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A bilevel multi-objective road pricing model for economic, environmental and health sustainability

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Abstract

We propose a bilevel multi-objective approach to optimise tolls in a road network. Multiple objectives have been considered at either the upper or lower level in the literature but not both. We consider three objectives at the upper level: minimising system travel time; total vehicle emissions; and negative health impacts, modelled as the level of pollutant uptake. For the lower level, we adopt a time surplus maximisation bi-objective user equilibrium model, assuming all users have two objectives: minimising travel time and toll. The complete bilevel optimisation problem is solved using a combination of a metaheuristic and a classical optimisation algorithm.

Keywords: Road pricing; bilevel optimisation; multi-objective optimisation; environmental sustainability; health sustainability

1. Introduction

Sustainable development in transport follows the concept of sustainability defined in the report of the United Nations World Commission on Environment and Development (1987) as meeting the needs of the present without compromising the ability of future generations to meet their own needs. This leads to what is often referred to as the

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“three-legged stool" of sustainability: economic, environmental and social sustainability (European Commission, 2004).

Improving sustainability is challenging because of the existence of many external costs associated with transport, including congestion, environmental costs associated with vehicle emissions and noise from traffic, costs of traffic accidents, etc. For example, while users pay for the internal costs (e.g. vehicle operating costs), the external costs (e.g. environmental and health impacts of greenhouse gases (GHG) and pollutants of vehicle emissions) remain unpaid by the users generating them. Most costs associated with environmental and social sustainability are external in nature. Jakob *et al*. (2006) estimate that in Auckland, New Zealand, the air pollution, accident and climate change costs constitute 58%, 36% and 6%, respectively, among these three externalities. The air pollution costs associated with private vehicles, including the health cost, agricultural damage and forest damage, amount to a total of over NZ$ 211 million in 2001. On the other hand, the annual cost of congestion at around the same time (in 1997 and 2002), was estimated at over NZ$ 700 million (NZIER Authoritative Analysis, 2008). This makes the costs associated with air pollution the second highest external cost only after congestion cost in Auckland.

To enhance sustainability in transport, congestion pricing is a policy instrument that has been applied in many cities around the world, e.g. Singapore, London and Stockholm, as part of an integrated strategy not only to reduce congestion but also to improve the environment in terms of air quality and hence reduce the negative impact of vehicle emissions on health (see Chin, 1996; Santos *et al*., 2010; Tonne et al., 2008; Johansson *et al*., 2009). To support transport policy analysis, it is important to be able to assess if such an integrated strategy can help achieve these three objectives. There are quite a few studies on the effect of congestion pricing on air quality, (e.g. Mitchell, 2005; Beevers and Carslaw, 2005a,b), but not many studies have quantified the effect of congestion pricing on all three aspects spatially. Tonne *et al*. (2008) and Johansson *et al*. (2009) are the only two studies that go one step further by assessing the health impacts of changes in air quality as a result of congestion pricing in London and Stockholm, respectively. Tonne *et al*. (2008) find that the Congestion Charging Scheme in London appears to have resulted in a modest benefit with regard to air pollution levels and associated life expectancy. Johansson *et al*. (2009) demonstrate the importance of not only assessing the effects on air quality limit values, but also quantifying the effect of air pollution on health so that actions to reduce air pollution can be justified.

From an optimisation point of view, to maximise the effectiveness of congestion pricing, it is natural to consider internalising the two major external costs of congestion and air pollution, including the costs associated with their impact on the environment and population health, by charging road users an appropriate toll. The classical formulation of toll optimisation in a road network is a bilevel optimisation problem (Yang and Lam, 1996; Chow, 2010), whereby the upper level represents the decisions of the planner or policy decision maker, and the lower level represents the decisions of the travellers. The upper-level decision maker decides on the tolls, and given the tolls, the travellers make their route choices based on their preferences concerning travel time and toll cost. Naturally, this should be a multi-objective decision problem at both levels. For instance, by optimising the tolls, the upper-level decision maker would want to minimise not only the total travel time, but also the vehicle emission levels such that the environmental and subsequent health impacts can be minimised. On the other hand, given the travel times and tolls on different routes for an origin-destination (O-D) pair, users might have different preferences as per their willingness to pay, yet it is only natural to think that all users would want to minimise travel time as well as monetary cost.

In this paper, we propose a bilevel multi-objective approach to optimising the tolls in a road network. Multiple objectives have been considered at either the upper or lower level in the literature, but not both simultaneously.

At the upper level, it is known that the tolls that minimise total travel time do not necessarily minimise the emission levels (Nagurney *et al*., 2010; Yin and Lawphongpanich, 2006), there exist trade-offs between these objectives and it is important to determine the efficient tolls such that neither the total travel time nor the total emissions can be reduced without increasing the other. In addition to minimising total travel time and total emissions, as highlighted in Johansson *et al*.'s study in Stockholm, the objective of minimising the adverse health impacts is also important for policy analysis. Yin and Lawphongpanich (2006) first apply two objectives at the upper level. These objectives are (1) to minimise system travel time; and (2) to minimise total CO emissions, while the lower level is a classical UE traffic assignment model. The non-dominated frontier of the two objectives is identified, consisting of all combinations of total travel time and total emissions such that neither of the two can be reduced without worsening the other. Yin and Lawphongpanich (2006) showed that the first-best pricing scheme does not necessarily lead to fewer emissions, consistent with the observations made by Nagurney *et al*. (2010). Recently, Chen and Yang (2012) proposed a bilevel optimisation model to determine efficient non-negative link toll schemes and link toll cum rebate schemes, with minimisation of the system travel time and total emissions as the two objectives at the upper level, as in Yin and Lawphongpanich (2006), but considering two types of link emission functions: increasing functions and non-monotone functions. The lower level is also a classical UE model.

For the lower level, it is important to model the variability of preferences among individuals in terms of their willingness to pay. Dial (1979) first considers bi-objective traffic assignment, minimising time and cost as the two objectives in a route choice model. However, in Dial's model (Dial, 1979), a simplification is made consisting of the addition of time and the toll cost in a linear choice function. As discussed in Wang *et al*. (2010), Dial's model only represents a special case of bi-objective user equilibrium (BUE), whereby traffic arranges itself in such a way that no individual trip maker can improve their toll or travel time without worsening the other component by unilaterally switching routes. Dial then further developed algorithms to optimise the tolls such that the system travel time could be minimised (Dial, 1999a,b), i.e. a single objective is considered at the upper level.

In this study, we consider three objectives at the upper level and two objectives at the lower level. At the upper level, we consider three objectives, representing performance measures of economic, environmental and health sustainability. At the lower level, we adopt a BUE model as proposed in Wang *et al*. (2010). We assume that all users have two objectives: (1) to minimise travel time; and (2) to minimise toll cost. This will allow flexibility in modelling route choice behaviour that cannot be captured with the conventional approach of UE. This proposed bilevel multi-objective pricing model can be applied to assess the effect of congestion pricing on travel times as well as air quality and its subsequent effect on health. It can also determine the efficient tolls and their limits in terms of these performance measures.

In this paper we describe our proposed bilevel multi-objective optimisation model (Section 2), present the proposed solution method combining a metaheuristic with a mathematical optimisation technique (Section 3), and then discuss some observations made from some tests on a small network (Section 4). In Section 5, we present our main conclusions.

2. The Model

We will first introduce some notation. Let $G = (N, A)$ be a network, where N is a set of nodes and $A \subseteq V \times V$ is a set of arcs (or links). Let $Z \subset \{(r, s): r \in N, s \in \mathrm{N}\}$ be a set of origin-destination pairs, and d_p be the demand for travel from the origin to the destination of the O-D pair p. For every O-D pair $p \in Z$, there exists a set K_p of paths from the origin of p to its destination. We denote by F_k the flow on path $k \in K_p$, i.e. the amount of traffic using the k^{th} path connecting the O-D pair p. Let $K = \cup_{p \in Z} K_p$ be the set of all paths. Let $\boldsymbol{F} = (F_k: k \in K) \in \mathbf{R}^{|K|}$ denote a vector of path flows. We say that $\boldsymbol{F}$ is *feasible* if $\boldsymbol{F} \geq 0$ and $\sum_{k \in Kp} F_k = d_p$ for all $p \in Z$. We are interested in the flow f_a on each link $a \in A$, and note that link flow can be computed from the path flow as

$$f_a = \sum_{p \in Z} \sum_{k \in K_p} \delta_a^k F_k \tag{1}$$

for all $a \in A$, where $\delta_a^k = 1$ if link a is on the k^{th} path between the O-D pair $p \in Z$ and 0 otherwise. Hence, $\boldsymbol{f} = (f_a: a \in A) \in \mathbf{R}^{|A|}$ defines a link flow vector. In what follows, we work with link and path flows as appropriate. Furthermore, let $\boldsymbol{\tau} = (\tau_{\mathrm{a}}: a \in A)$ be a link toll vector. The dependence of the link flow vector $\mathbf{f}$ on $\boldsymbol{\tau}$ indicates that users will react to tolls set by the policy decision makers, i.e. different tolls will result in different link flows.
In Section 2.1 we describe the three upper level objectives in terms of the above and in Section 2.2 we provide the BUE model at the lower level.

2.1. Upper Level

The upper level models the decision-making process of the policy decision makers. We assume that they have three objectives formulated corresponding to economic, environmental and health sustainability, respectively, as described below.

2.1.1. Objective 1 – Minimise total system travel time

As mentioned before, we denote by d_p the number of trips between the O-D pair $p \in Z$, by f_a the traffic flow on link a (in vehicles per time unit), and by $t_a(f_a)$ the travel time at traffic flow f_a on link a. The Bureau of Public Roads (1964) function will be applied to model the relation between travel time and traffic flow,

$$t_a\left(f_a\right) = t_a^0\left[1 + \alpha\left(\frac{f_a}{c_a}\right)^{\beta}\right] \tag{2}$$

where t_a^0 is the free-flow travel time on link a, c_a is the practical capacity of link a (in vehicles per time unit), and α, β are function parameters. We use $\alpha = 0.15$ and $\beta = 4.0$.

The objective of total travel time minimisation can now be expressed as

$$\min z_t\left(\mathbf{f}\left(\tau\right)\right) = \sum_{a \in A} f_a\left(\tau\right) t_a\left(f_a\left(\tau\right)\right) \tag{3}$$

2.1.2. Objective 2 – Minimise total vehicle emissions

The speed-flow function corresponding to Equation (2) is given by

$$v_a\left(f_a\right) = \frac{v_a^0}{1 + \alpha\left(\frac{f_a}{c_a}\right)^{\beta}} \tag{4}$$

where v_a^0 is the free-flow speed on link a.

We note that the emission functions that have been considered in the literature, in particular in the context of toll optimisation (Yin and Lawphongpanich, 2006), are non-decreasing functions of flow. In reality, depending on the road type, traffic mix (vehicle types, engine sizes, fleet age-mix, etc.), and possibly other factors, the emission functions are not necessarily non-decreasing (Transport Research Laboratory, 1999). We, therefore, identify two different emission functions for testing.

(1) The total CO emission function as adopted in Yin and Lawphongpanich (2006), from Alexopoulos and Assimacopoulos (1993):

$$e_a\left(f_a\right) = 0.2038 \cdot t_a\left(f_a\right) \cdot \exp\left[0.7962 \cdot \frac{l_a}{t_a\left(f_a\right)}\right] \tag{5}$$

where l_a denotes the length of link a. The length l_a is measured in kilometres for each link and the emissions e_a are in grams per hour.

(2) The total CO emission function as derived in Niemeier and Sugawara (2002):

$$e_a\left(v_a\left(f_a\right)\right)=\exp\Big[-0.0562\cdot\left(v_a-16\right)+0.002187\cdot\left(v_a-16\right)^2 \\ -7.609\cdot10^{-5}\cdot\left(v_a-16\right)^3+1.225\cdot10^{-6}\cdot\left(v_a-16\right)^4 \\ +1.837\cdot l_a\Big], \tag{6}$$

where v_a is a function of the flow f_a on link a following Equation (4). Note that v_a in Equation (6) is expressed in miles per hour and l_a is expressed in miles. A conversion factor of 1 mile = 1.609344 kilometres will be applied. Note that this function is neither decreasing nor increasing, neither convex nor concave.

The objective of total CO emission minimisation can then be expressed as

$$\min z_e\left(\mathbf{f}(\tau)\right)=\sum_{a\in A} f_a(\tau)e_a(f_a(\tau)) \text{ or} \tag{7}$$

$$\min z_e\left(\mathbf{f}(\tau)\right)=\sum_{a\in A} f_a(\tau)e_a(v_a(f_a(\tau))). \tag{8}$$

In both cases, the dependence of **f** on $\boldsymbol{\tau}$ indicates, once again, the variability of link flows depending on the toll vector τ set by the policy decision makers.

2.1.3. Objective 3 – Minimise negative impact on health

In order to assess the negative impact of vehicle emissions on health, we need to conduct two major steps.

(1) Estimation of the pollutant uptake by individuals on each used path

We adopt a three-stage approach to modelling the pollutant uptake by the travellers during their trip. This involves: (i) modelling the emission rates for each link based on the traffic flow, the average vehicle speed and the vehicle fleet composition; (ii) modelling the air pollutant concentrations from the emission rates and the surface meteorology; and (iii) modelling the pollutant uptake by travellers from the air pollution concentrations and the travel time along each link. Each stage is described in turn below. While this approach is in general applicable to any pollutant, we will concentrate on carbon monoxide (CO) in what follows.

(i) Stage 1 – From traffic flow, speed and fleet composition to vehicle emissions

Here we adopt the emission functions as identified in Equations (5) and (6). From these, we obtain the total vehicle emission rate of CO on each link, e_a, in grams per hour.

(ii) Stage 2 – From vehicle emissions to pollutant concentrations

Here, we adopt the *Site-Optimised Semi-Empirical* (SOSE) model as described in Dirks *et al*. (2002, 2003) to predict the CO concentration on link a, C_a, from the road emission rate and the average wind speed,

$$C_a=\frac{e_a(f_a)}{\Delta z\cdot\left(u+u_0\right)}+C_B, \tag{9}$$

where C_a is the estimated concentration of CO along link a; $e_a(f_a)$ is the total emission level of CO on link a; and Δz, u, u_0, C_B are calibrated model parameters. The parameter Δz is the "box height" defined as the height of a box above the road into which pollutants are assumed to be uniformly mixed, u is the horizontal wind speed, u_0 is the wind speed offset, included to avoid unrealistically high pollution concentration predictions in periods of very low wind speeds, and C_B is the background concentration.

(iii) Stage 3 – From CO concentration to CO dose

Here we adopt the approach proposed by Dirks *et al*. (2012) to predict the uptake of CO of a passive traveller based on the road emission rate, the time spent travelling on a link and the breathing rate of the traveller.

$$d_a = C_a \cdot t_a(f_a) \cdot \overline{\beta}_0 \tag{10}$$

where d_a is the dose of CO along link a; t_a is the travel time on link a; and $\overline{\beta}_0$ is the breathing rate at rest. Since dose is an additive function, the total dose for route k is simply the sum of the dose on each link along the route, i.e.

$$d_k = \sum_{a \in k} d_a \tag{11}$$

(2) Health impact assessment based on population exposure level

Based on the three-stage process above, we can determine the total individual CO dose on each used path. We then apply the following measures to assess the health impact of pollutant uptake on the population.

(i) Median individual CO dose

$$\min z_d\left(\mathbf{f}(\tau)\right) = \text{median}_{k \in K} d_k\left(\mathbf{f}(\tau)\right) \tag{12}$$

(ii) Maximum individual CO dose

$$\min z_d\left(\mathbf{f}(\tau)\right) = \max_{k \in K} d_k\left(\mathbf{f}(\tau)\right) \tag{13}$$

The dependence of the objective function z_d on toll τ is via the CO concentration function in Equation (9), which in turn depends on the emission function e_a, Equations (5) and (6), that is determined in part by link speed v_a, Equation (4), and thus link flow and toll. Note that the median CO dose is a proxy variable to measure the extent to which the population is exposed to CO emissions, while the maximum CO dose measures the worst case within the population.

2.2. Lower Level

The lower level models the route choice behaviour of travellers given the toll values from the upper level. We assume that road users wish to minimise: (1) their travel time; and (2) their toll cost. This leads to the definition of bi-objective user equilibrium (see Definition 2 in Wang and Ehrgott (2013)). In bi-objective user equilibrium problems, we consider two path cost functions, namely, travel time $C_k^{(1)}(\mathbf{F}) = T_k(\mathbf{f}) = \sum_{a \in k} t_a(f_a)$, where $t_a(f_a)$ is the travel time function (2) and toll $C_k^{(2)}(\mathbf{F}) = \tau_k = \sum_{a \in k} \tau_a$. Hence, both path costs are additive, link travel time and link toll are separable, and link toll does not depend on flow.

Definition 1 Feasible path flow vector **F*** is a bi-objective equilibrium flow, if whenever $\mathbf{C}_k(\mathbf{F^*}) \leq \mathrm{C}_{k'}(\mathbf{F^*})$ and $\mathbf{C}_k(\mathbf{F^*}) \neq \mathrm{C}_{k'}(\mathbf{F^*})$ for $k, k' \in K_p$ for any $p \in Z$ then $F_{k'} = 0$.

In Wang and Ehrgott (2013), we have shown that the bi-objective user equilibrium in Definition 1 is equivalent to the time surplus maximisation bi-objective user equilibrium (TSmaxBUE) model, also proposed in Wang and Ehrgott (2013), which we briefly review here. We assume that given an O-D pair p, users have an indifference function between toll and time. For any given path $k \in K_p$ with a specific toll τ_k, there is a limit on the travel time that a user would be willing to spend. We model this indifference function as a function $T_p^{max}: \mathbf{R} \to \mathbf{R}$ that is strictly decreasing, i.e. $T_p^{max}(\tau_k^1) < T_p^{max}(\tau_k^2)$ if $\tau_k^1 > \tau_k^2$. This takes into account that users would expect to spend less time in traffic if they need to pay a higher toll.

Given the indifference curves T_p^{max} for all $p \in Z$, we define the time surplus for path $k \in K_p$ as

$$TS_k(\mathbf{F}) := T_p^{max}(\tau_k) - T_k(\mathbf{f}) = T_p^{max}\left(\sum_{a \in k} \tau_a\right) - \sum_{a \in k} t_a(f_a) \tag{14}$$

Then, road users choose the path k* with maximum time surplus, i.e.

$$k^* = \text{argmax}\{TS_k(\mathbf{F}) : k \in K_p\} \tag{15}$$

The equilibrium state at the road user level, i.e. the lower level of our bilevel model is then defined by the TSmaxBUE condition.

Definition 2 The path flow vector **F*** is called a time surplus maximisation bi-objective user equilibrium flow if $F_k > 0 \Rightarrow TS_k^{max}(\mathbf{F}^*) \geq TS_{k'}^{max}(\mathbf{F}^*)$ for all $k, k' \in K_p$, or equivalently, if $T_k^{max}(\mathbf{F}) > TS_{k'}^{max}(\mathbf{F}) \Rightarrow F_{k'} = 0$.

In words, the TSmaxBUE condition states that:

“Under the *Time Surplus Maximisation equilibrium* condition traffic arranges itself in such a way that no individual trip maker can improve his/her time surplus by unilaterally switching routes,"

or alternatively

“Under the *Time Surplus Maximisation equilibrium* condition all individuals are travelling on the path with the highest time surplus value among all the efficient paths between each O-D pair."

In summary, our bilevel model requires the setting of a link toll vector τ that simultaneously minimises the three functions $z_t(\mathbf{f}(\boldsymbol{\tau}))$, $z_e(\mathbf{f}(\boldsymbol{\tau}))$ and $z_d(\mathbf{f}(\boldsymbol{\tau}))$ at the upper level, and the attainment of a bi-objective user equilibrium with respect to path cost functions $\sum_{a\in k} t_a(f_a(\boldsymbol{\tau}))$ and $\sum_{a\in k} \tau_a$ at the lower level. Thus, we deal with a bilevel model with a three-objective optimisation problem at the upper level and a bi-objective equilibrium problem at the lower level.

3. Solution Method

3.1. A Game Theoretical Approach

The bilevel model described in Section 2 resembles a Stackelberg leader-follower game with the upper level decisions belonging to the leader and the lower level ones belonging to the follower. In our road pricing model, the policy decision maker, acting as the leader, decides the tolls to be imposed on each link of the network with the aim to minimise the following three objectives: (1) the system travel time, z_t; (2) the total CO emissions (in grams per hour) over the network, z_e; and (3) the median or maximum CO dose to road users over different paths on the network, z_d. The decisions of policy makers on tolls must, however, take into account the behaviour of the followers, i.e. the network users, who respond to the imposed link tolls by configuring themselves over the network paths in a way that optimises their individual objectives. As explained above, we assume that road users maximise their time surplus among the available routes, considering their desire to minimise both toll cost and travel time.

The generic mathematical formulation of a bilevel optimisation problem that models a Stackelberg leader-follower game, as described above, is provided in Equations (16) – (19).

$$\min \Phi_u(x_u, x_l) \tag{16}$$

$$x_l \in \arg\min\{\phi_l(x_l) : \psi_l(x_l) \geqq 0, \vartheta_l(x_l) = 0\} \tag{17}$$

$$\Psi_u(x_u, x_l) \geqq 0 \tag{18}$$

$$\Xi_u(x_u, x_l) = 0 \tag{19}$$

In Equations (16) – (19), Φ_u and ϕ_l denote the objective functions at the upper and lower level, respectively. Similarly, Ψ_u; Ξ_u and ψ_l; ϑ_l are, respectively, the upper and lower level constraints. Decision variable vectors $\boldsymbol{x}_u = (x_1, ..., x_r)$ and $\boldsymbol{x}_l = (x_{r+1}, ..., x_n)$ comprise the upper and lower level decision variables that together form $\boldsymbol{x} = (\boldsymbol{x}_u, \boldsymbol{x}_l)$, which is the n-dimensional decision variable vector of the overall problem. It is important to note that the lower level problem is optimised with respect to $\boldsymbol{x}_l$ only, while $\boldsymbol{x}_u$ acts as a fixed parameter. Therefore, $\boldsymbol{x}_l$ can be considered to be a function of $\boldsymbol{x}_u$.

Relating the generic mathematical formulation (16) – (19) to the bilevel road pricing model of Section 2, the link toll vector $\boldsymbol{\tau} = (\tau_a : a \in A)$ constitutes the upper level decision variable vector $\boldsymbol{x}_u$ and the link flow vector $\mathbf{f}(\boldsymbol{\tau})$ constitutes the lower level decision variable vector $\boldsymbol{x}_l$. Furthermore, the upper level objective function vector is given by $\Phi_u(\boldsymbol{x}_u) = (z_t(\mathbf{f}(\boldsymbol{\tau})), z_e(\mathbf{f}(\boldsymbol{\tau})), z_d(\mathbf{f}(\boldsymbol{\tau})))$. The only constraints at the upper level are the non-negativity of the tolls $\boldsymbol{\tau} \geq 0$, whereas at the lower level, link flows must be non-negative ($\mathbf{f}(\boldsymbol{\tau}) \geq 0$) and path flows must satisfy demand ($\sum_{k\in K_p} F_k = d_p$ for all $p \in Z$).

At the lower level, however, we have described an equilibrium model in Section 2.2 that results from each road user maximising their own time surplus, Equation (14). This equilibrium model does not directly fit into the framework of the bilevel optimisation problem (16) – (19). We therefore replace the TSmaxBUE equilibrium model with an equivalent (unconstrained) optimisation problem, following the nonlinear complementarity approach optimising a gap function as described in (Lo and Chen, 2000).

Considering the function,

$$\phi(a,b) = \frac{1}{2}\left(\sqrt{a^2+b^2} - (a+b)\right)^2 \tag{20}$$

Lo and Chen (2000) have shown that path flow vector $\mathbf{F} = (F_k : k \in K)$ is an equilibrium flow vector with a (single) path cost function $C_k(\mathbf{F})$ that is to be minimised if and only if $\mathbf{F}$ is an optimal solution to the following unconstrained optimisation problem (21),

$$\begin{aligned} \min \sum_{p\in Z}\sum_{k\in K_p}\frac{1}{2}\left[\sqrt{(F_k)^2+\left(\eta_k-\pi_p\right)^2}-\left(F_k+\left(\eta_k-\pi_p\right)\right)\right]^2 + \\ \sum_{p\in Z}\frac{1}{2}\left[\sqrt{(\pi_p)^2+\left(\sum_{k\in K_p}F_k-d_p\right)^2}-\left(\pi_p+\left(\sum_{k\in K_p}F_k-d_p\right)\right)\right]^2. \end{aligned} \tag{21}$$

Here η_k is a variable denoting the cost of path k while variable π_p stands for the minimal cost of any path from the set K_p of paths connecting origin and destination of O-D pair $p \in Z$. Because Equation (21) requires a path cost function (to be minimised), but time surplus in Equation (14) is to be maximised, the path cost function considered here is $C_k(\mathbf{F}) = \sum_{a\in k} t_a(f_a) + T^{max}(0) - T^{max}(\tau_k)$, see Wang and Ehrgott (2013) for details. Notice that the first term in Equation (21) ensures the satisfaction of the requirements of the TSmaxBUE condition, i.e. the paths with time surplus less than the maximum carry zero flow, while the second term in Equation (21) accounts for the demand constraints.

3.2. Solution Algorithm

The bilevel multi-objective optimisation problem is solved using a combination of a metaheuristic and a classical optimisation algorithm. A multi-objective evolutionary algorithm (namely NSGA-II, see Deb et al. (2002)) handles the upper level problem, while the NCP problem (21) is solved with a quasi-Newton method. The algorithm was coded in MATLAB and the quasi-Newton method available with the fminunc routine of MATLAB was used. See Deb and Agrawal (1994) and Deb and Goyal (1996) for more details on the genetic operators. The steps of the evolutionary algorithm are as follows.

Step 1	Generate a random population of link toll vectors and set the generation count to 1.
Step 2	For each individual, solve the NCP using `fminunc`. Use the link flow vector obtained from the lower level problem to compute the three objective values of the upper level problem.
Step 3	Rank the individuals into fronts using the fast non-dominated sorting approach of NSGA-II and compute the crowding distance of each individual in objective space.
Step 4	Use the rank and the crowding distance lexicographically to populate a pool of parent solutions (these are solutions within the present generation which are allowed to undergo genetic operations of cross-over and mutation), via a tournament selection procedure. The size of this pool is taken to be half of the total population size.
Step 5	Parent individuals are selected at random from within the pool formed in Step 4 to generate a population of children, using either real valued cross-over or real valued mutation operations. The size of the child population is taken to be equal to the size of the random population generated in Step 1.
Step 6	For each member of the child population, the lower level NCP is solved again.
Step 7	The evaluated child population is merged with the population of individuals from which the parent pool was extracted, following which Step 3 is repeated.
Step 8	Use the ranks and crowding distance lexicographically to form the subsequent generation of individuals. The size of the new generation formed must be equal to the size of the random population generated at Step 1.
Step 9	Increment the generation count by 1.
Step 10	If the generation count reaches the termination criteria, then stop the algorithm, else continue from Step 4.

4. A Four Node Example

To test our model and algorithm, we consider a network with four nodes and eight links. The objective values of the initial and final populations of the genetic algorithm optimisation process for the four respective cases resulting from the possible combinations of dose function z_d and emission function z_e are investigated. The results obtained clearly indicate that the algorithm proceeds from a random distribution of objective vectors to an approximate non-dominated frontier. To provide more insight into the trade-offs between the objectives, and hence the necessity to consider all three objectives at the upper level, we investigate more detailed results for the four respective cases. These show that there is some trade-off between minimum total travel time and minimum CO emission of around 1-2% (with both emission functions). However, both the minimum maximal and minimum median dose of CO to individuals are obtained with considerably higher total travel time (4-5% increase) and total CO emission (5% up to 18%) than the minimum, depending on the emission function used. These results indicate the importance of considering individual health impact in the model.

5. Conclusions

In this paper, we have proposed a multi-objective bilevel pricing model for sustainable transport system development. The model consists of three objective functions representing economic, environmental and health sustainability at the upper or policy decision maker level, while the lower level is a bi-objective user equilibrium considering path cost functions travel time and toll cost. By re-casting the lower problem as a nonlinear complementarity problem, we were able to develop a solution method based on the combination of a multi-objective evolutionary metaheuristic with a mathematical optimisation technique to solve the resulting bilevel multi-objective optimisation problem. Our results on a small test problem show the conflicting nature of the upper level objectives, and demonstrate the need to consider bilevel multi-objective methods for policy decision making in the context of the sustainable development of transportation systems. Further work will consider the solution of the model on realistic networks.

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