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Optimal Design of the Spectrum Sensing Parameters in the Overlay Spectrum Sharing

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Abstract—In this paper, a novel approach is proposed to obtain the optimal operating point of spectrum sensing in overlay spectrum sharing systems. The objective is to maximize the secondary service achievable capacity subject to the primary service collision probability as well as the other system and service constraints. In the related literature the miss detection probability, as the main reason of collision, is often considered to model the impact of spectrum sensing on the achievable ergodic capacity of the secondary service. In this paper however we directly consider the collision probability constraint in finding the optimal ergodic capacity instead of considering the miss detection probability. We then propose a framework in which other opportunities which lie in the wireless channel fluctuation and power allocation are also extracted in favor of achieved capacity. In addition to the conventional One-Shot (O-S) scheme, we also propose four novel approaches to solve the optimization problem: *Modified-One-Shot (M-O-S) scheme*, *Multi-Shot (M-S) scheme*, *Conservative-Modified-One-Shot (C-O-S) scheme*, and *Restricted-Modified-One-Shot (R-O-S) scheme*. Our studies show that the proposed formulation results in a higher secondary service capacity even when compared to the cases with very low miss detection probability. In the proposed schemes in this paper, the main decision parameter is the average (over fading) received interference at the secondary service receiver due to the primary service transmission, I, which can be simply measurable in the secondary transmitter. Extensive numerical studies are conducted to investigate various system aspects. Our studies further suggest that for very low, moderate, and very high values of I, the proper schemes are C-O-S, M-S, and M-O-S, respectively.

Index Terms—Ergodic capacity, inaccurate spectrum sensing, overlay spectrum access, ROC, spectrum sharing.

1 Introduction

In spectrum sharing, the secondary service, opportunistically detects and uses under-utilized parts of the primary service's spectrum [1]. The under-utilized spectrum is referred to as spectrum holes or white spaces. The under-utilized portions of the spectrum in a specific time and location can be considered for secondary service access [2].

The secondary service may access to the under utilized spectrum adopting a variety of access strategies which are generally categorized into *overlay*, *underlay* and combined overlay underlay spectrum sharing, see, e.g., [3], [4]. In this paper, we focus on the overlay spectrum sharing. The performance of the overlay spectrum sharing is closely related to the sensing mechanism for detecting and monitoring the white spaces [5]. The spectrum sensing evaluates the spectrum status as *idle* or *busy*. The secondary service then starts/continues transmission in the idle states, and stops in the busy states.

Various spectrum sensing techniques are proposed in the literature (see, e.g., [5] and [6]). Among them here we focus on the spectrum sensing techniques based on energy detectors. However, the presented results in this paper can simply be extended to the other sensing techniques. Energy detector based spectrum sensing is the most popular technique due to its implementation simplicity, see, e.g., [7].

In practice, spectrum sensing is not always perfect and accurate. The performance of spectrum sensing is usually modelled through the following two parameter: *false alarm* and *miss detection* probabilities. False alarm referred to the case when the spectrum sensing evaluates the spectrum status as busy while it is actually idle. Similarly, miss detection referred to the case where the spectrum status evaluated as idle while it is in fact busy. In practice these two performance parameters are related through the receiver operating characteristic (ROC) curve. The ROC is the fundamental characteristic of a spectrum sensing mechanism [3]. A false alarm, if happens, may result in a degrading spectral efficiency since the available idle spectrum is not utilized by the secondary service. A miss detection may also result in collision between the primary and secondary service transmissions since the spectrum status is mistakenly evaluated as idle. Collisions degrade the primary service Quality-of-Service (QoS). Therefore, the spectrum sensing parameters should be selected in such a way that the QoS requirements of the primary services are also satisfied.

In spectrum sharing, the main design objective is to maximize the achievable capacity of the secondary service subject to the primary service QoS as well as other system and service constraints. In this paper, the QoS of the primary user is represented by a pair of parameters, (Q, ξ), where Q is the interference threshold constraint and ξ is the maximum allowable collision probability [3]. The interference threshold is the maximum allowed interference level imposed by the secondary service at the primary service receiver. Hereafter, we refer to (Q, ξ) as the *collision probability constraint*. A collision is experienced if the secondary service transmission imposes an interference level higher than Q at the primary service receiver. In fact, the availability of the spectrum is subject to satisfying the collision probability constraint for the primary service.

In the overlay spectrum sharing, one may consider reducing the miss detection probability in order to avoid collision. It is also assumed that the collision is experienced in the case of secondary transmission in miss detected spectrum. Therefore,

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by reducing the miss detection probability and keeping it below ξ, one can satisfy the collision probability constraint for the primary service, see, e.g., [8], [9], [10], and [11].

In the related literature, the aforementioned optimization problem is often solved in two disjoint steps. In the first step, spectrum sensing parameters are obtained, followed by the power allocation implementation in the next step. Spectrum sensing parameters are obtained based on the assumption made on the correspondence of the miss detection and the collision. This approach is widely utilized in the literature, e.g., [8], [11], [12]. We refer to this approach as the *One-Shot (O-S)* scheme which is analyzed in our previous work [4].

In practice however, a miss detection does not always result in a collision. For instance, in cases where the imposed interference due to the secondary transmission on a miss detected spectrum at the primary receiver is sufficiently low, a miss detection does not result in a collision. This can happen due to a deeply faded channel between the secondary transmitter and the primary receiver. This means that improving the sensing performance by reducing miss detection and false alarm probabilities does not necessarily result in higher achievable capacity. As a matter of fact, in addition to the spectrum sensing performance, experiencing collision in the primary receiver depends on the status of the channel fading between the secondary service transmitter and the primary service receiver as well as the secondary service power allocation. Failure to take into account the above facts can be considered as the main disadvantage of the O-S scheme.

To consider the impact of fading of the channel between the secondary service transmitter and the primary service receiver, and the secondary service power allocation, in this paper, we directly utilize the primary service collision probability constraint as a metric for spectrum availability. In other words, the spectrum is deemed available if the collision probability of the primary service is not increased due to the secondary access. Therefore, the collision probability provides a more comprehensive perspective when compared to the miss detection probability. Considering the collision probability constraint, we expect a higher secondary service achievable capacity since we would be able to exploit wireless fading as well as the secondary service power allocation.

1.1 Contribution of The Paper

In this paper, we consider the collision probability constraint at the primary receiver as the spectrum availability criterion. The main design objective is to maximize achievable ergodic capacity of the secondary service subject to the primary service collision probability and other system and service constraints. Utilizing this optimization problem the optimal ROC operating point as well as the power allocation strategy are obtained. Since the aforementioned optimization problem is usually solved in the secondary service, we develop our formulation primarily based on the average (over fading) received interference in the secondary receiver originated from the primary network, which can be easily measured at the secondary service receiver.

Finding exact solution of the aforementioned optimization problem is rather complicated due to the probabilistic constraint. Here we propose four different approaches to solve the optimization problem: *Modified-One-Shot (M-O-S) scheme*, *Multi-Shot (M-S) scheme*, *Conservative-Modified-One-Shot (C-O-S) scheme*, and *Restricted-Modified-One-Shot (R-O-S) scheme*. The main theme of the proposed schemes in this paper is to convert the collision probability constraint into a combination of the secondary service power allocation, and designing the spectrum sensing parameters, i.e., the miss detection probability.

We then investigate the performance of the proposed schemes through numerical studies. It is seen that although the O-S scheme performs well for moderate values of the average (over fading) interference at the primary receiver, I, its corresponding achievable capacity is significantly degraded in cases where I is either very low or very high. It is also seen that the proposed M-S scheme quickly converges and the M-O-S scheme overcomes the fast capacity decreasing slope which is seen in high values of I in the O-S scheme. The M-S, C-O-S, and R-O-S schemes also improve the secondary service capacity for small values of I. The C-O-S scheme suggests a peak power allocation constraint for the secondary service transmission power where it is adaptively changed through the collision probability constraint. In the contrary, the R-O-S provides an average secondary service transmission power constraint corresponding to the original collision probability constraint. In other words, our studies indicate the following rule of thumb for the appropriate scheme based on the value of I: For very low, moderate and very high I, the proper schemes are C-O-S, M-S, and M-O-S, respectively.

1.2 Related Works

Capacity of the secondary service in overlay spectrum sharing has been widely studied, see, e.g., [9], [11], [13], [14], and [15]. In [9] an infrastructure based secondary service is considered where the base station gathers the sensing information and makes decision on the spectrum availability. However, they have assumed a fixed rate for each secondary user, i.e., the power allocation had been simply ignored. Furthermore, the achievable rate due to transmission over the miss detected spectrum has also been ignored. In this paper, we show that taking into account the power allocation as well as channel fading during the miss detection, improves the capacity performance. The improvement in capacity performance in cases with low received interference I, is rather significant.

Investigating the fundamental sensing-throughput tradeoff is the subject of [11] in which through an optimization problem the suitable sensing time for maximizing throughput of the secondary service is obtained. However, in [11] the wireless channel fading is not taken into account. Furthermore, in that paper the collision probability constraint is simply substituted with the miss detection constraint. Here, we consider channel fading through the collision probability constraint.

Sensing-based spectrum sharing was proposed in [13] and [15] for maximizing the ergodic capacity of the secondary service. In [13] and [15] a priori knowledge of the channel between the secondary service transmitter and the primary service receiver is assumed although it may not be easily applicable to the overlay strategy in practice.

For maximizing the secondary service ergodic capacity, [14] proposes a dynamic programming approach subject to the miss detection constraint. However, this approach is particularly designed for specific spectrum sensing parameters.

Note that there is a trend in both research community and industry to improve the accuracy of the spectrum sensing mechanisms (e.g., [5] and [6] and references therein). However, the results presented in this paper provide instances in which by adopting appropriate schemes, one can push the system performance far beyond a system with accurate spectrum sensing. In the other words, if the corresponding interference created due to inaccurate sensing at the primary receiver is tolerable, then allocating larger sensing time to improve the sensing accuracy may degrade the achievable secondary service capacity without providing higher level of protection for primary system QoS. In practice this might happen because of time varying nature of the wireless channel between the secondary transmitter and primary receiver (cross channel). Utilizing collision probability constraint, C-O-S scheme as proposed in this paper exploits such cases, thus improves the achievable system capacity far beyond a spectrum sharing system with accurate spectrum sensing.

Comparing and contrasting the Bayesian- and capacity-based spectrum sensing procedure are the subject of the study conducted in [10]. The main objective of [10] is to maximize a weighted sum ergodic capacity of the primary and secondary services to obtain the best sensing threshold level for spectrum sensing procedure. They also investigated the impact of the location information on the network performance. The obtained results indicates that capacity-based spectrum sensing is more appropriate due to the corresponding larger capacity performance. According to the results in [10], in this paper we also consider the capacity-based spectrum sensing, however our study focuses on the secondary service capacity. Furthermore, we are interested in understanding the impact of the collision probability constraint on the capacity of the secondary service.

1.3 Organization of the Paper

The rest of this paper is organized as follows. In Section 2, the general assumptions and the system model are presented. The spectrum sensing is the subject of Section 3. Impact of the spectrum sensing parameters on the ergodic capacity of the secondary service is considered in Section 4. In Section 4, we also present sub-optimal schemes to obtain the proper ROC operating point as well as the power allocation. In Section 5, we present the numerical studies. Finally, concluding remarks are presented in Section 6.

2 System Model

Consider a spectrum sharing system with a primary and a secondary transceiver denoted by Tx_p and Rx_p, and, Tx_s and Rx_s, respectively (see, Fig. 1). The maximum Tx_s average transmit power is $\overline{P}_s$. In each time instant n, $g_{sp}[n]$ and $g_{ss}[n]$ denote the instantaneous channel power gains from Tx_s to Rx_p, and Rx_s, respectively. Similarly, we define $g_{ps}[n]$ and $g_{pp}[n]$ as the instantaneous channel power gains from Tx_p to Rx_s, and Rx_p, respectively. Therefore, the received signals at Rx_s and Rx_p are represented as

$$Y_s[n] = \sqrt{g_{ss}[n]} X_s[n] \mathbf{1}_s[n] + \sqrt{g_{ps}[n]} X_p[n] \mathbf{1}_p[n] + Z_s[n], \quad (1)$$

$$Y_p[n] = \sqrt{g_{sp}[n]} X_s[n] \mathbf{1}_s[n] + \sqrt{g_{pp}[n]} X_p[n] \mathbf{1}_p[n] + Z_p[n], \quad (2)$$

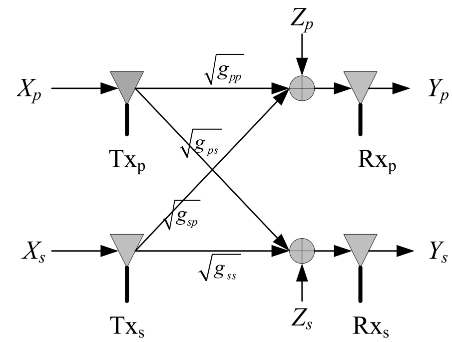

Fig. 1. The schematic of the spectrum sharing system [4].

where $X_s[n]$ and $X_p[n]$ are the transmitted signals from Tx_s and Tx_p at time n, respectively. Here, for the sake of simplicity, we assume that $X_p[n]$ ($X_s[n]$) is always on and it represents the actual transmitted signal of the primary (secondary) transmitter only in case that $\mathbf{1}_p[n] = 1 (\mathbf{1}_s[n] = 1)$. In (1) and (2), $Z_s[n]$ and $Z_p[n]$ are additive white Gaussian noise at Rx_s and Rx_p with variances σ_s^2, and σ_p^2, respectively.

We further assume that $\sigma_s^2 = N_0 B$, where B is the bandwidth of primary spectrum. In (1) and (2), $\mathbf{1}_s[n]$, and $\mathbf{1}_p[n]$ are indicators which demonstrate the activity of the secondary and primary services in time instant n, respectively. Based on this notation, if $\mathbf{1}_p[n] = 1$ ($\mathbf{1}_p[n] = 0$), the primary service is active (inactive) or the spectrum is busy (idle). Equivalently, if $\mathbf{1}_s[n] = 1$ ($\mathbf{1}_s[n] = 0$), the secondary service is active (inactive). However, in opportunistic spectrum access (OSA) $\mathbf{1}_s[n] = 1$, if and only if $\mathbf{1}_p[n] = 0$, otherwise, $\mathbf{1}_s[n] = 0$. We assume that $\{\mathbf{1}_p[n]\}_{n=1}^{\infty}$ is a stationary and ergodic random process. This on-off model has been successfully considered in the related literature, see, e.g., [16] and [4]. We also denote $\mathbf{P}\{\mathbf{1}_p = 1\} \triangleq p_b$, thus $\mathbf{P}\{\mathbf{1}_p = 0\} = 1 - p_b \triangleq p_i$.

Channel power gains $g_{ss}[n]$, $g_{sp}[n]$, $g_{pp}[n]$, and $g_{ps}[n]$ are assumed to be stationary and ergodic independent random processes with probability density (distribution) functions $f_{ss}(g_{ss})$ ($F_{ss}(g_{ss})$), $f_{sp}(g_{sp})$ ($F_{sp}(g_{sp})$), $f_{pp}(g_{pp})$ ($F_{pp}(g_{pp})$), and $f_{ps}(g_{ps})$ ($F_{ps}(g_{ps})$), respectively. For brevity of expositions, hereafter, the time index n is dropped.

3 Spectrum Sensing

In overlay spectrum sharing, the *spectrum sensing* estimates the status of the spectrum, i.e., the value of $\mathbf{1}_p$. The spectrum status is usually estimated based on the corresponding received power from the primary service at the sensors' locations. In this paper, we focus on cases where the spectrum sensing is implemented at the secondary service receiver. This assumption suits the downlink communication. Note that, in the related literature authors usually assume that the sensing procedure is implemented either in the secondary transmitter or in both secondary transmitter and receiver, see, e.g., [5].

Note that the objective of this paper is to investigate the impact of the primary service transmission on the secondary service performance. As far as it is related to the objective of this paper, assuming that the spectrum sensing is implemented at the receiver not only makes no difference in the results but also makes our analysis more tractable. This assumption has been successfully adopted in [9] as well as in our previous work in [4]. One may suggest that implementing spectrum sensing in Tx_s is more appropriate for protecting primary service against interference. For instance, consider a scenario in which Tx_s is close to Rx_p but Rx_s is far from Tx_p. In such cases, if the only considered protection mechanism for the primary system is the spectrum sensing, then sensing at Rx_s might result in an inaccurate sensing which may in return compromise the QoS of the primary system. In our proposed model however, the location of sensing does not make a difference as we additionally satisfy collision probability constraint. This introduces a new degree of freedom by which we develop joint power allocation and spectrum sensing. Intuitively, this approach is able to deal with an inaccurate sensing by opportunistic secondary service power allocation through exploiting the cross channel fluctuations. Interestingly, in this approach we only need $f_{sp}(g_{sp})$, and the instantaneous cross channel power gains are not required. $f_{sp}(g_{sp})$ can be easily obtained in many cases. Here we investigate this approach and show that using collision probability constraint enables us to provide robust primary service protection and achieve a higher capacity at the same time.

We further assume that energy detectors are utilized in the spectrum sensing. This approach is widely adopted in the related literature, see, e.g. [9] and references therein.

The performance of the spectrum sensing is evaluated by two important parameters, probability of false alarm, ϵ, and probability of miss detection, δ [3]. A false alarm is experienced in cases where the spectrum is actually idle, but the spectrum sensing mistakenly assesses the spectrum status as busy; therefore, $P\{\mathbf{1}_s = 0|\mathbf{1}_p = 0\} \triangleq \epsilon$. Miss detection occurs in cases where the spectrum is busy and it is mistakenly assessed as idle; therefore, $P\{\mathbf{1}_s = 1|\mathbf{1}_p = 1\} \triangleq \delta$.

In practice, however, the false alarm and miss detection probabilities are related to each other through the ROC curve, which is a fundamental attribute of each spectrum sensing procedure [3]. The ROC curve usually presents the detection probability, i.e., $1-\delta$, versus ϵ. In other words, for a given value of δ, utilizing ROC curve, the value of ϵ is uniquely obtained.

In case of miss detection, the transmission made by Tx_s imposes unexpected interference at the primary receiver Rx_p (see, Fig. 1). Such unexpected interference may result in QoS degradation in the primary service. Here, the primary service QoS constraint is represented by (Q, ξ), where Q is the maximum tolerable interference at Rx_p, i.e., *interference threshold*, and ξ is the maximum tolerable probability of collision at the primary service receiver, i.e., the *maximum allowable collision probability*. A collision is experienced by the primary receiver, if its received interference exceeds Q. Hereafter, (Q, ξ) is referred to as the *collision probability constraint*.

Assume that the spectrum sensing is conducted in a time slotted fashion with frame duration of T. The spectrum sensing is then performed within the observation interval with duration $0 \leq \tau \ll T$. Therefore, the secondary service can make transmission, if it is allowed, during the rest of the frame, i.e., $T - \tau$ seconds.

Assume that I is the average (over fading) received interference at the Rx_s by the primary service transmission, i.e., $I \triangleq \mathbf{E}_{\mathbf{g}_p}[g_{ps}P_p]$. The effective interference for spectrum sensing is in fact equal to the product of I and the probability of sensing the channel while it is in busy state

$$I_{sense} = \mathbf{P}\{\mathbf{1}_p = 1\}I = p_b I. \tag{3}$$

Generally, the ROC curve is presented by $1 - \delta = \mathbf{ROC}(\epsilon, \tau, I)$, where $\mathbf{ROC}(., \tau, I)$ returns the detection probability based on the false alarm probability for given values of τ and I. For instance, in energy detector based spectrum sensing, it is shown that [13]:

$$\mathbf{ROC}(\epsilon, \tau, I) = \mathbf{Q}\left(\left(\frac{\mathbf{Q}^{-1}(\epsilon)}{\sqrt{\tau\omega_s/2\pi} - \frac{I_{sense}}{N_0 B}}\right)\sqrt{\frac{\tau\omega_s/2\pi}{\frac{I_{sense}}{N_0 B}+1}}\right),$$

where $\omega_s/2\pi$ is the sampling rate and $\mathbf{Q}(x) \triangleq 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2}dt$[1].

Remark 1: Using $\mathbf{ROC}(\epsilon, \tau, I)$ we can obtain τ as

$$\tau = \frac{2\pi}{\omega_s}\left(\frac{\frac{I_{sense}}{N_0 B}\mathbf{Q}^{-1}(1-\delta)}{\mathbf{Q}^{-1}(1-\delta) - \frac{\mathbf{Q}^{-1}(\epsilon)}{\sqrt{\frac{I_{sense}}{N_0 B}+1}}}\right)^2.$$

Here we assume that τ is smaller than coherence time of the channel. In the cases where the τ is required to be larger than the channel coherence time, the sensing output is getting inaccurate. Note that in this paper we present methods in which the impact of spectrum sensing inaccuracy on the primary system performance can be managed in an optimization framework by adjusting the power allocation in the secondary service. The cost, of course, would be a decrease in the system throughput, see Section 4.

It is worth mentioning that given a bound on the miss detection probability, i.e., miss detection probability constraint, ζ, the false alarm probability is obtained using ROC curve. Usually it is assumed that the miss detection probability constraint is equivalent to the collision probability constraint, ξ. However, these two constraints are conceptually different.

In fact, miss detection probability is a physical layer parameter which is evaluated at Rx_s; however, collision probability is evaluated at Rx_p and depends on the secondary service resource allocation policies, e.g., power allocation.

Our treatment of the subject is to consider a combination of these two constraints in an optimization problem. Specifically, here we consider maximizing ergodic capacity of the secondary service as the optimization objective to bridge between the miss detection and collision probability constraints.

The secondary service average channel utilization factor, $\mathbf{U}\{\mathbf{1}_s = 1\}$, is obtained as

$$\begin{aligned}\mathbf{U}\{\mathbf{1}_s = 1\} &= \frac{T-\tau}{T}\mathbf{P}\{\mathbf{1}_p = 1\}\mathbf{P}\{\mathbf{1}_s = 1|\mathbf{1}_p = 1\} \\ &+ \frac{T-\tau}{T}\mathbf{P}\{\mathbf{1}_p = 0\}\mathbf{P}\{\mathbf{1}_s = 1|\mathbf{1}_p = 0\} \\ &= \frac{T-\tau}{T}\left((1-\epsilon)p_i + \delta p_b\right) \triangleq p_{i,\delta}. \end{aligned} \tag{4}$$

Similarly, one can show

$$\mathbf{U}\{\mathbf{1}_s = 0\} = \frac{\tau}{T} + \frac{T-\tau}{T}\left(\epsilon p_i + (1-\delta)p_b\right).$$

1. In the case of multiple secondary transceivers, there would be usually a medium access control (MAC) mechanism in place that manages the access and aligns the sensing time among the secondary nodes. A MAC mechanism implemented in the secondary network, the secondary nodes stay silent during sensing time; thus the energy detector only receives primary signals

In the following, we investigate the impact of the miss detection and false alarm probabilities on the secondary service achievable capacity.

4 Secondary Service Achievable Capacity

The capacity of the secondary service is

$$\begin{aligned} C_s(p_i) &= \frac{T-\tau}{T}(1-\epsilon)p_i B\mathbf{E}_{\mathbf{g}}\left[\log\left(1+\frac{g_{ss}P_s}{N_0B}\right)\right] \\ &+\frac{T-\tau}{T}\delta p_b B\mathbf{E}_{\mathbf{g}}\left[\log\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta p_b g_{ps}P_p}\right)\right] \\ &=\frac{T-\tau}{T}B\mathbf{E}_{\mathbf{g}}\left[\log\left(\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta p_b g_{ps}P_p}\right)^{\delta p_b}\times\right.\right. \\ &\left.\left.\left(1+\frac{g_{ss}P_s}{N_0B}\right)^{(1-\epsilon)p_i}\right)\right], \end{aligned} \tag{5}$$

where $\mathbf{g} \triangleq (\underbrace{g_{ss}, g_{sp}}_{\mathbf{g}_s}, \underbrace{g_{ps}, g_{pp}}_{\mathbf{g}_p})$, and $\mathbf{E}_x$ represents the expectation with respect to the random variable x. Here, P_s is the transmission power of Tx_s which is generally a function of $\mathbf{g}$. Similarly, P_p is the transmission power of Tx_p. A lower bound on the above expression is

$$\begin{aligned} &C_s(p_i) \\ &\geq \frac{T-\tau}{T}B\mathbf{E}_{\mathbf{g}}\left[\log\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta p_b g_{ps}P_p}\right)^{(1-\epsilon)p_i}\times\right. \\ &\left.\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta p_b g_{ps}P_p}\right)^{\delta p_b}\right] \\ &=\frac{T-\tau}{T}B\mathbf{E}_{\mathbf{g}}\left[\log\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta p_b g_{ps}P_p}\right)^{(1-\epsilon)p_i+\delta p_b}\right], \end{aligned}$$

where utilizing (4), it can be further reduced to

$$C_s(p_i) \geq p_{i,\delta}B\mathbf{E}_{\mathbf{g}}\left[\log\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta p_b g_{ps}P_p}\right)\right]. \tag{6}$$

In the spectrum sharing, the primary service adjusts its transmission power only based on its own channel power gain, g_{pp}. Note that for $a \geq 0$, $b \geq 0$, and $x \geq 0$, $h(x) = \log(1+\frac{a}{b+x})$ is a convex function. Therefore, considering the independency of the channel power gains and employing Jensen's inequality [17] on $C_s(p_i)$ in (6), it is seen that

$$C_s(p_i) \quad \geq \quad \mathbf{E}_{\mathbf{g}_s}\left[p_{i,\delta}B\log\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta p_b I}\right)\right], \tag{7}$$

where $I \triangleq \mathbf{E}_{\mathbf{g}_p}[g_{ps}P_p]$.

The secondary service accesses to the spectrum using OSA. In practice the secondary service does not know the channel power gain g_{sp}. Therefore, it is reasonable to assume that the Tx_s allocates power only based on the channel power gain g_{ss}. Thus, $C_s(p_i, I)$ is written as

$$C_s(p_i, I) \triangleq \mathbf{E}_{g_{ss}}\left[p_{i,\delta}B\log\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta p_b I}\right)\right]. \tag{8}$$

In this paper, our main objective is to maximize $C_s(p_i, I)$[2]. Due to the miss detected spectrum, the secondary service imposes interference at the Rx_p, and consequently a collision incident may occur. The collision probability is defined as

$$\begin{aligned} p_{col} &= \frac{T-\tau}{T}\mathbf{P}\{\mathbf{1}_s=1|\mathbf{1}_p=1\}\mathbf{P}\{g_{sp}P_s > Q\} \\ &= \frac{T-\tau}{T}\delta p_b\mathbf{P}\{g_{sp}P_s > Q\}. \end{aligned} \tag{9}$$

Here we show that actual primary system performance, indicated by the outage probability, is related to collision probability. The primary service outage probability is

$$\begin{aligned} p_{out} &= \mathbf{P}\left\{\frac{g_{pp}P_p}{N_0B+\frac{T-\tau}{T}\delta g_{sp}P_s} < \gamma|\mathbf{1}_p=1\right\} \\ &\overset{1}{=} 1-e^{-\frac{\gamma\mu_{pp}N_0B}{P_p}}\mathbf{E}_{g_{ss}}e^{-\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}g_{sp}P_s}{P_p}} \\ &\overset{2}{=} 1-e^{-\frac{\gamma\mu_{pp}N_0B}{P_p}}\int_{t>0}\mathbf{P}\left\{e^{-\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}g_{sp}P_s}{P_p}} > t\right\}dt \\ &= 1-e^{-\frac{\gamma\mu_{pp}N_0B}{P_p}}\int_{0<t<1}\mathbf{P}\left\{g_{sp}P_s < \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}\right\}dt \\ &= 1-e^{-\frac{\gamma\mu_{pp}N_0B}{P_p}} \\ &+e^{-\frac{\gamma\mu_{pp}N_0B}{P_p}}\int_{0<t<1}\mathbf{P}\left\{g_{sp}P_s > \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}\right\}dt \\ &= 1-e^{-\frac{\gamma\mu_{pp}N_0B}{P_p}} \\ &\overset{3}{+} e^{-\frac{\gamma\mu_{pp}N_0B}{P_p}}\int_{t\in\mathcal{Q}_-}\mathbf{P}\left\{g_{sp}P_s > \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}\right\}dt \\ &+e^{-\frac{\gamma\mu_{pp}N_0B}{P_p}}\int_{t\in\mathcal{Q}_+}\mathbf{P}\left\{g_{sp}P_s > \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}\right\}dt, \end{aligned}$$

where in (1) we assume that P_p is not a function of channel power gains g_{pp} and g_{sp}, (2) we use $\mathbf{E}_X[X] = \int_{t>0}\mathbf{P}(X > t)dt$, and in (3) we define set $\mathcal{Q}_-$ as $(0, e^{-\frac{T-\tau}{T}\delta\mu_{pp}Q}]$, in which if $t \in \mathcal{Q}_-$ then $Q \leq \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}$. We also define $\mathcal{Q}_+ = \overline{\mathcal{Q}_-}$ [3]. Thus

2. This is due to the fact that obtaining $C_s(p_i)$ in (6), the value of g_{ps} is required. Estimating g_{ps} however is far from practical; therefore, in this paper instead of maximizing $C_s(p_i)$ we maximize its lower band which can be obtained based on obtainable parameters in a practical scenario. As it is also shown, using this approach, we end up providing practical algorithms and scenarios in this paper. Estimating g_{ps} is challenging mainly because of the fact that it requires a direct/indirect signaling between the primary and secondary systems which is not always possible in most practical cases as the two systems are usually considered to be able to act independently. An alternative would be adopting sophisticated signal processing techniques for blind channel estimation; however, the accuracy of such techniques is often an increasing function of the channel observation time. Thus an acceptable level of estimation accuracy may compromise the secondary service capacity due to longer required channel observation time.

3. In fact Q_- represents the time interval in which the received interference at the primary receiver because of the secondary transmission does not have negative impact on the primary QoS. Q_+ on the other hand is the time interval in which the transmission activity of secondary system results in collision with the primary transmission.

if $t \in \mathcal{Q}_+$ there holds $Q > \frac{\ln \frac{1}{t}}{\frac{T-\tau}{T}\delta \frac{\gamma \mu_{pp}}{P_p}}$. Denoting $\Delta = e^{-\frac{\gamma \mu_{pp} N_0 B}{P_p}}$, one can write

$$\begin{aligned} p_{out} &= 1-\Delta \\ &+\frac{\Delta}{\frac{T-\tau}{T}\delta p_b}\int_{t\in\mathcal{Q}_-}\frac{T-\tau}{T}\delta p_b \mathbf{P}\left\{g_{sp}P_s > \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}\right\}dt \\ &+\frac{\Delta}{\frac{T-\tau}{T}\delta p_b}\int_{t\in\mathcal{Q}_+}\frac{T-\tau}{T}\delta p_b \mathbf{P}\left\{g_{sp}P_s > \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}\right\}dt \\ &\leq 1-\Delta+\frac{\Delta}{\frac{T-\tau}{T}\delta p_b}\int_{t\in\mathcal{Q}_-}\xi dt+\frac{B}{\frac{T-\tau}{T}\delta p_b}\int_{t\in\mathcal{Q}_+}dt, \end{aligned}$$

where we use the fact that for $t \in \mathcal{Q}_-$ we always have

$$\frac{T-\tau}{T}\delta p_b \mathbf{P}\left\{g_{sp}P_s > \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}\right\} \leq \xi,$$

and for $t \in \mathcal{Q}_+$ there holds

$$\frac{T-\tau}{T}\delta p_b \mathbf{P}\left\{g_{sp}P_s > \frac{\ln\frac{1}{t}}{\frac{T-\tau}{T}\delta\frac{\gamma\mu_{pp}}{P_p}}\right\} \leq 1.$$

Consequently,

$$p_{out} \leq 1-\Delta+\frac{\Delta}{\frac{T-\tau}{T}\delta p_b}\left(1-(1-\xi)e^{-\frac{T-\tau}{T}\delta\mu_{pp}Q}\right). \quad (10)$$

Equation (10) indicates that depending on the outage performance requirement of the primary service we can design/obtain appropriate collision probability constraint, (Q, ξ). Note that the above outage probability upper bound increases with increasing collision probability constraint.

The primary service collision probability constraint, (Q, ξ), i.e., $p_{col} \leq \xi$ sould be satisfied by the secondary service considering the secondary service power transmission constraint, $\overline{P}_s$. Therefore, the secondary service achievable capacity is obtained using the following optimization problem:

Problem $\mathcal{O}$:

$$\begin{aligned} \max \quad & C_s(p_i, I), & (11)\\ \text{s.t.} \quad & \mathbf{E}_{g_{ss}}\left[p_{i,\delta}P_s\right] \leq \overline{P}_s, & (12)\\ & \frac{T-\tau}{T}\delta p_b \mathbf{P}\left\{g_{sp}P_s > Q\right\} \leq \xi, & (13)\\ & P_s \geq 0, 0 \leq \delta \leq 1. & (14) \end{aligned}$$

Remark 2: In the formulation provided above the instantaneous value of g_{sp} is not required at the Tx_s; instead, the assumption adopted is availability of g_{sp} distribution information at the secondary service. In practice this information can be provided by empirical studies in the time of network design and does not require signaling between the two systems. This can be also obtained utilizing passive measurements such as kernel density de-convolution estimator [18]. Assume that Tx_s frequently receives data $r[n]$ as $h[n]+z[n]$, which $z[n]$ is AWGN with known variance. The problem is to estimate the density of random variable h by measuring r. We then apply Kernel-based technique and suggest the following estimation

$$\hat{f}(x) = \frac{1}{nb}\sum_{j=1}^{n} K\left(\frac{x-r[j]}{b}\right),$$

with a Kernel function $K : \mathbb{R} \to \mathbb{R}^+$ and a bandwidth parameter $b > 0$. The Fourier transform of $\hat{f}(x)$, is

$$\hat{\mathcal{F}}(\omega) = \int e^{ix\omega}\frac{1}{nb}\sum_{j=1}^{n}K\left(\frac{x-r[j]}{b}\right)dx = \frac{1}{n}\sum_{j=1}^{n}e^{ir[j]\omega}\mathcal{K}(b\omega),$$

where $\mathcal{K}(\omega)$ is Fourier transform of the Kernel function. Note that $\hat{\mathcal{F}}(\omega) = \hat{\mathcal{F}}_h(\omega)\mathcal{Z}(\omega)$, where $\hat{\mathcal{F}}_h(\omega)$ is Fourier transform of the estimated pdf of random variable h. Applying the inverse Fourier transform to $\hat{\mathcal{F}}_h(\omega)$ we can then show that the suggested pdf for time instance n is

$$\begin{aligned} \hat{f}_h^{[n]}(x) &= \frac{1}{2\pi}\int e^{-ix\omega}\frac{1}{n\mathcal{Z}(\omega)}\sum_{j=1}^{n}e^{ir[j]\omega}\mathcal{K}(b\omega)d\omega \\ &= \frac{n-1}{n}\hat{f}_h^{[n-1]}(x)+\frac{1}{n}\frac{1}{2\pi}\int e^{-ix\omega}\frac{1}{\mathcal{Z}(\omega)}e^{ir[n]\omega}\mathcal{K}(b\omega)d\omega. \end{aligned}$$

Thus, in each time slot n we only need to appropriately scale the estimated pdf in the previous time slot and add it up with new term derived from measuring $r[n]$ on-the-fly. This technique can be used to estimate $f_{sp}(g_{sp})$ based on the measured signal strength received from Rx_p. Note that in reality Rx_p needs to transmit feedback signals to Tx_p frequently, hence Tx_s can passively have access to noisy and/or outdated version of g_{sp}, assuming reciprocity. However, in the overlay spectrum sharing scenarios the primary service activity may undermine the accuracy of the proposed method, as Tx_s only receives noise when the primary is not active. In [19] a technique is proposed based on Markovian evolution of the spectrum occupancy joined with Bayes's rule for correcting the estimated pdf. Coupling Kernel density de-convolution estimator with [19] it is then possible to incorporate spectrum sensing outcome in pdf estimation.

Using Problem $\mathcal{O}$ the power allocation in the secondary service and the optimal value of δ are obtained so that the secondary service achievable capacity is maximized. Due to the probabilistic nature of constraint (13), obtaining the exact solutions of problem $\mathcal{O}$ is difficult. Furthermore, the above primal optimization problem might not be generally a convex optimization problem, thus various local subproblems can be obtained based on different approaches.

Problem $\mathcal{O}$ can be solved using a very simple scheme, namely, *One-Shot (O-S)* scheme. In this approach, problem $\mathcal{O}$ is solved in two disjoint steps. In the first step, spectrum sensing parameters are obtained and then in the next step the power allocation is conducted. This scheme is widely used in the related literature, see, e.g., [8] and [11]. It is shown in Section 4.1 and Section 5 that although for moderate values of I the O-S scheme performs well, the achievable capacity is significantly degraded in cases where I is either very low or very high.

To tackle the aforementioned issues, we propose four different approaches to solve $\mathcal{O}$: *Modified-One-Shot (M-O-S)* scheme, *Multi-Shot (M-S)* scheme, *Conservative-Modified-One-Shot (C-O-S)* scheme, and *Restricted-Modified-One-Shot (R-O-S)* scheme. The main theme of the proposed approaches in this paper is to convert the collision probability constraint into a combination of the secondary service power allocation and the spectrum sensing parameter design, i.e., the miss detection probability.

The M-O-S scheme overcomes the fast capacity decreasing slope which is seen in high values of I in the O-S scheme.

Moreover, the M-S, C-O-S, and R-O-S schemes improve the secondary service capacity for small values of I. The M-S scheme solves problem $\mathcal{O}$ through an iterative fast convergent algorithm. To satisfy the collision probability constraint, the C-O-S scheme suggests a peak power allocation constraint for the secondary service transmission power which is adaptively adjusted based on the collision probability constraint. Instead of the peak power constraint, the R-O-S introduces a constraint on the average transmission power of the secondary service to satisfy the collision probability constraint. We elaborate on the above mentioned schemes in the following.

4.1 One-Shot (O-S) Scheme

In this paper, we consider One-Shot scheme as a benchmark. This scheme is presented in our previous study in [4]. In the following, the O-S scheme is briefly described for easy reference. In O-S scheme, similar to almost all spectrum sensing mechanism, the secondary service fixes the miss detection probability as $\delta = \xi$. The false alarm probability is then obtained using the corresponding ROC curve [8], [11], and [4]. Then without considering the constraint in (13), the secondary service solves the following optimization problem:

Problem $\mathcal{O}^{O-S}$:

$$\max_{P_s \geq 0} \mathbf{E}_{g_{ss}} p_{i,\xi} B \left[\log \left(1 + \frac{g_{ss} P_s}{N_0 B + \frac{T-\tau}{T} \xi p_b I} \right) \right]$$
$$\text{s.t.} \quad \mathbf{E}_{g_{ss}} \left[p_{i,\xi} P_s \right] \leq \overline{P}_s.$$

In the above using (4) and the ROC curve, the value of $p_{i,\xi}$ can be obtained as

$$p_{i,\xi} = \frac{T-\tau}{T} \left(\left(1 - \mathbf{ROC}^{-1}(1-\xi, \tau, I) \right) p_i + \xi p_b \right),$$

where, for given τ and I, $\epsilon = \mathbf{ROC}^{-1}(1-\delta, \tau, I)$, and $\mathbf{ROC}^{-1}(.,.,.)$ is the inverse function of the ROC curve.

Utilizing the Lagrange multipliers approach the optimal power allocation of the secondary service is obtained as

$$P_s^* = \left(\frac{1}{\lambda} - \frac{N_0 B + \frac{T-\tau}{T} \xi p_b I}{g_{ss}} \right)^+, \quad (15)$$

where $(x)^+ = \max\{x, 0\}$, and parameter λ is the Lagrangian coefficient which is obtained from the following equation:

$$\int\limits_{g_{ss} \in \mathcal{G}_{ss,\xi}} \left(\frac{1}{\lambda} - \frac{N_0 B + \frac{T-\tau}{T} \xi p_b I}{g_{ss}} \right) f_{ss}(g_{ss}) dg_{ss} = \frac{\overline{P}_s}{p_{i,\xi}}. \quad (16)$$

In (16), $\mathcal{G}_{ss,\xi}$ can be defined as

$$\mathcal{G}_{ss,\xi} = \left\{ g_{ss} | g_{ss} \geq \lambda \left(N_0 B + \frac{T-\tau}{T} \xi p_b I \right) \right\}.$$

Substituting (15) into the objective function of optimization problem $\mathcal{O}^{O-S}$, the capacity of the secondary service is then obtained as

$$C_s^{O-S}(p_i, I) = p_{i,\xi} B \int\limits_{g_{ss} \in \mathcal{G}_{ss,\xi}} \log \left(\frac{g_{ss}}{\lambda \left(N_0 B + \frac{T-\tau}{T} \xi p_b I \right)} \right) dF_{ss}(g_{ss}). \quad (17)$$

As it is seen in this approach, a simple solution based on water-filling is obtained and it is only required that the miss detection probability to be set equal to ξ.

In the O-S scheme the secondary service has to adjust the actual miss detection probability so that it remains equal to ξ. The first issue arises for large enough values of the received interference, I; because, in the case of miss detection, a significantly high interference level is imposed at the Rx_s during the secondary service transmission period. As a result, referring to (17) we expect that for large enough values of the received interference, the achieved capacity will be significantly decreased. Indeed, this expectation is verified through the numerical results in Section 5. To tackle this drawback, we propose M-O-S approach.

The second issue with the respect to the O-S scheme is observed in cases where I is very small. In such cases, the spectrum sensing procedure restricts the secondary service accessability into a fraction of $p_{i,\xi}$ of the whole available frame. Moreover, during the miss detection state, Rx_s experiences a very small interference I; therefore, it might be reasonable to increase the access time of the secondary service. This opportunity is in fact ignored in the O-S scheme. It must be noted that, the Rx_p is able to tolerate a collision probability of ξ which may enable the secondary service to increase its access time. For instance, in cases where the wireless channel between the Tx_s and the Rx_p experiences a deep fade, the secondary service is able to access the spectrum without severely degrading the primary QoS. To exploit this issue, we propose three novel schemes: M-S, C-O-S, and R-O-S.

4.2 Modified-One-Shot (M-O-S) Scheme

In O-S scheme, the secondary service simply sets the miss detection probability equal to the collision probability constraint. As it is also mentioned in the above, this is the main cause of the performance degradation, especially in cases where I is high. Instead of this, one can choose the best miss detection probability that maximizes the obtained achievable capacity subject to the collision probability constraint. For this, we first maximize the secondary service achievable capacity subject to the transmission power constraint for a given miss detection probability. Then, the best miss detection probability which maximizes the obtained achievable capacity is chosen, in such a way that the collision probability constraint is satisfied. We refer to this scheme as Modified-One-Shot Scheme (M-O-S) scheme. We elaborate on this scheme in the following.

In this scheme, we set $\delta = \delta_0 \leq 1$. The optimal transmission power of the secondary service is then obtained similar to (15) as follows:

$$P_s^0 = \left(\frac{1}{\lambda_0} - \frac{N_0 B + \frac{T-\tau}{T} \delta_0 p_b I}{g_{ss}} \right)^+, \quad (18)$$

where λ_0 is obtained from

$$\int\limits_{g_{ss} \in \mathcal{G}_{ss,\delta_0}} \left(\frac{1}{\lambda_0} - \frac{N_0 B + \frac{T-\tau}{T} \delta_0 p_b I}{g_{ss}} \right) f_{ss}(g_{ss}) dg_{ss} = \frac{\overline{P}_s}{p_{i,\delta_0}}, \quad (19)$$

and $\mathcal{G}_{ss,\delta_0}$ is defined as

$$\mathcal{G}_{ss,\delta_0} \triangleq \left\{ g_{ss} | g_{ss} \geq \lambda_0 \left(N_0 B + \frac{T-\tau}{T} \delta_0 p_b I \right) \right\}. \quad (20)$$

Consequently, the achievable capacity of the secondary service is obtained similar to (17) as

$$C_s(p_i, I) = p_{i,\delta_0} B \int_{g_{ss}\in\mathcal{G}_{ss,\delta_0}} \log\left(\frac{g_{ss}}{\lambda\left(N_0B+\frac{T-\tau}{T}\delta_0 p_b I\right)}\right) dF_{ss}(g_{ss}). \tag{21}$$

For given values of p_i and I, $C_s(p_i, I)$ can adopt different values when δ_0 is changed. Here our objective is to find the optimal miss detection probability δ^* which maximizes $C_s(p_i, I)$. In cases where $C_s(p_i, I)$ is concave, the optimal miss detection probability can be simply obtained by taking derivation. In cases where it is not concave, numerical search methods such as bisection search can be employed to find the optimal miss detection probability. The obtained optimal miss detection probability is acceptable if and only if $\delta^* \leq \xi$. In cases where δ^* is larger than ξ, in the M-O-S scheme we simply set $\delta^* = \xi$. This can be formulated as

$$\delta^* = \min\left\{\xi, \max_{\delta_0} C_s(p_i, I)\right\}.$$

The corresponding obtained achievable capacity is denoted by $C_s^{M-O-S}(p_i, I) = C_s(p_i, I)|_{\delta_0=\delta^*}$. Note that, in this case $C_s^{O-S}(p_i, I) \leq C_s^{M-O-S}(p_i, I)$. In fact, for small values of I at Rx_s, M-O-S scheme reduces to O-S scheme since $\delta^* = \xi$. However, for large enough values of I, we have $\delta^* = \max_{\delta_0} C_s(p_i, I) < \xi$; therefore, $\delta^* p_b I < \xi p_b I$ which results in $C_s^{O-S}(p_i, I) < C_s^{M-O-S}(p_i, I)$.

4.3 Multi-Shot (M-S) Scheme

As it was mentioned, we expect that a deterministic approximation of the probabilistic constraint in (13) may result in a lower than optimal secondary service achievable capacity. In the following, we present a Multi-Shot (M-S) scheme to find a suboptimal solution of the problem $\mathcal{O}$. In this scheme, instead of replacing the constraint in (13), we in an appropriate way check its validity using the proposed scheme. The steps of the M-S scheme are presented in the following.

1) We set $\delta = \delta_0$, $0 \leq \delta_0 \leq 1$, thus $\epsilon_0 = \mathbf{ROC}^{-1}(1-\delta_0, \tau, I)$.

2) The allocated power is set as

$$P_s^0 = \left(\frac{1}{\lambda_0} - \frac{N_0B + \frac{T-\tau}{T}\delta_0 p_b I}{g_{ss}}\right)^+, \tag{22}$$

where λ_0 is obtained from (19). 3)The collision probability corresponding to the miss detection probability δ_0, i.e., P_{col,δ_0}, is then obtained as

$$\begin{aligned} P_{col,\delta_0} &= \frac{T-\tau}{T}\delta_0 p_b \times \\ &\int_{g_{ss}\in\mathcal{G}_{ss,\delta_0}} \bar{F}_{sp}\left(\frac{Q}{\frac{1}{\lambda_0} - \frac{N_0B+\frac{T-\tau}{T}\delta_0 p_b I}{g_{ss}}}\right) dF_{ss}(g_{ss}). \end{aligned} \tag{23}$$

For a predefined error ε, if $|P_{col,\delta_0} - \xi| < \varepsilon$, then δ_0 is a solution for the miss detection probability and the algorithm ends; otherwise, $\bar{\delta}$ is obtained from $P_{col,\delta_0} = \xi$, i.e., the collision probability constraint is satisfied[4].

4) $\delta_0 = \bar{\delta}$ and the algorithm is repeated from step 2 onwards.

Through the numerical results in Section 5, it is seen that the presented algorithm quickly converges to a fixed value miss detection probability which we call *suitable miss-detection*.

In the M-S scheme, we iteratively check the collision probability constraint in finding the solutions (step 3). Another approach to consider the collision probability constraint is to obtain an equivalent power allocation constraint which, if holds, results in the collision probability constraint being satisfied. Based on this approach, in the following subsections we propose C-O-S and R-O-S schemes. Furthermore, as it will be seen when compared to the M-S Scheme, the most important advantages of C-O-S and R-O-S are that the corresponding optimal[5] power allocation is obtained and then based on that the appropriate miss detection probability is calculated. We also expect that C-O-S and R-O-S schemes outperform O-S and M-O-S for some particular system parameters. This is because in C-O-S and R-O-S, the collision probability constraint is integrated into the power allocation which enables us to exploit certain fading conditions as discussed earlier.

4.4 Conservative-Modified-One-Shot (C-O-S) Scheme

Here we propose a new scheme based on a conservative interpretation of the collision probability constraint. Furthermore, the proposed interpretation of the collision probability constraint converts the generally non-convex collision probability constraint into a convex constraint which results in a convex optimization problem. The proper ROC point is then obtained using the new optimization problem with the convex constraint utilizing the M-O-S scheme. In the following, we present the details of the C-O-S scheme.

Assume that the miss detection probability is fixed, i.e., $\delta = \delta_0 \leq 1$. We also note that due to feasibility of the power allocation strategy, $\frac{T}{T-\tau}\frac{\xi}{\delta_0 p_b} \leq 1$. The collision probability constraint (13) is

$$p_{col} = \frac{T-\tau}{T}\delta_0 p_b \mathbf{E}_{g_{ss}}\left[\mathbf{P}\left\{g_{sp} > \frac{Q}{P_s}|g_{ss}\right\}\right]. \tag{24}$$

Instead of evaluating the above constraint, we consider the following constraint on the collision probability

$$\mathbf{P}\left\{g_{sp} > \frac{Q}{P_s}\right\} \leq \frac{T}{T-\tau}\frac{\xi}{\delta_0 p_b}, \quad \forall g_{ss}, \tag{25}$$

in which instead of the average, each instance of the collision probability for a given value of g_{ss} must satisfy the collision probability constraint. As it is seen, here instead of evaluating the collision probability constraint, a conservative version of

4. In fact the collision probability constraint represents the "available resource" for the secondary service. In the other words, the larger the collision probability constraint, the higher could be the secondary system transmission power without compromising QoS in the primary system. A higher secondary service transmit power also results in a higher achievable secondary service capacity. Therefore, as it is always done in resource allocation problems, to maximize the objective function the whole available resource is utilized. It means that in this particular case the power is allocated so that the actual collision probability becomes equal to the maximum tolerable collision probability in the primary system.

5. This power allocation is not in fact globally optimal.

this constraint is taken to account. Therefore, (17) is equivalently reduced to

$$F_{sp}\left(\frac{Q}{P_s}\right) \geq 1-\frac{T}{T-\tau}\frac{\xi}{\delta_0 p_b}, \quad \forall g_{ss}. \tag{26}$$

Since δ_0 is set so that $\frac{T}{T-\tau}\frac{\xi}{\delta_0 p_b} \leq 1$, the right hand side of (26) always adopts positive values. We also note that the probability distribution function is a monotonically increasing function, therefore (26) is equivalent to

$$\frac{Q}{P_s} \geq F_{sp}^{-1}\left(1-\frac{T}{T-\tau}\frac{\xi}{\delta_0 p_b}\right), \quad \forall g_{ss},$$

where $x = F_{sp}^{-1}(y)$ is the reverse function of the Probability Distribution Function $y = F_{sp}(x)$. As a result, one may interpret the collision probability constraint as a bound on the instantaneous transmission power of the secondary service as the following

$$P_s \leq \frac{Q}{F_{sp}^{-1}\left(1-\frac{T}{T-\tau}\frac{\xi}{\delta_0 p_b}\right)} \triangleq Q(\xi), \quad \forall g_{ss}. \tag{27}$$

The obtained constraint in (27) is in fact a peak power constraint which is a function of (Q,ξ). The optimization problem $\mathcal{O}$ is then reduced to

Problem $\tilde{\mathcal{O}}_0$:

$$\max_{P_s\geq 0} \mathbf{E}_{g_{ss}}\left[p_{i,\delta_0} B \log\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta_0 p_b I}\right)\right],$$
$$\text{s.t.} \quad \mathbf{E}_{g_{ss}}\left[p_{i,\delta_0}P_s\right] \leq \overline{P}_s,$$
$$P_s \leq Q(\xi), \quad \forall g_{ss},$$

which is a convex optimization problem. Using the obtained results in [20], the optimal power allocation strategy is

$$P_s^0 = \begin{cases} 0 & \text{if } g_{ss}\in \bar{\mathcal{G}}_{ss,\delta_0}, \\ \frac{1}{\lambda_0}-\frac{N_0B+\frac{T-\tau}{T}\delta_0 p_b I}{g_{ss}} & \text{if } g_{ss}\in \mathcal{G}_{ss,\delta_0}\bigcap \dot{\mathcal{G}}_{ss,\delta_0}, \\ Q(\xi) & \text{otherwise,} \end{cases} \tag{28}$$

where $\mathcal{G}_{ss,\delta_0}$ is defined similar to (20), and $\dot{\mathcal{G}}_{ss,\delta_0}$ is

$$\dot{\mathcal{G}}_{ss,\delta_0} = \left\{g_{ss} | g_{ss} \leq \frac{\lambda_0}{1-\lambda_0 Q(\xi)}\left(N_0B+\frac{T-\tau}{T}\delta_0 p_b I\right)\right\}.$$

Here, $\bar{\mathcal{G}}_{ss,\delta_0}$ represents the corresponding complementary set to $\mathcal{G}_{ss,\delta_0}$.

It is worth mentioning that only in cases where $\lambda_0 < \frac{1}{Q(\xi)}$ the power allocation strategy (28) is applicable; In other cases, for all fading realizations, $P_s^0 = 0$. This may result in the restriction of the achievable capacity of the secondary service for a specific set of system parameters.

The Lagrangian multiplier, λ_0, is obtained solving the following equation:

$$\int_{g_{ss}\in \mathcal{G}_{ss,\delta_0}\bigcap \dot{\mathcal{G}}_{ss,\delta_0}} \left(\frac{1}{\lambda_0}-\frac{N_0B+\frac{T-\tau}{T}\delta_0 p_b I}{g_{ss}}\right) dF_{ss}(g_{ss})$$
$$+Q(\xi)\mathbf{P}\left\{\bar{\dot{\mathcal{G}}}_{ss,\delta_0}\right\} = \frac{\overline{P}_s}{p_{i,\delta_0}}. \tag{29}$$

Consequently, the achievable capacity of the secondary service is obtained substituting the obtained allocated power P_s^0 into the objective function $\tilde{\mathcal{O}}_0$ as

$$C_s(p_i,I) = p_{i,\delta_0}B\int_{g_{ss}\in\bar{\dot{\mathcal{G}}}_{ss,\delta_0}} \log\left(1+\frac{Q(\xi)g_{ss}}{N_0B+\frac{T-\tau}{T}\delta_0 p_b I}\right)$$
$$+p_{i,\delta_0}B\int_{g_{ss}\in \mathcal{G}_{ss,\delta_0}\bigcap \dot{\mathcal{G}}_{ss,\delta_0}} \log\left(\frac{g_{ss}}{\lambda\left(N_0B+\frac{T-\tau}{T}\delta_0 p_b I\right)}\right) dF_{ss}(g_{ss}).$$

Now, similar to the proposed approach in Section 4.2 our objective is to find the optimal miss detection probability δ^* which maximizes $C_s(p_i,I)$. The obtained achievable capacity is denoted by $C_s^{C-O-S}(p_i,I)$.

4.5 Restricted-Modified-One-Shot (R-O-S) Scheme

Here we propose another scheme which interprets the collision probability constraint as a familiar secondary service transmission power constraint. Here we develop this scheme for the case of Rayleigh fading. Extending the results to other fading environments depends on the convexity of the probability distribution function of the channel power gain between Tx_s and Rx_p (for instance, the case of log-normal distribution is presented in the following.). The details of the proposed scheme named as R-O-S scheme is presented in the following.

Let the miss detection probability set as $\delta = \delta_0 \leq 1$. Assume that the channel power gain g_{sp} is distributed through an exponential distribution with mean $1/\mu_{sp}$. Therefore, the collision probability constraint (13) is reduced to

$$\mathbf{E}_{ss}\left[e^{-\mu_{sp}\frac{Q}{P_s}}\right] \leq \frac{T}{T-\tau}\frac{\xi}{\delta_0 p_b} \triangleq \hat{\xi}. \tag{30}$$

Our treatment of the subject is based on evaluating an upper bound on the left hand side of (30). Consider function $W(x) = \exp(-\frac{a}{x})$, where $x > 0$ and a is a positive real number. It is easy to verify that function $W(x)$ is concave for $x > \frac{a}{2}$. Setting parameter a equal to $\mu_{sp}Q$, it is seen that for $P_s > \frac{\mu_{sp}Q}{2}$,

$$\mathbf{E}_{ss}\left[e^{-\mu_{sp}\frac{Q}{P_s}}\right] \leq e^{-\mu_{sp}\frac{Q}{\mathbf{E}_{ss}[P_s]}}. \tag{31}$$

Consequently, an equivalent constraint to (30) is obtained through the following power allocation constraints

$$\mathbf{E}_{ss}[P_s] \leq \frac{\mu_{sp}Q}{\ln(\frac{1}{\hat{\xi}})}, \tag{32}$$

$$P_s > \frac{\mu_{sp}Q}{2}. \tag{33}$$

Expression (32) suggests that, in this specific case the transmission power of the secondary service may be restricted more than physical constraint i.e., P_s. This is the fact that we refer to this scheme as R-O-S. As a result, an equivalent optimization problem for allocating the transmission power of the secondary service regarding (32) and (33), can be obtained from $\mathcal{O}$ as follows

Problem $\check{\mathcal{O}}_0$:

$$C_s(p_i,I) = \max \mathbf{E}_{g_{ss}} p_{i,\delta_0} B\left[\log\left(1+\frac{g_{ss}P_s}{N_0B+\frac{T-\tau}{T}\delta_0 p_b I}\right)\right],$$
$$\text{s.t.} \quad \mathbf{E}_{g_{ss}}\left[P_s\right] \leq \min\left\{\frac{\overline{P}_s}{p_{i,\delta_0}}, \frac{\mu_{sp}Q}{\ln(\frac{1}{\hat{\xi}})}\right\},$$
$$P_s \geq \frac{\mu_{sp}Q}{2},$$

which is a convex optimization problem. It is easy to show that the optimal power allocation of $\check{\mathcal{O}}_0$ is

$$P_s^0 = \begin{cases} 0 & \text{if } g_{ss} \in \bar{\hat{\mathcal{G}}}_{ss,\delta_0} \bigcup \bar{\check{\mathcal{G}}}_{ss,\delta_0}, \\ \frac{p_{i,\delta_0}}{\lambda_0} - \frac{N_0B + \frac{T-\tau}{T}\delta_0 p_b I}{g_{ss}} & \text{if } g_{ss} \in \hat{\mathcal{G}}_{ss,\delta_0} \bigcap \check{\mathcal{G}}_{ss,\delta_0}, \end{cases} \quad (34)$$

where

$$\hat{\mathcal{G}}_{ss,\delta_0} = \left\{ g_{ss} | g_{ss} > \frac{\lambda_0}{p_{i,\delta_0}} \left(N_0B + \frac{T-\tau}{T}\delta_0 p_b I \right) \right\},$$

$$\check{\mathcal{G}}_{ss,\delta_0} = \left\{ g_{ss} | g_{ss} < \frac{\lambda_0}{p_{i,\delta_0} - \frac{\mu_{sp} Q \lambda_0}{2}} \left(N_0B + \frac{T-\tau}{T}\delta_0 p_b I \right) \right\}.$$

Here a non-zero power is allocated to the secondary service based on (34) if the Lagrangian multiplier λ_0 satisfies

$$\lambda_0 < \frac{2p_{i,\delta_0}}{\mu_{sp} Q},$$

otherwise no power is allocated for all fading realizations. The Lagrangian multiplier is obtained solving the following equation for λ_0

$$\int_{g_{ss} \in \hat{\mathcal{G}}_{ss,\delta_0} \bigcap \check{\mathcal{G}}_{ss,\delta_0}} \left(\frac{p_{i,\delta_0}}{\lambda_0} - \frac{N_0B + \frac{T-\tau}{T}\delta_0 p_b I}{g_{ss}} \right) dF_{ss}(g_{ss})$$
$$= \min \left\{ \frac{\overline{P}_s}{p_{i,\delta_0}}, \frac{\mu_{sp} Q}{\ln(\frac{1}{\xi})} \right\}.$$

Substituting the optimal power allocation in (34) for the obtained Lagrangian multiplier into $C_s(p_i, I)$, the achievable capacity of the secondary service for given miss detection probability is then obtained as

$$C_s(p_i, I) = p_{i,\delta_0} B \int_{g_{ss} \in \hat{\mathcal{G}}_{ss,\delta_0} \bigcap \check{\mathcal{G}}_{ss,\delta_0}} \log \left(\frac{p_{i,\delta_0} g_{ss}}{\lambda_0 \left(N_0B + \frac{T-\tau}{T}\delta_0 p_b I \right)} \right) dF_{ss}(g_{ss}).$$

In cases where $C_s(p_i, I)$ is concave, δ^* can be simply obtained by taking derivation. In cases where it is not concave, then numerical search methods such as bisection search can be adopted to find δ^*. The achieved capacity of the secondary service utilizing R-O-S scheme, $C^{R-O-S}(p_i, I)$, is then obtained by substituting δ_0 with δ^* in $C_s(p_i, I)$.

4.5.1 R-O-S Scheme for Log-Normal Distribution

Here, we investigate the impact of log-normal fading on the analysis in Section 4.5. Assume that channel power gains are drawn from a log-normal distribution. For log-normal random variable X with parameter (m, ν), the pdf is

$$f_X(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\log(x)-\eta)^2}{2\sigma^2}} 1_{x>0},$$

where η and σ are obtained through the following equations

$$\eta = \log \left(\frac{m^2}{\sqrt{\upsilon + m^2}} \right), \quad \sigma = \sqrt{\log \left(\frac{\upsilon}{1+m^2} \right)}.$$

TABLE 1
Simulation Parameters

Parameter	Value
μ_{ss}	10 dB
μ_{sp}	10 dB
Background noise power (N_0B)	0.1 Watt
$\overline{P}_s$	1 Watt
ξ	0.2
T	1 sec
τ	0.1 sec
p_b	0.7
p_i	0.3

The collision probability constraint in (30) can be evaluated as

$$\mathbf{E}_{ss} \left[\mathbf{Q} \left(\frac{\ln\left(\frac{Q}{P_s}\right) - \eta_{sp}}{\sigma_{sp}} \right) \right] \leq \frac{T}{T-\tau} \frac{\xi}{\delta_0 p_b} \triangleq \hat{\xi}. \quad (35)$$

Note that, function $\mathbf{Q}\left(\frac{\ln(\frac{Q}{x}) - \eta_{sp}}{\sigma_{sp}} \right)$ is a concave function for $x \geq \frac{e^{\eta_{sp}+1}}{Q}$. Thus, an upper bound for the left hand side of (35) is

$$\mathbf{Q} \left(\frac{\ln\left(\frac{Q}{\mathbf{E}_{ss}[P_s]}\right) - \eta_{sp}}{\sigma_{sp}} \right) \leq \hat{\xi}, \quad (36)$$

or,

$$\ln \left(\frac{Q}{\mathbf{E}_{ss}[P_s]} \right) \geq \sigma_{sp} \mathbf{Q}^{-1}\left(\hat{\xi}\right) + \eta_{sp}. \quad (37)$$

An equivalent constraint to collision probability constraint is obtained through the following power allocation constraints

$$\mathbf{E}_{ss}[P_s] \leq \frac{Q}{e^{\sigma_{sp}\mathbf{Q}^{-1}(\hat{\xi}) + \eta_{sp}}},$$

$$P_s \geq \frac{e^{\eta_{sp}+1}}{Q}.$$

Following the same line of argument presented in Section 4.5, the corresponding optimal power allocation and appropriate miss detection probability are then obtained.

5 Numerical Results

The main objective of this section is to investigate which of the proposed schemes in Section 4 is/are the most suitable for different set of system parameters. The parameters considered in this section are presented in Table 1. For the numerical results we consider a Rayleigh fading channel. Therefore, the channel power gain between the Tx_s and Rx_p (Rx_s) is an exponential random variable with mean value $1/\mu_{sp}$ ($1/\mu_{ss}$).

5.1 Convergence of the Multi-Shot Scheme

We investigate the convergence property of the M-S scheme. Here for different values of the imposed interference at the Rx_s from the Tx_p, collision probability constraint, and the interference threshold constraint, the speed of convergence of the M-S scheme is studied. We also have a brief look at the impact of the mean values of the channel power gains on the convergence property of this scheme.

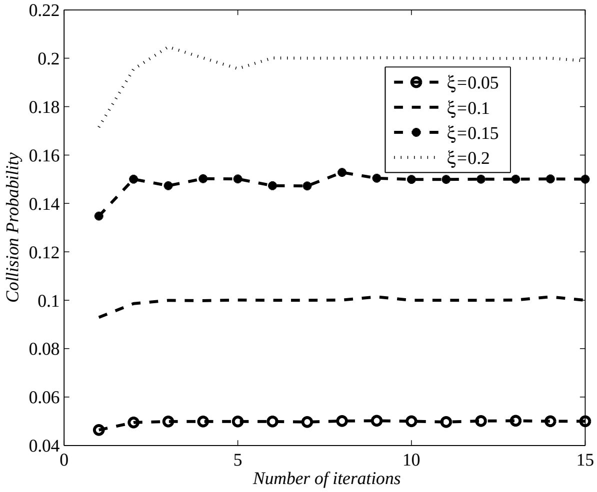

Fig. 2. Convergence of M-S scheme: Impact of the collision probability constraint for $I = 10$ W and $Q = 0.1$ W.

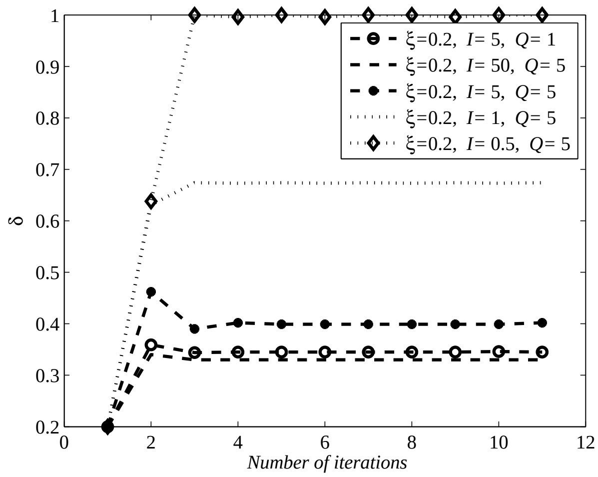

Fig. 3. Convergence of M-S scheme: Impact of Q and I.

5.1.1 Impact of the Collision Probability Constraint

As it is seen in Fig. 2, generally after 5 or 6 iterations the M-S scheme is stable and quickly converges to a fixed value. Notice that, the convergence is faster for smaller value of collision probability constraint. For small enough ξ, M-S and M-O-S schemes perform almost the same (see Fig. 8), thus the algorithm swiftly converges. However, large enough ξ results in broader search region for optimal miss detection probability which may result in a longer convergence time.

5.1.2 Impact of Q and I

Fig. 3 indicates that the proposed M-S scheme converges fast enough for different amounts of the interference threshold and received interference at Rx_s. This illustration also shows that for small enough values of I, the secondary service manages to have access to the spectrum almost all the time, i.e., high miss detection probability. This is mainly due to the fact that the secondary service is not able to recognize between busy and idle spectrum.

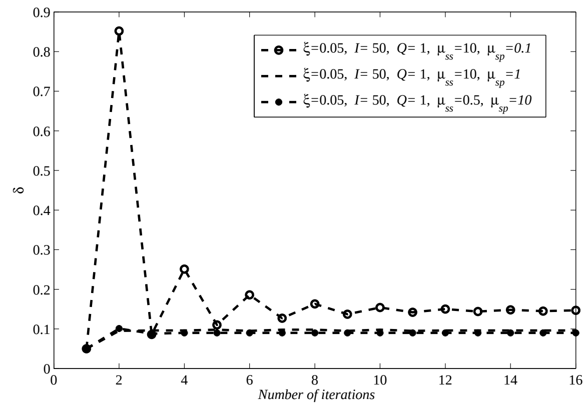

Fig. 4. Convergence of M-S scheme: Impact of μ_{ss} and μ_{sp}.

5.1.3 Impact of Channel Power Gains' Mean Values

In Fig. 4 we study the impact of μ_{ss} and μ_{sp} on the convergence of the M-S scheme. As it can be seen, in this case again the scheme quickly converges, however, for small values of μ_{sp} the speed of convergence is decreased. Smaller μ_{sp} amounts to stronger link between the Tx_s and Rx_p which partly results in equivalency between miss detection and collision probabilities. In this case, event $\mathbf{P}\{g_{sp}P_s > Q\}$ happens with high probability and does not affect collision probability.

5.2 Achievable Capacity of the Secondary Service

5.2.1 Impact of I

Fig. 5 indicates the achievable capacity of the secondary service in the case of O-S, M-O-S, M-S, C-O-S, and R-O-S schemes. Here we investigate the impact of the imposed interference at Rx_s due to the primary service transmissions, i.e., I.

It is seen that the achievable capacity in the O-S has a maximum for moderate values of I. Indeed, for $I \approx 10$ the spectrum sensing achieves its best performance, i.e., low false alarm probability. However, for small values of I, the secondary service fails to take advantage of the actually accessible spectrum since the spectrum sensing performance is limited to ξ. Therefore, although the imposed interference at the Rx_s is low, the secondary service is unable to access to the spectrum.

For high enough values of I, O-S scheme cannot gain a reasonable performance. This is again due to the spectrum sensing. The secondary service designates the miss detection and set it equal to ξ; however, increasing I correspondingly increases the imposed interference through $\xi p_b I$ thus the achievable capacity is decreased.

For high enough values of I, we expected that the M-O-S scheme outperforms the O-S scheme this is also confirmed in Fig. 5. The M-O-S scheme performs similar to the O-S scheme in terms of the achievable capacity for small and moderate values of I. It is also seen that by increasing I, the M-O-S scheme results in a higher achievable capacity. In the low interference regime, it is seen that the achievable capacity of the secondary service in the case of M-S scheme is higher than that of the O-S and the M-O-S schemes. Therefore, the M-S scheme can partially mitigate the poor performance of the O-S in small values of I. However, for the high enough values of I, its achieved performance is even worse than that of the

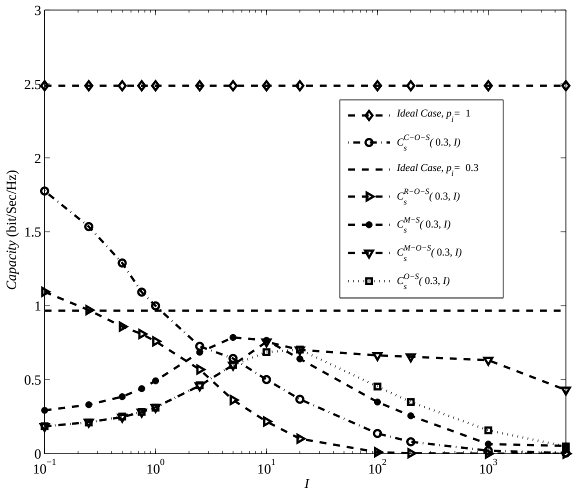

Fig. 5. Secondary service achievable capacity *vs.* I for O-S, M-O-S, M-S, C-O-S, and R-O-S schemes, and $\xi = 0.05$, $Q = N_0 B$.

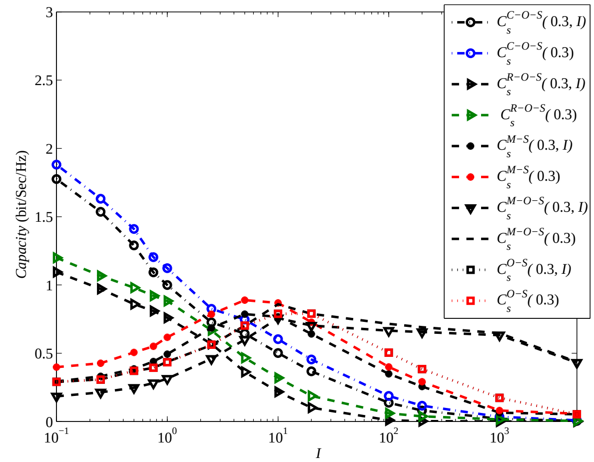

Fig. 6. $C_s(p_i, I)$ and $C_s(p_i)$ *vs.* I for O-S, M-O-S, M-S, C-O-S, and R-O-S schemes, and $\xi = 0.05$, $Q = N_0 B$.

O-S scheme. Fig. 5 also indicates that in the case of O-S, M-O-S and M-S schemes the achievable capacity of the secondary service never reaches the ideal case with $p_1 = 0.3$, i.e., zero miss detection and false alarm probabilities.

Surprisingly, it is also seen that in the case of C-O-S and R-O-S schemes, the achievable capacity of the secondary service outperforms the ideal case with $p_i = 0.3$ for small enough values of I. This is mainly due to the fact that, in low interference regime if the secondary service can appropriately access the busy part of the spectrum it can gain a meaningful performance. However, in the ideal case due to zero miss detection probability this opportunity is simply ignored. Note that both schemes cannot reach the ideal case of $p_i = 1$.

It must be noted that C-O-S scheme outperforms R-O-S for all interference values. For moderate and high values of I, the obtained achievable capacity in the case of C-O-S and R-O-S schemes approaches zero. This is mainly due to the inherent shortage of these schemes that is substituting actual collision probability constraint with transmit power constraint.

Based on the result presented it is concluded that the C-O-S fits for the cases with small level of interference. For moderate interference values the M-S is more appropriate. The M-O-S is also seen to outperform other schemes for high enough values of interference.

As mentioned before $\mathcal{O}$ maximizes $C_s(p_i, I)$ which is a lower-bound on $C_s(p_i)$. In Fig. 6 we study the gap between $C_s(p_i)$ and $C_s(p_i, I)$ versus I for different schemes. As it is seen, there is gap which is getting very small for large values of I. Note that in obtaining $C_s(p_i, I)$ unlike $C_s(p_i)$, g_{ps} is not used which makes the system much simpler.

5.2.2 Impact of Q

Fig. 7 shows the impact of the interference threshold Q on the secondary service achievable capacity in the case of O-S, M-O-S, M-S, C-O-S, and R-O-S schemes. Here, we set $I = 10$. It is known that, in such interference regime, the M-S scheme outperforms the others schemes. As it is seen, the obtained achievable capacity in the case of O-S and M-O-S schemes remain constant although the interference threshold is changed. This is due to the fact that both schemes ignore the collision probability constraint.

It is also seen that the performance of the M-S scheme steadily increases as the interference threshold is increased. Furthermore, utilizing the M-S scheme, the obtained achievable capacity of the secondary service is almost superior comparing to the other schemes. Fig. 7 also indicates that the R-O-S scheme poorly performs in comparison with the other schemes. Considering the results of Fig. 5, we expect that for $I = 10$ in the case of R-O-S scheme, the secondary service is not able to gain a proper achievable capacity. However, increasing Q makes the situation even worse. It is mainly due to the power allocation strategy in (34) in which power is not allocated to secondary service for all of the fading realizations.

The obtained performance considering C-O-S scheme is higher than that of R-O-C in the case of $Q < 0.4$. However, increasing Q beyond 0.45 tends the achievable capacity of the secondary service to zero considering the C-O-S scheme as the case. Surprisingly, for $0.05 < Q < 0.4$ the achieved performance of the C-O-S scheme outperforms the O-S and M-O-S schemes.

Based on the observed results in Fig. 5, one can see that the M-S scheme achieves a higher capacity for high interference threshold. For small values of Q depending the interference regime, the M-S, C-O-S, and M-O-S schemes outperform each other for different values of I. Since, in the practical systems usually the interference threshold Q is in the order of background noise level, we can claim that if I is small enough, the C-O-S will be the appropriate scheme. For the case that I adopts a moderate value, the M-S is the suitable scheme, and finally in the high interference regime, the M-O-S is the most appropriate scheme.

5.2.3 Impact of ξ

Fig. 8 demonstrates the achievable capacity of the secondary service versus ξ in the case of the O-S, M-O-S, M-S, C-O-S, and R-O-S schemes for a given imposed interference $I = 1$ and the interference threshold $Q = N_0 B$.

In the O-S scheme, increasing ξ, increases of the achievable capacity. Indeed, since the amount of I is moderate we expect

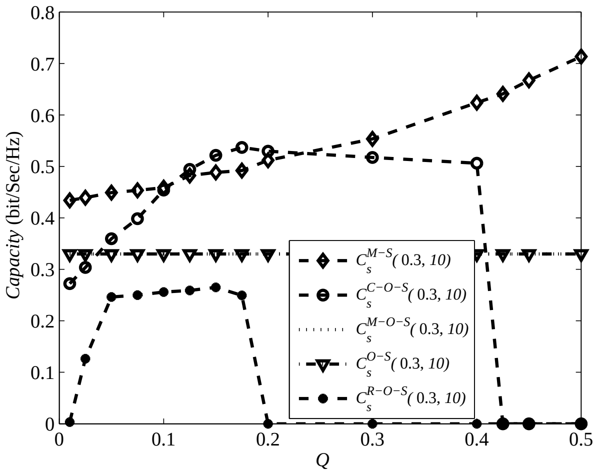

Fig. 7. Secondary service achievable capacity *vs.* Q for O-S, M-O-S, M-S, C-O-S, R-O-S schemes, for $\xi = 0.05$, $I = 10$ W.

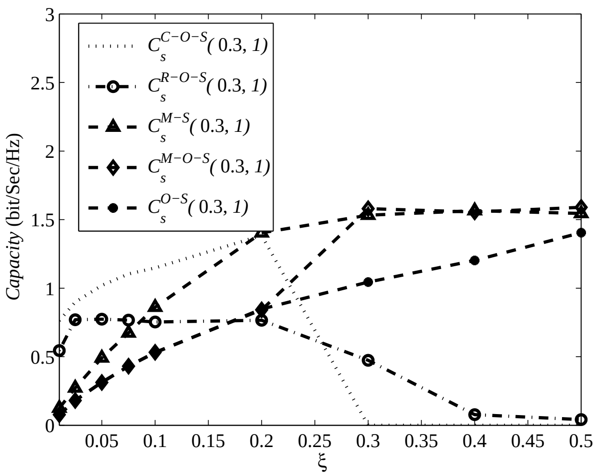

Fig. 8. Secondary service achievable capacity *vs.* ξ for O-S, M-O-S, M-S, C-O-S, R-O-S schemes, for $\xi = 0.05$, $Q = N_0 B$, $I = 1$ W.

that increasing ξ results in increasing the accessing time to the busy states of the spectrum and consequently the performance might be increased. The same performance is also anticipated for the M-O-S scheme. However, as it is indicated in Fig. 8, for the cases where $0 < \xi < 0.2$, $C^{O-S} = C^{M-O-S}$. For the case with $\xi > 0.2$, $C^{O-S} < C^{M-O-S}$. Indeed, for the moderate values of I, it is more appropriate for the secondary service to increase the miss detection probability thus longer access time to the busy state of the spectrum. The M-S scheme outperforms both the O-S and the M-O-S for small ξ; however, the achieved performance is similar to the M-O-S for higher values of ξ.

We also observed that for small enough values of ξ, the R-O-C achieves a reasonable performance. As it is seen in Fig. 8 for the case of $0 < \xi < 0.2$, the C-O-S scheme is the optimal scheme. However, increasing ξ makes C^{R-O-S} and C^{C-O-S} approach to zero. It must be noted that in practical situations parameter ξ is sufficiently small, therefore, for small values of I the C-O-S is the appropriate scheme. However, for moderate and large values of I the M-S and M-O-S schemes are the optimal schemes, respectively.

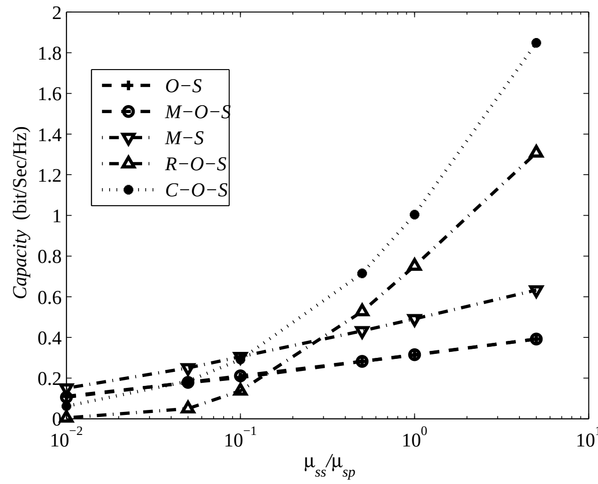

Fig. 9. Secondary service achievable capacity *vs.* μ_{ss}/μ_{sp} for O-S, M-O-S, M-S, C-O-S, and R-O-S schemes, for $Q = N_0 B$, $I = 1$ W.

5.2.4 Impact of μ_{ss} and μ_{sp}

Fig. 9 shows the achievable capacity of the secondary service versus μ_{ss}/μ_{sp} in the case of the O-S, M-O-S, M-S, C-O-S, and R-O-S schemes for a given imposed interference $I = 1$ and the interference threshold $Q = N_0 B$. Here, we set $\mu_{sp} = 10$.

As it is seen, in all schemes by increasing μ_{ss}/μ_{sp} the corresponding achievable capacity is also increased. This is because of the strengthening impact of g_{ss}. Notice that the rate of increment in the case of C-O-S and R-O-S schemes is larger than O-S, M-O-S and M-S schemes. Moreover, for small enough μ_{ss}/μ_{sp}, O-S, M-O-S and M-S achieve higher capacity comparing to C-O-S and R-O-S. In this case, M-S results in a higher achievable capacity. For large enough μ_{ss}/μ_{sp}, however, C-O-S has the highest achievable capacity. Similar results can also be anticipated for the case that μ_{sp} is changed.

5.3 Miss Detection Probability

In Fig. 10, the miss detection probability of the spectrum sensing is plotted versus I for given collision probability constraint (Q, ξ). As it is observed, for a small value of I, only the C-O-S scheme is able to access the whole busy state of the spectrum, i.e., $\delta = 1$. In C-O-S due to small received interference I, as it is also seen in Fig. 5, the secondary service is able to gain a meaningful gain on the capacity performance in the contrary to the other schemes. By increasing I, however C-O-S cannot adaptively decrease the miss detection probability; therefore, the achievable capacity is significantly decreased. Similar behavior with higher degree of achievable capacity reduction is also observed in the R-O-C scheme. In the contrary, M-S-O decreases the miss detection performance by increasing I, therefore it gains the highest achievable capacity among other schemes in the high interference regime.

Considering the results in Fig. 5 and 10 one may also propose utilizing a combination of the C-O-S and M-O-S schemes as follows. For a given collision probability constraint (Q, ξ), if $C^{C-O-S} > C^{M-O-S}$, choose the C-O-S scheme; otherwise utilize the M-O-S scheme. Utilizing such approach, we expect that for both small and large values of I, higher achievable capacity is obtained comparing to the other schemes.

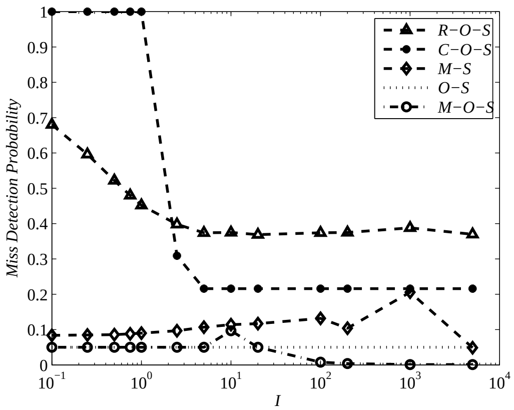

Fig. 10. Miss Detection Probability *vs.* I for O-S, M-O-S, M-S, C-O-S, and R-O-S schemes, for $Q = N_0 B$ and $\xi = 0.05$.

6 Conclusions

In this paper a novel approach was proposed to obtain the optimal ROC operating point of spectrum sensing in overlay spectrum sharing system. The objective was to maximize the secondary service achievable capacity subject to the primary service collision probability as well as other system and service constraints. In addition to conventional One-Shot (O-S) scheme, we also propose four different approaches to solve the optimization problem including *Modified-One-Shot (M-O-S) scheme*, *Multi-Shot (M-S) scheme*, *Conservative-Modified-One-Shot (C-O-S) scheme*, and *Restricted-Modified-One-Shot (R-O-S) scheme*. We then investigated the performance of the proposed schemes through numerical studies. Although the O-S scheme performed well for moderate values of the average (over fading) interference at the primary receiver I, its corresponding achievable capacity was significantly degraded in cases where I was either very low or very high. Moreover, the proposed M-S scheme quickly converged. It was also seen that the M-O-S scheme overcome the fast capacity decreasing slop which was seen in high values of I in the O-S scheme. The M-S, C-O-S, and R-O-S schemes also improved the secondary service capacity for small values of I. The C-O-S scheme suggested a peak power allocation constraint for the secondary service transmission power where it adaptively changed through the collision probability constraint. In the contrary, the R-O-S prepared an average over transmission power of the secondary service regarding the collision probability constraint. Our studies show that comparing to the cases with very low miss detection probability, utilizing the proposed formulation results in achieving even higher secondary service capacity. Furthermore, our studies suggested that for very low, moderate and very high values of I, the proper schemes were C-O-S, M-S, and M-O-S, respectively.

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