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Jumps and stochastic volatility in crude oil futures prices using conditional moments of integrated volatility

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Abstract

We evaluate alternative models of the volatility of commodity futures prices based on high-frequency intraday data from the crude oil futures markets for the October 2001–December 2012 period. These models are implemented with a simple GMM estimator that matches sample moments of the realized volatility to the corresponding population moments of the integrated volatility. Models incorporating both stochastic volatility and jumps in the returns series are compared on the basis of the overall fit of the data over the full sample period and subsamples. We also find that jumps in the returns series add to the accuracy of volatility forecasts. (JEL: G13, Q41)

Key words: stochastic volatility, commodity futures prices, crude oil futures

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1. Introduction

The volatility of commodity futures prices has become a topic of increasing interest in recent years for academic researchers, practitioners and those involved with the regulation of derivatives markets. Many commodity futures markets have become increasingly 'financialized' over the past decade as financial firms with no inherent exposure to the commodity have adopted a strategy of portfolio diversification into commodity futures as an asset class.

Although this trend has affected many commodity futures markets, it has had a marked impact on one of the most important markets: that for derivatives of crude oil, which is now the most heavily traded commodity futures contract by volume. Crude oil, as a key global commodity, has experienced considerable price level variation in the boom preceding the global financial crisis in 2008 and the ensuing Great Recession. A major oil price shock in 2008 was caused by constraints on the production of crude oil paired with low elasticity of demand (for details, see Hamilton (2009) and Kilian (2009)). This shock, while being caused by fundamentals, was clearly exacerbated by financial speculation and 'financialization' of commodities. Variation in oil price levels has been accompanied by wide variations in the volatility of returns. In the futures markets, returns exhibit heavy tails, autocorrelation, and volatility clustering, leading to significant challenges in modeling their first and second moments.

Both the International Monetary Fund (IMF) and the Federal Reserve Board (see Alquist et al. (2011) and IMF 2005 p. 67; 2007, p. 42) use futures prices as the best available proxy for the market expectations of the spot crude oil price.

Like many financial series, commodity futures prices are likely to exhibit random-walk behavior. Such behavior in crude oil futures prices implies that a model of prices or returns is not likely to beat the naïve model. However, even if returns are not forecastable, their volatility may be successfully modeled. In this paper, we employ various models of stochastic volatility in order to analyze the *uncertainty* of crude oil futures returns and to evaluate the forecastability of their volatility. The empirical analysis makes use of high-frequency (tick-by-tick) data from the futures markets, first aggregated to 10-minute intervals during the trading day. The intraday variation is then utilized to generate daily time series of prices, returns and realized volatility.

Our sample period of October 2001 to December 2012 is characterized by high frequency fluctuations and fat tails. This is an appropriate setting for our investigation of the role of jumps (modelled as *extreme events*). Before performing any model estimation, we employ non-parametric methods to identify the periods when these *extreme events* might have occurred. Our empirical findings are in line with these test results indicating a very high volatility during 2008.

The high frequency data allows us to test various models for oil futures returns using a straightforward Generalized Method of Moments (GMM) estimator that matches sample moments of the realized volatility to the corresponding population moments of the integrated volatility in the spirit of Bollerslev and Zhou (2002). These models are then compared, in terms of overall fit of the data and forecast accuracy statistics, over the full sample. The model with stochastic volatility and jumps is also tested over a subsample (January 2006–December 2012) to address structural stability (as in

Andersen, Benzoni and Lund (2002)). Key findings include the importance of both jumps and stochastic volatility in oil futures returns and the apparent unimportance of leverage as a modeled component.

The wider applicability of this method of estimation to other markets is outside the scope of this paper, but an interesting topic for future research.

2. Review of the literature

Schwartz (1997), Schwartz and Smith (2000), Casassus and Collin-Dufresne (2005) propose multi-factor models for energy prices where returns are only affected by Gaussian shocks only, but constrain volatility to be constant. Pindyck (2004) examines the volatility of energy spot and futures prices, estimating the standard deviation of their first differences. Askari and Khrichene (2008) fit jump-diffusion models to futures on Brent crude oil. Schwartz and Trolle (2009) propose a multifactor stochastic volatility model for pricing futures and options on light sweet crude oil trading on the NYMEX. Using daily data, they present evidence that taking account of stochastic volatility improves pricing, but they consider the inclusion of jumps to be less important. Vo (2009) estimates a multivariate stochastic volatility model using daily data on the West Texas Intermediate (WTI) crude oil futures contracts traded on the NYMEX and finds that stochastic volatility plays an important role.

Larsson and Nossman (2011) find evidence for stochastic volatility and jumps in both returns and volatility daily spot prices of WTI crude oil from 1989 to 2009.

The role of volatility as a measure of uncertainty of oil price futures is stressed by Bernanke (1983), Pindyck (1991) and Kellogg (2010) who show that this measure of uncertainty is extremely relevant for firms' investment decisions.

Our contribution lies in the use of the information on volatilty of oil futures returns provided by high frequency, intra-day data while focusing on the role of volatility as measure of variability and uncertainty of oil price forecasts.

3. Data description

We exploit the distributional information embedded in high-frequency (10-minute interval) intraday futures price quotations on crude oil in order to test for the presence of stochastic volatility and jumps in crude oil futures returns.

Light, sweet crude oil (West Texas Intermediate) began futures trading on the New York Mercantile Exchange (NYMEX) in 1983 and is the most heavily traded commodity future. Crude oil futures trade in units of 1,000 U.S. barrels (42,000 gallons), with contracts dated for 30 consecutive months plus long-dated futures initially listed 36, 48, 60, 72, and 84 months prior to delivery. Additionally, trading can be executed at an average differential to the previous day's settlement prices for periods of two to 30 consecutive months in a single transaction. Crude Oil Futures are quoted in dollars and cents per barrel.

The raw data used in this study are 10-minute aggregations of crude oil futures contract transactions-level data provided by TickData, Inc. For each 10-minute interval during the day trading session and for each traded contract, the open, high, low, close prices are recorded, along with the volume of trades in that interval. For the purpose of computing returns, the trading session's close price and the following trading session's close price are used to produce an estimated overnight (or over-the-weekend) return.

Industry analysts have noted that to avoid market disruptions, major participants in the crude oil futures market roll over their positions from the near contract to the next-near contract over several days before the near contract's expiration date. A continuous price series over contracts, which expire monthly, is created by hypothetically rolling over a position from the near contract to the next-near contract three days prior to expiration of the near contract.

The returns series and the realized volatility measures are displayed in Figure 1 and their descriptive statistics are given in Table 1. Both series exhibit excess kurtosis, while the realized volatility series has a large skewness coefficient. The Kolmgorov–Smirnov test for normality rejects its null for both series, while the Shapiro–Francia test (1972) for normality concurs with those judgements. The Box–Pierce portmanteau (or Q) test for white noise rejects its null for both series. The daily returns series exhibits significant ARCH effects at 1, 5, 10 and 22 lags, while no evidence of ARCH effects is found in the realized volatility series.

4. Estimation method

Following Bollerslev and Zhou (2002), who use continuously observed futures prices on oil, we build a conditional moment estimator for stochastic volatility jump-diffusion models based on matching the sample moments of realized volatility with population moments of integrated volatility. In this context, as Andersen and Benzoni (2008) have suggested, realized volatility serves as a non-parametric ex post estimate of the variation in returns. In this paper, realized volatility is computed as the sum of high-frequency (10-minute interval) intraday squared returns.

4.1. No-jump case

The returns on futures at time t over the interval $[t-k, t]$ can be decomposed as

$$r(t,k) = \ln F_t - \ln F_{t-k} = \int_{t-k}^{t} \mu(\tau)\, d\tau + \int_{t-k}^{t} \sigma(\tau)\, dW_\tau$$

The quadratic variation or integrated variance, which coincide in the no-jump case, can be expressed as

$$QV(t,k) = IV(t,k) = \int_{t-k}^{t} \sigma^2(\tau)\, d\tau$$

In discrete time, the corresponding sample realized variance (RV) can be described as

$$RV(t,k,n) = \sum_{j=1}^{n\cdot k} r\left(t-k+\frac{j}{n}, \frac{1}{n}\right)^2$$

$$RV(t,k,n) \longrightarrow^{p} IV(t,k) \quad \text{as } n \longrightarrow \infty$$

where n is the sampling frequency of 33 intervals per day when we derive the daily RV.

4.2. Integrated volatility and jumps

When we allow for discrete jumps, the returns on futures at time t over the interval $[t-k, t]$ can be decomposed as

$$
\begin{aligned}
r\left(t,k\right) &= \ln F_t - \ln F_{t-k} \\
&= \int_{t-k}^{t} \mu\left(\tau\right) d\tau + \int_{t-k}^{t} \sigma\left(\tau\right) dW_{\tau} + \int_{t-k}^{t} x\left(\tau\right) dN\left(\lambda\tau\right)
\end{aligned}
$$

In this case, integrated variance and quadratic variation do not coincide:

$$
\begin{aligned}
IV_{jumps}\left(t,k\right) &= \int_{t-k}^{t} \sigma^2\left(\tau\right) d\tau + \sum_{t-k\leq s\leq t} \left(x\left(s\right) dN\left(\lambda s\right)\right)^2 \\
&= QV\left(t,k\right) + \sum_{t-k\leq s\leq t} \left(x\left(s\right) dN\left(\lambda s\right)\right)^2
\end{aligned}
$$

Barndorff-Nielsen and Shephard (2004) proposed the Realized Bipower Variation as a consistent estimate of integrated volatility component in the presence of jumps:

$$
BV\left(t,k;n\right) = \tfrac{\pi}{2} \textstyle\sum_{i=2}^{n\cdot k} \left| r\left(t-k+\tfrac{ik}{n}, \tfrac{1}{n}\right)\right| \left| r\left(t-k+\tfrac{(i-1)k}{n}, \tfrac{1}{n}\right)\right|
$$

$$
\begin{gathered}
RV\left(t,k,n\right) - BV\left(t,k;n\right) \longrightarrow QV(t,k) - IV(t,k) \\
QV(t,k) - IV(t,k) = \sum_{t-k\leq s\leq t} \left(x\left(s\right) dN\left(\lambda s\right)\right)^2 \\
\text{as } n \longrightarrow \infty
\end{gathered}
$$

4.3. Data filtering

Given the high autocorrelation of the square root of the realized variance (RV) series, the analysis in this paper is performed on moving averages of the daily realized volatility series, using arithmetic weights over the current trading session and three previous trading sessions:

$$
MA4RV_t = 0.4RV_t + 0.3RV_{t-1} + 0.2RV_{t-2} + 0.1RV_{t-3}
$$

As we can notice from Figure 2 and Figure 3 , the autocorrelation in the first differences of the daily realized volatility is reduced significantly after applying the moving average transformation to the data.

5. Estimation results

We estimated four forms of the stochastic model: the basic (SV) model, the SV model incorporating leverage (in which the volatility is influenced by the level of returns), (SVLev); the SV model incorporating jumps in the returns process (SVJ), and the SV model incorporating both leverage and jumps in the return process (SVLevJ). As we discuss below, there is no empirical support for leverage, in that the parameter expressing the effect of leverage is never significantly different from zero. Thus, we present here our findings from the SV and SVJ models.

5.1. Stochastic Volatility model (SV)

We model the returns on futures on crude oil using the Heston (1993) model. For simplicity, we set the drift of the log price equal to zero.[2] This choice is consistent with Alquist et al. (2011) who find that a reasonable and parsimonious forecasting model for spot oil prices is the random walk without drift.

[2]As Bollerslev et al. (2002) suggest, a drift could be easily introduced in the futures returns equation.

$$
\begin{aligned}
dp_t &= d\ln(F_t) \\
&= \sqrt{V_t} dW_{1t} \\
dV_t &= \kappa\left(\theta - V_t\right) dt + \sigma\sqrt{V_t} dW_{2t} \\
E\left(dW_{1t} dW_{2t}\right) &= 0
\end{aligned}
$$

In this model, there are two orthogonal Wiener processes, dW_{1t} and dW_{2t}, driving the evolution of returns and volatility. Three estimated parameters appear in the model: κ, θ and σ.

We estimated this model over the full sample, imposing the six moment conditions implied by the model in the GMM procedure. As there are six moment conditions and three estimated parameters, there are three overidentifying restrictions that may be used to evaluate the model. As shown in Table 3, all six moment conditions are in accordance with the data, and the Hansen's J statistic indicates that the overidentifying restrictions are valid. The three estimated parameters of the model are very precisely estimated and take on sensible values from an analytical perspective.

5.2. Stochastic Volatility model with jumps in returns (SVJ)

$$
\begin{aligned}
dp_t &= d\ln(F_t) \\
&= \sqrt{V_t} dW_{1t} + x dPoisson\left(\lambda t\right) \\
dV_t &= \kappa\left(\theta - V_t\right) dt + \sigma\sqrt{V_t} dW_{2t}
\end{aligned}
$$

$$
E\left(dW_{1t} dW_{2t}\right) = 0
$$

$$x \sim N\left(0, \sigma_x^2\right)$$

In this extended model, the same two orthogonal Wiener processes appear, augmented by a Poisson process that captures jumps in returns. This gives rise to two additional parameters, λ and σ_x, governing the effects of the jump process.

λ is the indicator of the frequency of the jumps: it tells us, on average, how many times we have extreme events (jumps in this case for us are *extreme events*) within the sample. It is the parameter of the Poisson counting process that takes values:

$$\begin{cases} 1 & \text{when an extreme event happens} \\ 0 & \text{otherwise} \end{cases}$$

The Normally distributed, mean zero random variable x represents the magnitude of the jumps in returns, with the intensity of jumps controlled by the σ_x^2 parameter. The timing of jumps is a Poisson process, with parameter λ representing the mean and variance of that process. For jumps in returns to play a significant role in the model, both parameters must be significantly different from zero (and positive).

We estimated this model over the full sample, imposing the eight moment conditions implied by the data in the GMM procedure. As there are eight moment conditions and five estimated parameters, there are three overidentifying restrictions that may be used to evaluate the model. As shown in Table 4, six moment conditions are in accordance with the data while two of them are marginally rejected and the overall Hansen's J statistic indicates that the overidentifying restrictions are valid. All five estimated parameters of the model are very precisely estimated and take on sensible values from an analytical perspective.

In order to better motivate the concept of jumps in the futures returns process, we employ non-parametric methods to identify those periods when "*extreme events*" may have occurred. Following Tukey (1977), we consider extreme events to be those periods when the one-trading-day change in futures returns lay outside the bounds of the "adjacent values" of a conventional box plot. The adjacent values are defined using 1.5 times the inter-quartile range (IQR), or difference between the empirical 75th and 25th percentiles (p75, p25) of the series. The upper bound is defined as $p75 + 1.5$ IQR, while the lower bound is defined as $p75 - 1.5$ IQR.

When the one-trading-day changes defined by this criterion are computed for the full 2001-2012 sample of 2,864 trading days, we find 84 extreme events, as illustrated in Figure 6. That compares reasonably with the implications of our model's estimates. The parameter estimate $\widehat{\lambda} = 0.027722$ implies that 79 jumps should occur over the sample period. Our graphical method allows the 84 extreme events to be labeled and measured.

The longer sample (October 2001–December 2012) allows us to achieve accuracy of the estimates of the strongly persistent volatility parameters (as in Andersen, Benzoni and Lund (2002)) while our estimates reported in Table 6 show that jumps are also statistically significant when considering a smaller sample (January 2006–December 2012). This experiment allows us to assess the structural stability of the model as within the shorter sample λ, the frequency of extreme events, is significantly different from zero.

5.3. Does leverage matter?

As suggested by Alquist et al. (2011), there is no reason why oil producers should be concerned about the volatility of the price of oil. The data seem to suggest that there is no connection between the shocks affecting futures prices and the shocks affecting the corresponding volatility. In the financial asset pricing literature, when the so-called “leverage effect” is widely supported by the data, stock prices and volatility usually move in opposite directions.

Both the SV and SVJ models may be extended to incorporate a leverage effect, which introduces an additional parameter ρ, reflecting the importance of returns in the volatility equation. We estimated each of those extended models, and found values of ρ that could not be distinguished from zero at conventional significance levels. For brevity, we do not tabulate those estimates here. We therefore conclude that there is no evidence of a leverage effect in crude oil futures prices and returns.

6. Evaluating the performance of volatility forecasts

To evaluate the in-sample forecast performance of our stochastic volatility models, we simulated each estimated model using the point estimates displayed in Tables 3 and 4. Each simulation was repeated 100 times with new draws from the Normal distribution for the W_1, W_2 and x processes and from the Poisson distribution for the λ process. Descriptive statistics for realized volatility and the forecast series from the SV and SVJ models are presented in Table 7. As is evident, these dynamic forecasts do quite well at reproducing the mean of realized volatility, while exhibiting less variation than the observed series. In particular, although the filtered realized volatility series exhibit considerable skewness and kurtosis, the forecast series' third and fourth moments are not nearly as large.

Two measures of forecast accuracy were computed for each calendar year, 2002–2012: the root mean square forecast error (RMSE) and the mean absolute forecast error (MAE), each based on the differences between the forecasted values and actual values of realized volatility. These measures are based on the averages over the simulated forecast values. Figure 4 illustrates the RMSE values for each calendar year for the SV and SVJ models. As is apparent, the SVJ model produces a modest increase in forecast accuracy. It is also apparent that the forecast accuracy varies widely over the sample, with RMSE values considerably higher during 2003 and rising during the onset of the financial crisis in 2007–2008. The dotted line on the figure illustrates the average closing price of crude oil futures over the period. There is a weak negative correlation of -0.17 between the RMSE values and average futures prices.

Figure 5 illustrates similar statistics for the mean absolute error (MAE) criterion. On the basis of this criterion, the SVJ model, allowing for jumps in returns, also produces a modest improvement in forecast accuracy over the simpler SJ model. The variation in annual forecast accuracy is even greater for MAE than for RMSE, with marked deterioration in forecast accuracy during the onset of the financial crisis. In contrast, before and after the crisis, the volatility forecasts are considerably more accurate. Each series has a weak positive correlation of around $+0.20$ with the average closing price of crude oil futures.

7. Conclusions

We find that stochastic volatility models are effective in fitting the volatility of oil price futures returns. We find significant evidence of jumps in returns, and conclude that SV models incorporating jumps are more effective than models that do not take jumps into account. This conforms to the econometric evidence which suggests that the simple SV model is misspecified by omitting the statistically significant jump parameter. In-sample forecasting performance of models with jumps increases when kurtosis is high.

This result is also in line with our findings from a non-parametric method for extreme events where we identify the high volatility characterizing oil futures returns particularly around the year 2008 when a major oil shock took place.

Although this analysis is only a first step toward developing a deeper understanding of the movements of volatility of crude oil futures prices and returns, these findings are promising indications that analytically-based models of these important series are capable of capturing their salient characteristics.

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Table 1: Descriptive statistics for NYMEX Crude oil futures: October 2001–December 2012

October 2001–December 2012	returns	realized volatility
number of observations	2864	2864
maximum	5.1874	1.8519
minimum	−6.4649	.0004
mean	.0054	.0398
standard deviation	.2980	.0571
skewness	−0.7475	13.4236
kurtosis	128.1334	369.0318
Kolmogorov–Smirnov test (p)	0.0924 (0.000)	0.2509 (0.000)
Shapiro–Francia W' test for normality (p)	15.187 (0.000)	16.861 (0.000)
Portmanteau (Q) test for white noise (p)	81.66 (0.000)	9874.6 (0.000)
ARCH test, 1 lag (p)	56.084 (0.000)	0.962 (0.327)
ARCH test, 5 lags (p)	260.239 (0.000)	6.472 (0.263)
ARCH test, 10 lags (p)	398.555 (0.000)	6.907 (0.734)
ARCH test, 22 lags (p)	463.858 (0.000)	8.752 (0.995)

Table 2: Descriptive statistics for NYMEX Crude oil futures: January 2006–December 2012

January 2006–December 2012	returns	realized volatility
number of observations	1793	1793
maximum	1.814744	.4527819
minimum	−1.301274	.0004428
mean	.002283	.0322889
standard deviation	.2388315	.0427161
skewness	.0672244	4.132848
kurtosis	7.48734	25.49913
Kolmogorov–Smirnov test (p)	0.0559(0.000)	0.2462(0.000)
Shapiro–Francia W' test for normality (p)	9.445(0.000)	14.816(0.000)
Portmanteau (Q) test for white noise (p)	76.8734(0.000)	27919.8909(0.000)
ARCH test, 1 lag (p)	45.292(0.000)	285.709(0.000)
ARCH test, 5 lags (p)	217.345(0.000)	600.170(0.000)
ARCH test, 10 lags (p)	342.640(0.000)	657.012(0.000)
ARCH test, 22 lags (p)	400.061(0.000)	747.498(0.000)

Table 3: GMM estimates for SV model: 10/2001–12/2012

Moment conditions / Parameters	estimate	p-values
$E\left[\mathcal{V}_{t+1,t+2}\middle\| \mathcal{G}_t\right] - \mathcal{V}_{t+1,t+2}$	0.121083	0.6269
$E\left[\mathcal{V}^2_{t+1,t+2}\middle\| \mathcal{G}_t\right] - \mathcal{V}^2_{t+1,t+2}$	-0.061095	0.1826
$E\left[\mathcal{V}_{t+1,t+2}\mathcal{V}_{t-1,t}\middle\| \mathcal{G}_t\right] - \mathcal{V}_{t+1,t+2}\mathcal{V}_{t-1,t}$	-0.054228	0.1943
$E\left[\mathcal{V}^2_{t+1,t+2}\mathcal{V}_{t-1,t}\middle\| \mathcal{G}_t\right] - \mathcal{V}^2_{t+1,t+2}\mathcal{V}_{t-1,t}$	0.109775	0.4754
$E\left[\mathcal{V}_{t+1,t+2}\mathcal{V}^2_{t-1,t}\middle\| \mathcal{G}_t\right] - \mathcal{V}_{t+1,t+2}\mathcal{V}^2_{t-1,t}$	-0.047680	0.2656
$E\left[\mathcal{V}^2_{t+1,t+2}\mathcal{V}^2_{t-1,t}\middle\| \mathcal{G}_t\right] - \mathcal{V}^2_{t+1,t+2}\mathcal{V}^2_{t-1,t}$	-0.047577	0.2321
κ	0.071388	0.0000
θ	0.037591	0.0000
σ	0.064702	0.0000
J-statistic (3 df)	3.2332	0.3571

Table 4: GMM estimates for stochastic volatility model with jumps: 10/2001–12/2012

Moment conditions / Parameters	value	p-value
$E\left[V_{t+1,t+2}\middle\| G_t\right] - V_{t+1,t+2}$	0.000057	0.2481
$E\left[V^2_{t+1,t+2}\middle\| G_t\right] - V^2_{t+1,t+2}$	0.000037	0.1141
$E\left[V_{t+1,t+2}V_{t-1,t}\middle\| G_t\right] - V_{t+1,t+2}V_{t-1,t}$	0.000010	0.2015
$E\left[V^2_{t+1,t+2}V_{t-1,t}\middle\| G_t\right] - V^2_{t+1,t+2}V_{t-1,t}$	0.0000050	0.2124
$E\left[V_{t+1,t+2}V^2_{t-1,t}\middle\| G_t\right] - V_{t+1,t+2}V^2_{t-1,t}$	0.000000	0.8701
$E\left[V^2_{t+1,t+2}V^2_{t-1,t}\middle\| G_t\right] - V^2_{t+1,t+2}V^2_{t-1,t}$	0.000000	0.9205
$E\left[p_{t+1}\middle\| G_t\right] - p_{t+1}$	-0.000349	0.0739
$E\left[p^2_{t+1}\middle\| G_t\right] - p^2_{t+1}$	-0.000106	0.0816
κ	0.069872	0.000
θ	0.036719	0.000
σ	0.059591	0.000
λ	0.027722	0.0002
σ_x	0.284183	0.000
J-statistic (3 df)	4.4676	0.2152

Table 5: GMM estimates for SV model: 01/2006–12/2012

Moment conditions / Parameters	estimate	p-values
$E\left[\mathcal{V}_{t+1,t+2}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}_{t+1,t+2}$	0.072063	0.2493
$E\left[\mathcal{V}^2_{t+1,t+2}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}^2_{t+1,t+2}$	0.005898	0.2218
$E\left[\mathcal{V}_{t+1,t+2}\mathcal{V}_{t-1,t}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}_{t+1,t+2}\mathcal{V}_{t-1,t}$	−0.002365	0.4795
$E\left[\mathcal{V}^2_{t+1,t+2}\mathcal{V}_{t-1,t}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}^2_{t+1,t+2}\mathcal{V}_{t-1,t}$	0.049214	0.0392
$E\left[\mathcal{V}_{t+1,t+2}\mathcal{V}^2_{t-1,t}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}_{t+1,t+2}\mathcal{V}^2_{t-1,t}$	0.003468	0.3556
$E\left[\mathcal{V}^2_{t+1,t+2}\mathcal{V}^2_{t-1,t}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}^2_{t+1,t+2}\mathcal{V}^2_{t-1,t}$	−0.001151	0.5141
κ	0.044182	0.0004
θ	0.030416	0.0000
σ	0.050505	0.0000
J-statistic (3 df)	6.0091	0.1112

Table 6: GMM estimates for stochastic volatility model with jumps: 01/2006–12/2012

Moment conditions / Parameters	value	p-value
$E\left[\mathcal{V}_{t+1,t+2}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}_{t+1,t+2}$	0.000090	0.1713
$E\left[\mathcal{V}^2_{t+1,t+2}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}^2_{t+1,t+2}$	0.000052	0.0429
$E\left[\mathcal{V}_{t+1,t+2}\mathcal{V}_{t-1,t}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}_{t+1,t+2}\mathcal{V}_{t-1,t}$	0.000006	0.2199
$E\left[\mathcal{V}^2_{t+1,t+2}\mathcal{V}_{t-1,t}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}^2_{t+1,t+2}\mathcal{V}_{t-1,t}$	0.000003	0.3800
$E\left[\mathcal{V}_{t+1,t+2}\mathcal{V}^2_{t-1,t}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}_{t+1,t+2}\mathcal{V}^2_{t-1,t}$	−0.000003	0.4311
$E\left[\mathcal{V}^2_{t+1,t+2}\mathcal{V}^2_{t-1,t}\middle\vert\,\mathcal{G}_t\right]-\mathcal{V}^2_{t+1,t+2}\mathcal{V}^2_{t-1,t}$	−0.000001	0.4708
$E\left[p_{t+1}\middle\vert\,\mathcal{G}_t\right]-p_{t+1}$	−0.000036	0.1159
$E\left[p^2_{t+1}\middle\vert\,\mathcal{G}_t\right]-p^2_{t+1}$	−0.000002	0.1159
κ	0.043718	0.0005
θ	0.029839	0.0000
σ	0.048854	0.0000
λ	0.008076	0.0332
σ_x	0.146824	0.0000
J-statistic (3 df)	5.7855	0.1225

Table 7: Forecast summary statistics for SV, SVJ models, October 2001–December 2012

	mean	std.dev.	min.	max.	skewness	kurtosis
Filtered realized volatility	.0398104	.0436735	.0029656	.8551645	6.021913	73.34864
SV model forecast	.0383487	.0039587	.028021	.0797176	1.965016	16.55122
SVJ model forecast	.0373399	.0037802	.0277054	.0797176	2.468978	21.77815
Observations	2846					

Figure 1: Futures Returns and Smoothed Realized Volatility, October 2001–December 2012

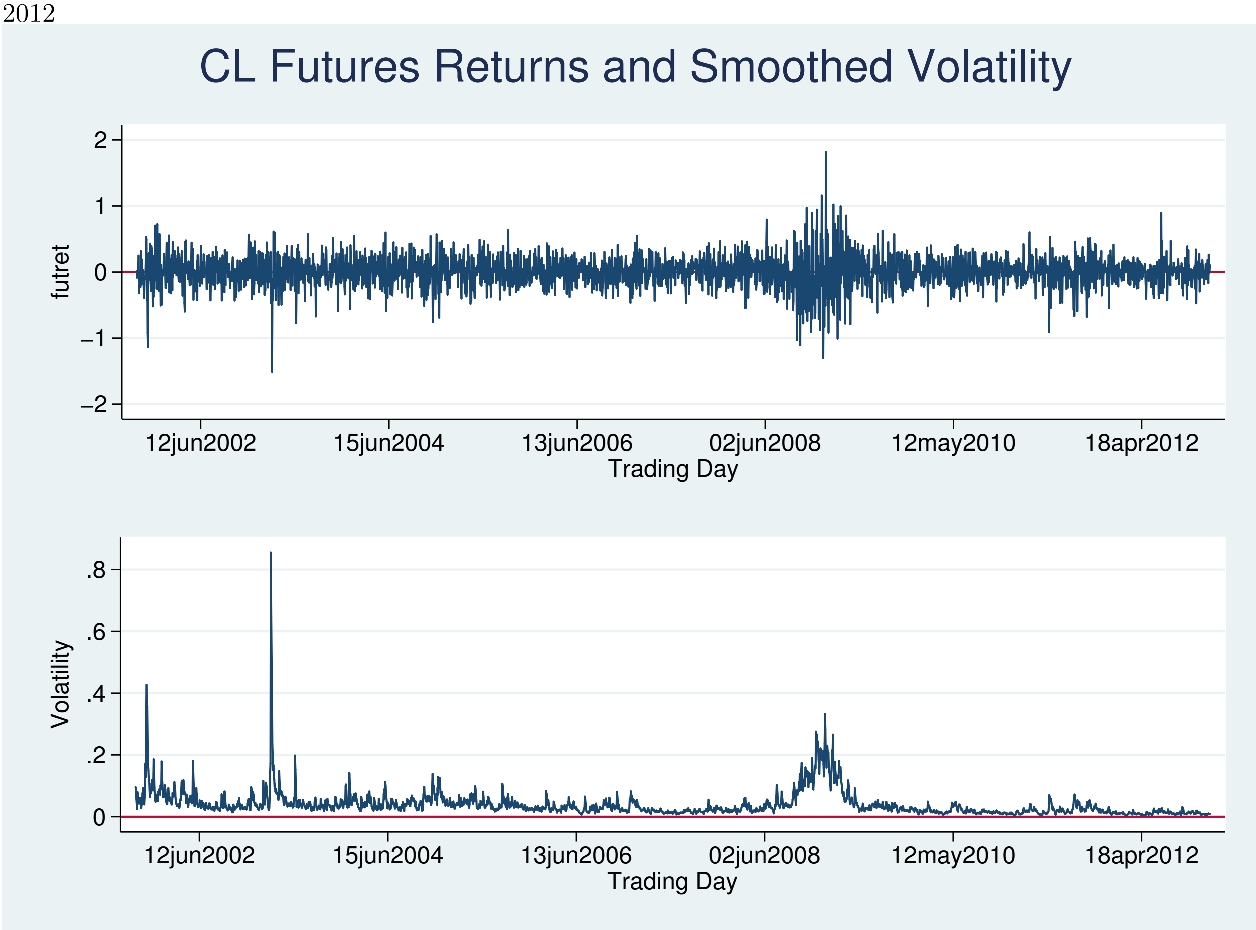

Figure 2: Autocorrelations of first differences of daily realized volatility, October 2001–December 2012

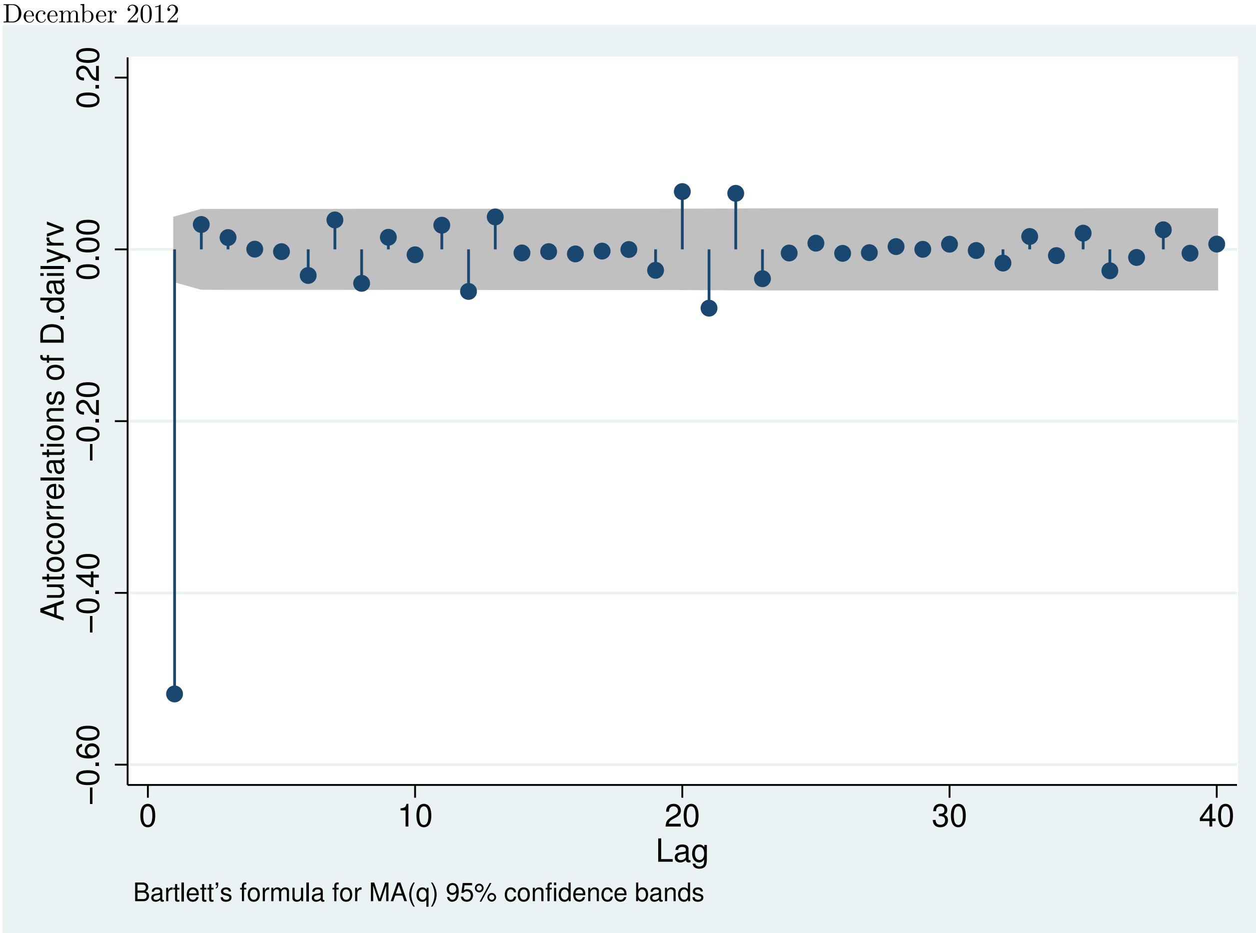

Figure 3: Autocorrelations of first differences of moving average daily realized volatility, October 2001–December 2012

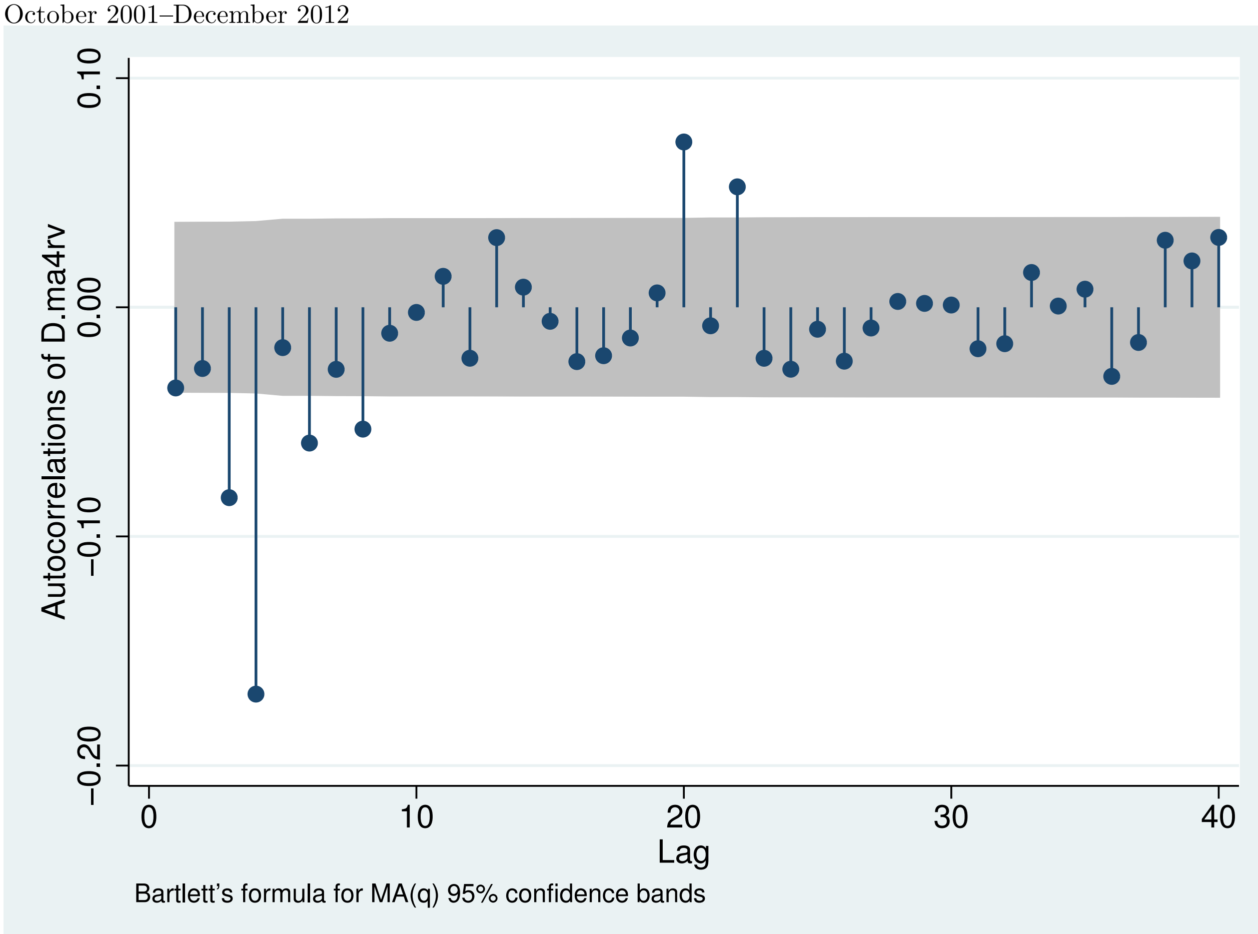

Figure 4: Root mean squared error (RMSE) of volatility forecasts from SV, SVJ models, annual averages 2002–2012

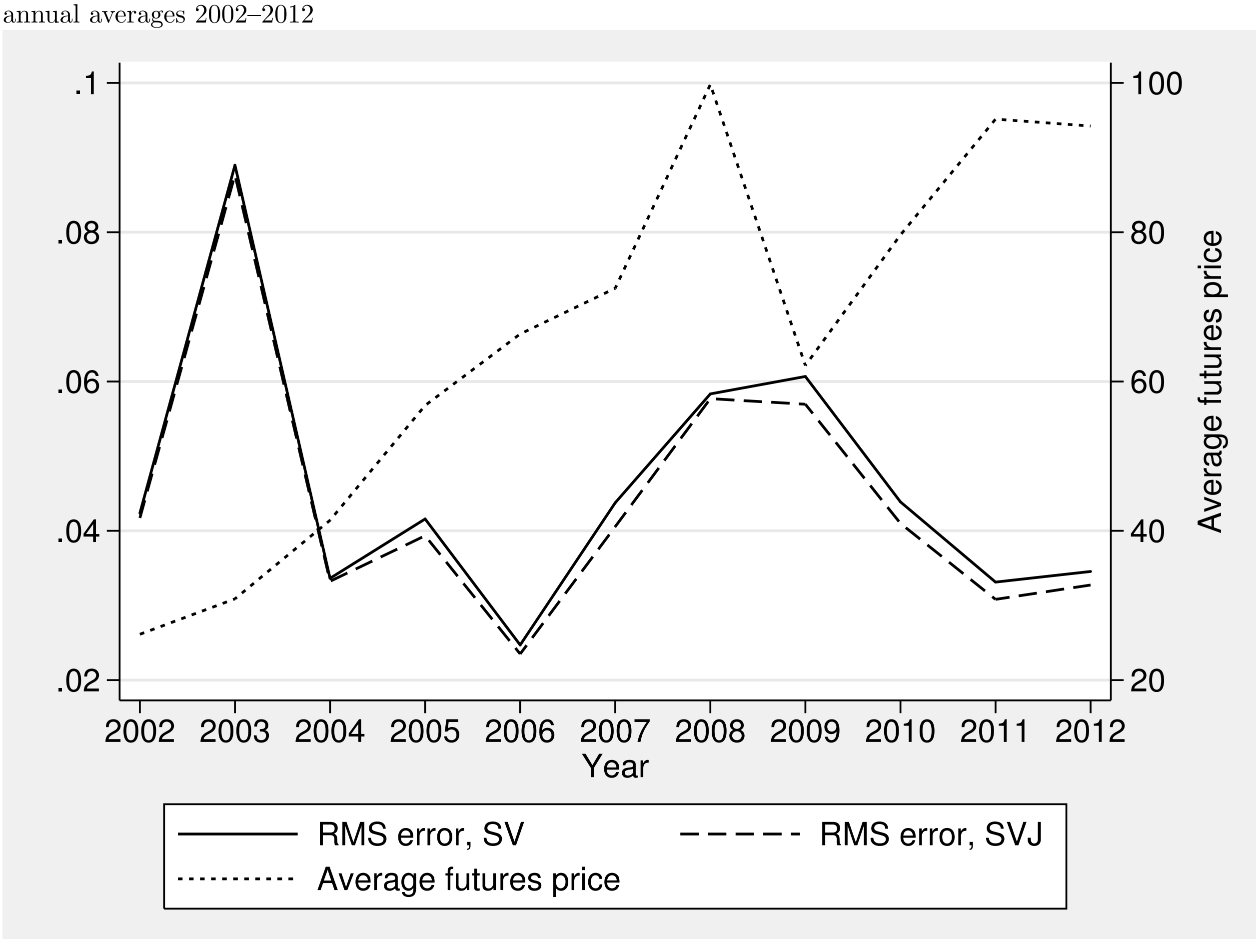

Figure 5: Mean absolute error (MAE) of volatility forecasts from SJ, SVJ models, annual averages 2002–2012

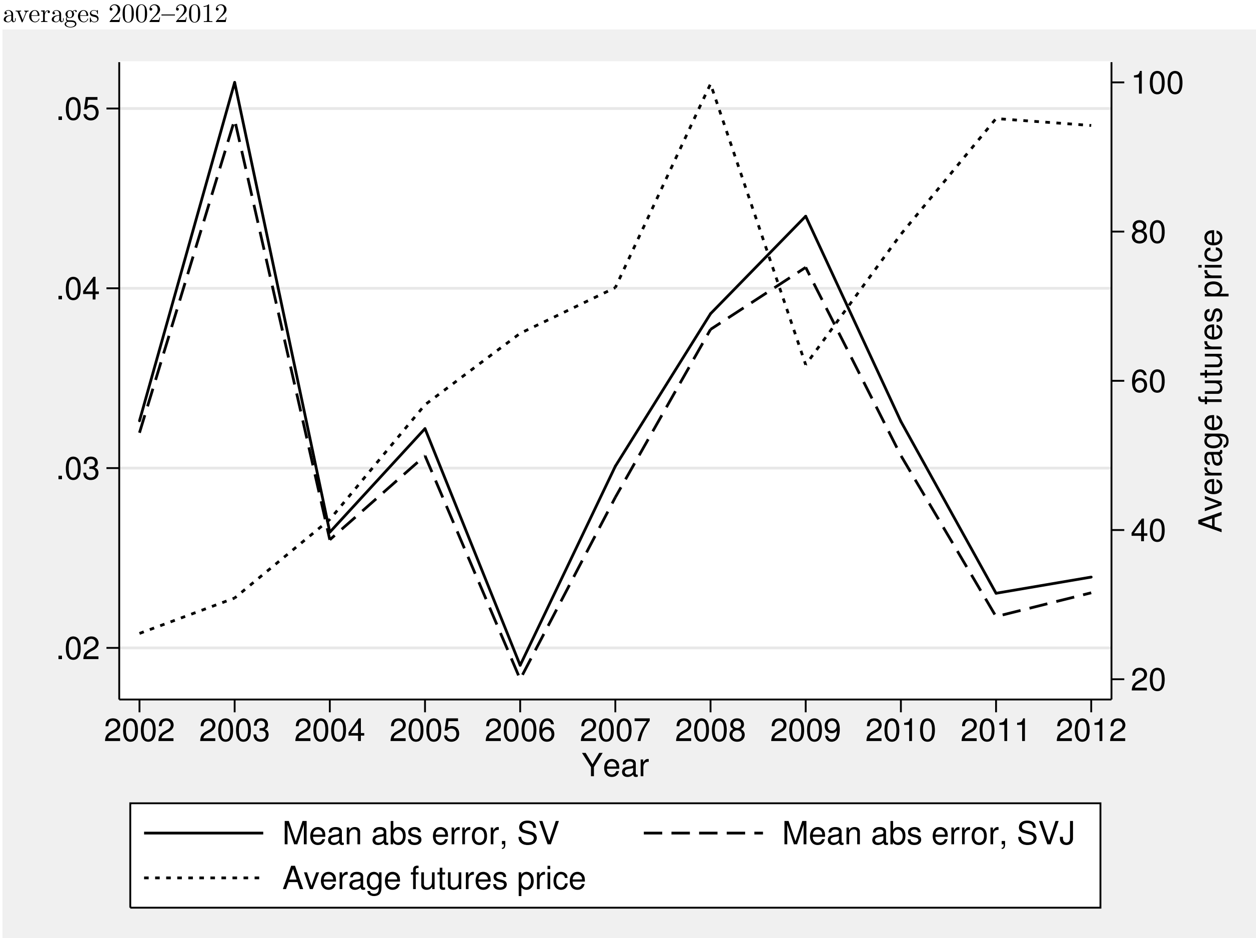

Figure 6: Test for 'extreme events' affecting the oil futures series (following Tukey (1977)), October 2001–December 2012