Article:

Le, TA and Navaie, K (2014) Downlink beamforming in underlay cognitive cellular networks. IEEE Transactions on Communications, 62 (7). 2212 - 2223. ISSN 0090-6778

https://doi.org/10.1109/TCOMM.2014.2323051

Downlink Beamforming in Underlay Cognitive Cellular Networks

Tuan Anh Le, *IEEE, Member*, and Keivan Navaie, *IEEE, Senior Member*

***Abstract*—We propose a novel scheme for downlink beamforming design in an underlay cognitive cellular system. The beamforming design is formulated as an optimization problem with the objective of keeping the cognitive base station transmit power as well as the induced interference on the primary users, below the predefined system thresholds. This is subject to providing a certain level of signal-to-interference-plus-noise ratio (SINR) to the secondary users. We then derive the corresponding semi-definite programming form for the formulated optimization problem and propose an iterative algorithm to obtain the beamforming vectors as the optimal solutions. Furthermore, we analytically show the convergence of the proposed iterative algorithm. Extensive simulations studies verify that the proposed algorithm quickly converges to the optimal solution. We then compare the proposed scheme with a benchmarking system based on the previous methods. Comparisons show that the proposed algorithm outperforms the benchmarking system and induces lower interference at the primary service receivers. It is also observed that comparing to the benchmarking system, the proposed algorithm offers higher sum rate. Simulation results further reveal that the proposed approach effectively works at relatively high SINR level required by secondary users and strict interference threshold set by the primary system while the benchmarking system fails to do so.**

***Index Terms*—Downlink beamforming, underlay cognitive cellular networks, interference management.**

I. Introduction

In a cognitive radio network, conditional usage of the primary system spectrum is granted to the secondary system. The secondary system[1] users are then allowed to communicate over the same spectrum without interrupting normal communication activities in the primary system [1]. Various cognitive transmission strategies have been developed to manage the access of the secondary system to the spectrum without interfering the primary users (PUs), see, e.g., [2] and references therein. One approach to the access strategy design is to utilize the spectrum during the time in which it is not in use by the primary system. In this approach which is referred to as overlay strategy, the secondary system needs monitor the amiability of the spectrum. In an alternative strategy the secondary system utilizes the spectrum while it is in use by the primary system subject to keeping the interference at the PU receivers below a predefined interference threshold. This approach is referred to as underlay strategy. In underlay access, the more efficient the cognitive system interference management, the higher is the system achievable throughput.

It has been shown that the transmit beamforming is an efficient technique to manage the interference in multi user wireless communication system, e.g., [3], [4], [5], [6], [7]. Beamforming employs an array of antennas to transmit radio frequency signals to multiple users over a shared channel. The phases and transmit power of the transmission across those antenna elements are controlled such that useful signal are constructively added up at a desired receiver while interfering signal are eliminated at unintended user terminals. Phases and power allocations across antenna elements corresponding to each user terminal are then represented by a complex vector which is referred to as the *beamforming vector*. In such systems, the design problem results in obtaining the optimal beamforming vectors.

Two common optimization strategies are usually adopted to design beamforming vectors for cellular networks. The first strategy is to minimize total transmit power while maintaining required levels of signal-to-interference-plus-noise ratios (SINRs) for mobile terminal users, see, e.g., [4], [8], [9]. The second strategy is based on maximizing the minimum SINR (or rate) among mobile users, subject to the transmit power constraint, see, e.g., [10], [11]. Needless to mention that these two optimization strategies are complementary and it is impossible to minimize the total transmit power while maximize the SINRs. This is because of the fundamental trade-off between the total transmit power and SINR in a multi user communication system [9].

For practical implementations, uplink-downlink duality is usually employed to derive iterative algorithms for downlink beamforming problem in cellular networks. One of the first iterative algorithms for the first aforementioned downlink optimization strategy is proposed in [3]. Further in [12] an additional per-antenna-power constraint is also added to the optimization problem and consequently an iterative algorithm is proposed to solve that problem in a single cell setting. Later similar problem is also considered in [5] for a multi cell setting without power constraint for each individual antenna elements, and an iterative algorithm is also proposed to obtain the optimal beamforming vectors.

In one of our recent works, [7], we also introduce a decentralized optimization problem for a multi-cell network. This optimization problem minimizes a linear combination of the two cost functions, capturing both the total transmit power of the base station (BS), and the corresponding weighted sum of the inter-cell interference. This is subject to maintaining the required SINR levels for all intra-cell users. In addition to deriving an iterative algorithm for this problem, in [7] we also

Manuscript received **December 12, 2013**.

T. A. Le and K. Navaie are with the School of Electronic and Electrical Engineering, University of Leeds, Leeds, UK, LS2 9JT. E-mail: {t.le, k.navaie}@leeds.ac.uk.

This work was supported by the UK Engineering and Physical Sciences Research Council (EPSRC) under grant EP/K/022725/1.

[1]In this paper the secondary system is also referred to as cognitive system.

propose a scheme to update the price for the interference-cost function such that the decentralized algorithm approaches the performance of its centralized counterpart.

Application of beamforming techniques in cognitive cellular systems has been recently investigated in the related literature, see, e.g., [13], [14]. The two aforementioned optimization strategies in cellular systems are adopted in underlay cognitive systems by introducing an additional constraint on the interference levels at PUs. An approach to solve these optimization problems is to transform them into rank-one-relaxation semi-definite-programming (SDP) form as in, e.g., [15], [16], [17]. The other technique is to recast the problem as a second-order-cone-programming (SOCP) form, as in [18]. In either case interior-point algorithms [4], [9], [19] are then adopted to obtain the optimal solution.

Conventionally in the related literature beamforming schemes are adopted in the underlay system with the main constraint of keeping the corresponding interference imposed at the PUs below a predefined threshold. Here, however we formulate the beamforming problem to further reduce the secondary system interference beyond the threshold. Further reduction of the imposed interference makes new radio resources available to be allocated to the the secondary system, thus results in higher secondary system throughput.

In this paper, we introduce a novel downlink optimization problem based on two slack variables which minimizes the cognitive BS transmit power and the induced interference on the PUs and keeps them below the predefined system thresholds. This is subject to providing a certain level of SINR required by the SUs. We first reformulate the proposed optimization problem to the SDP form. Using Lagrangian technique, we then show that the optimal solution to the proposed downlink optimization can be obtained by solving its corresponding dual-uplink problem, which is in fact a max-min optimization.

The corresponding max-min optimization consists of an inner and an outer subproblems. The allocated SUs transmit power vector in the dual uplink problem, acts as the optimization variable in the inner subproblem. In the outer subproblem, the optimization variables include Lagrange multipliers associated with the interference and power constraints in the original optimization problem. We then propose an iterative algorithm to solve the max-min optimization. The inner subproblem is solved by adopting the fixed-point approach [20]. The solutions to the outer subproblem are also obtained utilizing the subgradient-projection method [21]. Further we analytically investigate the proposed algorithm and show its convergence.

We carry out Monte-Carlo simulations to justify our proposed scheme and compare it against the existing beamforming schemes. We define a benchmarking system. As the benchmarking system we considered the method in [16] which has been widely used in the related literature. We also investigate the convergence of the proposed iterative algorithm using simulations. Simulation results confirm that the proposed iterative algorithm converges quickly to the optimal solution. Results also indicate that the proposed algorithm successfully implements both the transmit power and interference constraints. Comparisons against the benchmarking system also indicates that the resulting beamforming based on the proposed algorithm has significantly deeper nulls towards the PUs. This confirms our claim that the proposed algorithm make new radio resources available to be allocated to the secondary system. This can result in either having larger numbers of SUs at a given SINR level, or having a higher bit rate for the existing SUs in the network. Moreover, simulation results indicate that the proposed algorithm effectively works at relatively high SINR levels required by the SUs and low interference threshold set by PUs. However, the benchmark fails to maintain that interference threshold at much lower SINR levels.

The contributions of this paper can be summarized as the following:

- We propose a novel downlink optimization strategy for underlay cognitive cellular networks;
- The proposed optimization strategy is then transformed into SDP form which can be solved by convex optimization packages;
- For practical implementation purposes, we further derive an iterative algorithm to find solution to the proposed optimization problem;
- We then analytically show the convergence of the proposed iterative algorithm.

The rest of this paper is organized as follows. Section II describes the system model and introduces the downlink optimization problem for the cognitive radio network. Section III presents SDP form and the proposed iterative algorithm to obtain the solutions of the original problem introduced in Section II. Simulation results are presented and discussed in Section IV following by concluding remarks in Section V.

Notation: The standard Euclidean norm, the absolute value, the transpose, the complex conjugate, the complex conjugate transpose, and the trace operators are represented by the following notations, respectively: $\|\cdot\|$, $|\cdot|$, $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ and $\mathrm{Tr}\,(\cdot)$. A positive semi definite matrix is denoted as $\mathbf{Y} \succeq 0$. If all elements of a vector are non negative it is shown by $\mathbf{y} \succcurlyeq 0$. An identity matrix with a suitable size, and the expectation of a random variable are denoted by $\mathbf{I}$, and $\mathbb{E}\,(\cdot)$, respectively. Finally, the notation $(y_i)_{i=1}^{U}$ designates $\begin{bmatrix} y_1 & y_2 & \cdots & y_U \end{bmatrix}^T$.

II. System Model and Problem Formulation

Consider a cognitive cellular system consiting of a cognitive BS, U active SUs and K PUs as shown in Fig. 1. A secondary (cognitive) BS is supporting a set of U secondary users while not interfering with a set of K primary users. Let $\mathcal{S}_s = \{1, \cdots, U\}$ and $\mathcal{S}_p = \{1, \cdots, K\}$ be, respectively, the set of indices of SUs and PUs. We assume that the cognitive BS is equipped with M antenna elements and each SU or PU has a single antenna. The received signal at the SU i, $i \in \mathcal{S}_s$, is

$$y_i = \mathbf{h}_{s,i}^H \mathbf{w}_i s_i + \sum_{j \in \mathcal{S}_l, j \neq i} \mathbf{h}_{s,i}^H \mathbf{w}_j s_j + n_i, \tag{1}$$

where $\mathbf{h}_{s,i}^H \in \mathbb{C}^{1 \times M}$ is the channel of SU i as seen by the cognitive BS.

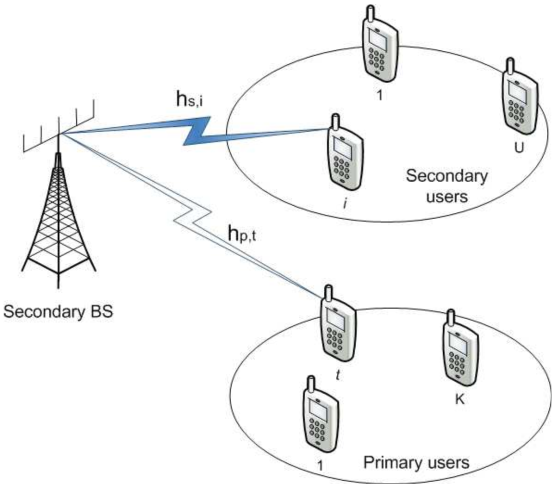

Fig. 1. Schematic diagram of the system model.

In the above, $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ and s_i are the beamforming vector and the data symbol associated to the SU i, respectively. Further, n_i is a zero mean circularly symmetric complex Gaussian noise with variance σ^2, i.e., $n_i \sim \mathcal{CN}(0,\sigma^2)$, at the SU i. Without loss of generality, the average energy in transmitting a symbol to the SU i is assumed to be unity, i.e., $\mathbb{E}\left(|s_i|^2\right) = 1$. Let $\mathbf{R}_{s,i} = \mathbb{E}\left(\mathbf{h}_{s,i}\mathbf{h}_{s,i}^H\right)$, we then express the SINR at any SU i as

$$\text{SINR}_i = \frac{\mathbf{w}_i^H \mathbf{R}_{s,i}\mathbf{w}_i}{\sum_{j\in\mathcal{S}_s, j\neq i}\mathbf{w}_j^H\mathbf{R}_{s,i}\mathbf{w}_j + \sigma^2}. \tag{2}$$

Let $\mathbf{h}_{p,t}^H \in \mathbb{C}^{1\times M}$ be the cross channel of PU t, $t\in\mathcal{S}_p$, as seen by the cognitive BS, and $\mathbf{R}_{p,t} = \mathbb{E}\left(\mathbf{h}_{p,t}\mathbf{h}_{p,t}^H\right)$. The total interference power that the cognitive BS induces on PUs can be written as $\sum_{t\in\mathcal{S}_p}\sum_{i\in\mathcal{S}_s}\mathbf{w}_i^H\mathbf{R}_{p,t}\mathbf{w}_i$.

At a required SINR by SUs, the lower the interference level imposed by the secondary system on the PUs, the larger is the number of SUs which can be served. Therefore, our objective is to design downlink beamforming vectors for the SUs such that the required level of SINR is maintained for every SU while the cognitive BS transmit power and the induced interference at the PUs' receivers are both minimized and kept below the given system thresholds.

Here to design the downlink beamforming vectors we formulate the following optimization problem based on two slack variables, α and β.

$$\begin{aligned}
\min_{\alpha,\beta,\mathbf{w}_i} \quad & \alpha+\beta \\
\text{s. t.} \quad & \frac{\mathbf{w}_i^H\mathbf{R}_{s,i}\mathbf{w}_i}{\sum_{j\in\mathcal{S}_s,j\neq i}\mathbf{w}_j^H\mathbf{R}_{s,i}\mathbf{w}_j+\sigma^2} \geq \gamma_i, \quad \forall i\in\mathcal{S}_s \\
& \sum_{t\in\mathcal{S}_p}\sum_{i\in\mathcal{S}_s}\mathbf{w}_i^H\mathbf{R}_{p,t}\mathbf{w}_i \leq \alpha I_{\text{m}}, \\
& \sum_{i\in\mathcal{S}_s}\mathbf{w}_i^H\mathbf{w}_i \leq \beta P_{\text{m}},
\end{aligned} \tag{3}$$

where γ_i is the required SINR level for SU i, I_{m} and P_{m} are fixed values. Note that in (3) α and β are treated in the same way. However, I_{m} and P_{m} can be adjusted to highlight the importance of keeping the interference below the acceptable level. It is worth mentioning that the optimization problem in (3) is a generalized version of the optimization proposed in our previous work in [7] with unity pricing. Later in Section IV, it is shown that the same result offered by [7] can be also achieved by substituting a particular set of parameters in (3).

III. Downlink Beamforming

In this section, we first reformulate problem (3) into a semidefinite programming (SDP). Then, we use Lagrangian method to derive an iterative algorithm to find the solutions of (3). We further show that the solution to the Lagrangian dual problem can be obtained by solving the corresponding dual-uplink problem of (3). Finally, we propose an iterative algorithm to find optimal downlink beamforming vectors to the original problem employing the fixed-point algorithm [20] and the subgradient-projection technique [21].

A. SDP Form

By setting $\mathbf{W}_i = \mathbf{w}_i\mathbf{w}_i^H \succeq 0$, rearranging the constraints, and using the fact that $\mathbf{x}^H\mathbf{Y}\mathbf{x} = \text{Tr}\left(\mathbf{Y}\mathbf{x}\mathbf{x}^H\right)$, the problem (3) is then rewritten in the following form:

$$\begin{aligned}
\min_{\alpha,\beta,\mathbf{W}_i} \quad & \alpha+\beta \\
\text{s. t.} \quad & f_i\left(\mathbf{W}_i\right) \geq 0, \ \forall i\in\mathcal{S}_s \\
& \alpha I_{\text{m}} - \sum_{t\in\mathcal{S}_p}\sum_{i\in\mathcal{S}_s}\text{Tr}\left(\mathbf{R}_{p,t}\mathbf{W}_i\right) \geq 0 \\
& \beta P_{\text{m}} - \sum_{i\in\mathcal{S}_s}\text{Tr}\left(\mathbf{W}_i\right) \geq 0 \\
& \mathbf{W}_i \succeq 0, \ \forall i\in\mathcal{S}_s,
\end{aligned} \tag{4}$$

where

$$f_i\left(\mathbf{W}_i\right) = \text{Tr}\left(\mathbf{R}_{s,i}\mathbf{W}_i\right) - \gamma_i\sum_{j\in\mathcal{S}_s,j\neq i}\text{Tr}\left(\mathbf{R}_{s,i}\mathbf{W}_j\right) - \gamma_i\sigma^2.$$

The optimization problem in (4) is in fact an instance of standard SDP form which can be solved by the existing optimization packages, e.g., CVX [22], to obtain $\mathbf{W}_i$. In transforming (3) into (4), we implicitly assume that $\mathbf{W}_i$ is rank-one, i.e., rank$\left(\mathbf{W}_i\right) = 1$. Later using semidefinite-relaxation technique, e.g., [16] and [8], we relax this condition, i.e., rank$\left(\mathbf{W}_i\right)$ is not required to be rank-one, to make (4) a convex optimization problem. If the obtained solution $\mathbf{W}_i$ to problem (4) is also rank-one than it means that this solution is also valid for the original problem (3). Otherwise, the randomization technique in [23] is adopted to generate a rank-one solution to the original problem (3) from the obtained $\mathbf{W}_i$.

Given a rank-one solution $\mathbf{W}_i$, it can be shown that the corresponding beamforming vector $\mathbf{w}_i$ is

$$\mathbf{w}_i = \sqrt{\varrho_i}\mathbf{b}_i,$$

where ϱ_i and $\mathbf{b}_i$ are the non-zero eigenvalue and its corresponding eigenvector of the rank-one matrix $\mathbf{W}_i$, respectively.

B. Uplink Downlink Duality

Here, we first adopt Lagrangian technique to transform the proposed downlink problem (3) to its corresponding uplink domain. We then find the optimal downlink beamforming vector as a linear function of its optimal uplink counterpart.

1) Dual Uplink Problem: Lagrangian function corresponding to (3) is given in (5) where $\lambda_i \geq 0$ is Lagrange multiplier associated with the ith SINR constraint, $\eta \geq 0$ is Lagrange multiplier associated with the interference constraint, and $\mu \geq 0$ is Lagrange multiplier associated with the transmit power constraint.

Straightforward mathematical manipulations results in

$$\begin{aligned} L(\alpha, \beta, \mathbf{w}_i, \lambda_i, \eta, \mu) &= \alpha\left(1-\eta I_{\mathrm{m}}\right)+\beta\left(1-\mu P_{\mathrm{m}}\right) \\ &+ \sum_{i \in \mathcal{S}_s} \lambda_i \sigma^2+\sum_{i \in \mathcal{S}_s} \mathbf{w}_i^H \mathbf{A}_i \mathbf{w}_i, \quad (6) \end{aligned}$$

where

$$\mathbf{A}_i=\sum_{j \in \mathcal{S}_s} \lambda_j \mathbf{R}_{s, j}-\lambda_i\left(1+\frac{1}{\gamma_i}\right) \mathbf{R}_{s, i}+\eta \sum_{t \in \mathcal{S}_p} \mathbf{R}_{p, t}+\mu \mathbf{I}. \quad (7)$$

If $\mathbf{A}_i \succeq 0$, $1-\eta I_{\mathrm{m}} \geq 0$, and $1-\mu P_{\mathrm{m}} \geq 0$, then Lagrange dual function is

$$g\left(\lambda_i, \eta, \mu\right)=\inf_{\alpha, \beta, \mathbf{w}_i} L(\alpha, \beta, \mathbf{w}_i, \lambda_i, \eta, \mu)=\sum_{i \in \mathcal{S}_s} \lambda_i \sigma^2, \quad (8)$$

otherwise,

$$g\left(\lambda_i, \eta, \mu\right)=\inf_{\alpha, \beta, \mathbf{w}_i} L(\alpha, \beta, \mathbf{w}_i, \lambda_i, \eta, \mu)=-\infty. \quad (9)$$

Therefore, the corresponding Lagrange dual problem is

$$\begin{aligned} \max_{\eta, \mu, p_i} \quad & \sum_{i \in \mathcal{S}_s} p_i \\ \text{s. t.} \quad & \mathbf{A}_i \succeq 0, \quad \forall i \in \mathcal{S}_s \\ & \eta \in \mathcal{S}_\eta,\ \mu \in \mathcal{S}_\mu, \end{aligned} \quad (10)$$

where $p_i=\lambda_i \sigma^2$, $\mathcal{S}_\eta \triangleq \{\eta: 1-\eta I_{\mathrm{m}} \geq 0\}$, and $\mathcal{S}_\mu \triangleq \{\mu: 1-\mu P_{\mathrm{m}} \geq 0\}$.

In the sequel, we introduce a lemma to find the solutions to the dual downlink problem (10).

Lemma 1: The solution to the dual downlink problem in (10) is the same as the solution to the following dual uplink problem.

$$\begin{aligned} \max_{\eta, \mu} \min_{p_i} \quad & \sum_{i \in \mathcal{S}_s} p_i \\ \text{s. t.} \quad & \max_{\|\hat{\mathbf{w}}_i\|=1} \frac{p_i \hat{\mathbf{w}}_i^H \mathbf{R}_{s, i} \hat{\mathbf{w}}_i}{\hat{\mathbf{w}}_i^H \mathbf{B}_i\left(\mathbf{p}^{-i}\right) \hat{\mathbf{w}}_i} \geq \gamma_i,\ \forall i \in \mathcal{S}_s, \\ & \eta \in \mathcal{S}_\eta,\ \mu \in \mathcal{S}_\mu, \end{aligned} \quad (11)$$

where $\hat{\mathbf{w}}_i$ is the dual uplink beamforming vector for the SU i, $\mathbf{p}^{-i}=\left(p_j\right)_{j=1, j \neq i}^U$, and

$$\mathbf{B}_i\left(\mathbf{p}^{-i}\right)=\sum_{j \in \mathcal{S}_s, j \neq i} p_j \mathbf{R}_{s, j}+\eta \sigma^2 \sum_{t \in \mathcal{S}_p} \mathbf{R}_{p, t}+\mu \sigma^2 \mathbf{I}. \quad (12)$$

Proof: See Appendix A. ∎

We have transformed the original downlink problem, (3), into its uplink counterpart, (11), by introducing Lemma 1. Details of the steps to solve the problem in (11) are given in Section III-C. At this point, it is observed that $\hat{\mathbf{w}}_i$, and p_i are the direct of transmission, and power allocation for SU i, respectively. In the following, we obtain the expression for an optimal downlink beamforming vector as a linear function of its uplink counterpart.

2) Downlink Beamforming Vector: Once an optimal uplink beamforming vector is obtained, the corresponding downlink beamforming vector can be then attained as follows.

Corollary 1: The optimum downlink beamforming vector for user i, i.e., $\mathbf{w}_i^\star$, is

$$\mathbf{w}_i^\star=\epsilon_i \hat{\mathbf{w}}_i^\star, \quad (13)$$

where $\hat{\mathbf{w}}_i^\star$ is the corresponding optimum dual uplink beamforming vector and ϵ_i is the scaling factor associated with the SU i.

Proof: See Appendix B. ∎

In the following, we obtain ϵ_i for $i \in \mathcal{S}_s$. First, we rewrite the ith SINR constraint in (3) as

$$k_i\left(\mathbf{w}_i\right)=\sum_{j \in \mathcal{S}_s, j \neq i} \gamma_i \mathbf{w}_j^H \mathbf{R}_{s, i} \mathbf{w}_j+\gamma_i \sigma^2-\mathbf{w}_i^H \mathbf{R}_{s, i} \mathbf{w}_i \leq 0.$$

Let $\lambda_i^\star>0$ be Lagrange multiplier at the optimal point. Using the complementary slackness condition, i.e., $\lambda_i^\star k_i\left(\mathbf{w}_i^\star\right)=0$ [19, Chapter 5], results in

$$\sum_{j \in \mathcal{S}_s, j \neq i} \gamma_i \mathbf{w}_j^{\star H} \mathbf{R}_{s, i} \mathbf{w}_j^\star+\gamma_i \sigma^2-\mathbf{w}_i^{\star H} \mathbf{R}_{s, i} \mathbf{w}_i^\star=0. \quad (14)$$

Substituting $\mathbf{w}_i^\star$ in (14) with (13) yields

$$\epsilon_i^2 \hat{\mathbf{w}}_i^{\star H} \mathbf{R}_{s, i} \hat{\mathbf{w}}_i^\star-\sum_{j \in \mathcal{S}_s, j \neq i} \gamma_i \epsilon_j^2 \hat{\mathbf{w}}_j^{\star H} \mathbf{R}_{s, i} \hat{\mathbf{w}}_j^\star=\gamma_i \sigma^2. \quad (15)$$

Let us denote $\mathbf{m}=\left(\gamma_i \sigma^2\right)_{i=1}^U$, $\mathbf{q}=\left(\epsilon_i^2\right)_{i=1}^U$ and define the $U \times U$ matrix $\mathbf{G}$ with the (i, j)th entry, i.e., $\forall i, j \in \mathcal{S}_l$, as

$$[\mathbf{G}]_{i, j}=\begin{cases} \hat{\mathbf{w}}_i^{\star H} \mathbf{R}_{s, i} \hat{\mathbf{w}}_i^\star, & \text{if} \quad i=j \\ -\gamma_i \hat{\mathbf{w}}_j^{\star H} \mathbf{R}_{s, i} \hat{\mathbf{w}}_j^\star, & \text{if} \quad i \neq j. \end{cases} \quad (16)$$

We then write (15) as

$$\mathbf{G q}=\mathbf{m}, \quad (17)$$

where $\mathbf{m} \succcurlyeq 0$. The scaling factor $\epsilon_i, i \in \mathcal{S}_s$, can be obtained through (17). A feasible solution for $\mathbf{q}$ exists if all elements of $\mathbf{q}$ are nonnegative. To investigate the the existence of such solution which depends on the structure of $\mathbf{G}$ we need the following definition. We also present the following theorems for easy reference.

***Definition* Z-matrix** [24], [25]: A matrix $\mathbf{A} \in \mathbb{R}^{K \times K}$ is called a *Z-matrix* if all of its off-diagonal elements are nonpositive.

Theorem 1: [24, Chapter 6, Theorem 2.3]: If all the diagonal elements of a matrix $\mathbf{A} \in \mathbb{R}^{K \times K}$ are positive and there exists a positive diagonal matrix $\mathbf{D}$ such that $\mathbf{AD}$ is *strictly diagonally dominant*, i.e.,

$$a_{ii} d_{ii}>\sum_{j=1, j \neq i}^K\left|a_{ij}\right| d_{jj}, \quad i=1, \cdots, K, \quad (18)$$

$$L(\alpha,\beta,\mathbf{w}_i,\lambda_i,\eta,\mu)=\alpha+\beta+\sum_{i\in\mathcal{S}_s}\lambda_i\left[\sum_{j\in\mathcal{S}_s}\mathbf{w}_j^H\mathbf{R}_{s,i}\mathbf{w}_j+\sigma^2-\mathbf{w}_i^H\mathbf{R}_{s,i}\mathbf{w}_i\left(1+\frac{1}{\gamma_i}\right)\right]$$
$$+\eta\left[\sum_{t\in\mathcal{S}_p}\sum_{i\in\mathcal{S}_s}\mathbf{w}_i^H\mathbf{R}_{p,t}\mathbf{w}_i-\alpha I_{\mathrm{m}}\right]+\mu\left[\sum_{i\in\mathcal{S}_s}\mathbf{w}_i^H\mathbf{w}_i-\beta P_{\mathrm{m}}\right], \tag{5}$$

then all the principal minors of $\mathbf{A}$ are also positive. In (18), a_{ii}, and d_{ii} denote the (i,i)th entry of matrices $\mathbf{A}$, and $\mathbf{D}$, respectively.

Theorem 2: [25, Theorem 3.11.10]: For a Z-matrix, $\mathbf{A}\in\mathbb{R}^{K\times K}$, the following statements are equivalent:

- All principal minors of $\mathbf{A}$ are positive;
- $\mathbf{A}^{-1}$ exists and is nonnegative, i.e., all elements are nonnegative.

The main result regarding the existence of the solution is presented in the following lemma.

Lemma 2: If $\mathbf{G}$, defined in (16), satisfies

$$\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^{\star}>\gamma_i\sum_{j\in\mathcal{S}_s,j\neq i}\hat{\mathbf{w}}_j^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_j^{\star},\ \forall i\in\mathcal{S}_s, \tag{19}$$

then there exists a unique feasible solution to (17) in the form of $\mathbf{q}=\mathbf{G}^{-1}\mathbf{m}$.

Proof: If $\mathbf{G}$ satisfies the conditions in (19), then $\mathbf{GI}$ is a strictly diagonally dominant matrix. Therefore, according to Theorem 1, all principal minors of $\mathbf{G}$ are positive. Considering (16) indicates that $\mathbf{G}$ is a Z-matrix, therefore according to Theorem 2, $\mathbf{G}^{-1}$ exists and all its elements are nonnegative. Since all elements of vector $\mathbf{m}$ in (17) are also nonnegative, then

$$\mathbf{q}=\mathbf{G}^{-1}\mathbf{m}\succcurlyeq 0. \tag{20}$$

■

C. Proposed Iterative Algorithm

The dual uplink problem (11) can be considered as two independent optimization problems, i.e., inner and outer optimization problems. The inner problem is a minimization problem over the set of variable p_i and the outer problem is a maximization problem over the set of variables μ and η.

Hence, the optimal solution to the dual uplink problem, (11), can be obtained by iteratively solving the inner minimization on p_i and the outer maximization on η and μ. For given η and μ, the inner problem can be solved using a fixed-point algorithm [20]. Finally, the optimal solutions to the outer problem can be obtained by using subgradient-projection algorithm [21].

1) The Inner Problem: Let us consider the following subproblem of (11) with fixed values of η and μ

$$\begin{aligned}f(\eta,\mu)=\min_{p_i}\quad&\sum_{i\in\mathcal{S}_s}p_i\\ \text{s. t.}\quad&\max_{\|\hat{\mathbf{w}}_i\|=1}\frac{p_i\hat{\mathbf{w}}_i^H\mathbf{R}_{s,i}\hat{\mathbf{w}}_i}{\hat{\mathbf{w}}_i^H\mathbf{B}_i\left(\mathbf{p}^{-i}\right)\hat{\mathbf{w}}_i}\geq\gamma_i,\ \forall i\in\mathcal{S}_s.\end{aligned} \tag{21}$$

We denote $\hat{\mathbf{w}}_i^{\star}$ as the optimal solution to the optimization problem in the left hand side of the constraint. In fact, $\hat{\mathbf{w}}_i^{\star}$ is the dominant eigenvector, i.e., the eigenvector associated with the maximum eigenvalue, of matrix $\mathbf{B}_i^{-1}\left(\mathbf{p}^{-i}\right)\mathbf{R}_{s,i}$. Hence, the ith constraint of (21) can be written as

$$p_i\geq\gamma_i\frac{\hat{\mathbf{w}}_i^{\star H}\mathbf{B}_i\left(\mathbf{p}^{-i}\right)\hat{\mathbf{w}}_i^{\star}}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^{\star}}. \tag{22}$$

We also denote $\mathbf{p}=(\mathbf{p}_i)_{i=1}^U$, $\mathbf{d}(\mathbf{p})=\left(d_i\left(\mathbf{p}^{-i}\right)\right)_{i=1}^U$ and

$$d_i\left(\mathbf{p}^{-i}\right)=\gamma_i\frac{\hat{\mathbf{w}}_i^{\star H}\mathbf{B}_i\left(\mathbf{p}^{-i}\right)\hat{\mathbf{w}}_i^{\star}}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^{\star}}. \tag{23}$$

The optimization problem (21) is then rewritten in a compact form as

$$\begin{aligned}f(\eta,\mu)=\min_{p_i}\quad&\sum_{i\in\mathcal{S}_s}p_i\\ \text{s. t.}\quad&\mathbf{p}\succcurlyeq\mathbf{d}(\mathbf{p}).\end{aligned} \tag{24}$$

The optimal solution to (24), can be obtained through the following iterative expression [20]:

$$\mathbf{p}(n+1)=\mathbf{d}(\mathbf{p}(n)) \tag{25}$$

where n indicates iteration index.

In order to show the convergence of the above sequence $\{\mathbf{p}(n)\}$, we first introduce the following lemma.

Lemma 3: Function $\mathbf{d}(\mathbf{p})$, with elements defined in (23), is a standard-interference function[2].

Proof: See Appendix C. ■

Then using the contraction mapping [26], we continue with the derivation of a condition that ensures the existence of a fixed point[3] $\mathbf{p}^{\star}$ for equation (25) in the following Lemma.

Lemma 4: Equation (25) has a fixed point $\mathbf{p}^{\star}$, if

$$c=\sqrt{U-1}\max_i\left(\sum_{t\in\mathcal{S}_s,t\neq i}\gamma_t\frac{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_t^{\star}}{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_t^{\star}}\right)\in[0,1), \tag{26}$$

i.e., $0\leq c<1$.

Proof: See Appendix D. ■

If $\mathbf{d}(\mathbf{p})$ is a standard interference function, and equation (25) has a fixed point $\mathbf{p}^{\star}$, then according to the results in [20] that fixed point is unique and the iterations $\{\mathbf{p}(n)\}$ generated by (25) eventually converge to $\mathbf{p}^{\star}$ from any initial vector $\mathbf{p}(0)$.

[2]A function is called *standard interference* if it satisfies the positivity, monotonicity, and scalability criteria, see, Appendix C.

[3]$\mathbf{p}(n+1)=\mathbf{d}(\mathbf{p})$ has a fixed point $\mathbf{p}^{\star}$, if $\mathbf{p}^{\star}=\mathbf{d}(\mathbf{p}^{\star})$ [20].

Therefore, under the condition stated in Lemma 4 the iteration in (25) is guaranteed to converged to $\mathbf{p}^\star$. The convergence speed of the iteration is characterized in the following lemma.

Lemma 5: For any initial vector $\mathbf{p}(0)$, the number of iterations n to obtain the accuracy of $\| \mathbf{p}(n) - \mathbf{p}^\star \| \leq \zeta$ is

$$n \leq \frac{\ln \zeta - \ln \| \mathbf{p}(0) - \mathbf{p}^\star \|}{\ln c} \tag{27}$$

where c is defined in (26).

Proof: Under the condition that $c \in [0, 1)$, the sequence $\{\mathbf{p}(n)\}$ generated by $\mathbf{p}(n+1) = \mathbf{d}(\mathbf{p}(n))$ converges linearly to $\mathbf{p}^\star$ such that [26]

$$\| \mathbf{p}(n) - \mathbf{p}^\star \| \leq c^n \| \mathbf{p}(0) - \mathbf{p}^\star \| . \tag{28}$$

The iteration obtains the accuracy of ζ if

$$c^n \| \mathbf{p}(0) - \mathbf{p}^\star \| \leq \zeta. \tag{29}$$

Using (28), (29) following with straightforward mathematical manipulations result in (27). ■

Remark 1: Since $\zeta, c \in [0, 1)$ and $\| \mathbf{p}(0) - \mathbf{p}^\star \| \geq 0$, it can be verified from (27) that n is a monotonic function of U and SINR level at SUs. Later in Section IV, this statement is confimed by simulation results.

2) The Outer Problem: Having solved the inner problem, the outer problem can be stated as

$$\begin{aligned} \max_{\eta,\mu} \quad & f(\mu, \eta) \\ \text{s. t.} \quad & \eta \in \mathcal{S}_\eta,\ \mu \in \mathcal{S}_\mu, \end{aligned} \tag{30}$$

where $f(\mu, \eta)$ is defined in (24). We show that the objective function is concave regarding to μ at a given value of η and vice versa. Then the projection subgradient method [21] is adopted to find the optimal solutions for μ and η. We also need to introduce the following lemma.

Lemma 6: For a given $\eta = \eta_0$, $f(\eta_0, \mu)$ is a concave function of μ and its subgradient is $\sum_{i \in \mathcal{S}_s} \mathbf{w}_i^H \mathbf{w}_i$. For a given $\mu = \mu_0$, the function $f(\eta, \mu_0)$ is concave in η and $\sum_{i \in \mathcal{S}_s} \mathbf{w}_i^H \mathbf{w}_i$ is its subgradient.

Proof: See Appendix E. ■

To obtain μ we propose the following iteration

$$\mu(n+1) = \mathcal{P}_{\mathcal{S}_\mu} \left\{ \mu(n) + \tau_\mu \sum_{i \in \mathcal{S}_s} \mathbf{w}_i^H \mathbf{w}_i \right\} \tag{31}$$

where $\mathcal{P}_{\mathcal{S}_\mu}$ is the Euclidean projection on the constraint set $\mathcal{S}_\mu = \{\mu : 1 - \mu P_\mathrm{m} \geq 0\}$ and τ_μ is the step size.

As it is seen in Lemma 6, $f(\eta_0, \mu)$ is a concave function of μ thus the Euclidean projection of the subgradient of $f(\eta_0, \mu)$ on the constraint set $\mathcal{S}_\mu$ stated in (31) is guaranteed to converge to the global optimum of $f(\eta_0, \mu)$ [21].

Similarly, η can be found using the following convergent iteration.

$$\eta(n+1) = \mathcal{P}_{\mathcal{S}_\eta} \left\{ \eta(n) + \tau_\eta \sum_{i \in \mathcal{S}_s} \mathbf{w}_i^H \mathbf{w}_i \right\}, \tag{32}$$

where $\mathcal{P}_{\mathcal{S}_\eta}$ is the Euclidean projection on the constraint set $\mathcal{S}_\eta = \{\eta : 1 - \eta I_\mathrm{m} \geq 0\}$ and τ_η is the step size.

3) The Proposed Algorithm: The proposed iterative algorithm is summarized in Algorithm 1.

Algorithm 1 Iterative Algorithm

1: Define: a set of SUs, $\mathcal{S}_s$, with their corresponding SINR requirements, a set of PUs, $\mathcal{S}_p$, I_m, P_m and an iteration stopping criteria, δ.
2: $n = 1$.
3: Initialize $\mathbf{p}(n) \succcurlyeq 0$, $\mu(n) > 0$, $\eta(n) > 0$.
4: For all $i \in \mathcal{S}_s$, find $\hat{\mathbf{w}}_i(n)$ as the dominant eigenvector of the matrix $\mathbf{B}_i^{-1}\left(\mathbf{p}^{-i}(n)\right)\mathbf{R}_{s,i}$, calculate $d_i\left(\mathbf{p}^{-i}(n)\right) = \gamma_i \frac{\hat{\mathbf{w}}_i^H(n)\mathbf{B}_i\left(\mathbf{p}^{-i}(n)\right)\hat{\mathbf{w}}_i(n)}{\hat{\mathbf{w}}_i^H(n)\mathbf{R}_{s,i}\hat{\mathbf{w}}_i(n)}$, calculate $\mathbf{G}(n)$ using (16) and form $\mathbf{d}(\mathbf{p}(n)) = \left(d_i\left(\mathbf{p}^{-i}(n)\right)\right)_{i=1}^U$.
5: **if** condition (19) is satisfied for all SU i with its associated $\hat{\mathbf{w}}_i(n)$ **then**
6: go to step 10
7: **else if** condition (19) is not satisfied for a SU i **then**
8: either reduce the target SINR so that $\tilde{\gamma}_i < \frac{\hat{\mathbf{w}}_i(n)\mathbf{R}_{s,i}\hat{\mathbf{w}}_i(n)}{\sum_{j \in \mathcal{S}_s, j \neq i} \hat{\mathbf{w}}_j(n)\mathbf{R}_{s,i}\hat{\mathbf{w}}_j(n)}$, or remove SU i from $\mathcal{S}_s$. Then go to step 2.
9: **end if**
10: $\mathbf{w}_i(n) = \epsilon_i \hat{\mathbf{w}}_i(n)$, where ϵ_i is found as the square root of the i-th entry of the vector $\mathbf{q}(n) = (\mathbf{G}(n))^{-1}\mathbf{m}$.
11: $\mu(n+1) = \mathcal{P}_{\mathcal{S}_\mu}\left\{\mu(n) + \tau_\mu \sum_{i \in \mathcal{S}_s} \mathbf{w}_i(n)^H \mathbf{w}_i(n)\right\}$.
12: $\eta(n+1) = \mathcal{P}_{\mathcal{S}_\eta}\left\{\eta(n) + \tau_\eta \sum_{i \in \mathcal{S}_s} \mathbf{w}_i(n)^H \mathbf{w}_i(n)\right\}$.
13: $\mathbf{p}(n+1) = \mathbf{d}(\mathbf{p}(n))$.
14: $n = n + 1$.
15: Repeat steps 4–14 until $\| \mathbf{p}(n+1) - \mathbf{p}(n) \| \leq \delta$.
16: $\hat{\mathbf{w}}_i^\star = \hat{\mathbf{w}}_i(n+1)$, calculate $\mathbf{G}(n+1)$ using (16).
17: The optimal downlink beamforming vector for SU i is $\mathbf{w}_i^\star = \epsilon_i \hat{\mathbf{w}}_i^\star$ where ϵ_i is found as the square root of the i-th entry of the vector $\mathbf{q}(n+1) = (\mathbf{G}(n+1))^{-1}\mathbf{m}$.

IV. Simulations

Here we consider a cognitive cellular system, described in Section II. To compare against the proposed optimization scheme, we consider a popular optimization strategy in cognitive systems, that minimizes the transmit power of the secondary BS subject to SINR constraint for every SU, as well as the interference constraint for each PU. In particular, we compare our proposed algorithm against a benchmarking system in [16] which develops the aforementioned strategy in SDP form.

A. Simulation Setup

We randomly drop SUs and PUs and use Monte-Carlo simulations over various number of user distributions. Fig. 2 illustrates an instance of the simulated user distribution consisting of one cognitive BS and four randomly located users (two PUs and two SUs). The channel covariance matrices from the secondary BS to SU i , i.e., $\mathbf{R}_{s,i}$, and to PU t, i.e., $\mathbf{R}_{p,t}$, are

$$\mathbf{R}_{s,i} = \xi_{s,i}\mathbf{R}(\theta_{s,i}, \sigma_a), \tag{33}$$

$$\mathbf{R}_{p,t} = \xi_{p,t}\mathbf{R}(\theta_{p,t}, \sigma_a), \tag{34}$$

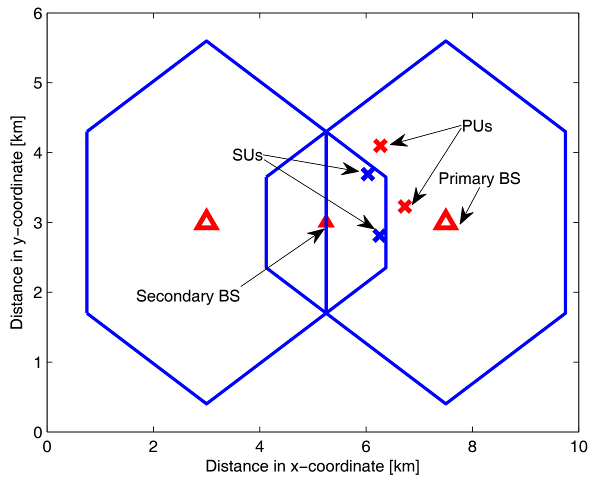

Fig. 2. An instance of the considered simulation scenario.

where $\xi_{s,i}$ or $\xi_{p,t}$ represents the channel gain coefficient, $\theta_{s,i}$ or $\theta_{p,t}$ is the angle of departure, σ_a is the standard deviation of the angular spread, and the (m,n)th entry of $\mathbf{R}(\theta, \sigma_a)$ is, [8], [27]:

$$e^{\frac{j2\pi\Delta}{\lambda}[(n-m)\sin\theta]}e^{-2\left[\frac{\pi\Delta\sigma_a}{\lambda}\{(n-m)\cos\theta\}\right]^2}. \tag{35}$$

In (33) and (34), $\xi_{s,i}$ and $\xi_{p,t}$ capture the distance-dependent path-loss according to $34.5{+}35\log_{10}(l)$, where l is the distance in meters with $l \geq 35$m, a log-normal shadow fading with 8dB standard deviation and a Rayleigh component for the multi-path fading channel. In (35), $\sigma_a = 2^\circ$ and the antenna spacing at the BS $\Delta = \lambda/2$, where λ is the carrier wavelength. The cell radius of the cognitive BS, the noise power spectral density, the noise figure at each user receiver and antenna gain are assumed to be 1.3km, -174dBm/Hz, 5dB and 15dBi, respectively.

B. Convergence Behavior

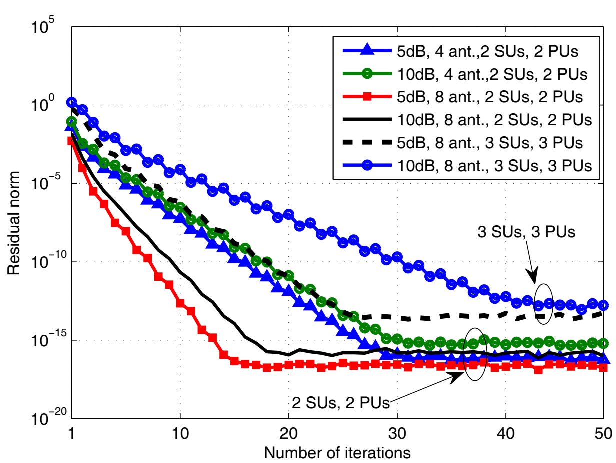

Fig. 3. Convergence behavior of the proposed iterative algorithm.

In Fig. 3, the residual norm of $\parallel \mathbf{p}(n) - \mathbf{p}^\star \parallel$ is plotted versus number of iterations n to show the convergence speed of the proposed iterative algorithm to the optimal solution $\mathbf{p}^\star$. Fig. 3 confirms the statement in Remark 1, i.e., the convergence speed of the proposed algorithm is a monotonic function of number of SUs and required SINR level at SUs. It can be seen from the figure that at the same target SINR and number of antenna, the proposed algorithm converges faster with less number of SUs. On the other hand, with the same number of SUs and the same number of antenna elements, the lower target SINR, the quicker the convergence is.

Results shown in Fig. 3 further reveal that the convergence speed of the proposed algorithm is also a monotonic function of the number of antenna elements. Finally, the figure indicates that the proposed algorithm has a fast convergence speed. With two SUs and two PUs, for instance, the algorithm approaches the optimal solution, with the accuracy of around 10^{-16} after 19 iterations and around 10^{-15} after 31 iterations with 8 and 4 antenna elements, respectively.

C. Comparison on Transmit Power and ICI

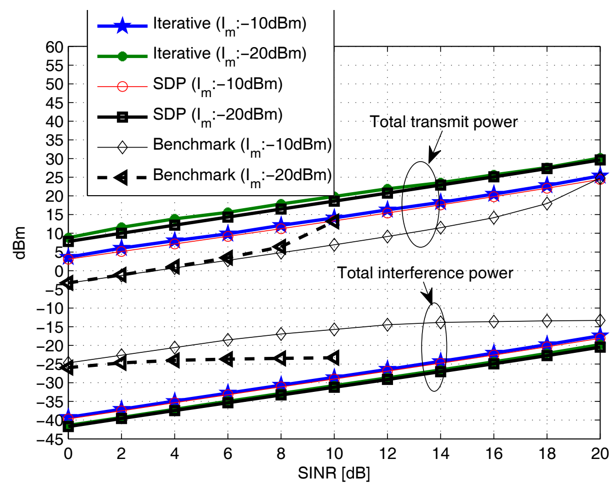

Fig. 4. Total transmit power of the cognitive BS and total interference imposed on PUs versus equal SINR levels at SUs. The power constraint P_m in problem (3) is set to 35dBm. The number of antenna elements at the cognitive BS is 6.

In Fig. 4 illustrates the total transmit power of the cognitive BS and total interference imposed on PUs versus equal SINR levels at SUs for the proposed approach and the benchmark with different interference constraints I_m. In the proposed approach, the power constraint P_m in problem (3) is set to 35dBm. Solution to optimization problem (3) is obtained by the proposed iterative algorithm and CVX [22] for the SDP form in (4).

As it is observed, the proposed algorithm in this paper satisfies all the interference constraints required by the primary system, i.e., -10dBm and -20dBm, as well as the power constraint at the BS. It is further seen that the stricter interference constraint in the primary system, the higher is the required transmit power of the cognitive BS. This is an effect of narrowing down the feasibility region in the optimization (3). Fig. 4 indicates that the solution to the optimization problem (3) obtained by the iterative algorithm is the same as that offered by SDP algorithm. The proposed approach outperforms the benchmark at high required SINR level by SUs and stricter interference threshold given by PUs. For instance, the benchmarking system fails to maintain the interference threshold of -20dBm after the required SINR of 10dB while the proposed scheme effectively works up to 20dB.

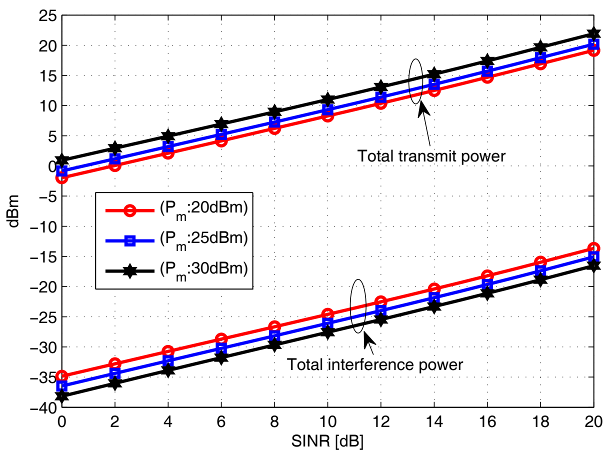

Fig. 5. Total transmit power of the cognitive BS and total interference imposed on PUs versus equal SINR levels at SUs. The interference constraint I_{m} in problem (3) is set to -10dBm. The number of antenna elements at the cognitive BS is 6.

In Fig. 5, the performance, i.e., total transmit power and total interference power, of the proposed approach is shown versus equal SINR levels at SUs with fixed interference constraint $I_{\mathrm{m}} = -10$dBm and various levels of transmit power constraint. The figure indicates that the proposed algorithm forces the total transmit power and total interference power well below the given constraints. This shows the effectiveness of introducing the slack variables α and β in the optimization problem (3). Fig. 5 also shows that the proposed approach imposes lower total interference on PUs when the transmit power constraint increases. This is an effect of enlarging the feasibility region of problem (3).

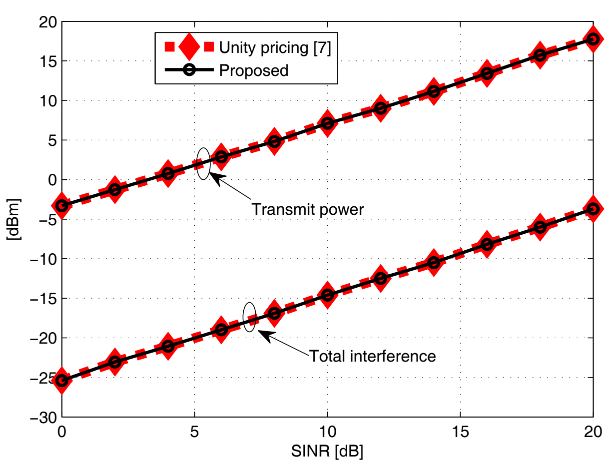

Fig. 6. Total transmit power of the cognitive BS versus equal SINR levels at SUs. The number of antenna elements at the cognitive BS is 6.

Fig. 6 shows the transmit power and total interference power of the proposed scheme and the unity-pricing strategy introduced in [7]. We set $I_{\mathrm{m}} = 0$dBm and $P_{\mathrm{m}} = 0$dBm in problem (3). Fig. 6 indicates that the proposed algorithm provides the same performance as that of the scheme in [7] with unity pricing. This is because of the fact that by setting $I_{\mathrm{m}} = 1$ and $P_{\mathrm{m}} = 1$ in (3), the proposed optimization (3) becomes an epigraph form [19] of the unity-pricing problem introduced in [7].

D. Comparison on Radiation Patterns

In order to have an insight on the interference management ability of the two systems, we investigate their actual radiation patterns. We repeat the experiment described in Example 1 of [16]. In that experiment, there are three SUs located at $-5°$, $10°$ and $25°$ relative to the BS's array broadside. The noise variance is set to 0.1 while the SINR threshold values are set to 1 for SUs. In addition, there are two PUs located at $30°$ and $50°$ relative to the BS's array broadside with their corresponding interference tolerable values of 0.001 (-30dBW) and 0.0001 (-40dBW). We then implement the proposed algorithm in [16] for those PUs and SUs with the total interference threshold level I_{m} of 0.0011 (-29.6dBW). It is worth emphasizing that in our proposed optimization problem, a threshold is put on the total interference imposed on all PUs while in the benchmark, the interference threshold is set for each PU.

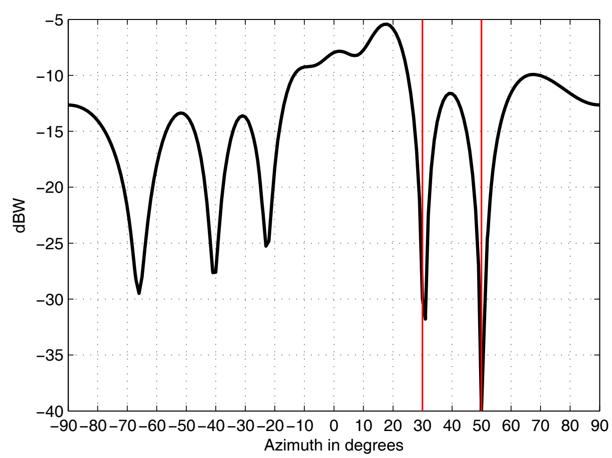

Fig. 7. Reproduction of the radiation pattern of the BS for the benchmark [16, Fig. 3]. The number of antenna elements is 8. The required transmit power is 19.05dBm

Figs. 7 and 8 show the radiation patterns of the BS for the benchmark and proposed scheme, respectively. Comparing Figs. 7 and 8 it is observed that both schemes are capable of shaping interference, i.e., providing nulls, at the angles that those PUs are located. It also can be also seen that the proposed algorithm significantly outperforms the benchmark in terms of controlling interference towards PUs, i.e., around 140dB deeper nulls in comparison with the benchmark are provided by the proposed scheme. The improvement is achieved with the cost of an increase in the transmit power

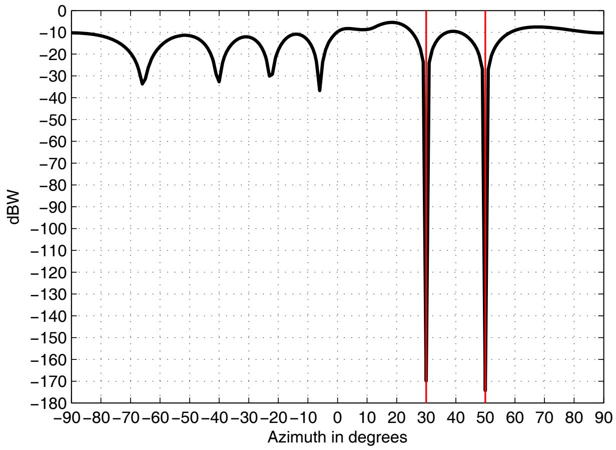

Fig. 8. Radiation pattern of the BS for the proposed iterative algorithm. The number of antenna elements is 8. The required transmit power is 19.81dBm

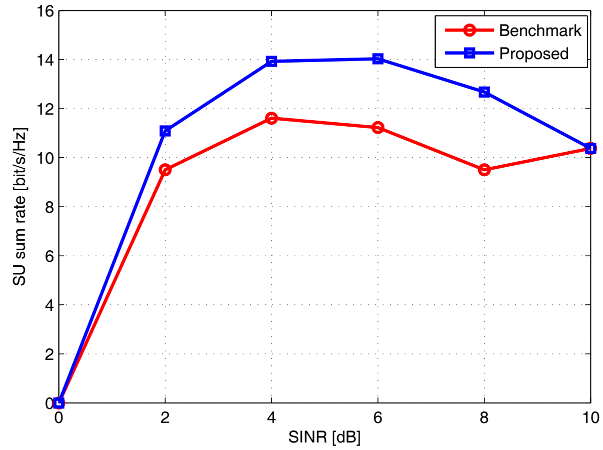

Fig. 9. Sum rate obtained by SUs versus equal SINR levels at SUs. The constraints $I_{\rm m}$ and $P_{\rm m}$ in problem (3) are set to -30dBm and 35dBm, respectively. The number of antenna elements at the cognitive BS is 6.

from 19.05dBm to 19.81dBm. The superior performance of the proposed strategy gainst the benchmark can be explained as follows. First, by putting one constraint on the total interference, the feasibility region of the proposed optimization problem is larger than that of the benchmark. Second, by using slack variable α, the proposed optimization forces the total interference well below the predefined threshold.

E. Comparison on SUs' Sum Rate

In the following, we compare the proposed algorithm against the benchmark in terms of secondary users' sum rate. We need to protect a set of two PUs located at 30° and 50° relative to the cognitive BS's array broadside with the distance of 1.3km to the cognitive BS. In the meantime, we try to serve a set of ten candidate SUs located at -5°, 10°, 25°, 40°, 55°, 70°, -20°, -35°, -50° and -65° relative to the cognitive BS's array broadside. The distance from SUs to the BS is 0.13km. At a given SINR level, we start implementing the proposed and benchmark approach with one SU and keep increasing the number of SUs until the interference threshold is exceeded. The sum rate shown in Fig. 9 is calculated as $U\log 2(1+\text{SINR})$ where U is the number of admitted SUs.

The results shown in Fig. 9 indicates that the proposed algorithm obtains higher sum rate than the benchmark in the SINR range from 2 to 8dB. This is due to the fact that the proposed approach can provide deeper nulls to ward the PUs, hence, it can serve more SUs than its counterpart at a given SINR level. The two approaches offer the same performance at 10dB of SINR since at that point the interference gap between them is not significant, i.e., see Fig. 4. However, it is worth mentioning that the benchmark fails to operate, i.e., maintaining the $I_{\rm m}$ and $P_{\rm m}$ constraints, after 10dB while the proposed approach still works effectively at higher SINR levels.

V. Conclusion

In this paper, we proposed a novel optimization problem to design downlink beamforming vectors for a cognitive cellular network. For the proposed optimization problem, we then derived the corresponding SDP form and developed an iterative algorithm to find the solutions. Simulation results confirmed that the proposed iterative algorithm has a fast convergence speed. The results also indicated that the proposed algorithm guarantees the transmit power and interference constraints. Comparisons against the benchmark approach showed significantly lower interference levels are shaped by the proposed algorithm towards primary users. This advantage leads to the better performance in terms of higher secondary users' sum rate offered by the proposed scheme at the SINR range from 0 to 8 dB. Simulation results revealed that the proposed approach effectively works up to SINR level of 20 dB, required by secondary users, and interference threshold of -20 dBm, set by primary users, while the benchmark fails to do so beyond SINR level of 10 dB.

Appendix A
Proof of Lemma 1

Proof: Using (7), we can rewrite the problem (10) as

$$\begin{aligned}\max_{\eta,\mu}\max_{p_i} \quad & \sum_{i\in\mathcal{S}_s} p_i \\ \text{s. t.} \quad & \mathbf{K}_i \succeq \lambda_i\left(1+\frac{1}{\gamma_i}\right)\mathbf{R}_{s,i},\ \forall i\in\mathcal{S}_s, \\ & \eta\in\mathcal{S}_\eta,\ \mu\in\mathcal{S}_\mu,\end{aligned} \tag{36}$$

where $\mathbf{K}_i = \sum_{j\in\mathcal{S}_s}\lambda_j\mathbf{R}_{s,j} + \eta\sum_{t\in\mathcal{S}_p}\mathbf{R}_{p,t} + \mu\mathbf{I}$. Let $\hat{\mathbf{w}}_i^\star$ be the optimal solution to the left-hand side of the SINR constraints in problem (11). Substituting $\hat{\mathbf{w}}_i^\star$ into the SINR constraints in (11) and rearranging the terms using (12) yields

$$\hat{\mathbf{w}}_i^{\star H}\left(\mathbf{K}_i - \lambda_i\left(1+\frac{1}{\gamma_i}\right)\mathbf{R}_{s,i}\right)\hat{\mathbf{w}}_i^\star \le 0. \tag{37}$$

To obtain (37), we use the fact that $p_i = \lambda_i\sigma^2 > 0$. From (37), we can write

$$\mathbf{K}_i \preceq \lambda_i\left(1+\frac{1}{\gamma_i}\right)\mathbf{R}_{s,i}. \tag{38}$$

Therefore, the problem (11) can be rewritten as

$$\begin{aligned}\max_{\eta,\mu}\min_{p_i} \quad & \sum_{i\in\mathcal{S}_s} p_i \\ \text{s. t.} \quad & \mathbf{K}_i \preceq \lambda_i\left(1+\frac{1}{\gamma_i}\right)\mathbf{R}_{s,i},\ \forall i\in\mathcal{S}_s, \\ & \eta\in\mathcal{S}_\eta,\ \mu\in\mathcal{S}_\mu.\end{aligned} \tag{39}$$

By changing the maximization to minimization in the inner subproblem and reversing the inequality direction of the constraints of the problem (36), we can obtain the problem (39).

Furthermore, it can be verified that the constraints in both problems hold with equality at the optimal solutions. Therefore, (36) and (39) have the same solution. This points to the conclusion that (10) and (11) have the same solution. ∎

Appendix B
Proof of Corollary 1

Proof: Consider the left hand side of the SINR constraint in (11), i.e.,

$$\max_{\|\hat{\mathbf{w}}_i\|=1}\frac{p_i\hat{\mathbf{w}}_i^H\mathbf{R}_{s,i}\hat{\mathbf{w}}_i}{\hat{\mathbf{w}}_i^H\mathbf{B}_i\left(\mathbf{p}^{-i}\right)\hat{\mathbf{w}}_i}.$$

The optimal solution to the problem denoted as $\hat{\mathbf{w}}_i^\star$, is the dominant eigenvector, i.e., the eigenvector associated with the maximum eigenvalue, of matrix $\mathbf{B}_i^{-1}\left(\mathbf{p}^{-i}\right)\mathbf{R}_{s,i}$. We can write

$$\mathbf{B}_i^{-1}\left(\mathbf{p}^{-i}\right)\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^\star = \chi_i\hat{\mathbf{w}}_i^\star, \tag{40}$$

where χ_i is the corresponding dominant eigenvalue.

The gradient of $L(\alpha,\beta,\mathbf{w}_i,\lambda_i,\eta,\mu)$ in (5), i.e., the Lagrangian of the optimization problem (3), with respect to $\mathbf{w}_i$ vanishes at the optimal points $\lambda_i^\star$ and $\mathbf{w}_i^\star$. Therefore, setting the gradient of $L(\alpha,\beta,\mathbf{w}_i^\star,\lambda_i^\star,\eta,\mu)=\mathbf{0}$, using algebra and the fact that $p_i^\star=\lambda_i^\star\sigma^2$, we have

$$\mathbf{B}_i^{-1}\left(\mathbf{p}^{-i}\right)\mathbf{R}_{s,i}\mathbf{w}_i^\star = \frac{p_j^\star}{\gamma_i}\mathbf{w}_i^\star. \tag{41}$$

Comparing (40) and (41) leads to the conclusion stated in Corollary 1. ∎

Appendix C
Proof of Lemma 3

Proof: $\mathbf{d}(\mathbf{p})$ is a standard-interference function because it satisfies the following criteria for all $\mathbf{p}\succcurlyeq 0$:

1. Positivity: Since $\mathbf{R}_{s,i}\succeq 0$ and $\mathbf{B}_i(\mathbf{p}^{-i})$ is positive definite, $\forall i\in\mathcal{S}_l$, it can be verified from (23) that $\mathbf{d}(\mathbf{p})\succ 0$, i.e., all elements of vector $\mathbf{d}(\mathbf{p})$ are non-negative, $\forall\mathbf{p}\succcurlyeq 0$.

2. Monotonicity: If $\mathbf{p}\succcurlyeq\mathbf{p}'$, i.e., element-wise inequality, then, using (23), it can be shown that:

$$d_i(\mathbf{p}^{-i})-d_i(\mathbf{p'}^{-i})=\frac{\sum_{j\in\mathcal{S}_l,j\neq i}\left(p_j-p_j'\right)\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,j}\hat{\mathbf{w}}_i^\star}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^\star}\geq 0,$$

for all $i\in\mathcal{S}_l$. Therefore $\mathbf{d}(\mathbf{p})\succcurlyeq\mathbf{d}(\mathbf{p}')$.

3. Scalability: For all $\delta>1$, let us consider

$$\delta d_i(\mathbf{p}^{-i})=\frac{\sum_{j\in\mathcal{S}_l,j\neq i}\delta p_j\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,j}\hat{\mathbf{w}}_i^\star+\delta\hat{\mathbf{w}}_i^{\star H}\mathbf{C}_i\hat{\mathbf{w}}_i^\star}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^\star} \tag{42}$$

where

$$\mathbf{C}_i=\sigma^2\left(\sum_{t\in\mathcal{S}_p}\eta\mathbf{R}_{p,t}+\mu\mathbf{I}\right),$$

is a positive definite matrix. Since $\delta>1$,

$$\delta\hat{\mathbf{w}}_i^{\star H}\mathbf{C}_i\hat{\mathbf{w}}_i^\star > \hat{\mathbf{w}}_i^{\star H}\mathbf{C}_i\hat{\mathbf{w}}_i^\star. \tag{43}$$

Using (42) and (43), it can be seen that

$$\delta d_i(\mathbf{p}^{-i}) > \frac{\sum_{j\in\mathcal{S}_l,j\neq i}\delta p_j\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,j}\hat{\mathbf{w}}_i^\star+\hat{\mathbf{w}}_i^{\star H}\mathbf{C}_i\hat{\mathbf{w}}_i^\star}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^\star},$$

which implies $\delta d_i(\mathbf{p}^{-i})>d_i(\delta\mathbf{p}^{-i})$, for all $i\in\mathcal{S}_l$. Therefore, $\delta\mathbf{d}(\mathbf{p})\succ\mathbf{d}(\delta\mathbf{p})$, i.e., element-wise inequality. ∎

Appendix D
Proof of Lemma 4

Proof: Using (23), we can write

$$\begin{aligned}\|\,\mathbf{d}(\mathbf{p})-\mathbf{d}(\mathbf{p}')\,\|^2 &= \sum_{i\in\mathcal{S}_s}\left(d_i(\mathbf{p}^{-i})-d_i(\mathbf{p'}^{-i})\right)^2 \\ &= \sum_{i\in\mathcal{S}_s}\left(\gamma_i\sum_{t\in\mathcal{S}_s,t\neq i}(p_t-p_t')\left(\frac{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_i^\star}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^\star}\right)\right)^2.\end{aligned} \tag{44}$$

Applying Cauchy-Schwarz inequality on (44), we get to (45), then using algebra, we obtain (51), details are given at the top of the next page. From (51), we can write

$$\begin{aligned}\|\,\mathbf{d}(\mathbf{p})-\mathbf{d}(\mathbf{p}')\,\|^2 &\leq c^2\sum_{i\in\mathcal{S}_s}(p_i-p_i')^2 \\ &= c^2\,\|\,\mathbf{p}-\mathbf{p}'\,\|^2\end{aligned} \tag{52}$$

where

$$c\triangleq\sqrt{U-1}\max_i\left(\sum_{t\in\mathcal{S}_s,t\neq i}\gamma_t\frac{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_t^\star}{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_t^\star}\right). \tag{53}$$

From (52), we have

$$\|\,\mathbf{d}(\mathbf{p})-\mathbf{d}(\mathbf{p}')\,\|\leq c\,\|\,\mathbf{p}-\mathbf{p}'\,\|. \tag{54}$$

According to [26, Chapter 3], if (54) holds for $c\in[0,1)$, then $\mathbf{d}(.)$ is a contraction mapping and $\mathbf{d}(\mathbf{p})$ has a unique fixed point $\mathbf{p}^\star$. It can be easily verified from (53) that $c=0$ is satisfied for $U=1$, i.e., one user per cell. Furthermore, by setting $c<1$ in (53), one can arrive at (26) in Lemma 4. ∎

Appendix E
Proof of Lemma 6

Proof: Using the same technique to prove Lemma 1, i.e., presented in Appendix A, we transform $f(\eta_0,\mu)$ into the downlink domain as

$$\begin{aligned}f(\eta_0,\mu)=\min_{\mathbf{w}_i} \quad & \mu\sum_{i\in\mathcal{S}_s}\mathbf{w}_i^H\mathbf{w}_i, \\ \text{s. t.} \quad & g(\mathbf{w}_i)\geq\gamma_i, \quad \forall i\in\mathcal{S}_s,\end{aligned} \tag{55}$$

where

$$g(\mathbf{w}_i)\triangleq\frac{\mathbf{w}_i^H\mathbf{R}_{s,i}\mathbf{w}_i}{\sum_{j\in\mathcal{S}_s,j\neq i}\mathbf{w}_j^H\mathbf{R}_{s,i}\mathbf{w}_j+\sigma^2+\eta_0\sigma^2\sum_{t\in\mathcal{S}_p}\mathbf{R}_{p,t}}.$$

$$\begin{aligned}
\| \mathbf{d}(\mathbf{p}) - \mathbf{d}(\mathbf{p}') \|^2 &\leq (U-1) \sum_{i\in\mathcal{S}_s} \gamma_i^2 \left(\sum_{t\in\mathcal{S}_s, t\neq i} (p_t - p_t')^2 \left(\frac{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_i^\star}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^\star} \right)^2 \right) && (45)\\
&= (U-1) \sum_{i\in\mathcal{S}_s} \gamma_i^2 \left(\sum_{t\in\mathcal{S}_s} (p_t - p_t')^2 \left(\frac{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_i^\star}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^\star} \right)^2 - (p_i - p_i')^2 \right) && (46)\\
&= (U-1) \sum_{i\in\mathcal{S}_s} \sum_{t\in\mathcal{S}_s} \gamma_i^2 (p_t - p_t')^2 \left(\frac{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_i^\star}{\hat{\mathbf{w}}_i^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_i^\star} \right)^2 - \sum_{i\in\mathcal{S}_s} \gamma_i^2 (p_i - p_i')^2 && (47)\\
&= (U-1) \sum_{t\in\mathcal{S}_s} \sum_{i\in\mathcal{S}_s} \gamma_t^2 (p_i - p_i')^2 \left(\frac{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_t^\star}{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_t^\star} \right)^2 - \sum_{i\in\mathcal{S}_s} \gamma_i^2 (p_i - p_i')^2 && (48)\\
&= (U-1) \sum_{i\in\mathcal{S}_s} (p_i - p_i')^2 \left(\sum_{t\in\mathcal{S}_s} \gamma_t^2 \left(\frac{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_t^\star}{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_t^\star} \right)^2 - \gamma_i^2 \right) && (49)\\
&= (U-1) \sum_{i\in\mathcal{S}_s} (p_i - p_i')^2 \sum_{t\in\mathcal{S}_s, t\neq i} \gamma_t^2 \left(\frac{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_t^\star}{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_t^\star} \right)^2 && (50)\\
&\leq (U-1) \sum_{i\in\mathcal{S}_s} (p_i - p_i')^2 \left(\sum_{t\in\mathcal{S}_s, t\neq i} \gamma_t \frac{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,i}\hat{\mathbf{w}}_t^\star}{\hat{\mathbf{w}}_t^{\star H}\mathbf{R}_{s,t}\hat{\mathbf{w}}_t^\star} \right)^2 . && (51)
\end{aligned}$$

Let $\mathbf{w}_{i,1}^\star$, and $\mathbf{w}_{i,2}^\star$, respectively, be the optimal beamforming vectors for $f(\eta_0, \mu_1)$ and $f(\eta_0, \mu_2)$, where μ_1 and μ_2 are two positive numbers. Consider

$$\begin{aligned}
f\left(\eta_0, \frac{\mu_1+\mu_2}{2}\right) &= \min_{\{\mathbf{w}_i : g(\mathbf{w}_i)\geq\gamma_i\}} \frac{\mu_1+\mu_2}{2} \sum_{i\in\mathcal{S}_s} \mathbf{w}_i^H\mathbf{w}_i\\
&\geq \frac{1}{2}\mu_1 \sum_{i\in\mathcal{S}_s} \mathbf{w}_{i,1}^{\star H}\mathbf{w}_{i,1}^\star + \frac{1}{2}\mu_2 \sum_{i\in\mathcal{S}_s} \mathbf{w}_{i,2}^{\star H}\mathbf{w}_{i,2}^\star\\
&= \frac{1}{2} f(\eta_0, \mu_1) + \frac{1}{2} f(\eta_0, \mu_2).
\end{aligned}$$

The above inequality confirms that function $f(\eta_0, \mu)$ is concave in μ. Now we consider

$$\begin{aligned}
f(\eta_0, \mu_2) - f(\eta_0, \mu_1) &= \mu_2 \sum_{i\in\mathcal{S}_s} \mathbf{w}_{i,2}^{\star H}\mathbf{w}_{i,2}^\star - \mu_1 \sum_{i\in\mathcal{S}_s} \mathbf{w}_{i,1}^{\star H}\mathbf{w}_{i,1}^\star\\
&\leq \mu_2 \sum_{i\in\mathcal{S}_s} \mathbf{w}_{i,1}^{\star H}\mathbf{w}_{i,1}^\star - \mu_1 \sum_{i\in\mathcal{S}_s} \mathbf{w}_{i,1}^{\star H}\mathbf{w}_{i,1}^\star\\
&= (\mu_2 - \mu_1) \sum_{i\in\mathcal{S}_s} \mathbf{w}_{i,1}^{\star H}\mathbf{w}_{i,1}^\star.
\end{aligned}$$

Therefore,

$$f(\eta_0, \mu_2) \leq f(\eta_0, \mu_1) + (\mu_2 - \mu_1) \sum_{i\in\mathcal{S}_s} \mathbf{w}_{i,1}^{\star H}\mathbf{w}_{i,1}^\star.$$

This fact points to a conclusion that $\sum_{i\in\mathcal{S}_s} \mathbf{w}_i^H\mathbf{w}_i$ is a subgradient of $f(\eta_0, \mu)$.

Following the same line of arguments, the second statement of the lemma can be proven. ■

REFERENCES

[1] J. Mitola, "Cognitive radio: An intergrated agent architecture for software defined radio," Ph.D. dissertation, Royal Institute of Technology (KTH), Stockholm, Sweden, Dec. 2000.

[2] M. G. Khoshkholgh, K. Navaie, and H. Yanikomeroglu, "Access strategies for spectrum sharing in fading environment: Overlay, underlay and mixed," *IEEE Trans. Mobile Computing*, vol. 9, no. 12, pp. 1780–1793, Dec. 2010.

[3] F. Rashid-Farrokhi, K. J. R. Liu, and L. Tassiulas, "Transmit beamforming and power control for cellular wireless systems," *IEEE JSAC*, vol. 16, no. 8, pp. 1437– 1450, Oct. 1998.

[4] A. B. Gershman, N. D. Sidiropoulos, Shahhazpanahi, M. Bengtsson, and B. Ottersten, "Convex optimization-based beamforming: From receive to transmit and network designs," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 62–75, May 2010.

[5] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1748–1759, May 2010.

[6] F. Gross, *Smart Antennas for Wireless Communications*. McGraw-Hill, 2005.

[7] T. A. Le and M. R. Nakhai, "Downlink optimization with interference pricing and statistical CSI," *IEEE Trans. Commun.*, vol. 61, no. 6, pp. 2339–2349, Jun 2013.

[8] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using Semidefinite optimization," in *Proc. 37th Annu. Allerton Conf. Commun., Control, and Computing*, 1999, pp. 987 – 996.

[9] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via Conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161– 176, Jan. 2006.

[10] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Transactions on Signal Processing*, vol. 51, no. 9, pp. 2381–2401, Sep. 2003.

[11] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18–28, Jan. 2004.

[12] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2646–2660, Jun. 2007.

[13] S. Yiu, M. Vu, and V. Tarokh, "Interference reduction by beamforming in cognitive networks," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM 2008)*, Dec. 2008, pp. 1–6.

[14] Y. Huang, Q. Li, W.-K. Ma, and S. Zhang, "Robust multicast beamforming for spectrum sharing-based cognitive radios," *IEEE Trans. Signal Processing*, vol. 60, no. 1, pp. 527– 533, Jan. 2012.

[15] I. Wajid, M. Pesavento, Y. C. Eldar, and D. Ciochina, "Robust downlink beamforming with partial channel state information for conventional and cognitive radio networks," *IEEE Trans. Signal Processing*, vol. 61, no. 14, pp. 3656– 3670, Jul. 2013.

[16] Y. Huang and D. P. Palomar, "Rank-constrained separable Semidefinite programming with applications to optimal beamforming," *IEEE Trans. Signal Processing*, vol. 58, no. 2, pp. 644–678, Feb. 2010.

[17] G. Zheng, K.-K. Wong, and B. Ottersten, "Robust cognitive beamforming with bounded channel uncertainties," *IEEE Trans. Signal Processing*, vol. 57, no. 12, pp. 4871– 4881, Dec. 2009.

[18] L. Zhang, Y.-C. Liang, Y. Xin, and H. V. Poor, "Robust cognitive beamforming with partial channel state information," *IEEE Trans. Wireless Communications*, vol. 8, no. 8, pp. 4143– 4153, Aug. 2009.

[19] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[20] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE JSAC*, vol. 13, no. 7, pp. 1341–1348, Sep. 1995.

[21] N. Z. Shor, *Minimization Methods for Non-Differentiable Functions*. Springer-Verlag, 1985.

[22] M. Grant and S. Boyd, *cvx Users' Guide for cvx version 1.2 (build 711)*, 2009, software availabe at http://www.stanford.edu/∼boyd/cvx/download.html.

[23] H. Dahrouj and W. Yu, "Multicell interference mitigation with joint beamforming and common message decoding," *IEEE Trans. Commun.*, vol. 59, no. 8, pp. 2264–2273, Aug. 2011.

[24] A. Berman and R. Plemmons, *Nonnegative Matrices in Mathematical Sciences, 2nd ed.* SIAM Classics in Applied Mathematics, 1994.

[25] R. Cottle, J. S. Pang, and R. Stone, *The Linear Complimentarity Problem*. Society for Industrial and Applied Mathematics, 1992.

[26] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation*. Prentice-Hall, 1989.

[27] T. A. Le and M. R. Nakhai, "An iterative algorithm for downlink multi-cell beam-forming," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM 2011)*, Dec. 2011, pp. 1–6.