mathematical notation in the later part of the section.

2.1 Definition of vulnerability, serviceability and criticality

Many authors suggest that there is no single definition suitable for vulnerability but it must be defined in the context of an event (Jenelius et al., 2006; Einarsson and Rausand, 1998; Holmgren, 2004; Berdica, 2002). Laurentius (1994) described vulnerability as "*susceptibility for rare, big risks*" while Holmgren (2004) defined it as "*sensitivity to threats and hazards*".

The term *risk* can be considered as containing two components – the probability of an event occurring and the consequences arising due to the event (Berdica 2002). It is well known that the probabilities are difficult to estimate as they are based on historic information which assumes the circumstances around an event remain the same at all times. On the other hand it is very important to be able to assess the consequences of an event as they affect daily life, business and economy. In this paper we aim to focus on the consequences rather than the probabilities.

Taylor et al (2006) considered the vulnerability from the point of view of reduced accessibility, which is very similar to the definition followed by Jenelius et al., (2006). In this paper we follow the definition of Berdica (2002), "*vulnerability as the susceptibility to incidents that can result in considerable reductions in road network serviceability*" which has been widely used since then. Berdica (2002) also defines similar term *robustness* as the "*ability to cope with disturbing incidents*" which relates to the term vulnerability by definition, i.e. a network which is less robust is considered as more vulnerable. Furthermore, *serviceability* of a link is defined as the *possibility to use that link during a given period* which then relates to the possibility of partial degradation of roads. Finally, if the consequences of a link being affected are great then the link is considered *critical* to the network.

Based on the definitions, a number of vulnerability measures have been developed which are outlined in the next few paragraphs.

2.2   Specification of vulnerability indices

This section specifies a series of indicators to measure the vulnerability of road network, to facilitate investigating their suitability in case of an urban network set out in a later section.

The literature on vulnerability indices can be broadly grouped into the distance-based and the cost-based approaches. Indices based on travel distance are relevant to sparse regional networks where if a link is blocked drivers may need to take longer detours to reach their destinations. On the contrary, in dense urban networks usually several alternative routes are available, and moreover, it is well known that drivers often prefer quicker routes which need not necessarily be shorter in distance terms. Hence the indices such as Hansen's Index

(Taylor et al 1996) and the Efficiency Measure (Latora & Marchiori 2001) which are based purely on distance are considered unsuitable for measuring vulnerability of dense urban road networks and for this reason we omit them from further discussion in this paper.

Now, consider a network of links serving Origin-Destination (OD) demand represented by $Q = \{....,q_k,...\}$ where q_k is the demand for a particular OD pair k. Let A be the set of links indexed $a = 1,2,...A$. The link travel times **t** are assumed to be function of link flows. Thus if x_a denotes the flow on link a ($a = 1,2,...A$) with **x** the A-vector of flows across all links, then the travel time t on link a is denoted $t_a(\mathbf{x})$. Let N be the number of nodes on the network. With this basic notation we now specify the measures of vulnerability as defined by various authors.

2.2.1 Travel cost-based vulnerability measures

M1: Change in generalised cost measure (Taylor et al., 2006)

The generalised cost, a measure of disutility of travel usually measured in terms of a combination of distance travelled and time spent, is used as a measure of accessibility. The index T_a measures the change in generalised cost between when a link is intact and when the link is removed and is defined as follows;

$$T_a = \sum_k q_k \Delta c_{ka} \tag{1}$$

where,

$\Delta c_{ka} = c_k - \tilde{c}_{ka}$ change in generalised cost for OD pair k when link a fails

c_k = least path cost for OD pair k

$\tilde{c}_{ka}$ = least path cost for OD pair k when link a has failed

M2: Network efficiency measure (Nagurney and Qiang, 2007)

The Nagurney and Qiang transportation network efficiency measure $\varepsilon(G,Q)$ for a given network of topology G and origin destination demand Q is defined as

$$\varepsilon(G,Q) = \sum_k \frac{q_k}{c_k} / n_k \tag{2}$$

where,

n_k = number of OD pairs

Eq (2) represents the average number of trips per unit cost and represents the efficiency of the network in terms of traffic to cost ratio. The higher the traffic handled per unit cost, the more efficient the network is.

M3: The importance measure (Jenelius et al., 2006)

The importance measure assumes that all drivers are forced on a more expensive route when an event causes the disruption or closure of a link or a group of links. Their behaviour is described by the user equilibrium principle where the route choice is meant to minimise personal travel cost. Following from (1) the basis for the measure is the change in the cost of travel and this is defined as below.

The importance of a '*non – cut*' link *a*, which is where a link closure will only cause a *finite* increase in travel cost with regard to the whole network and is defined as

$$Importance\ (a) = \frac{\sum_k w_k \widehat{\Delta c}_{ka}}{\sum_k w_k}$$

(3)

where

w_k = weight assigned to OD pair k that reflects its significance

$\widehat{\Delta c}_{ka} = -\Delta c_{ka}$

In this paper the authors assume that the weight assigned to OD pair is equal to the demand for travel which provides the 'social and economic' context to the vulnerability index (Jenelius et al 2006) and it is this index that we report as indicator M3.

M4: Network Robustness Index (Scott et al., 2006)

The network robustness index (NRI) is defined as "the change in travel time cost associated with rerouting all traffic in the system should that segment become unstable". This index is based on capacities of individual links and considers the rerouting options for the origin destination pair using the link. It then uses the travel time to measure the cost of rerouting the traffic should a link be completely removed. It assumes that the disruption will cause a complete closure of the link and that drivers follow user equilibrium in route choice. The system cost of travel for when all the links are intact is also calculated and the difference is the NRI.

The system travel cost for the base case when all the links are intact denoted as *C* (an aggregate of flow-based link travel times) which is defined as

$$C = \sum_s t_s x_s \tag{4}$$

Then the Network Robustness Index for link *a* is defined as,

$$NRI_a = \tilde{C}_a - C \tag{5}$$

where,

$\tilde{C}_a$ = total cost of travel on the network when link *a* is removed

2.2.2 Critical review of cost-based vulnerability indices

The set of indices M1 to M4 reflect the overall accessibility of a network as measured by either the change in generalised cost on the network, or in efficiency terms e.g. the demand (trips) per unit cost. They are all useful measures to reflect on the network-wide impact of a link closure. However, the measures M1- M3 seem too restrictive in implicitly assuming that all trips are routed through the *least cost path* for each OD pair which is appropriate for a sparse regional network wherein alternative routes usually are too long or may not even exist in some cases. On the other hand, in dense urban networks travel patterns are much more complex as each OD pair is usually served by more than one route. Usually drivers will be able to find an alternative route if a road link on their regular route is affected. That is to say that the extent to which the affected road can influence the travel on the rest of the road network due to possible traffic diversions plays a significant role in measuring the vulnerability in an urban network. Thus the modelling method needs to be based on *multi-path* routing rather than the *least cost-based* routing. Furthermore, a given road may be partially available to service the traffic demand even after an event as opposed to a complete link failure as assumed by the measures in the literature. The least cost-based structure of the measures M1-M3 means it is very difficult to adopt them to model the effect of partial loss of road capacity. Finally, it is noted that M4 considers the effect of rerouting of traffic on the network, therefore it is free from the least cost path effect. We introduce a new measure of vulnerability based on a multi-path routing approach which considers the link flow, travel time akin to M4, but also taking into account the serviceability, hierarchy of each link as described further in Section 3.

2.3 Review of traffic diversion plans

Modelling the performance of a network under degraded conditions is essential in transport planning for developing mitigating measures. It helps the planners understand how the network can absorb disturbances, and also how to deal with and adapt so that the road network retains essentially the same function, structure and identity (Walker et al., 2004). In the face of network link failures, road users are either to make detours, change their mode and destination or reduce trip activities (Erath et al., 2009). In this paper we focus on rerouting of traffic rather than modal shift and destination choice. Many studies e.g. Zhou (2008) and Wei & Perugu (2009) use computer simulation technique to evaluate the traffic diversion plans which include traffic signal timing optimisation on diverted routes but exclude route choice of network traffic. Our approach differs from these studies and focuses on rerouting of traffic involving route choice of drivers.

It is known that the route choice behaviour of drivers in such situations follows various equilibriums. These include stochastic user equilibrium; used to represent uncertainties (Asakura, 1999), user equilibrium (Scott et al., 2006) and Probit-based stochastic user equilibrium (Sumalee and Kurauchi, 2006). There is another behaviour known as the Probabilistic User Equilibrium (Lo and Tung, 2003) which is similar to the user equilibrium but is based on past experience with a variability in travel time and aims at minimising user travel cost. In this research we aim to minimise the overall travel time on the network by

following the system optimal principle (Wardrop 1952). We believe that this is a reasonable assumption because in a degraded network drivers are unlikely to have perfect knowledge of network travel times and hence tend to look for guidance on the routes to their destinations.

3.   The new vulnerability index

This section introduces a new measure of vulnerability which considers the serviceability, hierarchy of road links. It also considers the possibility of detouring when a road link is partially or fully blocked due to an event.

M5: Network Vulnerability Index

The Network Vulnerability Index (NVI) takes into account the serviceability and importance of each road link on the network and is defined as below:

$$NVI = \sum_{i=1}^{|A|} \left[\left(\frac{x_i^{before}}{r_i^{before}} \right) t_i^{before} \right] - \sum_{i=1}^{|A|} \left[\left(\frac{x_i^{after}}{r_i^{after}} \right) t_i^{after} \right] \tag{6}$$

where,

r_i = serviceability of link *i,* that is the total available capacity of link *i* / standard hourly link capacity per lane for the given type of road

$|A|$ = number of elements in set *A* i.e. the number of links on the network

In equation (6) r_i the *serviceability* of link *i* is calculated by dividing the total available capacity of the link with the standard hourly maximum flow rate (i.e. capacity) *per lane* for a given type of road. The total available capacity of a link is obtained by summing the capacity of all *operational* lanes available. The standard hourly flow rate per lane is adopted from the DfT's Transport Analysis Guidance (DfT, 2012b) which depends on the *hierarchy* of a road and is taken as a surrogate to represent the importance of a link. The value of *r* reduces when a road link is affected by an event such as flood as the effective capacity available after the event is reduced. The above index means a higher loss of capacity on a major dual carriageway road carrying large volume of traffic is likely to have a greater significance to the value of M5. Equally a higher loss of capacity on a two lane undivided road will also affect the value of M5 significantly. The index also allows for partial degradation of a link i.e. some proportion of the initial capacity might be lost after an incident thus allowing the modellers a scope to build several possible 'what-if' scenarios. When a link fails completely, the affected link is removed from the network and the calculation of M5 is completed as before.

4.   Urban area of York: geography, road network, data needed and the methodology

This section introduces the geography of York, identifies the important road links of the network, and then describes the steps involved in computing the vulnerability indices M1-M5.

4.1 Geography of York

The city of York, with an estimated population of about 198,081 (2011 census) lies on the confluence of the Rivers Ouse and Foss in the Vale of York in North Yorkshire in England (See Figure 1). The city and its surrounding areas are bounded by the Pennines and North York moors on the west and east respectively. The city and the Vale of York is known for its valuable agricultural land, rich history and buildings and attracts tourists annually with traffic volumes increasing significantly as compared to the national average. However, most of the city is located in floodplains. This combined with heavy rainfall events and snow melting from the Pennines makes York prone to perennial flooding from the Rivers Ouse and Foss even though the city has extensive flood defences. The city experienced its recent worst flood since 1625 in the year 2000, when River Ouse reached 5.5m above its normal level of 5m Above Ordinance Datum (AOD) and burst its banks ((Dennis et al., 2003)). This came with disruptions to transport network, destruction of property and a huge blow to the local economy as the city came to a halt for almost three days. It is estimated that about 540 properties were flooded, 320 seriously at risk and 18,700 hectares of agricultural land was affected.
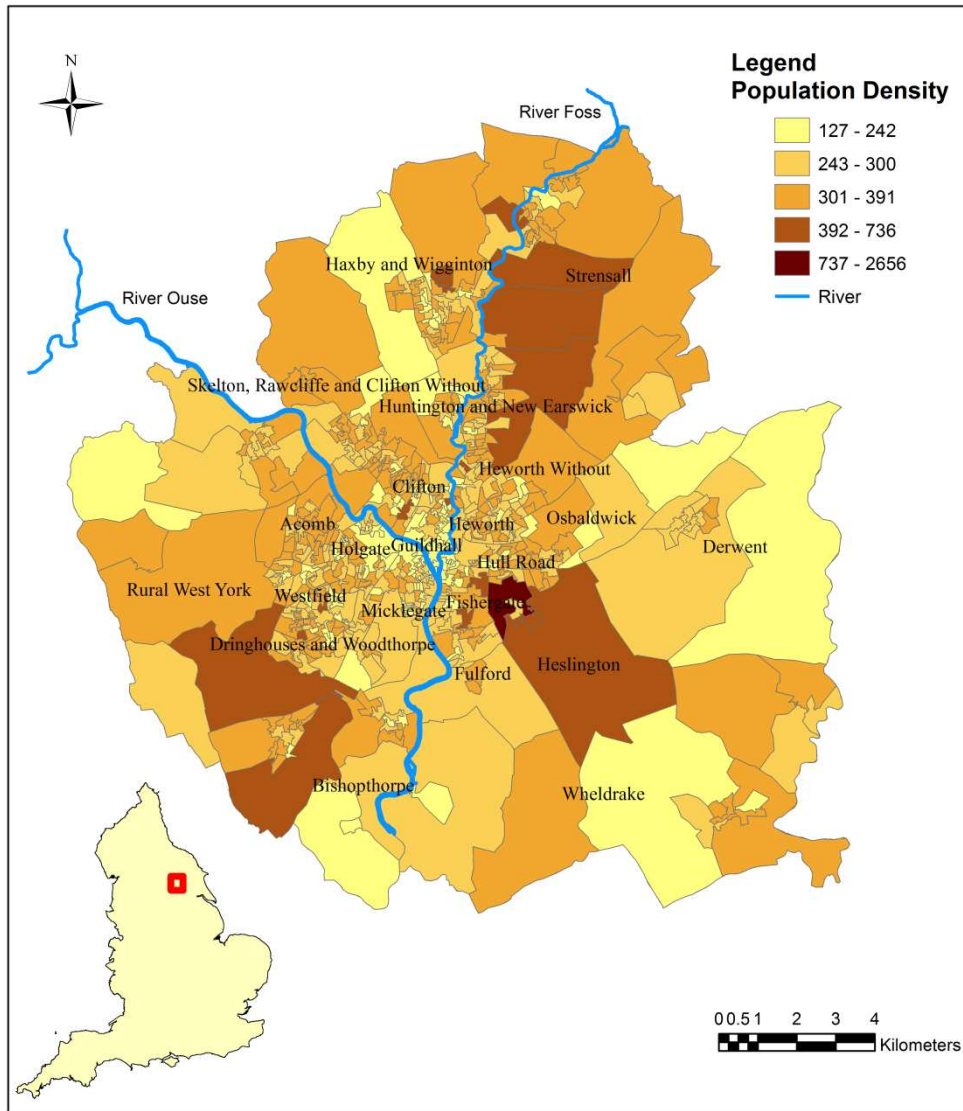
Figure 1: Location of York showing density of wards and output areas

Source: UK data service, 2001 census

4.2   Road network

The city centre is surrounded by an inner ring road (A1036), and seven major radial roads (e.g.A19 and A59, A1079 etc.) connect the city centre with the outer ring road (A64/A1237) and beyond thus providing access within, to and from the city to neighbouring towns and cities (See Figure 2). In order to identify critical road links, one can rigorously put every link of the network to a test e.g. as in El-Rashidy & Grant-Muller (2014), Scott et al (2006) which is computationally intensive especially in large-scale congested networks as described in Chen et al (2012). In contrast one can also learn from the history and use the information available to make informed decisions after putting them to further testing. The city of York got flooded several times in the past affecting different parts the transport network. The Environment Agency (2009) has compiled a list of roads (See Table 1) identifying them as prone to frequent flooding which we use as the starting point in our paper to illustrate the

consequences and thus identify the most critical road links in the network. Figure 2 identifies the flood prone road links.

Table 1: Roads prone to flooding

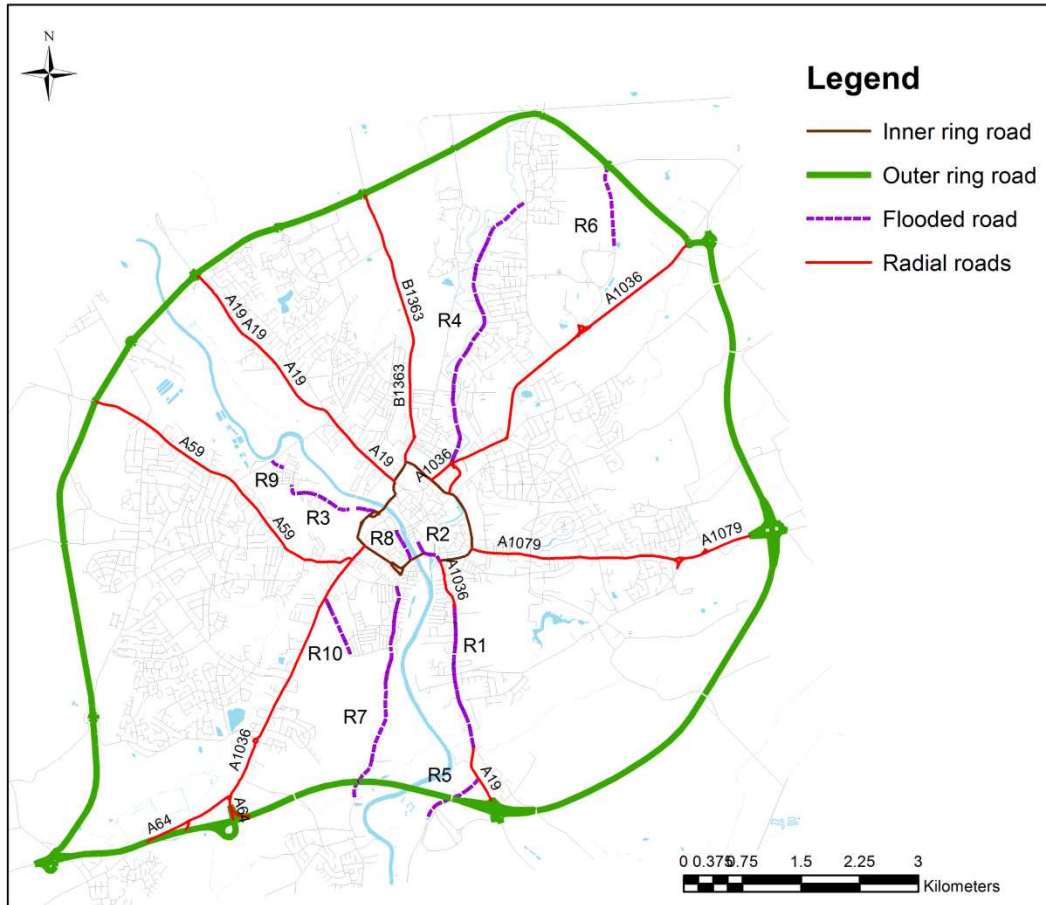| Road	Coded as	Description
Fulford Road	R1	Fulford Road is a major A road in York and forms part of the A19 which runs from Seaton Burns in the north to Doncaster in the south. It is also one of the main radial routes in the city and had an estimated annual traffic volume of about 7,352,000 vehicle kilometers (DfT, 2012c). It also has an estimated average daily flow of about 17061 vehicles per hour (DfT, 2012c).
Tower Street	R2	Tower Street is located at the end of the Bishopgate street bridge and has both A and B classification. The A section forms part of the A1036 road and the B section, the B1227 road.
Leeman Road	R3	Leeman Road is located near Railway station. It links the Station Road with Water End and provides the access to the west from the train station.
Huntington Road	R4	Huntington Road is one of the minor roads that connects the ring road A1237 with the city centre.
Naburn Road	R5	Naburn Lane is a B road which forms part of the B1227 taking traffic from the city onto the A1 motorway.
Monk Cross Link	R6	This road is also a minor road, linking the A1237 and the radial route Malton road/A1036. It serves as a by-pass for traffic avoiding the interchange at intersection of the A1237/164 roads.
Bishopthorpe Road	R7	This road is an A-road that forms part of the A59 that runs from Wallasey in Merseyside to York.
Skeldergate Road	R8	This a local street that connects the Low Ousegate and Bishopgate St. bridges at the west side of River Ouse.
Salisbury Terrace	R9	This road connects to the Water End bridge and proves the east side connection to the railway station.
Knavesmire	R10	Knavesmire Road is a minor road that links the A1036 Tadcaster road and the Bishopthorpe road.

Figure 2: Road network and flood prone road links

4.3   Data needed for computing vulnerability indices

This section describes the model set up and the steps involved in computing the vulnerability indices. As noted earlier in section 2.2.2, urban networks are much more complex as the drivers have many alternative routes available to travel from an origin to a destination. More so as the route choice of drivers depends on the expected generalised cost of travel based on the travel time as well as the distance to their destination. Wardrop's (1952) principle of user equilibrium, which states that *the cost of travel on all used routes for each OD pair will be equal in a congested network*, offers a useful benchmark to compare the costs between network intact and network degraded conditions. In order to solve for user equilibrium route choice of drivers, we have set up a network model using SATURN software (Van Vliet & Hall 2004) which requires two main input data items – (i) coded road network of York and (ii) a trip matrix of York representing the demand for travel between OD pairs. Entire road network of York has been coded as road links and junctions (Figure 3). Travel time on road links is assumed to vary with the flow level following the standard Bureau of Public Roads style speed – flow relationship. The city of York has been divided into 219 zones based on census wards and each zone is connected to the nearest road by a dummy link to allow the incoming /outgoing trips to/from the zone. The OD matrix thus has 219 zones to it, with a possible maximum of 219×219 OD pair combinations.
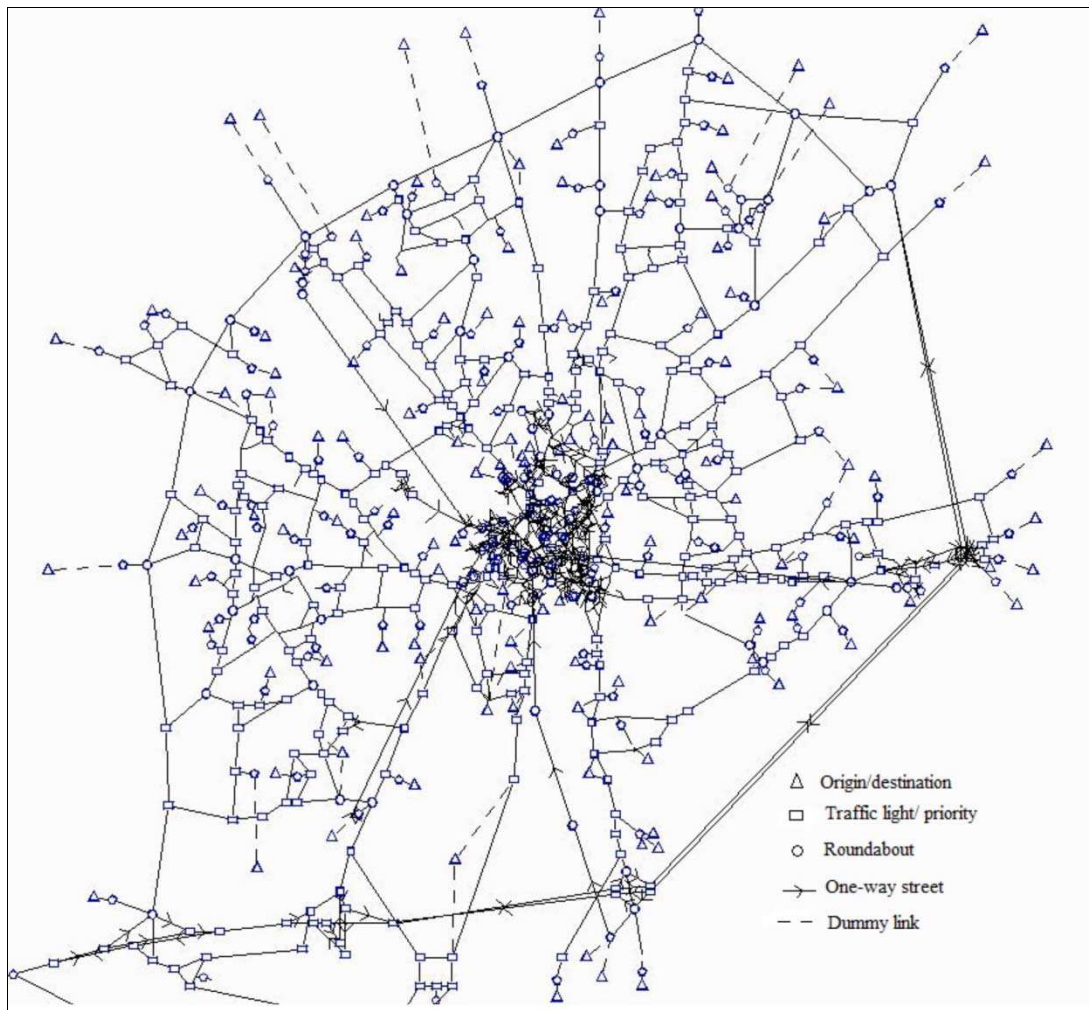
Figure 3: Modelled road network

4.4   Method for computing vulnerability indices

Computing the vulnerability indices M1, M2, M3 requires the knowledge of travel cost by the least cost path which can be obtained by 'skimming' (or simply adding) the link costs along the shortest path for each OD pair. But as the measures M4, M5 are based on multi-path approach, they require the knowledge of link flows and link travel times for all network links. Besides, M5 also requires the information on the number of lanes available for operations both when the network is intact and when it is degraded. The steps involved in computing M1-M3 and M4-M5 are listed separately as below:

Procedure for computing the measures M1,M2,M3:

1. Set up a network based model and assign the OD demand to shortest cost path.
2. Produce the skim cost matrix.
3. Repeat the steps 1,2 above with the network link degraded i.e. after removing the affected critical link.
4. Apply the equations (1), (2), (3) to compute measures M1, M2, M3 respectively for the particular critical link removed.
5. Repeat the steps 3, 4 from the above for *each* of the critical links.

Procedure for computing the measures M4,M5:

1. Set up a network based model and assign the OD demand to multiple routes based on user equilibrium principle.
2. Extract the link flows and link travel times from the assignment.
3. Repeat the steps 1,2 above with the network link degraded i.e. after removing the affected critical link.
4. Apply the equations (4)-(5), (6) to compute M4,M5 respectively for the particular critical link removed.
5. Repeat the steps 3, 4 above for *each* of the critical links.

4.5 Method for preparing a traffic diversion plan

Steps involved in preparing a traffic diversion plan are outlined as below:

1. Assign the traffic to the network when it is intact to establish the normal routing pattern.
2. Identify the origin destination (OD) pairs which normally would use a critical link. This step is carried out by using the *Select Link Analysis* technique which enumerates all OD pairs that use a given link.
3. Then remove the affected link and reassign the traffic to the remaining network. This step establishes the routes for all OD pairs if the affected link is not available.
4. Identify those routes in Step 3 used by the OD pairs noted in Step 2.
5. Compare the routes in Step 4 with those in Step 1 to identify where the diversions are necessary.

5 Numerical results

The main purpose of this section is to illustrate the indices M1-M5 described in sections 2,3 with the urban network of York introduced earlier. Our modelling approach is similar to that of many others e.g. Jenelius et al (2006) and considers the demand to be inelastic i.e. the demand remains constant even if the OD costs change. This is a reasonable assumption when a single link fails at a time especially as we are considering the peak hour travel involving daily commuting, but when many links fail simultaneously, there may be an uncertainty associated with the traffic demand which may need an approach as described in Kurauchi et al (2009). However, in this paper we follow a deterministic approach and assume that the demand is fixed, which has the advantage of comparable total traffic flows across various scenarios involving different link failures and even across alternative vulnerability measures.

5.1 Analysis of vulnerability with fully blocked roads

The procedure described in Section 4.4 for computing the measures M1 to M5 was applied to the urban network York, which resulted in a series of values for indices which are spread widely in terms of their scale. For example, M1 is in the order of hundreds of thousands, and M2, M3 are less than ten while the others fall in the middle somewhere (See Table A1 in

Appendix A). In order to bring them to a similar scale and to build a sense of comparison, we have worked out percentage values of each index relative to its base case i.e. when the network is intact. The exact specification of the indices expressing each as a proportion relative to the respective base value is given in the Appendix B. While building the reference set, it is noted that the base value of each indicator needed is identical whichever link it pertains to (See Table A2 in Appendix C) because the base value refers to the entire network and not to any particular link within the network. Finally the proportionate values of indices M1 to M5 (See Table 2) considering failure of links R1 to R10 each in turn have been arrived at by applying the equations A1 to A5 shown in Appendix B.

Table 2: Proportionate values of vulnerability indices for critical roads (rank in brackets)

Index	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
M1	-64% (=1)	-23% (=4)	-32% (=2)	-23% (=4)	-11% (=8)	-18% (=5)	-30% (=3)	-14% (=7)	-16% (=6)	-11% (=8)
M2	84% (=1)	88% (=4)	86% (=2)	87% (=3)	91% (=7)	88% (=4)	89% (=5)	89% (=5)	90% (=6)	91% (=7)
M3	64% (=1)	23% (=4)	32% (=2)	23% (=4)	11% (=8)	18% (=5)	30% (=3)	14% (=7)	16% (=6)	11% (=8)
M4	28% (=1)	10% (=4)	13% (=2)	9% (=5)	1% (=9)	7% (=6)	11% (=3)	3% (=8)	6% (=7)	1% (=9)
M5	-6% (=1)	-1% (=4)	-3% (=2)	-2% (=3)	-2% (=3)	-1% (=4)	0% (=5)	-2% (=3)	-1% (=4)	-1% (=4)

In order to interpret the results in Table 2 we need to consider the meaning and sense of each of the measures. While M1,M2,M3 are based on a change in the *OD costs* by the least cost path, M4,M5 compute the *total cost* of travel on the entire network based on the link costs and link flows. It is also noted that while increasing values of M1,M2,M5 make the network less vulnerable, decreasing values of M3, M4 make the networks robust. In order to help comparing the results, we have introduced a simple ranking of criticality (shown within the brackets) identifying the relative position in terms of vulnerability (1= most vulnerable,..,9 = least vulnerable). Firstly, it is interesting to note that all indices M1 to M5 consistently identify the link R1 as being the most critical road link. It is also noted that the second most critical link (R3) is also consistently identified across. Furthermore, link R2 is also consistent at fourth place across the measures, the other rankings however vary depending on the indicator chosen. Secondly, it is noted that the values of M3 and hence the rankings have turned out to be identical to M1 (except the sign) as the weights used by both the indices are identical. Thirdly, M5 recognises the serviceability and hierarchy of each link – thus a link with low serviceability and high importance makes it a highly critical ink (e.g. R1). Equally, the measure is also able to identify a minor road (e.g. R3) as critical to the network as it considers the serviceability of each link. Finally the significance of results in Table 2 means that Fulford Road (R1) in the south and Leeman Road (R3) in the west are the two most important roads in York and if affected due to an event they need to be recovered ahead of any other road. In Section 5.3 later we discuss the diversion plans for the two most critical links identified.

5.2 Analysis of vulnerability with partial loss of road capacity and implications to network planning/operations

The main purpose of this section is to illustrate the use of the new M5 when road links are partially damaged after an event. For example, a road link could get partially flooded and probably a lane becomes unusable due to standing water on the carriageway. Equally, it could be due to a broken-down vehicle partially occupying a lane or could be even closing of a lane due to road works by the local authorities leaving the rest of the carriageway for the motorists to use. In order to illustrate the index with a partial loss of capacity, we have developed two test scenarios relating a flood category to the road capacity. The first scenario assumes a minor flood which when occurs, 20% of the road capacity would be lost thus reducing the available capacity of the link to 80%. Second scenario considers a moderate flood which could cause a 50% reduction to the capacity of the affected road. The loss to road capacity in each scenario was applied to each of the critical road links R1 to R10 in turns and the measure M5 was recomputed. The results of these tests for the two test scenarios are summarised in Table 3.

Table 3: M5 with partial loss of road capacity (rank in brackets)

Test scenario	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
20% loss of capacity	-0.23% (=1)	-0.02% (=7)	-0.05% (=4)	-0.08% (=2)	-0.04% (=5)	-0.04% (=5)	-0.05% (=4)	-0.02% (=7)	-0.06% (=3)	-0.03% (=6)
50% loss of capacity	-2.25% (=1)	-0.03% (=8)	-0.08% (=4)	-0.32% (=2)	-0.29% (=3)	-0.06% (=5)	-0.06% (=5)	-0.04% (=7)	-0.11% (=4)	-0.05% (=6)

The results in Table 3 present a consistent pattern of M5 with respect to the loss of capacity i.e. the higher the loss of capacity, the greater the absolute value of M5. For example, a 20 % loss of capacity to R1 results in an M5 of -0.23%, a 50% loss causes an M5 of -2.25% and a fully blocked R1 gives an M5 of -6% (as shown earlier in Table 2). This pattern also holds for all other roads R2 to R10. However, the main implication of the results in Table 3 means that as M5 takes into account the effect of rerouting of traffic, a road which is relatively more(less) critical at a given loss of capacity may turn out to be less(more) critical when it is completely blocked. To give an example, R4 is the second most critical road (M5 = -0.32%) when half the capacity is lost, but when the road is completely blocked it is less critical than R3 which has a higher M5 of -3% (See Table 2). When a given road is blocked either partially or fully, drivers' ability to find an alternative path (in comparable generalised cost terms) will be affected depending on the serviceability of the link as well as the level of congestion at which all other network links are operating. M5 is precisely designed to effectively capture these two factors viz., the serviceability and the congestion in a multi-path environment. The local authorities could perform a series of 'what-if' tests with varying capacity levels representing different real life situations and identify the critical links thus increasing their preparedness to face the events such as floods or simply while preparing alternative diversion plans before closing roads for maintenance. The following section

outlines the traffic diversion plans prepared and summarises the effect of detours due to various links failing.

5.3 Traffic diversion plans and the effect of detours

Steps 1-5 described in Section 4.5 have been applied and a diversion plan each has been developed for all the flood prone roads analysed. Each diversion plan will have a large number of OD pairs to divert, therefore identifying the optimal locations to show diversion signs is not a trivial problem. In our research we have followed a simple heuristic involving identifying the affected OD pairs with high demand and then identifying the diversion as necessary. Figure 4 shows the diversion plan for Fulford Road (R1) and Figure 5 for Leeman Road (R3). For keeping the paper to the journal length we do not show the plans for the other roads. These two roads have been shown as they are the two top ranking critical roads as identified by the M5 and the plans shown are based on the heuristic as indicated. Although the plans show one diversion each, many such plans for other OD pairs put together will form a complete set. In order to illustrate the effect of detours we have worked out the difference in generalised cost between degraded and intact networks for all the OD pairs which is summarised in Table 4. The first row of the table shows the average cost of detour associated with each link failure considering all OD pairs and the second showing the median cost and so on. These summaries have been worked out for all roads R1 to R10.

Table 4: Descriptive statistics of traffic diversion effects associated with link failures#

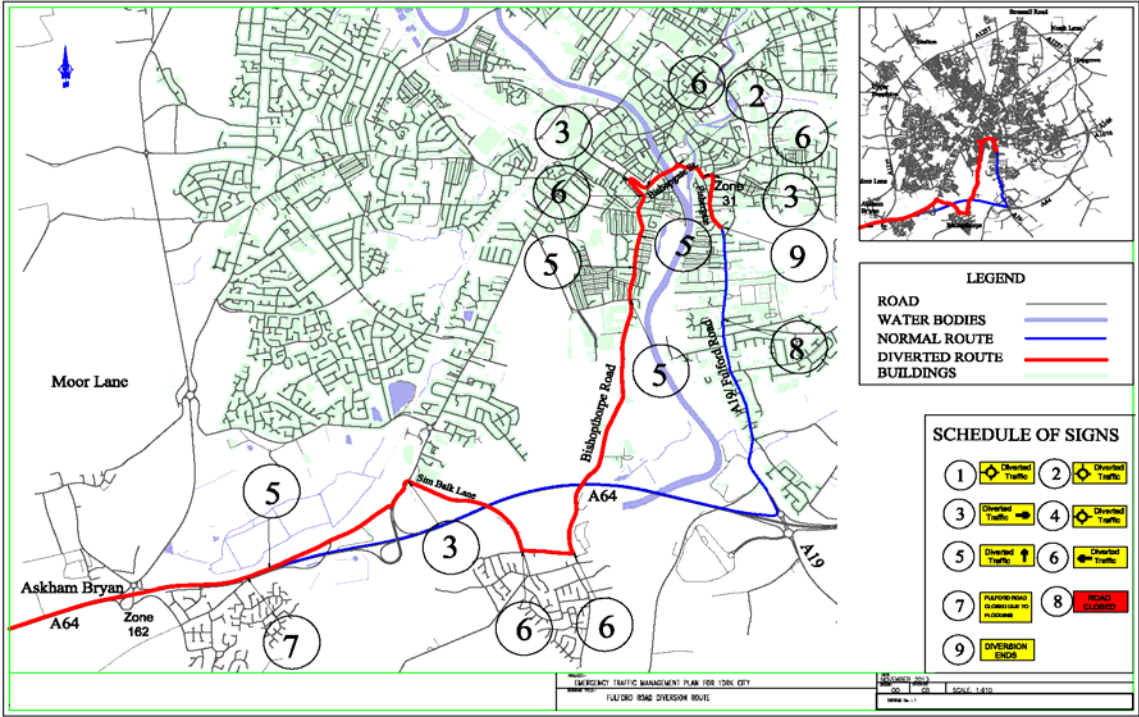| Statistic	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
Mean	570.57	268.01	374.28	186.59	75.05	136.25	213.56	100.21	180.52	74.19
Median	82.62	97.37	247.86	90.54	67.35	96.82	86.65	60.39	83.23	66.58
Standard Deviation	2012.41	932.38	501.09	587.50	46.93	263.66	1114.12	316.54	672.96	46.16
Maximum	27972.43	7352.07	5568.21	9388.64	384.79	4105.99	14609.36	5015.79	7109.40	405.55
Minimum	24.74	13.16	107.34	58.14	8.63	49.65	25.86	3.06	43.20	3.90

In generalised cost units

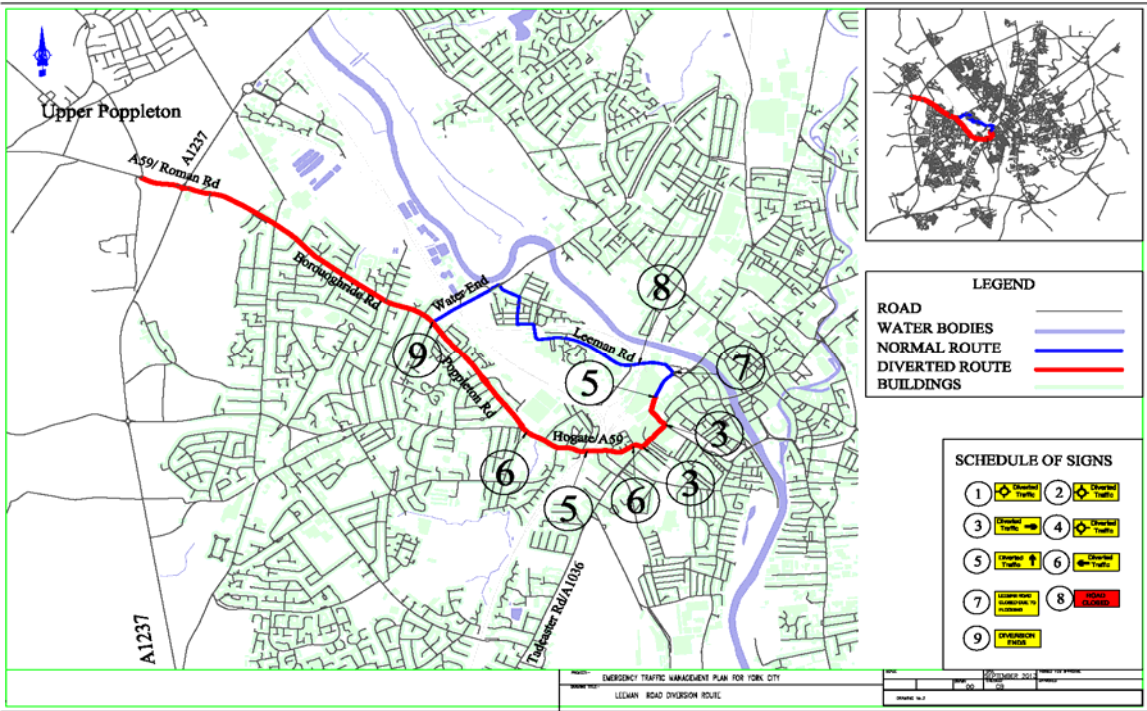**Figure 4: Diversion plan for Fulford Road (R1)**

Figure 5: Diversion plan for Leeman Road (R3)

6.   Concluding remarks

Transport networks are vulnerable to natural disasters such as floods, earthquakes, forest fires etc. which may result in reduced accessibility to certain parts of the land or in the extreme they may remain completely cut off from the rest until the affected links are revived. Analysis of vulnerability of road networks has been a growing research field over the past decade and attracted the attention of researchers in formulating methods resulting in a series of indices measuring the vulnerability of road networks to failures of links in sparse regional networks. This research considered the question of vulnerability of dense urban road networks due to events resulting in partial or complete loss of capacity of links. Denser urban networks present new challenges as lack of accessibility may not be the primary focus any more, but the importance of the affected road link weighs higher. This research applied a series of indices from the literature to the urban network of York in England and compared the outcomes with a new index M5. The main conclusions from this work are as below:

Vulnerability indices based on distance are suitable for sparse regional networks and less so for the urban networks. This is because drivers in urban networks weigh their travel time relatively higher than they do for the distance. In other words, drivers usually follow routes which are quicker even though they are longer in distance terms. Secondly, the indices based on OD costs tend to implicitly assume that all traffic is routed through the least cost OD path and hence are suitable for sparse networks. However, in dense urban networks drivers follow more than one path and hence all network link characteristics are of primary importance to the vulnerability analysis.

The new M5 considers the serviceability in terms of loss of capacity of road links relative to their base values adopted from the hierarchy of road system reflecting their importance. The new M5 means a road with a relatively higher loss of capacity has a greater significance to the vulnerability measure than a similar road with relatively lower loss of capacity will have. Formulation of M5 also means that the loss of capacity of a road can be analysed in continuous terms thus allowing for analysis on the basis of partial loss to the capacity. This means M5 can be applied in general to any situation involving loss of capacity e.g. due to road works, traffic incidents, which may have major implications to road network planning/operations. M5 also allows for several 'what-if' scenarios to be built so that road links can be ranked in terms of their significance which may be extremely useful in developing alternative plans in case of major disruptions.

Analysis of vulnerability does not stop at computing a numerical measure of vulnerability. This research paper also illustrates the use of M5 by developing an outline of a traffic diversion plan when the road network in York is affected by floods.

Appendix A: Vulnerability measures as set out in the literature

Table A1: Values of indices for critical roads

Index	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
M1	-288996	-101661	-145925	-101798	-51585	-79914	-136031	-62574	-72349	-51562
M2	3.73	3.91	3.79	3.85	4.02	3.91	3.93	3.96	3.99	4.03
M3	9.18	3.23	4.63	3.23	1.64	2.54	4.32	1.99	2.30	1.64
M4	1705	617	813	570	92	408	700	212	349	87
M5	-14349	-3016	-6671	-6009	-5620	-2477	-1226	-5131	-2629	-1630

Appendix B: Formulae used for calculating the proportionate values of indices

$$M1 = \frac{\sum_k q_k \Delta c_{ka}}{\sum_k q_k c_k}$$

A(1)

A(1) indicates the change in total cost relative to the base cost with the network intact.

$$M2 = \sum_k \frac{q_k}{\tilde{c}_k} \Bigg/ \sum_k \frac{q_k}{c_k}$$

A(2)

A(2) indicates the trips per unit cost in degraded network relative to the base level trips per unit cost when the network is intact.

$$M3 = \frac{\sum_k w_k \widehat{\Delta c_{ka}}}{\sum_k w_k} \Bigg/ \frac{\sum_k w_k c_k}{\sum_k w_k}$$

A(3)

A(3) computes importance of link *a* relative the base level of importance. A(3) is a general form of A(1) and is identical to A(1) when the weight adopted is equal to the OD demand.

$$M4 = \tilde{C}_a - C/C$$

A(4)

A(4) measures the change in total cost of travel on the network relative to the total cost in the base case when the network is intact.

$$M5 = \left[\sum_{i=1}^{|A|} \left[\left(\frac{x_i^{before}}{r_i^{before}} \right) t_i^{before} \right] - \sum_{i=1}^{|A|} \left[\left(\frac{x_i^{after}}{r_i^{after}} \right) t_i^{after} \right] \right] \Bigg/ \sum_{i=1}^{|A|} \left[\left(\frac{x_i^{before}}{r_i^{before}} \right) t_i^{before} \right]$$

A(5)

A(5) measures the NVI relative to the base case when the network is intact.

Appendix C: Reference values of vulnerability measures when the network is intact

Table A2: Base values of indices for critical roads

Index	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
M1	451118	451118	451118	451118	451118	451118	451118	451118	451118	451118
M2	4.43	4.43	4.43	4.43	4.43	4.43	4.43	4.43	4.43	4.43
M3	14.33	14.33	14.33	14.33	14.33	14.33	14.33	14.33	14.33	14.33
M4	6172	6172	6172	6172	6172	6172	6172	6172	6172	6172
M5	260050	260050	260050	260050	260050	260050	260050	260050	260050	260050

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