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Optimized Low Complexity Sensor Node Positioning in Wireless Sensor Networks

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***Abstract*—Localization of sensor nodes in wireless sensor networks (WSNs) promotes many new applications. Longer life time is imperative for WSNs, this requirement constrains the energy consumption and computation power of the nodes. In order to locate sensors at a low cost, the received signal strength (RSS)-based localization is favored by many researchers. RSS positioning does not require any additional hardware on the sensors and does not consume extra power. A low complexity solution to RSS localization is the linear least squares (LLS) method. In this paper, we analyze and improve the performance of this method. Firstly, a weighted least squares (WLS) algorithm is proposed which considerably improves the location estimation accuracy. Secondly, reference anchor optimization using a technique based on the minimization of the theoretical mean square error (MSE) is also proposed to further improve performance of LLS and WLS algorithms. Finally, in order to realistically bound the performance of any unbiased RSS location estimator based on the linear model, the linear Cramer-Rao bound (CRB) is derived. It is shown via simulations that employment of the optimal reference anchor selection technique considerably improves system performance, while the WLS algorithm pushes the estimation performance closer to the linear CRB. Finally, it is also shown that the linear CRB has larger error than the exact CRB, which is the expected outcome.**

***Index Terms*—Localization, Received signal strength (RSS), Cramer-Rao bound.**

I. Introduction

Wireless sensor networks (WSNs) consists of many small (up to several hundred) of low powered sensing nodes [1]. These nodes can be capable of sensing temperature, humidity, light intensity etc. In location aware WSNs, these nodes aside from sensing environmental conditions can also locate themselves. Thus promoting many new applications in the wireless communications industry. These applications may include firefighter tracking, cattle/wild life monitoring and logistics [2]. One way to locate the nodes is to use global positioning system (GPS), however deploying a GPS chip on every sensor node is expensive and energy consuming. Moreover, GPS assisted nodes can only be located when a guaranteed line of sight (LoS) is present with the navigational satellite. Hence nodes can be located using local positioning systems.

Various techniques can be found in literature to locate wireless sensor nodes. Location algorithms, which are based on the absolute distance between nodes are known as range based algorithms. On the other hand, algorithms that do not require determining the actual inter-node distance for localization are called range-free positioning algorithms [3], [4]. Range free algorithms are based on the number of hops for communications between two nodes as a distance metric. Range based algorithms are however more accurate than range free algorithms.

In the context of range based algorithm, distance can be estimated between nodes by making use of the angle of the impinging signal, this technique is more commonly known as the angle of arrival (AoA) technique [5], [6]. Apart from being very sensitive to errors due to multipath, AoA is not favored for low complexity WSN localization as an array of antennas or microphones are required on the sensor nodes to estimate the angle of the incoming signal. This increases the complexity and cost of the system. Absolute distance can be estimated using either the delay or attenuation of the signal. Systems capitalizing on the delay are more commonly known as time of arrival (ToA) systems. ToA localization, although more accurate, requires highly accurate clocks and hence are high in complexity [7], [8], [9]. On the other hand, received signal strength (RSS) based systems require no additional hardware and hence are more suitable for WSNs [10], [11], [12], [13].

For location estimation via RSS (and ToA) the so called trilateration technique is used. A number of nodes, usually high in resources and with known locations known as anchor nodes (AN) are used to estimate the locations of target nodes (TN). The location of ANs can be determined using GPS or they can be placed at predetermined positions. Readings from the TN is received at the ANs and are transmitted to a central station for processing.

Due to the non linear nature of the localization problem, location estimation via RSS (and also for ToA) can be achieved using maximum likelihood (ML) techniques [14], [15], [16] that commonly operate in an iterative fashion. Generally, a close initial estimate of location is required for the ML algorithm. Furthermore, the ML technique due to its iterative nature is high in complexity. On the other hand, location can also be estimated employing a low complexity linear least squares (LLS) approach [17].

In this paper we analyze and propose improvement to the performance of the LLS RSS location estimator. The LLS technique does not require a close initial estimate and is of low complexity as it does not require multiple iterations. The basic concept behind the LLS technique is that instead of using individual readings from ANs, readings from AN pairs are first formulated (subtracted from each other) to linearize the non linear system of equations. Generally, a reference node has to be chosen and paired with all other ANs. However, random selection of an AN as a reference can cause performance

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degradation. Other techniques to linearize the system includes averaging the readings from all ANs and then pairing them with individual AN. Finally, pairing each AN with every other AN can be used for linearization. The system performance can be optimized by choosing an optimal reference AN and pairing it with all other ANs .

For ToA systems, the authors in [18] have formulated a technique to choose an optimal reference AN, however no such study has been done for RSS localization. In this paper we devise a technique for optimal reference AN selection using the RSS systems. In order to further improve the performance, the correlation between the (now linear) RSS readings is used and a weighted least squares (WLS) algorithm is proposed. For optimized performance the optimal AN selection for the WLS method is also given in the paper.

In order to compare the MSEs of estimators the Cramer-Rao bound has been extensively used as a benchmark. For ML algorithms, the CRB on location estimated has been derived for ToA in [19], [20] and for RSS systems in [12]. However, since the LLS method is note based on individual readings, the CRB given in [12] does not tightly bound the performance of the LLS-RSS estimator. For ToA LLS technique the CRB is given in [18]. In this paper we derive the linear CRB to tightly bound the performance of the LLS and WLS algorithm based on the RSS system.

To sum up, the main contributions of this paper are as follows:

- WLS algorithm for the linear model is proposed.
- Optimal anchor selection for both LLS and WLS methods is proposed.
- Linear CRB for RSS systems is derived.

Simulation results show that the linear CRB is significantly larger than the exact CRB and is thus more realistic in lower bounding the performance of RSS systems using the linear model. It is shown via simulations that the performance of the LSS estimator improves considerably when the optimal reference AN is used. The system performance is further improved using the WLS algorithm with optimal AN selection.

The rest of the paper is organized as follows. Section II presents the problem statement and the system model. In Section III, the linear RSS model and the LLS solution is presented. In section IV, the WLS algorithm is proposed. In section V, the optimal reference AN selection technique is presented. In section VI, linear CRB is derived. Finally, in section VII, we discuss the simulation results which are followed by conclusions.

II. System Model

For future use, we define the following notations.

$\mathcal{R}^n$ is the set of n dimensional real numbers. $Tr(\mathbf{M})$ and $\det(\mathbf{M})$ represent the trace and determinant of the matrix $\mathbf{M}$ respectively. $(.)^T$ is the transpose operator. $E(.)$ refers to the expectation operator. $(\mathbf{M})_{ij}$ refers to the element at the i^{th} row and j^{th} column of matrix $\mathbf{M}$. $\mathcal{N}(\mu, \sigma^2)$ represents the normal distribution with mean μ and variance σ^2. $\mathbf{1}_{N\times N}$ represents the $(N \times N)$ matrix of all ones.

A two dimensional (2-D) network is considered, consisting of a TN which has unknown coordinates $\boldsymbol{\theta} = [x, y]^T$ $\left(\boldsymbol{\theta} \in \mathcal{R}^{\mathbf{2}}\right)$ that are to be estimated, and M ANs with known locations $\boldsymbol{\theta}_i = [x_i, y_i]^T \left(\boldsymbol{\theta}_i \in \mathcal{R}^2\right)$ for $i = 1, ..., M$. The received power at the ANs due to random shadowing is log-normally distributed. This model is based on empirical results obtained in [21], [22]. Thus the distance d_i between the TN and the i^{th} AN, is related to the path-loss at the i^{th} AN, $\mathscr{L}_i$, and the path-loss exponent (PLE), α_i, as [23]

$$\mathscr{L}_i = \mathscr{L}_0 + 10\alpha_i \log_{10} d_i + w_i, \tag{1}$$

where $\mathscr{L}_0$ is the path-loss at the reference distance d_0 ($d_0 < d_i$, and is normally taken as 1 m) and w_i is a zero-mean Gaussian random variable with known variance representing the log-normal shadowing effect, i.e. $w_i \sim \left(\mathcal{N}\left(0, \sigma_i^2\right)\right)$. The PLEs are assumed to be known via prior channel modeling or accurate estimation [25]. The path-loss is calculated as

$$\mathscr{L}_i = 10 \log_{10} P - 10 \log_{10} P_i \tag{2}$$

where P is the transmit power at the TN and P_i is the received power at the i^{th} AN. The distance d_i is given by

$$d_i = \sqrt{\left(x - x_i\right)^2 + \left(y - y_i\right)^2}.$$

The observed path-loss (in dB) from d_0 to d_i, $z_i = \mathscr{L}_i - \mathscr{L}_0$, can be expressed as

$$z_i = f_i(\boldsymbol{\theta}) + w_i, \qquad i = 1, ..., M \tag{3}$$

where $f_i\left(\boldsymbol{\theta}\right) = \gamma\alpha_i \ln d_i$ and $\gamma = \frac{10}{\ln 10}$. In a vector form, we have

$$\mathbf{z} = \mathbf{f}\left(\boldsymbol{\theta}\right) + \mathbf{w}, \tag{4}$$

where $\mathbf{z} = \left[z_1, ..., z_M\right]^T$ is the vector of the observed path loss. $\mathbf{f}\left(\boldsymbol{\theta}\right) = \left[f_1\left(\boldsymbol{\theta}\right), ..., f_M\left(\boldsymbol{\theta}\right)\right]^T$ is the actual path-loss vector and $\mathbf{w} = \left[w_1, ..., w_M\right]^T$ is the noise vector.

Since the noise is Gaussian and assuming independence of the noise components, the joint conditional probability density function (pdf) of $\mathbf{z}$ is given by

$$p\left(\mathbf{z} \mid \boldsymbol{\theta}\right) = \prod_{i=1}^{M} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left\{-\frac{\left(z_i - f_i\left(\phi\right)\right)^2}{2\sigma_i^2}\right\}. \tag{5}$$

Thus, the maximum likelihood (ML) estimate of (5) is equivalent to the nonlinear least square (NLS) solution of the cost function

$$\varepsilon\left(\boldsymbol{\theta}\right) = \left(\mathbf{z} - \mathbf{f}\left(\boldsymbol{\theta}\right)\right)^T \left(\mathbf{z} - \mathbf{f}\left(\boldsymbol{\theta}\right)\right). \tag{6}$$

The solution to (6) is obtained using high complexity iterative techniques such as the Gauss-Newton or Levenberg-Marquardt techniques [15], [16]. Due to its iterative nature, the ML techniques can converge to local minimum instead of global minimum if given an initial seed that is far from the actual node location. Hence a close initial guess is essential to the reliability of the ML technique. In addition to the high complexity of the ML method, it can suffer from various other challenging issues detailed in [26].

In order to bypass the close initial estimate requirement and high complexity of the ML method, location coordinates can be estimated using a low complexity linear least squares technique explained in the next section.

III. Linear model

The idea behind the LLS is to first linearize the RSS measurements and then use ordinary least squares (OLS) to estimate the unknown parameters. This idea was first introduced for ToA systems in [24] and analyzed for the same in [18]. However, for RSS measurements the the linearization is somewhat different due to additional parameters such as the PLEs. The non-linear system of path-loss equations can be linearized as follows. From (3), it can be readily shown that

$$E\left(\frac{1}{\beta_i}\exp\left(\frac{2z_i}{\gamma\alpha_i}\right)\right) = d_i^2,$$

where $\beta_i = \exp\left(\frac{2\sigma_i^2}{(\gamma\alpha_i)^2}\right)$. Similarly choosing a reference AN, it can be shown

$$E\left(\frac{1}{\beta_r}\exp\left(\frac{2z_r}{\gamma\alpha_r}\right)\right) = d_r^2$$

where $\beta_r = \exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right)$. For linearization, the square of each distance equation is subtracted from the square of a reference distance equation d_r^2. This results in a linear system which is represented in matrix form as

$$\mathbf{b} = \mathbf{A}\boldsymbol{\theta} + \mathbf{v}, \tag{7}$$

where $\mathbf{b} = [b_1, ..., b_N]^T$, is the observation vector and is given by

$$\mathbf{b} = \begin{bmatrix} \delta_r - \delta_1 - \kappa_r + \kappa_1 \\ \delta_r - \delta_2 - \kappa_r + \kappa_2 \\ \vdots \\ \delta_r - \delta_N - \kappa_r + \kappa_N \end{bmatrix}$$

for $\delta_r = \frac{1}{\beta_r}\exp\left(\frac{2z_r}{\gamma\alpha_r}\right)$ and $\delta_i = \frac{1}{\beta_i}\exp\left(\frac{2z_i}{\gamma\alpha_i}\right)$. While

$$\kappa_r = x_r^2 + y_r^2 \text{ and } \kappa_i = x_i^2 + y_i^2$$

for $i \neq r$, $i = 1, ..., N$ and $N = M - 1$; and $\mathbf{A}$ is the $N \times 2$ data matrix

$$\mathbf{A} = 2\begin{bmatrix} x_1 - x_r & y_1 - y_r \\ x_2 - x_r & y_2 - y_r \\ \vdots & \vdots \\ x_N - x_r & y_N - y_r \end{bmatrix}.$$

$\mathbf{v}$ is the noise vector which has zero mean and variance given by

$$\begin{aligned} \breve{\sigma}_i^2 &= E\left[\left(\delta_r - \delta_i - d_r^2 + d_i^2\right)^2\right] \\ &= d_i^4\exp\left(\frac{4\sigma_i^2}{(\gamma\alpha_i)^2}\right) - d_i^4 + d_r^4\exp\left(\frac{4\sigma_r^2}{(\gamma\alpha_r)^2}\right) - d_r^4 \end{aligned} \tag{8}$$

and covariance

$$\begin{aligned} &E\left[\left(\delta_r - \delta_i - d_r^2 + d_i^2\right)\left(\delta_r - \delta_j - d_r^2 + d_j^2\right)\right] \\ &= \left\{d_r^4\exp\left(\frac{4\sigma_r^2}{(\gamma\alpha_r)^2}\right) - d_r^4\right\}. \end{aligned} \tag{9}$$

The solution to the LLS problem in is obtained by minimizing the cost function

$$\varepsilon_{LLS}(\boldsymbol{\theta}) = (\mathbf{b} - \mathbf{A}\boldsymbol{\theta})^T(\mathbf{b} - \mathbf{A}\boldsymbol{\theta})$$

and is given as [27]

$$\hat{\boldsymbol{\theta}}_{LLS} = \mathbf{A}^\dagger\mathbf{b}, \tag{10}$$

where $\mathbf{A}^\dagger$ is Moore–Penrose pseudoinverse i.e. $\mathbf{A}^\dagger = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$. The LLS can be implemented in three different ways

1) LLS-ref: In this implementation, d_r is the distance of the TN from a reference AN as shown above.

2) LLS-avg: Instead of choosing a reference distance, d_r is taken as the average of all distances from the ANs. Thus in this case, $d_r^2 = \frac{1}{N}\sum_{i=1}^N d_i^2$.

3) LLS-comb: In this case, combination of all pairs of ANs is considered and subtracted from each other. This results in $M \times \left(\frac{M-1}{2}\right)$ equations. This technique is studied for ToA case in [28]. The elements of data matrix $\mathbf{A}$ are now given by

$$\mathbf{A} = 2\begin{bmatrix} x_1 - x_2 & y_1 - y_2 \\ \vdots & \vdots \\ x_1 - x_N & y_1 - y_N \\ x_2 - x_3 & y_2 - y_3 \\ \vdots & \vdots \\ x_{N-1} - x_N & y_{N-1} - y_N \end{bmatrix}$$

Similarly element of vector $\mathbf{b}$ are given as $\mathbf{b}_{ij} = \left[\ \delta_i - \delta_j - \kappa_i + \kappa_j\ \right]$ for for $i, j = 1, ..., M$ and $i < j$. It should be noted that the number of equations increase considerably for large number of ANs. Hence LLS-comb is not favorable for large number of ANs.

We will compare the performance of all variants of the LLS algorithm in the simulation section.

IV. Weighted least squares algorithm

For the LLS solution obtained in (10), no knowledge about the reliability of each measurement is used. If this information is present, links that are more reliable are given more weightage than others. Thus utilizing the information present in the covariance matrix, a weighted least square (WLS) algorithm is proposed in this section.

For a given covariance matrix $\mathbf{C}(\boldsymbol{\theta})$ the WLS solution is obtained by minimizing the cost function

$$\varepsilon_{WLS}(\boldsymbol{\theta}) = (\mathbf{b} - \mathbf{A}\boldsymbol{\theta})^T\mathbf{C}(\boldsymbol{\theta})^{-1}(\mathbf{b} - \mathbf{A}\boldsymbol{\theta}),$$

where the elements of $\mathbf{C}(\boldsymbol{\theta})$ are given by (8) and (9). It is however noted that the elements of the $\mathbf{C}(\boldsymbol{\theta})$ are dependent on the actual distance of the target node from the anchors, which is unknown, hence the estimated distance is used to estimated the covariance matrix $\mathbf{C}\left(\hat{\boldsymbol{\theta}}\right)$. The WLS estimate is obtained as follows

$$\hat{\boldsymbol{\theta}}_{WLS} = \mathbf{A}^\ddagger\mathbf{b}^\ddagger, \tag{11}$$

where $\mathbf{A}^\ddagger = \left\{\mathbf{A}^T\left[\mathbf{C}\left(\hat{\boldsymbol{\theta}}\right)\right]^{-1}\mathbf{A}\right\}^{-1}\mathbf{A}^T$ and $\mathbf{b}^\ddagger = \left[\mathbf{C}\left(\hat{\boldsymbol{\theta}}\right)\right]^{-1}\mathbf{b}$.

It is noted that similar to LLS, the WLS algorithm can also be implemented in three different modes i.e. WLS-ref, WLS-avg and WLS-comb. It is however seen that the covariance matrix is different for the three implementations. For WLS-ref, the diagonal and non diagonal terms of $\mathbf{C}(\boldsymbol{\theta})$ are given by (8) and (9). For WLS-avg, where the reference anchor is the mean of all anchors, the $M \times M$ covariance is matrix is given below.

$$\mathbf{C}(\boldsymbol{\theta}) = \operatorname{diag}\left\{d_1^4 \exp\left(\frac{4\sigma_1^2}{(\gamma\alpha_1)^2}\right) - d_1^4 +, ..., +d_N^4 \exp\left(\frac{4\sigma_N^2}{(\gamma\alpha_N)^2}\right) - d_N^4\right\} + \mathbf{1}_{M\times M}\left\{\overline{d}_r^4 \exp\left(\frac{4\overline{\sigma}_r^2}{(\gamma\overline{\alpha}_r)^2}\right) - \overline{d}_r^4\right\}, \quad (12)$$

where $\overline{d}_r^4 = \frac{1}{M}\sum_{i=1}^M d_i^4$, $\overline{\sigma}_r^2 = \frac{1}{M}\sum_{i=1}^M \sigma_i^2$ and $\overline{\alpha}_r = \frac{1}{M}\sum_{i=1}^M \alpha_i$.

For the WLS-comb, development of the of the $\left(\frac{M^2-M}{2}\right) \times \left(\frac{M^2-M}{2}\right)$ covariance matrix becomes slightly complicated. As for WLS-ref and WLS-avg, the non-diagonal elements are same, however this is does now hold for WLS-comb for which the diagonal terms are given as

$$\tilde{\sigma}_i^2 = E\left[\left(\delta_i - \delta_j - d_i^2 + d_j^2\right)^2\right] = d_i^4 \exp\left(\frac{4\sigma_i^2}{(\gamma\alpha_i)^2}\right) - d_i^4 + d_j^4 \exp\left(\frac{4\sigma_j^2}{(\gamma\alpha_j)^2}\right) - d_j^4 \quad (13)$$

for $i, j = 1, ..., M$ and $i < j$.

On the other hand, the non-diagonal terms are given by

$$E\left[\left(\delta_i - \delta_j - d_i^2 + d_j^2\right)\left(\delta_k - \delta_l - d_k^2 + d_l^2\right)\right]$$

for $i, j = 1, ..., M$ and $i < j$
and $k, l = 1, ..., M$ and $k < l$

$$= \left\{d_i^4 \exp\left(\frac{4\sigma_i^2}{(\gamma\alpha_i)^2}\right) - d_i^4\right\} \quad \text{for } i = k.$$

$$= \left\{d_j^4 \exp\left(\frac{4\sigma_j^2}{(\gamma\alpha_j)^2}\right) - d_j^4\right\} \quad \text{for } j = l.$$

$$= -\left\{d_i^4 \exp\left(\frac{4\sigma_i^2}{(\gamma\alpha_i)^2}\right) - d_i^4\right\} \quad \text{for } i = l.$$

$$= -\left\{d_j^4 \exp\left(\frac{4\sigma_j^2}{(\gamma\alpha_j)^2}\right) - d_j^4\right\} \quad \text{for } j = k.$$

$$= 0 \quad \text{for } i \neq l \text{ and } j \neq k.$$

V. Optimal reference anchor node selection

Generally, the performance of LLS-avg and LLS-comb is slightly better than LLS-ref implementation due to the averaging effect of all ANs. Similarly, the performance of WLS-avg and WLS-comb is better than WLS-ref. However, in its basic form, LLS/WLS-ref randomly selects a reference AN. This could at times lead to degraded system performance as the accuracy of the location estimate depends on factors such as the true distance d_r from the TN, shadowing noise variance σ_r^2 and the PLE α_r of a particular reference AN. In this section, a technique to select the optimal reference AN is proposed. The optimal reference AN is chosen to be the AN that minimizes the MSE of the location estimates. Thus

$$\boldsymbol{\theta}_{i_{opt}} = \arg\min_{\boldsymbol{\theta}_i}(MSE). \quad (16)$$

where

$$MSE\left(\hat{\boldsymbol{\theta}}\right) = Tr\left\{E\left[\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0\right)\left(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0\right)^T\right]\right\}, \quad (17)$$

where $\hat{\boldsymbol{\theta}}$ is the estimated location via LLS or WLS and $\boldsymbol{\theta}_0$ is the true location coordinates. The theoretical MSE is given for the LLS and WLS algorithm in the following subsections.

Theoretical MSE for LLS

For LLS, the estimated location $\hat{\boldsymbol{\theta}}$ is given by $\hat{\boldsymbol{\theta}}_{LLS} = \mathbf{A}^\dagger\mathbf{b}$ while $\boldsymbol{\theta}_0$ can be represented by $\boldsymbol{\theta}_0 = \mathbf{A}^\dagger\mathbf{b}_0$, where $\mathbf{b}_0$ represents the noise free observation vector and is given by

$$\mathbf{b}_0 = \begin{bmatrix} d_r^2 - d_1^2 - \kappa_r + \kappa_1 \\ d_r^2 - d_2^2 - \kappa_r + \kappa_2 \\ \vdots \\ d_r^2 - d_2^2 - \kappa_r + \kappa_N \end{bmatrix}.$$

Putting elements of $\hat{\boldsymbol{\theta}}_{LLS}$ and $\boldsymbol{\theta}_0$ in (17) and after some manipulation we obtain

$$MSE\left(\hat{\boldsymbol{\theta}}_{LLS}\right) = Tr\left\{\mathbf{A}^\dagger\mathbf{K}\left(\mathbf{A}^\dagger\right)^T\right\}, \quad (18)$$

where

$$\mathbf{K} = E\left(\mathbf{b}\mathbf{b}^T\right) - 2E(\mathbf{b})\mathbf{b}_0^T + \mathbf{b}_0\mathbf{b}_0^T$$

where $E(\mathbf{b}) = \mathbf{b}_0$. The diagonal and off diagonal elements of $E\left(\mathbf{b}\mathbf{b}^T\right)$ are given by (14) and (15) respectively.

Theoretical MSE for WLS

For the MSE of the WLS algorithm we use the estimated $\hat{\boldsymbol{\theta}}_{WLS}$ (11) in (17) to obtain the following MSE expression.

$$\begin{aligned} MSE\left(\hat{\boldsymbol{\theta}}_{WLS}\right) &= Tr\Bigg\{\left[\mathbf{A}^\ddagger\mathbf{C}(\boldsymbol{\theta})^{-1}E\left(\mathbf{b}\mathbf{b}^T\right)\left[\mathbf{C}(\boldsymbol{\theta})^{-1}\right]^T\left(\mathbf{A}^\ddagger\right)^T\right] \\ &- 2\left[\mathbf{A}^\ddagger\mathbf{C}(\boldsymbol{\theta})^{-1}\mathbf{b}_0\mathbf{b}_0^T\left(\mathbf{A}^\dagger\right)^T\right] + \left[\mathbf{A}^\dagger\mathbf{b}_0\mathbf{b}_0^T\left(\mathbf{A}^\dagger\right)^T\right]\Bigg\}. \end{aligned} \quad (19)$$

It is noted that the theoretical MSE depends on the actual distances which are unknown, hence their estimates are used to estimate the MSE in (18) and (19). Once the optimal AN is selected, it is used again in the LLS solution (10) or WLS solution (11) to provide the final estimate of the TN location. This will be referred to as LLS-opt and WLS-opt respectively.

The following results were obtained via simulations.

$$\left\{E\left(\mathbf{b}\mathbf{b}^T\right)\right\}_{ii} = \kappa_r^2 + \kappa_i^2 + \frac{d_r^4}{\beta_r^2}\exp\left(\frac{8\sigma_r^2}{(\gamma\alpha_r)^2}\right) + \frac{d_i^4}{\beta_i^2}\exp\left(\frac{8\sigma_i^2}{(\gamma\alpha_i)^2}\right) - \frac{2d_r^2}{\beta_r}\exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right) - \frac{2d_i^2}{\beta_i}\exp\left(\frac{2\sigma_i^2}{(\gamma\alpha_i)^2}\right) - 2\kappa_r\kappa_i - \frac{2d_i^2d_r^2}{\beta_i\beta_r}\exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right)\exp\left(\frac{2\sigma_i^2}{(\gamma\alpha_i)^2}\right) + \frac{2d_r^2\kappa_i}{\beta_r}\exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right) + \frac{2d_i^2\kappa_r}{\beta_i}\exp\left(\frac{2\sigma_i^2}{(\gamma\alpha_i)^2}\right). \tag{14}$$

$$\left\{E\left(\mathbf{b}\mathbf{b}^T\right)\right\}_{ij} = \kappa_r^2 + \frac{d_r^4}{\beta_r^2}\exp\left(\frac{8\sigma_r^2}{(\gamma\alpha_r)^2}\right) - \frac{d_j^2d_r^2}{\beta_j\beta_r}\exp\left(\frac{2\sigma_j^2}{(\gamma\alpha_j)^2}\right)\exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right) - \frac{d_i^2d_r^2}{\beta_i\beta_r}\exp\left(\frac{2\sigma_i^2}{(\gamma\alpha_i)^2}\right)\exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right) + \frac{d_i^2d_j^2}{\beta_i\beta_j}\exp\left(\frac{2\sigma_i^2}{(\gamma\alpha_i)^2}\right)\exp\left(\frac{2\sigma_j^2}{(\gamma\alpha_j)^2}\right) - \frac{2d_r^2\kappa_r}{\beta_r}\exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right) + \frac{d_r^2\kappa_j}{\beta_r}\exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right) + \frac{d_j^2\kappa_r}{\beta_j}\exp\left(\frac{2\sigma_j^2}{(\gamma\alpha_j)^2}\right) + \frac{d_r^2\kappa_i}{\beta_r}\exp\left(\frac{2\sigma_r^2}{(\gamma\alpha_r)^2}\right) + \frac{d_i^2\kappa_r}{\beta_i}\exp\left(\frac{2\sigma_i^2}{(\gamma\alpha_i)^2}\right) - \frac{d_i^2\kappa_j}{\beta_i}\exp\left(\frac{2\sigma_i^2}{(\gamma\alpha_i)^2}\right) - \frac{d_j^2\kappa}{\beta_j}\exp\left(\frac{2\sigma_j^2}{(\gamma\alpha_j)^2}\right) - \kappa_r\kappa_i - \kappa_r\kappa_j + \kappa_i\kappa_j. \tag{15}$$

Result 1. Equal PLEs and equal distances : In case of equal PLEs and equal distances of the TN from all ANs i.e $\alpha_i = \alpha$, $d_i = d\ \forall\ i$, the AN with the smallest noise variance σ_i^2 is selected as the reference AN.

Result 2. Equal PLEs and equal noise variance : For equal PLEs and equal noise variance from all ANs i.e $\alpha_i = \alpha$, $\sigma_i^2 = \sigma^2\ \forall i$, the AN with the shortest distance d_i from the TN is selected as the reference AN.

Result 3. Equal distance and equal noise variance: For equal noise variance and equal distances of the TN from all ANs i.e $\sigma_i^2 = \sigma^2$, $d_i = d\ \forall i$, the AN with the largest PLE α_i is chosen as the reference AN.

VI. Performance bound

The CRB lower bounds the MSE performance of any unbiased estimator. For 2-D TN location, the CRB on the estimation MSE is given by

$$MSE\left(\hat{\boldsymbol{\theta}}\right) \geq \frac{[\mathbf{I}(\boldsymbol{\theta})]_{11} + [\mathbf{I}(\boldsymbol{\theta})]_{22}}{\det[\mathbf{I}(\boldsymbol{\theta})]}, \tag{20}$$

where $[\mathbf{I}(\boldsymbol{\theta})]$ is the Fisher information matrix (FIM), and its elements are given by

$$[\mathbf{I}(\boldsymbol{\theta})]_{ij} = -E\left[\frac{\partial^2 \ln p(\mathbf{p} \mid \boldsymbol{\theta})}{\partial\boldsymbol{\theta}_i\partial\boldsymbol{\theta}_j}\right]. \tag{21}$$

To lower bound the ML algorithms, the elements of the FIM are given by

$$[\mathbf{I}(\boldsymbol{\theta})] = \begin{bmatrix} \sum_{i=1}^{M}\frac{\gamma^2\alpha_i^2(x-x_i)^2}{d_i^4\sigma_i^2} & \sum_{i=1}^{M}\frac{\gamma^2\alpha_i^2(x-x_i)(y-y_i)}{d_i^4\sigma_i^2} \\ \sum_{i=1}^{M}\frac{\gamma^2\alpha_i^2(x-x_i)(y-y_i)}{d_i^4\sigma_i^2} & \sum_{i=1}^{M}\frac{\gamma^2\alpha_i^2(y-y_i)^2}{d_i^4\sigma_i^2} \end{bmatrix}. \tag{22}$$

The CRB as obtained from the FIM in (22) only tightly bounds the performance of ML type algorithms. Since the LLS method is different from the ML approach, the exact CRB for RSS-based localization in [12] does not accurately predict the performance of estimators based on the linear model. Unlike the conventional CRB, which is based on the observations taken from individual ANs, the linear CRB is based on the observations

$$p_i = \frac{1}{\beta_r}\exp\left(\frac{2z_r}{\gamma\alpha_r}\right) - \frac{1}{\beta_i}\exp\left(\frac{2z_i}{\gamma\alpha_i}\right).$$

Clearly, $\frac{1}{\beta_{r,i}}\exp\left(\frac{2z_{r,i}}{\gamma\alpha_{r,i}}\right)$ represents a log-normal distribution; a closed form expression for the difference of two log-normal random variables is however not known. Although the summation of two log-normal random variables can be approximated by another log-normal random variable [29], [30], p_i can be approximated by a Gaussian random variable i.e.

$$p_i \sim \mathcal{N}\left(\mu_i\breve{\sigma}_i^2\right)$$

where

$$\mu_i = d_r^2 - d_i^2 \tag{23}$$

and

$$\breve{\sigma}_i^2 = d_r^4\exp\left(\frac{4\sigma_r^2}{(\gamma\alpha_r)^2}\right) - d_r^4 + d_i^4\exp\left(\frac{4\sigma_i^2}{(\gamma\alpha_i)^2}\right) - d_i^4. \tag{24}$$

In vector form,

$$p(\mathbf{p} \mid \boldsymbol{\theta}) \sim \mathcal{N}(\boldsymbol{\mu}(\boldsymbol{\theta}), \mathbf{C}(\boldsymbol{\theta})), \tag{25}$$

where $\boldsymbol{\mu}(\boldsymbol{\theta}) = [\mu_1(\boldsymbol{\theta}), \mu_2(\boldsymbol{\theta}), ..., \mu_N(\boldsymbol{\theta})]^T$ is the vector constituting the means, and $\mathbf{C}(\boldsymbol{\theta})$ is the $N \times N$ covariance matrix whose elements are given by (9) and (12).

In order to prove the validity of the Gaussian assumption, the empirical cumulative distribution function (CDF) of p_i and the theoretical Gaussian CDF are plotted in Fig. 1. It is observed that even for a relatively large variance of $\sigma_i^2 = \sigma_r^2 = 6$, the empirical CDF closely fits the Gaussian CDF. The plot shows two cases, for $d_r > d_i$ and for $d_r < d_i$. It is clear that for both cases the Gaussian assumption holds true.

For the multivariate Gaussian distribution in (25), the elements of the FIM are given by[1]

$$[\mathbf{I}(\boldsymbol{\theta})]_{ij} = \left(\frac{\partial\boldsymbol{\mu}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_i}\right)^T \mathbf{C}^{-1}(\boldsymbol{\theta})\left(\frac{\partial\boldsymbol{\mu}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_j}\right) + 0.5Tr\left(\mathbf{C}^{-1}(\boldsymbol{\theta})\frac{\mathbf{C}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_i}\mathbf{C}^{-1}(\boldsymbol{\theta})\frac{\mathbf{C}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_j}\right). \tag{26}$$

[1]In this paper, the linear CRB is derived for the the LLS-ref model, for other variants similar procedure can be followed.

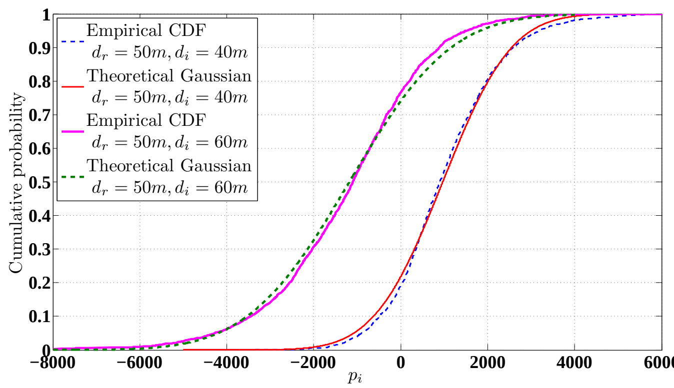

Figure 1. Empirical CDF of p_i and theoretical Gaussian CDF. $\sigma_i^2 = \sigma_r^2 = 6$.

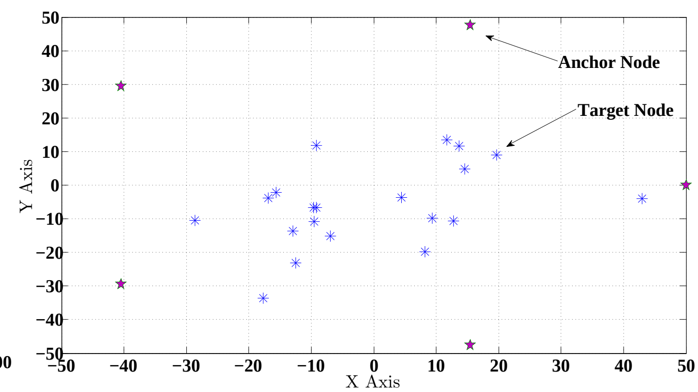

Figure 2. Network deployment

where
$$\frac{\partial \boldsymbol{\mu}_{i,j}(\boldsymbol{\theta})}{\partial x} = 2(x - x_r) - 2(x - x_{i,j})$$
and
$$\frac{\partial \boldsymbol{\mu}_{i,j}(\boldsymbol{\theta})}{\partial y} = 2(y - y_r) - 2(y - y_{i,j}). \tag{27}$$

The derivatives of $\mathbf{C}(\boldsymbol{\theta})$ are given by (28) and (29).

VII. Simulation Results

For performance comparison, we consider a circular deployment of 5 ANs around the origin of a 2-D coordinate system with radius R. To evaluate the average performance at various TN positions, 20 TNs are randomly deployed inside the network. For simplicity, the noise variance associated with all ANs is kept the same i.e $\sigma_i^2 = \sigma_r^2 = \sigma^2$. A different PLE value (given by vector $\boldsymbol{\alpha}$) is given to each AN, while the root mean square error (RMSE) is compared when the shadowing noise variance σ^2 in the path-loss is increased. The simulations are run independently η times. The network AN and TNs deployment is shown in Fig. 2.

In Fig. 3, we analyze the performance of LLS-opt and LLS-ref. For LLS-ref, the RMSE is given while choosing each AN as a reference AN at a time for all 20 TNs. It is seen that the selection of some ANs as reference ANs exhibits better performance than others, this is primarily due to larger PLE value for that particular AN. However, since the simulations show the average performance for all 20 TNs, a larger PLE does not guarantee a particular AN to be an optimal reference AN, since it also depends on the actual distance from the TN. On the other hand, the performance of LLS-opt supersedes that of LLS-ref.

In Fig. 4, we compare the results obtained for the theoretical MSE for LLS and WLS to the simulation for both algorithm respectively. It can be seen that that theoretical MSEs accurately predicts the performance of the LLS and WLS algorithms.

In Fig.5, performances of the variants of LLS and WLS are compared with LLS-opt and WLS-opt. The linear CRB is also plotted for comparison. For LLS-ref and WLS-ref, we randomly select AN-3 as the reference AN. As expected

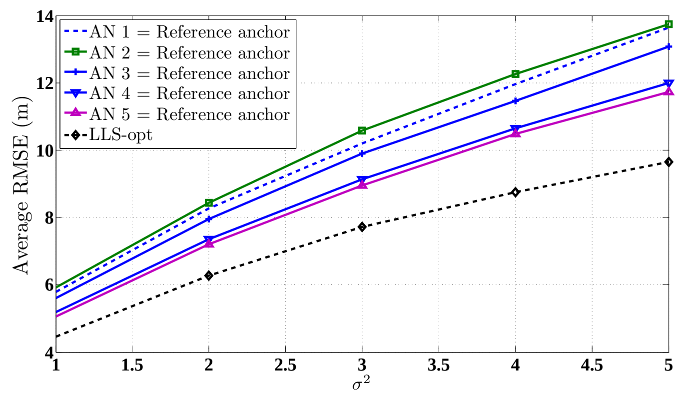

Figure 3. Performance comparison between LLS-ref for each AN as reference AN and LLS-opt. $R = 50$ m, $\eta = 900$, $M = 5$, $\boldsymbol{\alpha} = [2.4, 2.6, 2.8, 3, 3.2]^T$.

performance of LLS-avg and LLS-comb exceeds that of LLS-ref. However, the performance the LLS-opt surpasses all the other three LLS implementations. Interestingly, WLS-ref with reference AN-3 outperforms LLS-opt. However, the three variants of WLS algorithms perform similarly. While the WLS-opt performs better and approaches linear CRB.

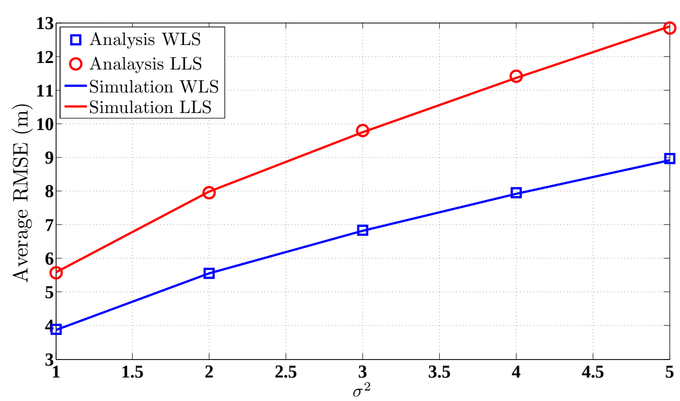

Figure 4. Performance comparison between theoretical MSE for LLS and WLS with simulation. $R = 50$ m, $\eta = 900$, $M = 5$, $\boldsymbol{\alpha} = [2.4, 2.6, 2.8, 3, 3.2]^T$.

$$\frac{\partial \mathbf{C}(\boldsymbol{\theta})}{\partial x} = \mathrm{diag}\left\{4d_1^2\,(x-x_1)\left[\exp\left(\frac{4\sigma_1^2}{(\gamma\alpha_1)^2}\right)-1\right]+,...,+4d_N^2\,(x-x_N)\left[\exp\left(\frac{4\sigma_N^2}{(\gamma\alpha_N)^2}\right)-1\right]\right\}+\mathbf{1}_{N\times N}\left\{4d_r^2\,(x-x_r)\left[\exp\left(\frac{4\sigma_r^2}{(\gamma\alpha_r)^2}\right)-1\right]\right\} \tag{28}$$

$$\frac{\partial \mathbf{C}(\boldsymbol{\theta})}{\partial y} = \mathrm{diag}\left\{4d_1^2\,(y-y_1)\left[\exp\left(\frac{4\sigma_1^2}{(\gamma\alpha_1)^2}\right)-1\right]+,...,+4d_N^2\,(y-y_N)\left[\exp\left(\frac{4\sigma_N^2}{(\gamma\alpha_N)^2}\right)-1\right]\right\}+\mathbf{1}_{N\times N}\left\{4d_r^2\,(y-y_r)\left[\exp\left(\frac{4\sigma_r^2}{(\gamma\alpha_r)^2}\right)-1\right]\right\} \tag{29}$$

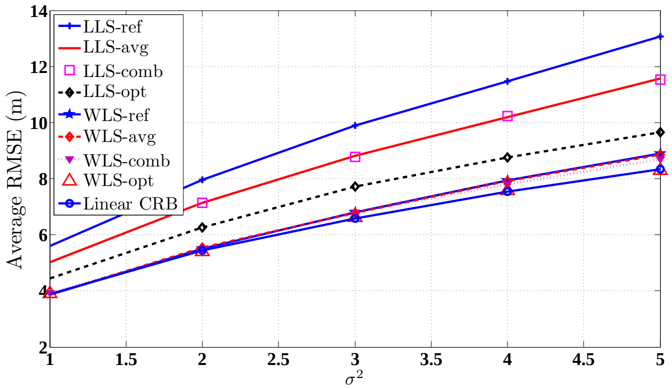

Figure 5. Performance comparison between different LLS and WLS implementations, linear CRB. $R = 50$ m, $\eta = 900$, $M = 5$, $\boldsymbol{\alpha} = [2.4, 2.6, 2.8, 3, 3.2]^T$.

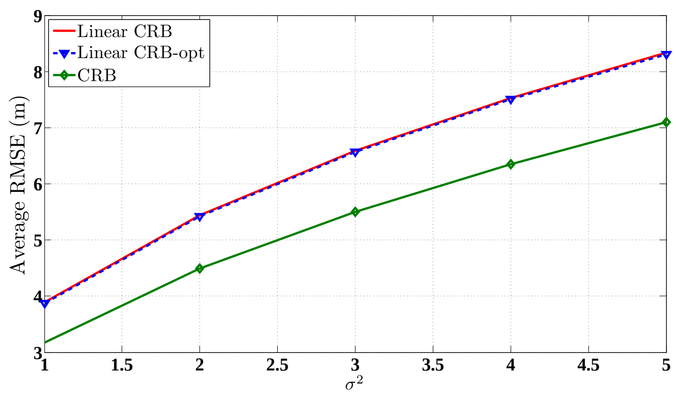

Figure 6. Performance comparison between linear CRB, linear CRB with optimal reference anchor and CRB. $R = 50$ m, $\eta = 900$, $M = 5$, $\boldsymbol{\alpha} = [2.4, 2.6, 2.8, 3, 3.2]^T$.

In Fig.6, the CRB is compared with the linear CRB and as expected the performance the linear CRB shows larger error than the exact CRB. Thus the linear CRB is a more realistic bound for the linear RSS estimator. On the other hand, the linear CRB changed little with optimal reference anchor selection.

VIII. Conclusions

The RSS based LLS localization algorithm is a low complexity technique for node positioning for WSN positioning. In this paper, we have carried out a performance analysis and proposed improvements to the LLS method. The linear model was introduced and modified for three different LLS variants. Performance was improved with a WLS algorithm that uses the information present in the covariance matrix of the observations. Further performance improvement was achieved with an optimal reference AN selection technique. The performance of the WLS method was shown to be close to the linear CRB which we have also derived. The linear CRB was shown to have larger error than the conventional CRB and thus realistically bounded the MSE of RSS location estimators operating on the linear model.

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