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Development of a relational model of disability

Age-specific rates of particular disability types are important for planning purposes and are a valuable input to estimates of populations with different disabilities. However, survey estimates of schedules of disability rates for the constituent countries of the UK display evidence of sampling variability and sub-national disability schedules are either extremely unreliable as a result of small sample sizes or are unavailable for reasons of disclosure protection. This paper develops and evaluates a method to smooth sampling variability in national schedules of disability using a technique that has applicability to sub-national estimation of age-specific disability rates. Relational models accurately quantify the relationship between national schedules of limiting long term illness (LLTI) (census) and particular disability types (Health Survey for England) using 2 or 3 parameters depending on disability type. These parameters are used to adjust LLTI schedule to represent different disability schedules smoothing sampling fluctuations. The specification of the relational model depends upon the complexity of the age-specific relationship between LLTI and a particular disability type. For some disability types a simple Brass relational model (2 parameters) provides a good fit but for others a modified version of the Ewbank relational model (3 parameters) is required.

Keywords: Relational models, disability, Limiting long term illness, schedules,

1. Introduction

This paper extends relational models beyond their original application, the estimation of mortality to the estimation of schedules of various disability types. The England curve of age-specific limiting long term illness (LLTI) rates from the census (2001) is used as the 'standard' schedule which is adjusted to represent disability schedules of various disability types, smoothing the variability in national age-specific rates of disability (Health Survey for

England (2000/01)). Whilst disability schedules share a similar age pattern to that of LLTI (see figure 1) with low rates across the younger ages that rise with age there are differences in levels of curves as well as in the speed of the increase in rates with age. The question we address in this paper is whether relational models are sufficiently flexible to accurately represent disability schedules (England) despite these differences.

The relational model of disability that is proposed is motivated by an important information gap; the lack of disability estimates locally either because these are very unreliable due to small sample sizes once disagggreagated by age, or because the release of local information is suppressed for reasons of disclosure protection (Purdam et al. 2008). The use of a curve of age specific rates of LLTI from the census as the standard schedule is proposed because age-specific LLTI rates are reliably available from the census for sub-national areas. Thus census LLTI schedules could act as a proxy for the level of disability in a neighbourhood with adjustments informed by relational models fitted for England as a whole. We do not evaluate sub-national relational models of disability here as this is undertaken in a separate paper (Marshall 2012) which validates the use of relational models of disability for sub-national areas. We return to this application of the techniques developed here in the discussion.

Estimates of the population with disabilities that distinguish disability type and severity are important for planning purposes to inform the provision of specialist services, equipment, and support (Field 1987; Siegel 2002). Disability schedules (curves of age-specific disability rates) are useful partly because the nature of disability service provision varies and is structured by age (Marshall 2009), but also because many disability types follow the same general age pattern: low rates across the younger ages that rise with age reaching the highest levels at the oldest ages (see Figure 1). Knowledge of the local population age structure and how it is changing provides an indication of the size of the local disabled population and how it might change as a result of demographic processes of mortality, fertility, and migration.

<<Figure 1 about here>>

In the UK, national survey estimates of disability distinguishing disability type and severity are subject to sampling variability once disaggregated by age resulting in a ragged curve due to sampling error, particularly at the oldest ages (see figure 1). Alternative data sources on disability that enumerate the total population are not subject to the issue of sampling error to the same extent but they do suffer from other weaknesses. For example, the 1991 and 2001 censuses record the numbers of people who are limited in work or everyday activities due to an illness, disability, or health problem but do not provide any information on the nature or severity of the limiting condition. Administrative sources such as disability registers and statistics of benefit claimants are compromised because they do not count those disabled people who do not register or use disability benefits (Macfarlane and Head 1999; Bajekal, et al. 2003).

Relational models comprise a (reliable) standard schedule of rates and a mathematical rule that maps the standard schedule to another schedule in a population where information may be incomplete or unreliable (Preston, et al. 2001). Relational models were originally developed for the estimation of mortality schedules (Brass 1971) and a key advantage of the approach is that the complexity of the mortality age pattern is captured in the standard schedule and a small number of parameters then quantify the deviation from this standard. Relational models require fewer parameters than mathematical mortality functions and can flexibly reproduce sets of model life tables using two suitably chosen parameters and a standard schedule (Keyfitz 1982; Preston, et al. 2001).

The original Brass (1971) relational model is based on a logit transformation of l(x), the probability of surviving to age x in the population of interest.

$$\mathrm{Y}(\mathrm{x})\frac{1}{2}\ln\left[\frac{\mathrm{l}(\mathrm{x})}{1-\mathrm{l}(\mathrm{x})}\right] \qquad \mathbf{1}$$

The logit transformation of l(x) is valuable because the relationship between two logit mortality schedules turns out to be remarkably linear (Newall 1988). On the basis of this linear relationship, Brass proposed a simple relational formula involving two parameters, α and β, to predict Y(x) from the logit of $l^{s}(x)$, $Y^{s}(x)$, in the standard population:

$$\mathrm{Y}(\mathrm{x}) = \alpha + \beta * \mathrm{Y}^{s}(\mathrm{x}) \qquad \mathbf{2}$$

When α=0 and β=1 then $\mathrm{Y}(\mathrm{x})$ and $\mathrm{Y}^{s}(\mathrm{x})$ are identical. Altering α affects the level of mortality in the population of interest, whilst altering β influences the relationship between mortality at adulthood and childhood. For more on the impact of changes in the values of α and β see section 3.2 and Zaba (1979: p80).

Two features determine the success of the relational approach, these being the appropriateness of the standard schedule and the relational rule (Preston, et al. 2001). The relational approach can be used successfully with any standard, but it is most effective if the standard is close to that of the population being modelled (Keyfitz 1982). There have been several extensions to the Brass relational function allowing more accurate representations of mortality particularly at the oldest and youngest ages. For example, Zaba (1979) and Ewbank, et al. (1983) propose relational models with two additional parameters that significantly improve the fit compared with the Brass relational model (Newall 1988). Murray, et al. (2003) note the difficulty associated with the empirical estimation of parameters in the Ewbank et al. (1983) and Zaba (1979) relational models and develop an alternative model which uses two additional age-specific correction factors based on mortality levels among children and adults, relative to the standard.

Relational models have been developed for particular countries (e.g. for Peru by Kamara and Lamsana (2001)) and have also been successfully extended to other demographic characteristics. For example, Brass (1981) developed a relational model for fertility schedules based on the Gompertz function, noting the utility of this approach in terms of its simplicity and the quality of fit of model rates. Zaba (1985; 1987) developed a relational model for schedules of immigration and emigration that involves three parameters. Booth (2006) documents the utilisation and development of these relational models of migration and of fertility in her excellent review of techniques of demographic forecasting (1980-2005).

Relational models are particularly appropriate for estimation of disability schedules because of the strong age pattern of prevalence rates which, like mortality schedules, are low at the youngest ages and rise with age. This pattern holds over time, place and for many disability types (see Figure 1). In this paper, the LLTI schedule for England (2001 census) is used as a reliable 'standard' schedule which is then adjusted to represent the schedules of particular disability types (Health Survey for England 2000/2001). Figure 2 compares the logit rates of LLTI (census) with logit schedules of locomotor (mobility) disability (Health Survey for England) at each single year of age illustrating the approximately linear relationship that is fundamental to the relational approach.

Although the relationship illustrated in figure 2 supports the use of relational models of disability it is important to note some differences between relational models of mortality and disability. Relational model of mortality predict the logit of survivorship rates whilst the relational model of disability proposed here predicts logits of disability prevalence rates. Survivorship rates start at 1 at birth and decrease to approach zero at the oldest age as all people die eventually. In contrast disability rates start close to 0 at the youngest ages and rise to between 0.7 and 0.2 depending on disability type. An additional difference is that whilst survivorship rates are monotonically decreasing, the increase in disability rates need not be monotonic as people can recover from disability. For example, Figure 1 shows that the rate of increase in LLTI stops around retirement age (60-65) before increasing again throughout the older ages. Features such as this 'retirement kink' in the LLTI schedule are likely to be preserved in the relational model disability schedule and we return to discuss this particular feature in the discussion.

After this introduction the paper is divided into five sections. First, the data sources that are used in the paper are discussed. Second, the four relational models that are fitted are defined. Third, the approach to evaluating the success of each model is outlined. Fourth, the findings of the model evaluation are stated. Fifth, the findings are discussed and finally, conclusions are drawn.

2. Data

The analysis in this paper combines data from two sources. The 2001 census provides reliable LLTI schedules and the Health Survey for England (2000/2001) provides detailed information on disability, distinguishing the nature of the disability.

2.1 The census of population (UK)

The census has been carried out since 1801, during which time sporadic questions on health and disability have been asked (Charlton 2000). The main advantage of the census as a source of data on disability is its coverage of the total population (all ages, households, and institutions) and the fine geographical detail at which data are reliably available. In 2001 the census included a question on limiting long term illness that records any illnesses, health problems, or disabilities that limit an individual in their daily activities. A very similar question had been asked in 1991. The question on LLTI features a prompt for elderly people to include problems that are due to old age. This is useful because it is known that the elderly tend to discount some health problems as being a result of ageing (Bajekal, et al. 2003). There is some undercount in the census which is larger in some areas of the country and for certain population groups (Cook 2004). However, these problems are small compared to the uncertainty associated with sample data and we do not address this undercount in this paper.

In terms of the general utility of self-reported limiting long term illness, a large body of work supports the validity of self-assessed health (Mitchell 2005) with LLTI found to be most strongly associated with general health perceptions, more serious health conditions (Manor, et al. 2001) and physical limitations rather than with psychological health (Cohen, et al. 1995). There are strong relationships between LLTI and other health outcomes including all cause and cause-specific mortality (Charlton, et al. 1994; Bentham, et al. 1995; Idler &

Benyamini 1997) as well as sickness benefits claims from different health conditions (Bambra and Norman 2006; Norman and Bambra 2007).

The census data on LLTI is downloaded from tables ST16 and ST65 which record the population with (and without) LLTI with age and sex detail for the household and institutional populations respectively[1].

2.2 The Health Survey for England

The Health Survey for England (HSE) was set up in 1991 to monitor the health of the private household population in England and the progress towards targets laid out in the Health of the Nation strategy (DoH 2007). The survey follows a multistage, stratified probability sampling design. The sample size was increased from around 4,000 to 16,000 in 1994 enabling analysis for Health Authority Regions and between socio-economic groups. From 1995 a sample of 4,000 children between the ages of 2 and 15 were included in the sample (Bajekal 2000). Each year of the HSE has a particular focus, with a module measuring disability included in 1995, 2000, 2001, and 2005. In this paper the data on disability in 2000 and 2001 are combined to increase sample sizes and to overlap the data collection date of the 2001 census, a feature that is particularly useful for the models that combine HSE and census data. The 2000 survey focused on disability amongst the elderly with a boosted sample of elderly people including the elderly living in residential and care homes along with a reduced sample of the general population (Bajekal and Prescott 2003). It should be noted that the HSE in 2000 does not give a complete coverage of the disabilities in communal establishments, missing younger people in institutions and covering only care and residential homes (and not,

[1] 2001 census standard tables can be downloaded from the Nomis website - https://www.nomisweb.co.uk/Default.asp

for example, hospitals). However, rates of LLTI are almost identical in the household and total population below the age of 65 in the UK and its constituent countries. This implies that the lack of coverage of the institutional population under the age of 65 in the HSE00/01 is not likely to be a problem in this analysis.

Disability is measured according to five domains: locomotion (mobility), personal care, sight, hearing, and communication. A person is classed as having no disability or a disability at a lower or higher level for each of the five domains based on their answers to questions on ability to perform everyday tasks (see appendix I). The highest score for any of the five types of disabilities is taken as the overall disability score. A score of 1 indicates a lower severity disability, a score of 2 indicates a higher severity disability and a score of 0 indicates no disability. In this paper model rates are produced for overall disability and each disability type with the exception of communication disability which is omitted because it does not display the strong age pattern necessary for the relational models developed here. Severity of disability is not distinguished and so the rates of disability include those with either a higher or lower severity disability. The HSE allows respondents to take into account the use of aids for hearing and sight disabilities, however, for the other domains the use of aids to perform tasks are not allowed. Data are collected using face-to-face computer assisted personal interviewing and the disability module applies to all people aged 10 or over. Proxy answers are not permitted for adults but parents answer for children under the age of 13 (Bajekal and Prescott 2003).

2.3 Data preparation

The analysis of the merged 2000-01 HSE datasets requires the use of two types of weights to ensure that estimates are representative of the target population. First, child weights are needed to compensate for the sample design at these ages which involves limiting the number of children interviewed in each household to two. Second, the HSE in 2000 includes weights to account for the oversampling of the elderly and institutional population. In order to compensate for the lack of older people living in institutions in 2001, the weights associated with people living in institutions in 2000 are doubled.

All models use rates by single year of age up to the age of 84 with an age of 88 to represent all those aged over 84. The use of 88 as the upper age limit is based upon the average age of the population aged over 84 (as calculated using the 2001 census Sample of Anonymised Records[2]). census tabulations of LLTI are only released with quinary age detail (from the age of 20 upwards) and in order to generate single year estimates, these five year rates are smoothed using an Excel based tool developed by Popgroup[3] users specifically for this purpose. The Excel smoothing tool and more information on the smoothing approach are available at: http://www.ccsr.ac.uk/popgroup/about/manuals.html

3. Models

3.1 Notation

The subscript notation that is used in the model specifications throughout this paper is detailed in Table 1.

<<Table 1 about here>>

3.2 Brass relational model

The Brass relational model used in this analysis is defined below:

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[2] The SARs are samples of individual records from the 2001 (and 1991) Censuses.

[3] For more information on POPGROUP see www.ccsr.ac.uk/popgroup/

Let:

p_{xd} = prevalence of disability d at age x in England (HSE00/01)

p_{xl} = prevalence of LLTI (l) at age x in England (census01)

Then:

$$\frac{1}{2}\log_e\left(\frac{p_{x.d}}{1-p_{x.d}}\right)=\alpha+\beta\left(\frac{1}{2}\log_e\left(\frac{p_{x.l}}{1-p_{x.l}}\right)\right)+e_x \qquad \mathbf{3}$$

The impact of varying values of α and β on the LLTI schedule are illustrated in Figure 3. A negative value of α shifts the LLTI schedule downwards whilst a positive value shifts it upwards. A value of β above 1 decreases rates of LLTI at the youngest ages and increases them at the oldest ages with the converse being true for values of β below 1.

<<Figure 3 about here>>

3.3 Ewbank relational model

Ewbank, et al. (1983) develop a more complex relational rule with four parameters that allows more twisting of the reference schedule at the oldest and youngest ages. This four parameter system is an extension of Brass' two parameter relational model. The Ewbank model that is fitted to derive local disability schedules is as below:

First, define function T (which comprises the two additional parameters λ and κ) as:

$$T(p_{xl};\lambda)=\frac{\left(\frac{p_{xl}}{1-p_{xl}}\right)^{\lambda}-1}{2\lambda} \text{ if } p_{xl}\geq 0.5 \qquad \mathbf{4}$$

$$T(p_{xl};\kappa)=\frac{1-\left(\frac{1-p_{xl}}{p_{xl}}\right)^{\kappa}}{2\kappa} \text{ if } p_{xl}<0.5 \qquad \mathbf{5}$$

Let:

$\omega = 1$ if $p_{xl} \geq 0.5$ and 0 otherwise

$\chi = 1$ if $p_{xl} < 0.5$ and 0 otherwise

Then:

$$\frac{1}{2}\left(\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right)\right) = \omega\left(\alpha + \beta\ T(p_{xl} : \lambda)\right) + \chi\left(\alpha + \beta\ T(p_{xl} : \kappa)\right) + e_x \qquad \mathbf{6}$$

The Ewbank relational model has several useful features for the purposes of this paper. First, it allows more flexibility in the adjustment of LLTI schedules than the Brass model which is likely to be necessary for disability types that deviate most from the LLTI age pattern. Second, the transformation T approaches the logit transformation (and thus a Brass relational model) as λ and κ tend to zero. This can be shown by expanding the transformation T into a series. This property of 'nested' models is useful for the model comparison and is also exploited to develop a 'Reduced Ewbank' model. Third, the transformation introduces the biggest changes at the most extreme ages where the logit transformation is most likely to be unsatisfactory. Finally, λ only affects estimates at the oldest ages and κ only affects estimates at the youngest ages. This is because the first term of equation 6 (involving λ) only applies when LLTI prevalence is less than or equal to 0.5 and the second term (involving κ) only applies when LLTI rates are greater than 0.5 (Ewbank, et al. 1983). In practice this means κ applies to the age of 10 and 76 and λ to the ages of 77 and above. This is not altogether different to the cut-off for a Ewbank model of mortality. For example, if the UK lifetable (2008-10) published by the Office for National Statistics were selected as our standard then survivorship probabilities would pass 0.5 at the age of 80.

3.4 Reduced Ewbank model

A criticism of the Ewbank model in the literature is the difficulty in estimating the additional parameters which complicate the application of this model (Murray, et al. 20003). Congdon (1993) discusses the problem of overparameterisation when fitting relational models, where a range of parameter estimates are associated with a similar model fit, and recommends a model with fewer parameters when overparameterisation occurs. A reduced version of the Ewbank model (from here on known as the Reduced Ewbank model) is developed in this paper to provide a more flexible alternative to the Brass model whilst avoiding the issues identified by Murray, et al. (2003) and Congdon (1993). There are some parallels between the reduced Ewbank model in this paper and that adopted by Kamara and Lansana (2001) for the estimation of mortality schedules in Peru where the four parameter relational model developed by Zaba (1979) is altered to derive a simpler relational model also involving three parameters.

There is a strong argument that α and β should remain in the Reduced Ewbank model as both are included in the Brass model and the full Ewbank model. This means that either κ or λ might be dropped from the model. Clearly setting one of these parameters to zero would remove the effect of β for at least part of the model (see equations 4 to 6) as the function T would equal zero. However, we know that the function T approaches a logit transformation when κ and λ tend to zero and so it is proposed that where one of these variables is dropped the T function should be replaced by a logit transformation. The Reduced Ewbank model is shown in equations 7 and 8.

Let:

$\omega = 1$ if $p_{xl} \geq 0.5$ and 0 otherwise

$\chi = 1$ if $p_{xl} < 0.5$ and 0 otherwise

Parameter kappa dropped from the model

$$\frac{1}{2}\left(\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right)\right) = \varpi\ (\alpha + \beta T(p_{xl} : \lambda)) + \chi\left(\alpha + \beta\left(\frac{1}{2}\log_e\left(\frac{p_{xl}}{1-p_{xl}}\right)\right)\right) + e_x \qquad \mathbf{7}$$

Parameter lambda dropped from the model

$$\frac{1}{2}\left(\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right)\right) = \omega\left(\alpha + \beta\left(\frac{1}{2}\log_e\left(\frac{p_{xl}}{1-p_{xl}}\right)\right)\right) + \chi\left(\alpha + \beta\ T(p_{xl} : \kappa)\right) + e_x \qquad \mathbf{8}$$

3.5 Piecewise relational model (sight disability)

Examination of the sight disability schedules (see figure 1) shows that age pattern is flat and very low up to the age of 60 with a steady increase in prevalence occurring thereafter. Epidemiological research confirms the rarity of sight disabilities at the younger and working ages and reveals that the causes are often congenital in nature (Munier, Gunning et al. 1998; Rahi and Dezateux 1998). Sight disabilities occur with increasing frequency at the oldest ages with causes linked to the ageing process. Macular degeneration, glaucoma and cataracts account for three quarters of sight problems for those aged over 80 (Munier, Gunning et al. 1998).

The shape of the sight schedule may be better modelled with a piecewise approach using an average prevalence rate up to the age of 60 and a relational model above the age of 60 where an age pattern emerges. This approach acknowledges the rarity of the mainly congenital sight disabilities under the age of 60 and the more typical disability age pattern at the older ages as sight problems that stem from the aging process emerge.

The piecewise Brass relational model is defined below:

Let:

$\gamma_r = 1$ if $x_r > 59$ and 0 otherwise

$\upsilon_r = 1$ if $x_r \leq 59$ and 0 otherwise

$$\frac{1}{2}\log_e\left(\frac{p_{xd}}{1-p_{xd}}\right) = \upsilon_. \delta + \gamma_. \left((\alpha + \beta\left(\frac{1}{2}\log_e\left(\frac{p_{xl}}{1-p_{xl}}\right)\right)\right) + e_x \qquad \mathbf{9}$$

In the model specification above the parameter δ is constrained as below:

$$\delta = \frac{1}{2}\log\left(\frac{p_{10-60}}{1-p_{10-60}}\right) \qquad \mathbf{10}$$

The α and β parameters have the same interpretation as in previous relational models adjusting the level and shape of the LLTI schedule to estimate the sight schedule, however in this model they only have an effect over the age of 60. The piecewise model is less desirable than the other relational models that apply across the whole age range in that it might lead to discontinuities in the model schedules on either side of the break in the piecewise function. However, such an approach may be required for sight disability which deviates most from the shape of the LLTI curve.

3.6 Model estimation

All of the relational models are fitted using non-linear regression using a method of weighted least squares regression (WLS) to estimate parameters. Other estimation procedures exist including ordinary least squares regression (OLS) and maximum likelihood estimation (ML). For relational models of mortality, the literature is not clear as to which approach should be used. Preston et al. (2000) recommend the OLS approach; Congdon (1993) fits relational models of mortality using WLS, whilst Stewart (2004), after evaluating various methods by which relational models of mortality could be fitted (including OLS and WLS), concludes that ML is most appropriate. As the estimation of disability represents a new application of relational models then an assessment of the performance of each model fitting procedure is worthwhile. Marshall (2012) evaluates the model fit of Brass relational models of disability where parameters are estimated using OLS, WLS and ML. Whilst OLS and WLS give a similar and good fit to the observed data, ML was found to overestimate rates at the oldest ages and on the basis of this it is not used in this paper.

The statistical computer package STATA is used to fit the relational models using the nl (non-linear regression) command. Starting values of 1 are given to all parameters. However, experimentation with other starting values did not alter the final parameter estimates.

3.7 Weights

Congdon (1993) notes that a model which uses the logit of a proportion as the dependent variable should use weights at each age x (w_x) based on:

$$w_x = p_{x.d}(1 - p_{x.d})N_x \qquad \textbf{11}$$

Where:

$p_{x.d}$ = the prevalence of disability d at age x in England

N_x = the population at age x in England

Proportions that are close to 0 or 1 or derived from small samples are given less weight during the model fitting process. Conversely, proportions that are near 0.5 or are based on larger sample sizes are given greater weight. Weights are calculated by single year of age wherever possible in order that they are most responsive to the single year rates that are modelled. However, for the least prevalent disabilities (personal care, sight, and hearing) weights are based upon quinary age groups because of the instability in the age pattern of single year weights for these disability types.

The weights for each disability are lowest at the youngest ages and increase with age before falling slightly at the very oldest ages (80+). Although the patterns of weights may seem counterintuitive in that rates of disability are generally least reliable at the older ages (see figure 1), examination of logit schedules, which comprise the dependant variable in a relational model, suggests such a weighting scheme is not unreasonable (see figure 4). Many of the logit schedules display the greatest fluctuations at the youngest ages with stability increasing at the middle and early older ages. It should be noted however, that the impact of the weights on model schedules is very slight compared to those derived from unweighted analysis (see Marshall (2012)).

<<Figure 4 about here>>

4. Comparing models

The procedure to select the most appropriate relational model for each disability type (for males and females) involves two stages. The first determines whether the improvement in model fit, compared to the next simplest model, reaches statistical significance. Second, the stability of parameter estimates from the chosen model is assessed and if evidence of overparameterisation is discovered the model is discarded.

4.1 Improvement in model fit

It is almost always the case that a more complicated model will fit the data better (have a lower residual sum of squares) than a simpler one. So, for example, the Ewbank model will generally fit the data better than the Reduced Ewbank model which in turn will have a lower residual sum of squares than the Brass model. The extra sum of squares F test (for more information see Motulsky and Christopoulos (2004)) is appropriate for non-linear regression models and is based upon the difference in residual sum of squares (from here on referred to as sum of squares) from two models and controls for the number of data points and the number of parameters in each model. It uses this information as shown in Table 2 to calculate a ratio that follows an F distribution under the null hypothesis that there is no evidence to accept the more complicated model (i.e. the residual sum of squares in each model are identical after accounting for improvements attributable to additional parameters). We can use the F-ratio to calculate an associated p-value that gives the probability that the improvement in model fit associated with the more complicated model (after accounting for improvements attributable to additional parameters) is actually a result of the sampling process rather than any 'real' improvement. For the purposes of this research a threshold of $p=0.05$ is used to determine whether the more complex model gives a better fit than the simpler model.

<<Table 2 about here>>

As the two versions of the Reduced Ewbank model have the same number of parameters, then the decision as to whether k or l is dropped is made on the basis of the model with the lowest residual sum of squares.

4.2 Model stability

In addition to the extra sum of squares F test it is also important to check parameter estimates for signs of overparameterisation, where a range of parameter values are associated with a similar model fit. The symptoms of overparameterisation are standard errors that 'explode' estimates of parameters outside 'normal' ranges and high estimated correlations between parameters (virtual collinearity). A useful test of overparameterisation is to compare estimates of α and β from a Reduced or full Ewbank model to those from a Brass model that is restricted to the ages not seriously affected by the additional parameters (λ/κ). If the model is not overparameterised then we would expect estimates of α and β to be similar in each model.

5. Findings

Before displaying results from the model comparison outlined in the previous section it is worth noting the generally good fit of the relational models to the observed disability schedules. R squared statistics for all relational models are above 0.9 for overall, locomotor and personal care (females) disability and are around or above 0.85 for personal care (males) sight (females) and hearing disability. A less good fit is observed for sight disability (males) where the R-squared falls below 0.8. Model fit appears to be slightly better for females than males and for overall disability and the most common disability type (locomotor disability). The improvement in R-squared values in the Reduced Ewbank and Ewbank models compared to the Brass model is greatest for sight disability, in particular for males.

<<Table 3 about here>>

Table 4 displays the results of the extra sum of squares F tests for males and females under the Brass, Reduced Ewbank, and Ewbank models. For males, the Reduced Ewbank models offer an improvement in fit over the Brass model that is statistically significant for all disability types with the exception of hearing disability. The additional improvement in model fit under the Ewbank model does not achieve statistical significance for any of the disability types. For females, the Reduced Ewbank model gives a better fit than the Brass model for all disability types. However, it is only for personal care and hearing disability that the full Ewbank model does not provide a statistically significant improvement over its reduced counterpart.

<<Table 4 about here>>

The Ewbank models for overall, locomotor, and sight disability (females) show clear evidence of overparameterisation. The $\hat{\beta}$ parameter estimate is much higher than in the Brass models and many of the parameter estimates have very high standard errors. Examination of estimated correlations between parameters from the Ewbank model reveals virtual collinearity (estimated correlation between the $\hat{\beta}$ and $\hat{\kappa}$ parameters and the $\hat{\beta}$ and $\hat{\iota}$ parameters is greater than 0.999 for the models of overall and locomotor disability). Examination of the parameter statistics from the Brass and Reduced Ewbank models recommended by the extra sum of squares F test suggest that overparameterisation may be less of an issue for these models. All parameter estimates achieve statistical significance and appear to be within ‘normal’ ranges. There are however, strong estimated correlations between the $\hat{\beta}$ and $\hat{\kappa}$ parameters in Reduced Ewbank models (estimated correlation ranges between 0.89 to 0.96) and this is noted as one cause for concern for the Reduced Ewbank models.

<<Table 5 about here>>

In situations where a model is overparameterised there are compelling statistical reasons to prefer a more parsimonious model (Congdon 1993). For example, one of the potential uses of the parameters from these models is to adjust ward or district LLTI curves using relational parameters to obtain schedules of various disability types in the absence of direct survey estimates. Confidence in the robustness of these parameter estimates as a measure of the underlying relationship between LLTI and disability is essential for such an application. The disability types that were modelled using the Reduced Ewbank model yield much more stable parameter estimates than the full Ewbank model (see Table 6). All parameters are now significant, within normal ranges and the estimated correlation between parameters is less of an issue (estimated correlation between parameters ranges between 0.89 to 0.96). The reduction in the quality of model fit associated with these Reduced Ewbank models is very slight, as illustrated by the R squared statistics in Table 3.

<<Table 6 about here>>

Whilst our parameter estimates appear more stable in the Reduced Ewbank model, evidence of overparamterisation remains particularly where the κ parameter is retained. Estimates of α and β in a Reduced Ewbank model differ to those in a Brass model that is restricted to the ages in which the additional κ parameter should have little effect. This suggests that the κ and β are not acting independently and confirms the high correlation between these parameters noted earlier. The issue of overparamterisation is as a potential weakness of relational models of disability, however, the stability of the reduced Ewbank models is much improved compared to the full Ewbank models. Encouragingly, comparison of modelled rates from a reduced Ewbank (κ) model and a Brass model, demonstrates that the impact of κ is exactly as we would expect; it improves model fit at the youngest ages where the fit of the Brass model is poor (see figure 5). The tendency of the same form of Reduced Ewbank model to be selected for both males and females for each disability type is another encouraging sign of the stability of Reduced Ewbank models. This suggests that a similar weakness of the Brass model is addressed in a consistent way for both female and male disability schedules.

<<Table 7 about here>>

<<Figure 5 about here>>

A remaining question is whether there are benefits to modelling the sight disability schedule using the piecewise model. The R^2 statistics in table 3 confirms that sight disability is least well modelled and as the sight disability curve remains flat until the age of 60 it could be that piecewise approach in equation 10 offers a more appropriate approach than the other relational models. However, the residual sums of squares associated with the piecewise Brass relational model (5.03 for males 4.07 and for females) are higher than for the Reduced Ewbank model (4.54 for males and 3.63 for females). We do not select the piecewise Brass relational model because it does not appear to offer an improvement in model fit compared to a Reduced Ewbank model.

The models recommended by this findings section are shown in Table 8 below. The model schedules themselves, along with the observed survey rates, are displayed in figures 6 and 7. It is encouraging that model fit appears to be reasonable for each disability type across the age range.

<<Table 8 about here>>

<<Figures 6 to 8 about here>>

5. Discussion

The findings of the previous section illustrate the success of relational models in representing disability schedules by adjusting the level and shape of LLTI curves. We now examine the model schedules (see Figures -) commenting on their robustness in light of other research on disability. We also return to potential application of the relational models developed here to produce local estimates of disability.

Interestingly, the rates of overall disability exceed census LLTI rates at the very oldest ages for both males and females. Although this seems a counterintuitive finding, as LLTI includes a broader range of limiting conditions than the Health Survey for England measure of disability, it is supported in the literature. Bajekal and Prescott (2003) note a ‘crossover’ effect in their report on disability in the 2001 Health Survey for England where survey rates of disability exceeded survey rates of LLTI at the oldest ages. This crossover effect is attributed to older people under-reporting limiting longstanding illness because they consider activity limitation a normal consequence of ageing. Additionally, surveys with a health focus, such as the Health Survey for England, tend to give higher levels of illness and disability than other surveys, such as the census, where information on a range of topics is collected. This might also contribute to the crossover in census rates of LLTI and model rates of overall disability at the oldest ages (Bajekal, et al. 2003).

A key difference between the male and female model schedules is a kink in many of the model disability schedules for males between the ages of 60 and 65 where the increase in rates with age slows before increasing again. This kink is found in the male LLTI curve and is preserved for many of the male disability schedules during the modelling process. The LLTI retirement kink is a feature noted by other researchers. For example, Bellaby (2006) finds a tailing off in the increase in LLTI after retirement ages, particularly for those in manual occupations, and a similar result from clinical assessments of health using standardised methods (e.g. forced expiratory volume (FEV1), blood pressure (allowing for control by medication), and body mass index). Westerlund, et al. (2009) report a retirement related improvement in self-reported health, particularly for those in poor work environments, in a longitudinal study of employees of the French national gas and electric company. A comparison of the observed and model disability schedules (see Figure 5) suggests that the transfer of the retirement kink to specific disability types appears reasonable. Additionally the relational approach appears to have the flexibility to suppress the kink for disabilities where it might be less appropriate (e.g. sight disability).

The relational model specification developed in this paper is partly motivated by the data availability in the UK and potential application of relational models to address an information gap; the lack of sub-national estimates of disability. For sub-national areas survey data on specific disabilities are either very unreliable due to small sample sizes or unavailable for reasons of disclosure protection. Traditionally relational models involve a fixed standard schedule and varying parameter estimates but here sub-national schedules could be derived from varying standards (local census LLTI schedules) with fixed parameter estimates (from national relational models). The underlying assumption is that the relationship between LLTI and disability schedules remains constant between areas. Marshall (2012) tests this assumption using national relational parameter estimates to generate estimates of HSE disabilities for the nine regions in England and districts in Scotland. These relational model estimates successfully capture the variability in the HSE observed regional disability prevalence and district crude rates of disability taken the Scottish Household Survey (which contains a limited set of disability questions some of which are comparable to those in the HSE). This research provides evidence to suggest that the relational models developed in this paper have not only applicability to smooth national disability schedules but also as a means to fill a local disability information gap. More generally, relational models have applicability to the estimation of schedules for other disability types, combinations of disabilities, severity of disability, or of other health problems that display a strong mortality-like age pattern.

6. Conclusion

This paper illustrates that relational models can accurately capture the relationship between age-specific rates of LLTI and various disability types. The Brass or Reduced Ewbank relational models have sufficient flexibility to adjust the census LLTI schedule to give an accurate representation of schedules of overall, locomotor, personal care, hearing, and sight disability from the Health Survey for England. This is valuable as relational models offer a means to generate a more reliable set of age-specific rates in situations where there is evidence of instability in rates directly estimated from survey data. The local availability of reliable census LLTI schedules provides a means to derive local disability schedules (where direct estimates are unavailable) using relational parameters estimated at higher geographies.

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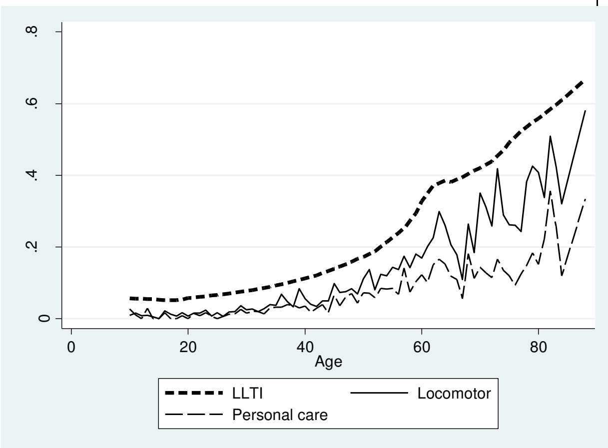
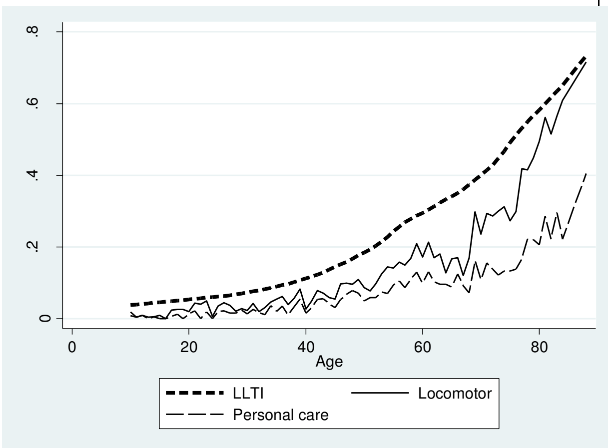
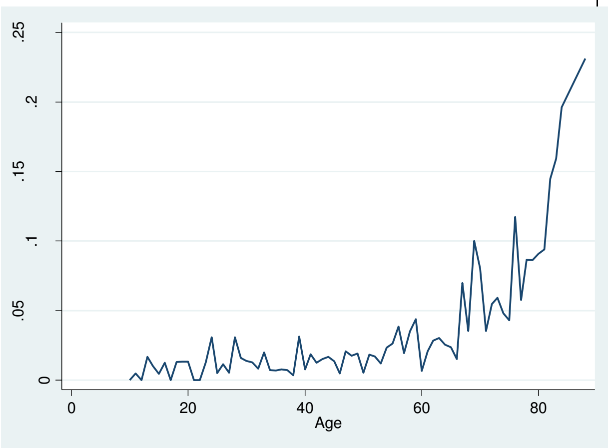
LLTI, locomotor and personal care disability schedules (England: Males – 2000/2001)



LLTI, locomotor and personal care disability schedules (England: Females – 2000/2001)



Sight disability (Females - England)



Hearing disability (Females - England)

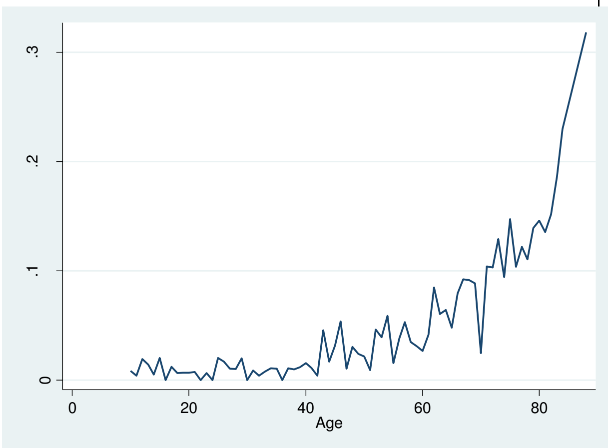


Figure 1: Limiting Long Term Illness (LLTI) and selected disability schedules for England

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Figure 2: Scatterplot of the relationship between the logit LLTI schedule and the logit mobility disability schedule (Males – England)

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Impact of altering values of α	Impact of altering values of β

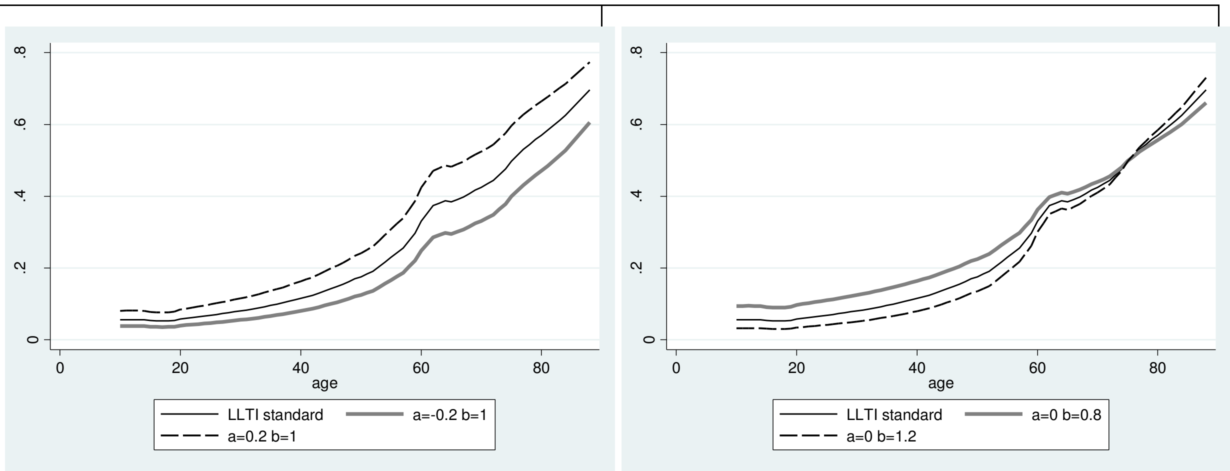


Figure 3: Brass relational model - Impact of altering values of α and β on the LLTI standard schedule (Males – England)

Source: Authors own calculations using data from the census (2001)

Locomotor (males)	Sight disability (males)

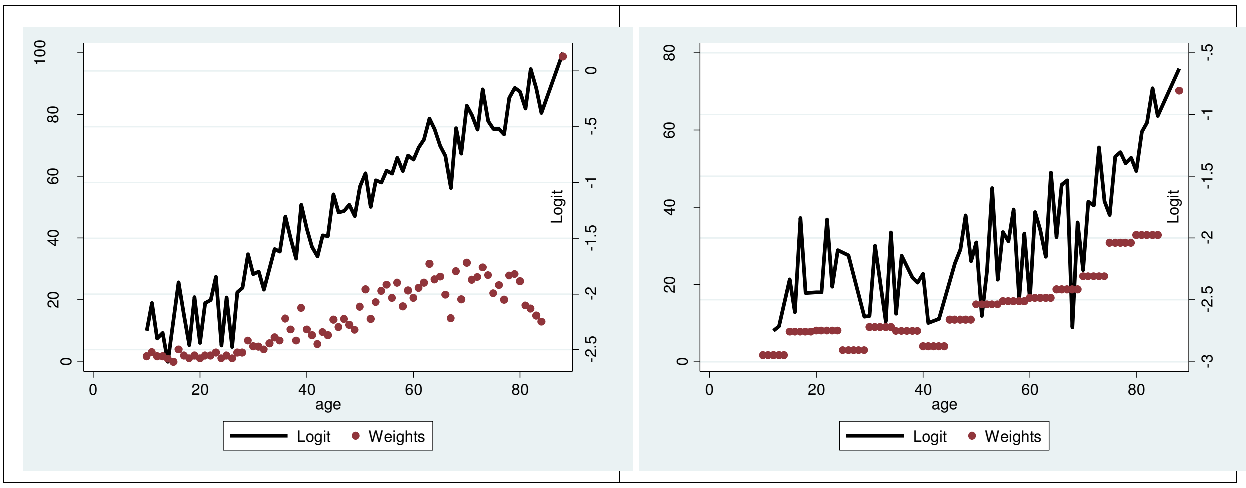


Figure 4: comparison of age pattern of weights and fluctuations in logit schedules of sight and locomotor disability (males)

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

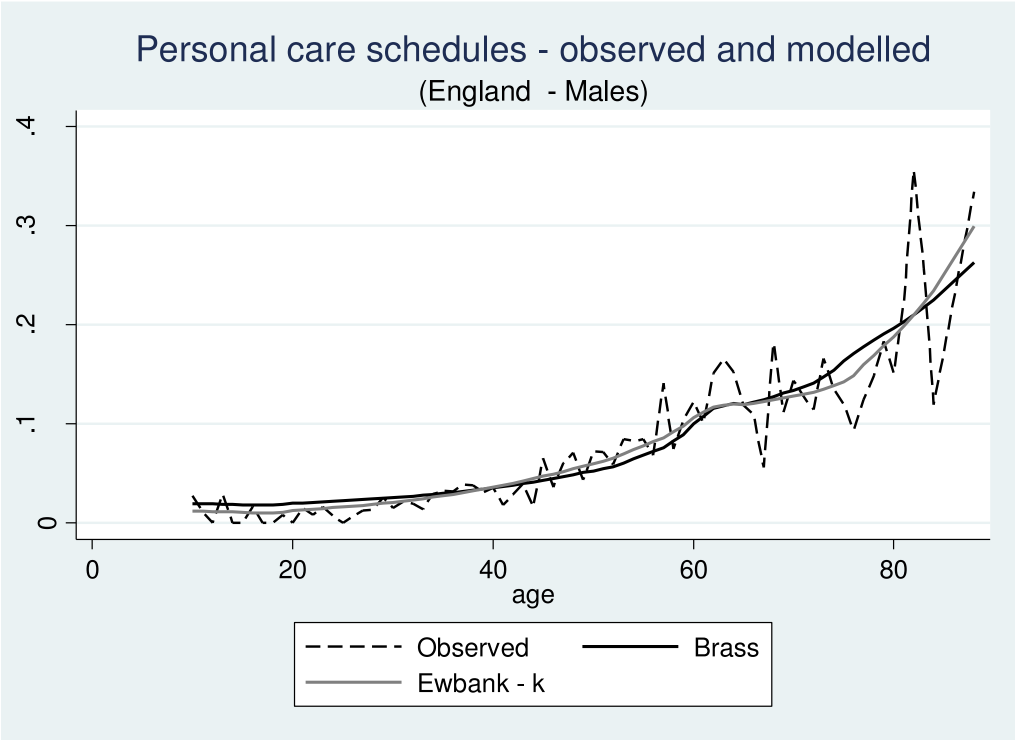


Figure 5: Brass and reduced Ewbank model schedules – Personal care disability (Males)

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

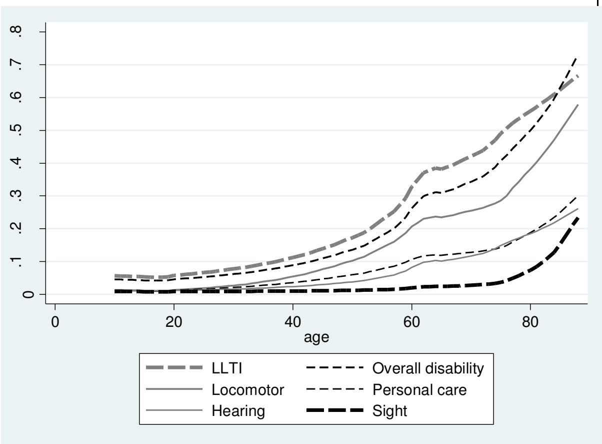
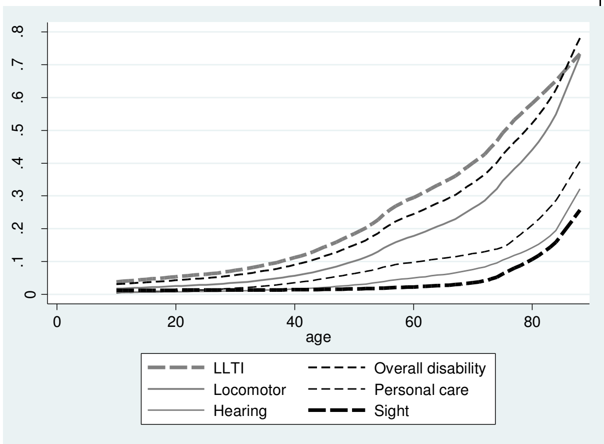
Males	Females
	

Figure 6: LLTI schedule (census) and model disability schedules (Males and females – England)

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

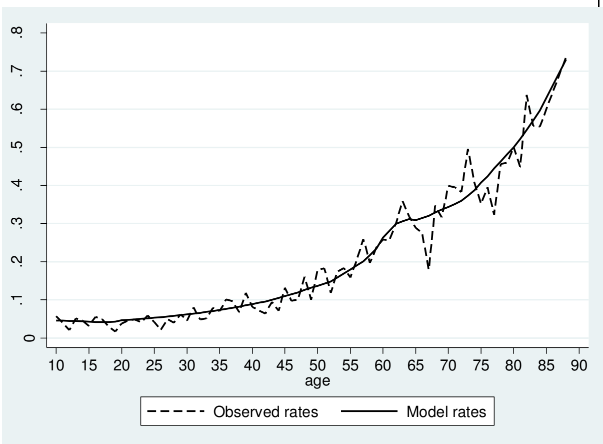
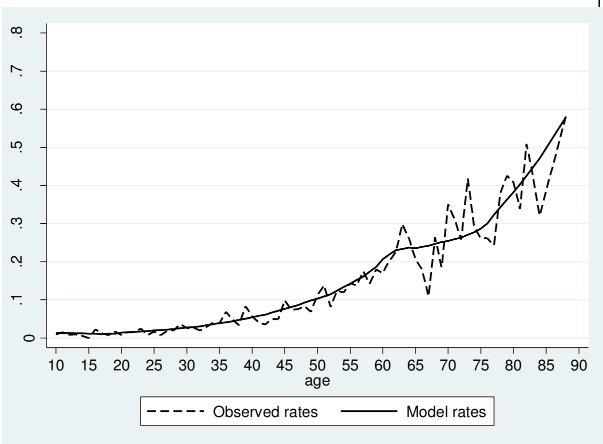
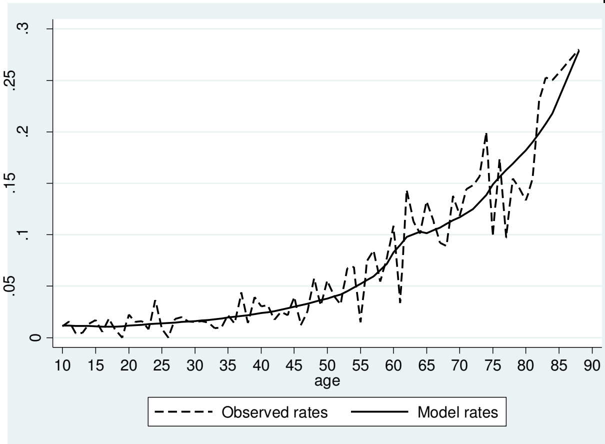
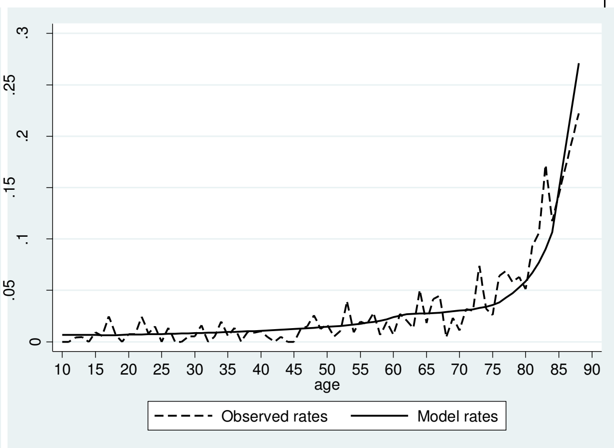
Overall disability	Locomotor
	
Hearing	Sight
	

Figure 7: Observed and model disability schedules – Overall, locomotor, hearing and sight disability (Males)

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Box 1: Limiting long term illness question – census 2001.

Do you have any long standing illness, health problem or disability which limits your daily activities or the work that you can do? Include problems which are due to old age. (Yes/No)

Source: 2001 Census household questionnaire. Available at

http://www.statistics.gov.uk/census2001/pdfs/H1.pdf

Table 1: Subscript notation

Notation	Range	Notes
i = individual	1 … N	
x = age	x = 10, 11, 12, … , 84, 88	88 is the age used to represent the 85+ age group
d = disability type	d = 1 … 5	Overall disability, locomotor, personal care, hearing and sight
l = limiting long term illness	0=no llti 1=llti	

Table 2: Extra sum of squares F test - calculations

Model	Sum of squares (SS)	Degrees of freedom (df)
Null hypothesis	SS_{null}	DF_{null}
Alternative hypothesis	SS_{alt}	DF_{alt}
Difference	$SS_{null}-SS_{alt}$	$DF_{null}-DF_{alt}$
Relative difference	$(SS_{null}-SS_{alt})/SS_{alt}$	$(DF_{null}-DF_{alt})/DF_{alt}$
Ratio ($F_{DF1-DF2,DF2}$)	$\frac{(SS_{null}-SS_{alt})/SS_{alt}}{(DF_{null}-DF_{alt})/DF_{alt}}$	

Source: Motulsky and Christopoulos (2004)

*Note the null sum of squares relates to the simpler model (e.g. Brass) and the alternative to a more complex model (e.g. reduced Ewbank).

Table 3: R^2 statistics for relational models

Males		
Disability	R^2	

	Brass	Reduced Ewbank	Ewbank	Piecewise
Overall disability	0.93	0.95	0.95	n/a
Locomotor	0.91	0.93	0.93	n/a
Personal care	0.80	0.83	0.83	n/a
Hearing	0.84	0.84	0.84	n/a
Sight	0.61	0.78	0.77	
Females				
Disability	R^2			
	Brass	Reduced Ewbank	Ewbank	Piecewise
Overall disability	0.97	0.98	0.98	n/a
Locomotor	0.96	0.97	0.97	n/a
Personal care	0.91	0.93	0.93	n/a
Hearing	0.86	0.87	0.87	n/a
Sight	0.82	0.85	0.87	

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Table 4: Residual sums of squares and F ratio p-values (from extra sum of squares F test)

MALES

Disability	Brass	Reduced Ewbank[2]		Ewbank		Total SOS
	SOS	SOS	F ratio p value[1]	SOS	F ratio p value[1]	
Disability	1.47	1.12 (l)	<0.0001 (l)	1.12	n/a	22.30
Locomotor	1.91	1.60 (k)	0.0003 (k)	1.58	0.27	22.24
Personal care	2.37	2.00 (k)	0.0004 (k)	1.95	0.51	11.45
Hearing	2.94	2.92 (l)	0.46 (l)	2.93	0.92	18.12
Sight	8.17	4.54 (k)	<0.0001 (k)	4.49	0.42	20.93
FEMALES						
Disability	**Brass**	**Reduced Ewbank[2]**		**Ewbank**		**Total SOS**
	SOS	**SOS**	**F ratio p value[1]**	**SOS**	**F ratio p value[1]**	
Disability	1.00	0.72 (l)	<0.0001 (l)	0.59	0.0001	30.91
Locomotor	1.32	1.05 (l)	<0.0001 (l)	0.90	0.0005	33.78
Personal care	1.44	1.13 (k)	<0.0001 (k)	1.14	n/a	16.03
Hearing	3.19	3.00 (l)	0.04 (l)	3.00	0.71	22.85
Sight	4.46	3.63 (k)	0.0001 (k)	3.30	0.01	4.46

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

1 The F test compares a model with the next simplest (in terms of number of parameters). The reduced Ewbank model is always compared to the Brass model. The Ewbank model is compared to the Brass model or to the reduced Ewbank model if the reduced Ewbank model offers a better fit than the Brass model.

2 The parameter that is kept in the reduced Ewbank model (k or l) is determined by which of the models has the lowest residual sum of squares.

Table 5: Parameter statistics from the relational models selected through the extra sum-of squares F test (see table 4)

Males							
	Parameter	Parameter estimate	Std. Err.	t	P>t	95% Confidence interval	
Overall disability	a	-0.17	0.02	-7.09	<0.0000	-0.22	-0.12
	b	0.96	0.03	29.06	<0.0000	0.90	1.03

	l	1.14	0.05	21.2	<0.0000	1.04	1.25
Locomotor	a	-0.45	0.02	-19.18	<0.0000	-0.49	-0.40
	b	1.74	0.16	10.81	<0.0000	1.42	2.06
	k	0.54	0.02	24.9	<0.0000	0.50	0.59
Personal care	a	-0.89	0.03	-32.14	<0.0000	-0.95	-0.83
	b	1.34	0.26	5.16	<0.0000	0.82	1.86
	k	0.54	0.04	12.63	<0.0000	0.46	0.63
Hearing	a	-0.85	0.03	-28.05	<0.0000	-0.92	-0.79
	b	0.97	0.05	19.28	<0.0000	0.87	1.07
Sight	a	-1.61	0.07	-21.58	<0.0000	-1.76	-1.46
	b	2.92	0.44	6.65	<0.0000	2.05	3.80
	k	-0.64	0.12	-5.38	<0.0000	-0.87	-0.40
Females							
	Parameter	Parameter estimate	Std. Err.	t	P>t	95% Confidence interval	
Overall disability	a	-0.14	0.02	-5.91	<0.0000	-0.19	-0.09
	b	120.40	143.59	0.84	0.41	-165.84	406.63
	k	-0.11	0.07	-1.66	0.10	-0.25	0.02
	l	-0.09	0.06	-1.58	0.12	-0.21	0.03
Locomotor	a	-0.33	0.03	-11.83	<0.0000	-0.39	-0.28
	b	562.76	1911.15	0.29	0.77	-3247.95	4373.48
	k	-0.05	0.09	-0.58	0.56	-0.24	0.13
	l	-0.04	0.08	-0.57	0.57	-0.20	0.11
Personal care	a	-0.89	0.02	-39.9	<0.0000	-0.93	-0.84
	b	1.37	0.11	12.94	<0.0000	1.16	1.58
	k	0.54	0.02	26.54	<0.0000	0.50	0.58
Hearing	a	-1.06	0.04	-26.63	<0.0000	-1.14	-0.98
	b	0.95	0.07	14.55	<0.0000	0.82	1.08
	l	0.93	0.06	14.63	<0.0000	0.80	1.05
Sight	a	-1.34	0.06	-21.42	<0.0000	-1.46	-1.21
	b	5.83	4.35	1.34	0.19	-2.85	14.52
	k	-0.57	0.23	-2.48	0.02	-1.03	-0.11
	l	-0.47	0.23	-2.06	0.04	-0.92	-0.02

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Table 6: Parameter statistics from the reduced Ewbank models selected because of evidence of overparameterisation in Ewbank models

	Parameter	Parameter estimate	Std. Err.	t	P>t	95% Confidence interval	
Overall disability – females	a	-0.13	0.02	-6.61	<0.0001	-0.17	-0.09
	b	0.99	0.03	36.12	<0.0001	0.93	1.04
	l	0.95	0.03	37.17	<0.0001	0.90	1.01
Locomotor - females	a	-0.30	0.02	-12.39	<0.0001	-0.35	-0.25

	b	1.06	0.04	28.73	<0.0001	0.99	1.14
	l	0.94	0.03	32.55	<0.0001	0.88	1.00
Sight – females	a	-1.32	0.07	-19.27	<0.0001	-1.46	-1.19
	b	1.56	0.23	6.85	<0.0001	1.11	2.02
	k	-1.13	0.21	-5.50	<0.0001	-1.54	-0.72

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

Table 7: Relational model parameters (α and β) in Reduced Ewbank models compared to Brass models restricted to the age range where the additional Reduced Ewbank parameter have little effect

MALES			
	Relational parameter	Reduced Ewbank model	Age restricted Brass model[1]
Overall disability	a	-0.17	-0.17
	b	0.96	0.97
Locomotor	a	-0.45	-0.32
	b	1.74	1.09
Personal care	a	-0.89	-0.80
	b	1.34	0.82
Sight	a	-1.61	-1.43

	b	2.92	0.94
FEMALES			
	Relational parameter	Reduced Ewbank model	Age restricted Brass model
Overall disability	a	-0.13	-0.16
	b	0.99	0.96
Locomotor	a	-0.30	-0.33
	b	1.06	1.03
Personal care	a	-0.30	-0.76
	b	1.06	0.89
Hearing	a	-1.21	-1.02
	b	1.72	1.05
Sight	a	-1.32	-1.24
	b	1.56	0.99

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)

1 Brass models are restricted to the ages of 20 and above if the comparison is with a Reduced Ewbank model involving κ. Brass models are restricted to the ages of 10 to 79 if the comparison is with a Reduced Ewbank model involving λ.

Table 8: Relational models recommended for each disability type (males and females)

Disability type	Males	Females
Overall disability	Reduced Ewbank (l)	Reduced Ewbank (l)
Locomotor disability	Reduced Ewbank (k)	Reduced Ewbank (l)
Personal care disability	Reduced Ewbank (k)	Reduced Ewbank (k)
Hearing disability	Brass	Reduced Ewbank (l)
Sight disability	Reduced Ewbank (k)	Reduced Ewbank (k)

Appendix I: Health survey for England - disability module

Disability Type	Survey Question	Response	Disability score
Locomotor	What is the furthest you can walk on your own without stopping and without discomfort?	Only a few steps More than a few steps but less than 200m More than 200m	2 1 0
	Can you walk up and down a flight of 12 stairs without resting?	Not at all Only if hold on and take rests Yes	2 1 0
	Can you, when standing, bend down and pick up a shoe from the floor?	No Yes	1 0
Personal care	Can you get in and out of bed on your own?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you get in and out of a chair on your own?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you dress and undress yourself on you own?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you wash your face and hands on your own?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you feed yourself, including cutting up food?	Only with someone to help With some difficulty Without difficulty	2 1 0
	Can you get to and use the toilet on your own?	Only with someone to help With some difficulty Without difficulty	2 1 0
Seeing	Can you see well enough to recognise a friend at a distance of four metres (across the road)? If no can you see well enough to recognise a friend at a distance of one metre (at arms length)	Cannot recognise a friend at 1m Can recognise a friend at 1m but not at 4m Can recognise a friend at 4m	2 1 0
Hearing	Is your hearing good enough to follow a TV programme at a volume others find acceptable? If not, can you follow a TV	Cannot follow a TV programme even with the volume turned up Can follow a TV programme with the volume turned up	2 1

	programme with volume turned up?	Can follow a TV programme at normal volume	0
Communication	Can you speak without difficulty?	Yes No	1 0
	Do you have problems communicating with other people?	Difficulty communicating with close relatives Difficulty communicating with other people No communication problem	2 1 0

Disability Scores in the Health Survey for England (2001).

Source: Disability report: Health Survey for England 2001 (Bajekal and Prescott 2003)

Appendix II – Extra sum of squares F test (all models)

Disability	Brass	Reduced Ewbank (k)		Reduced Ewbank (l)		Ewbank (k and l)		Total SOS
Males								
	SOS	SOS	F ratio p value[1]	SOS	F ratio p value[1]	SOS	F ratio p value[1]	
Disability	1.47	1.32	n/a	1.12	<0.0001	1.12	n/a	22.30
Locomotor	1.91	1.60	0.0003	1.85	n/a	1.58	0.27	22.24
Personal care	2.37	2.00	0.0004	2.33	n/a	1.95	0.51	11.45
Hearing	2.94	3.28	n/a	2.91	0.46	2.93	0.92	18.12
Sight	8.17	4.54	<0.0000	5.36	n/a	4.49	0.42	20.93
Females								
Disability	**Brass**	**Reduced Ewbank (k)**		**Reduced Ewbank (l)**		**Ewbank (k and l)**		**Total SOS**
	SOS	SOS	F ratio p value[1]	SOS	F ratio p value[1]	SOS	F ratio p value[1]	
Disability	1.00	1.19	n/a	0.72	<0.0000	0.59	0.0001	30.91
Locomotor	1.32	1.33	n/a	1.05	<0.0000	0.90	0.0005	33.78
Personal care	1.44	1.13	<0.0000	1.37	n/a	1.14	n/a	16.03
Hearing	3.19	3.84	n/a	3.00	0.04	3.00	0.71	22.85
Sight	4.46	3.63	0.0001	3.88	n/a	3.30	0.01	4.46

Sums of squares and F ratio p-values for all relational models

Source: Authors own calculations using data from the Health Survey for England (2000/2001) and the census (2001)