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CONTROL OF FLEXIBLE MANIPULATOR SYSTEMS

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Abstract

This paper presents an investigation into the development of open-loop and closed-loop control strategies for flexible manipulator systems. Shaped torque inputs, including gaussian shaped and lowpass (Butterworth and elliptic) filtered input torque functions, are developed and used in an open-loop configuration and their performance studied in comparison to a bang-bang input torque through experimentation in a single-link flexible manipulator system. Closed-loop control strategies using both collocated (hub angle and hub velocity) and non-collocated (end-point acceleration) feedback are then proposed. A collocated proportional-derivative (PD) control is first developed and its performance studied through experimentation. The collocated control is then extended to incorporate, additionally, non-collocated feedback through a proportional-integral-derivative (PID) configuration. The performance of the hybrid collocated and non-collocated control strategy thus developed is studied through experimentation. Experimental results verifying the performance of the developed control strategies are presented and discussed.

Key words: *Flexible manipulator, open-loop control, closed-loop control, PD control, PID control.*

CONTENTS

1 Introduction

The term 'control', in general, can characterise an open-loop strategy based on a more or a less accurate model of the system under investigation or a feedback law making use of the control deviation. In some cases, it may also characterise a combination of these two categories; i.e., an open-loop strategy for the gross motion with an underlying feedback accounting for small deviations.

Two main approaches can be distinguished when considering the control of flexible manipulator systems. The first approach involves the development of a mathematical model through computation of the necessary geometric, kinematic or kinetic quantities on the basis of assuming rigid body structure. In adopting such an approach, an investigation to reveal the accuracy of the identified parameters is required, so that a satisfactory model is obtained. Alternatively, necessary measurements to yield information on the deflections have to be carried out in addition to the movements of the joints. The second approach accounts, in addition to the factors in the first approach, for deviations caused by the elastic properties of the manipulator and thus requires additional measurements, for example by strain gauges, optical sensors, accelerometers, etc. These measurements are to compensate for deviations caused by elasticities and thus are used to improve the control performance.

Due to the elastic properties of the system, the development of a mathematical description and subsequent control of a flexible manipulator is a complicated task. A considerable amount of basic research has been carried out on the modelling and control of flexible manipulators for the last two decades. The control problem, to achieve high performance, is to acquire the ability to dampen the oscillations of the structure. This is made difficult by the presence of a large (infinite) number of modes of vibration in the structure which are, in general, lightly damped. The modes become significant in two ways. Firstly, because the oscillations themselves prolong the settling time or, equivalently, give greater dynamic errors. Secondly, attempts to actively control some modes result in instability of other (generally high-frequency) modes, referred to as observation and control spillover.

Vibration control techniques for flexible manipulator systems are generally classified into two categories: passive and active control. Active control utilises the principle of wave interference. This is realised by artificially generating anti-source(s) (actuator(s)) to destructively interfere with the unwanted disturbances and thus result in reduction in the level of vibrations. Active control of flexible manipulator systems can in general be divided into two categories: open-loop and closed-loop control. Open-loop control involves altering the shape of actuator commands by considering the physical and vibrational properties of the flexible manipulator system. The approach does not account for changes in the system once the control input is developed. Closed-loop control differs from the open-loop control in that it uses measurements of the system's state and change the actuator input accordingly to reduce the system oscillation.

Passive control utilises the absorption property of matter and thus is realised by a fixed change in the physical parameters of the structure, for example adding viscoelastic materials to increase the damping properties of the flexible manipulator. Furthermore, it has been reported that the control of vibration of a flexible manipulator by passive means is not sufficient by itself to eliminate structural deflection (Book et al., 1986). On the other hand, if only active control is used, then due to actuator and sensor dynamics destabilization of modes near the bandwidth of the actuator or sensor may result (Aubrun, 1980; Aubrun and Margulies, 1982). To avoid such destabilization a certain amount of passive damping will be required to be employed, thus using hybrid control, i.e., a combination of active and passive control methods. Combined active/passive control strategies have been proposed previously where low-frequency modes of vibration are controlled by active means and the modes with frequencies just above the actively controlled modes are controlled by passive means (Lane, 1984; Lane and Dickerson, 1984; Plunkell and Lee, 1970).

Several methods of passive vibration control of flexible manipulator systems have been developed over the years. These mainly include methods of implementation of a constrained viscoelastic damping layer to provide energy dissipation medium (Lane, 1984; Lane and Dickerson, 1984; Plunkell and Lee, 1970; Lane and Dickerson, 1983; Kerwin, 1959; Nashif and Nicholas, 1970) and the utilisation of composite materials in the construction of a flexible manipulator to provide higher strength and stiffness-to-weight ratios and large structural damping than a metallic flexible manipulator (Plunkell and Lee, 1970; Alberts et al., 1986; Choi and Ghandhi, 1988; Choi et al., 1990a; Choi et al., 1990b; Thompson and Sung, 1986; Trovik, 1980; Tzou, 1980). Observations have shown that although passive damping provides a sharp increase in damping at higher frequency modes, the lower frequency modes still remain uncontrolled. Moreover, the addition of viscoelastic material and constraining layer leads to an increase in the size and dynamic load of the system (Tzou, 1987; Tzou, 1988).

Recently, open-loop control methods have been considered in vibration control where the control input is developed by considering the physical and vibrational properties of the flexible manipulator system. Although, the mathematical theory of open-loop control is well established, few successful applications in the control of distributed parameter flexible manipulator systems have been reported (Athans and Ealb, 1966; Cesari, 1983; Citron, 1969; Dellman et al., 1956; Lee, 1960; Saga and White, 1977; Singh et al., 1989). The method involves development of suitable forcing functions so that to reduce the vibrations at resonance modes. The methods developed include shape command methods, computed torque technique and bang-bang control. The shaped command methods attempt to develop forcing functions that minimise residual vibrations and the effect of parameters that affect the resonance modes (Aspinwall, 1980; Meckl and Seering, 1985; Meckl and Seering, 1985; Meckl and Seering, 1987; Meckl and Seering, 1988; Meckl and Seering, 1990; Singer and Seering, 1988; Singer and Seering, 1989; Singer and Seering, 1990; Singer and Seering, 1992; Swigert, 1980; Wang, 1986). Common problems of concern encountered in these methods include long move (response) time, instability due to un-reduced modes and controller robustness in case of large change of the manipulator dynamics.

In the computed torque approach, depending on the detailed model of the system and desired output trajectory, the joint torque input is calculated using a model inversion process (Alberts et al., 1990; Bayo, 1988; Bayo and Moulin, 1989; Moulin and Bayo, 1991). The technique suffers from several problems, due to, for instance, model inaccuracy, uncertainty over implementability of the desired trajectory, sensitivity to system parameter variation and response time penalties for a causal input.

Bang-bang control involves the utilisation of single and multiple switch bang-bang control functions (Onsay and Akay, 1991). Bang-bang control functions require accurate selection of switching time, depending on the representative dynamic model of the system. Minor modelling error could cause switching error and thus result in a substantial increase in the residual vibrations (Sangveraphunsiri, 1984). Although, utilisation of minimum energy inputs has been shown to eliminate the problem of switching times that arise in the bang-bang input (Jayasuriya and Choupra, 1991), the total response time, however, becomes longer (Meckl and Seering, 1990; Onsay and Akay, 1991).

Effective control of a system always depends on accurate real-time monitoring and the corresponding control effort. Initial discussions of the feedback control of a flexible manipulator and the usefulness of optimal regulator as applied to this problem date back to the early 1970s (Neto, 1972). It is known in the conventional approach that compensation can alter the first vibrational mode by either adding some damping or extending the bandwidth of the system (Ogata, 1970). Compensation, however, will limit the performance of the manipulator because inputs with frequency contents above the first vibrational mode could still cause vibration. Various modern control designs have been proposed during the last two decades for flexible manipulator systems with different types of vibration measuring systems.

When the free motion of a system consists mainly of a limited number of clearly separable modes then it is possible to control these modes directly using the so called independent modal space control (IMSC) method, where the controller is designed for each mode independent of other modes (Baz and Poh, 1990; Baz and et al., 1989; Baz et al., 1992; Gould and Murray-Lasso, 1966; Lindberg and Longman, 1984; Meirovitch and

Baruh, 1982; Sinha and Kao, 1991). The modal space control has been used for the suppression of the manipulator's flexible motion in a three-link log loading manipulator with which considerable improvement has been achieved over the conventional joint-based collocated controller (Karkkainen and Halme, 1985). Although, initial investigations at the use of IMSC lack a consideration of the location of the actuator (Meirovitch et al., 1983), later investigations have shown that the actuator placement is important for the suppression of spillover and, thus, methods for the optimal placement of sensors and actuators have been developed (Lindberg and Longman, 1984; Kondoh et al., 1990; Omatu and Seinfeld; 1986; Schulz and Heimbold, 1983).

An appreciable amount of work carried out in the control of flexible manipulator systems involves the utilisation of strain gauges, mainly to measure mode shapes (Sangveraphunsiri, 1984; Hastings and Book, 1985; Hastings and Ravishankar, 1988; Sakawa et al., 1985). There are two essential components involved in measuring the modal response using strain gauges. The first is a method of measurement of the modes of vibration of the flexible manipulator. The second is the development of a computational technique for distinguishing the different modes in the overall deflection of the flexible manipulator. Once the modal information is available a control loop can be closed for each mode either to damp or to actively drive the manipulator in a manner which reduces the vibration. It appears that the strain gauge measurement is very simple and relatively inexpensive to use. However, the technique may place more stringent requirements on the dynamic modelling and control tasks. Strain gauges have the disadvantage of not giving a direct measurement of manipulator displacement, as they can only provide local information. Thus, displacement measurement by using strain gauges requires more complex and possibly time consuming computations which can lead to inaccuracies (Hastings, 1988).

To solve the problem due to displacement measurement, as encountered in using strain gauges only, attempts have been made to develop schemes that incorporate end-point measurements as well (Cannon and Schmitz, 1984; Kotnik et al., 1988; Schmitz, 1985). Some researchers have proposed an approach which utilises local or global measurement of the flexible displacement of the manipulator to control the system vibration (Harishima and Ueshiba, 1986; Ramasrishnan, 1985; Wang, et al., 1989). In this method the deflection of the manipulator is detected (measured), using, for example, CCD camera or laser beam, relative to a rotating reference frame X-Y fixed to the hub of the manipulator. However, as an end-point position control system has smaller stability margins than a collocated control, it is necessary to include a collocated rate feedback (hub velocity) to obtain acceptable performance of the closed-loop system (Schmitz, 1985). By using the end-point sensor, more accurate end-point positioning can be accomplished, but the resulting controller is less robust to plant uncertainties than the corresponding collocated design.

The difficulty in maintaining stability and performance robustness, due to the spillover effects from unmodelled modes that occur when a high-order system is controlled by a low-order controller, is of major concern in the control of flexible systems. To improve robustness it is typically required that the controller bandwidth be sufficiently reduced (Nesline and Zarchan, 1984). Studies have shown that most robust control techniques that ensure stability in the presence of parameter errors can only increase damping by a limited amount (Darato, 1987; Kosul et al., 1983). If the inherent damping is very low, this increase may be insufficient to adequately improve the response. Moreover, the controllers rely on accurate system models. This makes the controller very sensitive to modelling errors, leading to a degradation in system performance and, in some cases, instability. It is evident that, in using either global or local displacement measurement a device is required to be attached on the manipulator, affecting the behaviour of the manipulator (Mace, 1991).

Both feedforward and feedback control structures have been utilised in the control of vibration of flexible manipulator systems (Dougherty et al., 1982; Dougherty et al., 1983; Hendrichfreise et al., 1987; Rattan et al., 1990; Shchuka and Goldenberg, 19891 Wells and Schueller, 1990; Yamada, Nakagawa, 1985). These include combined feedforward and feedback methods based on control law partitioning schemes which use end-point position signal in an outer loop controller to control the flexible modes and the inner loop to control the rigid body motion independent of the dynamics of the manipulator. Although, the pole-zero cancellation property of the feedforward control speeds up the system response, it increases overshoot and oscillation. However, it is found that, in contrast to many high-order compensators, systems with feedforward control incorporating proportional and derivative (PD) feedback are not highly sensitive to plant parameter variations.

In the investigations carried out on the control of flexible manipulator systems the only non-collocated sensor/actuator pairs that have successfully been employed include the motor torque with either the manipulator strain or global/local end-point position. However, practical realisation of both methods have associated short-term and long-term drawbacks. If a state-space description of the closed-loop dynamics is available, it is possible to use acceleration feedback to stabilise a rigid manipulator (Studenny and Belanger, 1986). Investigations on the control of a flexible manipulator system using acceleration feedback to design the compensator and the end-point position feedback using a design based on a full-state feedback observer have shown that the controller using end-point position feedback exhibits a relatively slow and rough response in comparison with the acceleration feedback controller; the difference becoming more noticeable with increasing slewing angle (Kotnik et al., 1988). Moreover, acceleration feedback produces relatively higher overshoot. The use of acceleration feedback appears to have intuitive appeal from an engineering design viewpoint, particularly due to the relative ease of implementation and low cost. Moreover, in sensing acceleration for control implementation, all sensing and actuation equipment is structure mounted. This implies that issues such as camera positioning or field of view are not of major concern which are important considerations, specifically, in large scale applications such as telerobotics. Furthermore, applications to multi-link flexible manipulators could benefit from such methods to a greater extent. Some researchers have also proposed adaptive control methods to compensate for parameter variations (Chen and Menq, 1990; Feliu et al., 1990; Yang et al., 1991). However, these approaches utilise optical methods of global/local end-point sensing for obtaining the feedback signal.

To tackle the various problems associated with the design approaches discussed above, both open-loop and closed-loop control methods are developed and investigated in this paper. In open-loop control, gaussian shaped input torque and filtered input torque are used. As referred to above, the only non-collocated sensor/actuator pairs that have successfully been employed include the motor torque with either the manipulator strain or global/local end-point position. However, practical realisation of both methods have associated short term and long term drawbacks. It is proposed here to devise a control strategy that uses both the collocated (hub angle and hub velocity) and non-collocated (end-point acceleration) feedback. Initially, a collocated controller is developed. This is then extended to incorporate, additionally, non-collocated feedback.

The single-link flexible manipulator system shown schematically in Figure 1 is utilised to verify the performance of the control strategies experimentally. This consists of an aluminium type flexible manipulator of physical dimensions and characteristics given in Table 1, driven by a high torque printed-circuit armature type motor. The measurement sensors consist of an accelerometer at the end-point of the manipulator for measurement of end-point acceleration, a shaft encoder and a tachometer, both at the hub of the manipulator, for measurement of hub angle and hub velocity respectively and four strain gauges located along the manipulator length. The outputs of these sensors as well as a voltage proportional to the current applied to the motor are fed to an IBM-AT compatible PC through a signal conditioning circuit and an anti-aliasing filter for analysis and calculation of the control signal.

2 Open-loop control

The aim of this investigation is to develop methods to reduce motion induced vibration in flexible manipulator systems during fast movements. The assumption is that the motion itself is the main source of system vibration. Thus, torque profiles which do not contain energy at system natural frequencies do not excite structural vibration and hence require no additional settling time. The procedure for determining shaped inputs that generate fast motions with minimum residual vibration has previously been addressed (Meckl and Seering, 1988; Bayo, 1988). The torque input needed to move the flexible manipulator from one point to another without vibration must have several properties: (a) it should have an acceleration and deceleration phase, (b) it should be able to be scaled for different step motions, and (c) it should have as sharp a cutoff frequency as required. These three properties of the required input torque will allow driving the manipulator system as quickly as possible without exciting the resonances.

In this section two types of open-loop shaped input torque are developed on the basis of extracting the energies around the natural frequencies so that the vibration in the flexible manipulator system is reduced during and after the movement. In the first approach a gaussian shaped input torque is developed and its various properties studied so that to enable selection of a specific torque profile for a particular manipulator system. In the second approach, the extraction of energy at the system resonances is based on filter theory. The filters are used for pre-processing the input to the plant, so that no energy is ever put into the system near its resonances.

2.1 Gaussian shaped torque input

A gaussian shaped input torque, i.e. the first derivative of the gaussian distribution function is examined here. The application of this function has in the past been shown in the form of an acceleration profile, utilised to develop input torque profile through inverse dynamics of the system (Bayo, 1988). The method includes nonlinear terms and imposes heavy computational load (Bayo and Moulin, 1989). Moreover, several problems are associated with the techniques that invert the plant. First, a trajectory must be selected. If the trajectory is impossible to follow, the plant inversion fails to give usable results. Often a poor trajectory is selected to guarantee that the system can follow it, thus defeating the purpose of the input (Bayo, 1988). Second, a detailed model of the system is required which is not easy to obtain for machines. Third, the plant inversion is not robust to variations in the system parameters. The method presented in this paper does not require these complicated processes. The behaviour of the function as an input torque profile for the system is investigated by adopting a much simpler method of developing an input torque profile for a flexible manipulator system. Variation of frequency distribution, duty cycle and amplitude of the gaussian shaped input torque with various parameters are studied. This enables to generate appropriate input trajectory to move the flexible manipulator for a given position with negligible vibration.

The gaussian distribution function can be written as

$$P(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{\left[\frac{-(t-\mu)^2}{2\sigma^2}\right]} \tag{1}$$

where, σ and μ represent the standard deviation and the mean respectively and t is an independent variable. Considering the first derivative of $P(t)$ as a system torque input with t representing time and μ and σ as constants for a given torque input, equation (1) yields the torque $\tau(t)$ as

$$\tau(t) = \frac{(t-\mu)}{\sqrt{2\pi}\sigma^3} e^{\left[\frac{-(t-\mu)^2}{2\sigma^2}\right]} \tag{2}$$

The gaussian shaped torque input thus obtained is shown in Figure 2.

To study the effects of μ and σ on various properties of the driving torque in equation (2), these are varied and the corresponding torque obtained. Figure 3 shows the cutoff frequencies of the gaussian input torque as a function of σ with μ as a parameter. The cutoff frequency is obtained from the autopower spectrum of the developed torque profile, where at the cutoff frequency the power level of the gaussian torque input reduces to two thirds of its peak value. It is noted that for a given value of μ the cutoff frequency increases with a decrease in the value of σ. For a given value of σ, on the other hand, the cutoff frequency increases with an increase in the value of μ. Figure 4 shows the variation of the duty cycle of the gaussian input torque as a function of σ; the duty cycle corresponds to the time between points A and B in Figure 2. This is useful in the selection of the value of σ for an allowed period of movement. Figure 5 shows the variation of the amplitude of the gaussian input torque with the value of σ. This is useful in allowing to keep the maximum amplitude of the developed torque profile within a particular range so that to avoid actuator saturation and structural damage. Note that the last two properties, namely, the duty cycle and amplitude shown in Figures 4 and 5 are independent of μ. With the set of information given in Figures 3, 4 and 5, it is possible to select suitable parameters for the gaussian torque input and generate the torque profile accordingly prior to exciting the manipulator.

To investigate the effectiveness of the gaussian shaped input torque on the performance of the flexible manipulator system, the experimental set-up shown in Figure 6 was utilised. The first and second natural frequencies of the flexible manipulator under consideration, as found experimentally through frequency response measurements, are at 12.016 and 35.397 Hz respectively. A gaussian shaped input torque was thus developed with a cutoff frequency at 10 Hz, $\sigma = 0.15$ and $\mu = 10$. The performance of the manipulator was studied experimentally with this gaussian shaped input torque in comparison to a bang-bang input torque for a similar angular displacement, keeping the peak torque at a similar level for both cases.

Figure 7 shows the bang-bang and gaussian shaped input torques utilised, where the negative sides of both the input torques are scaled down so that to offset friction. It is noted that the gaussian shaped input torque has a smooth start and stopping behaviour. This is an important characteristic for (vibrationless) movement of the system. The bang-bang input torque does not have such a characteristic. The duty cycle for the gaussian shaped input torque is found to be almost double that of the bang-bang input torque. However, as noted in Figure 8 the angular displacement of the manipulator reaches a steady-state level faster with the gaussian shaped input torque than with the bang-bang input torque. Moreover, the movement is much smoother with the gaussian shaped input torque. Due to the properties of the gaussian shaped input torque, high frequency components are reduced in amplitude substantially. This enhances the process of convergence, since the angular correction needs to be performed only once. It is important to mention here that due to system inertia and smooth rise of the gaussian shaped input torque, it takes a few moments before starting the movement of the manipulator.

Figure 9 shows the end-point acceleration. It is noted that the end-point acceleration is remarkably improved with the gaussian shaped input torque. The maximum end-point acceleration is of the order of 10 times smaller with the gaussian shaped input torque as compared to that with the bang-bang input torque. Figure 10 shows the effects of the input torques on the vibration of the flexible manipulator. It is noted that a considerable amount of reduction at vibrating modes is achieved with the gaussian shaped input torque as compared to that with the bang-bang input torque.

It follows from the results presented above that application of a gaussian shaped input torque results in a remarkable improvement in vibration reduction during and after a movement of the manipulator as compared with a bang-bang input torque for similar angular movement. A higher level of improvement is observed in the case of end-point acceleration than the strain gauge response. This is significant as the positional accuracy of the end-point is much more important as the payload is normally fastened to the end-point.

2.2 *Filtered torque input*

The gaussian shaped inputs developed above were aimed at providing faster motions and minimising spectral energy for higher frequencies covering all the systems natural modes. An alternative strategy would be to start with a single cycle of a square wave, which is known to give optimal response, and filter out any spectral energy near the natural frequencies. The filters thus designed are not intended for use with the system in a closed loop.

The filters that will be used are for pre-processing the input to the plant so that no energy is ever put into the system near its resonances. Note that real-time processing requirement is imposed. The simplest method to remove energy at system natural frequencies is to pass the square wave through a low-pass filter. This will attenuate all frequencies above the filter cutoff frequency. The most important consideration is to achieve a steep roll-off rate at the cutoff frequency so that energy can be passed for frequencies nearly up to the lowest natural frequency of the flexible manipulator. There are various types of filter, namely, Butterworth, Elliptic and Chebyshev, which can be employed. Here a Butterworth and an Elliptic filter are used. These filters have the desired low-pass frequency response in magnitude, allow for any desired cutoff rate and are physically realisable.

The magnitude of the frequency response of a Butterworth filter is given by (Jackson, 1989)

$$|H(j\omega)|^2 = \frac{1}{1+(\omega/\omega_c)^{2n}} = \frac{1}{1+\varepsilon(\omega/\omega_p)^{2n}} \tag{3}$$

where, n is a positive integer signifying the order of the filter, ω_c is the filter cutoff frequency (-3db frequency), ω_p is the passband edge frequency and $1/(1+\varepsilon^2)$ is the band edge value of $|H(j\omega)|^2$. Note that $|H(j\omega)|^2$ is monatomic in both the passband and stopband. The order of the filter required to meet an attenuation δ_2 at a specified frequency ω_s (stopband edge frequency) is easily determined from equation (3) as

$$n = \frac{\log[(1/\delta_2^2)-1]}{2\log(\omega_s/\omega_c)} = \frac{\log(\delta_1/\varepsilon)}{\log(\omega_s/\omega_p)} \tag{4}$$

where, by definition, $\delta_2 = 1/\sqrt{1+\delta_1^2}$. Thus, the Butterworth filter is completely characterised by the parameters n, δ_2, ε and the ratio ω_s/ω_p.

Equation (4) can be employed with arbitrary δ_1, δ_2, ω_c and ω_s to determine the required filter order n, from which the filter design is readily obtained. The Butterworth approximation results from the requirement that the magnitude response be maximally flat in both the passband and the stopband. That is, the first $(2n-1)$ derivatives of $|H(j\omega)|^2$ are specified to be equal to zero at $\omega = 0$ and at $\omega = \infty$.

The sharpest transition from passband to stopband for given δ_1, δ_2 and n is achieved by an elliptic filter design. In fact, the elliptic design is optimum in this sense. The magnitude response of an elliptic filter is equi-ripple in both the passband and stopband. The squared magnitude response of an elliptic filter is of the form (Zverev, 1967)

$$|H(j\omega)|^2 = \frac{1}{1+\varepsilon^2 U_n^2(\omega/\omega_c)}$$

where $U_n(\omega)$ is a Jacobian elliptic function of order n and ε is a parameter related to the passband ripple. It is known that most efficient designs occur when the approximation error is equally spread over the passband and stopband. Elliptic filters allow this objective to be achieved easily, thus, being most efficient from the viewpoint of yielding the smallest-order filter for a given set of specifications. Equivalently, for a given order and a given set of specifications, an elliptic filter has the smallest transition bandwidth.

The filter order required for a passband ripple δ_1, stopband ripple δ_2, and transition ratio ω_p/ω_s is given as

$$n = \frac{K(\omega_s/\omega_c)K\left(\sqrt{1-(\varepsilon^2/\delta_1^2)}\right)}{K(\varepsilon/\delta_1)K\left(\sqrt{1-(\omega_p/\omega_s)^2}\right)}$$

where $K(x)$ is the complete elliptic integral of the first kind, defined as

$$K(x) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-x^2\sin^2\theta}}$$

$\delta_2 = 1/\sqrt{1+\delta_1^2}$ and $\delta_1 = 10\log_{10}(1+\varepsilon^2)$. Values of the above integral are given in tabulated from in a number of text books (Dwight, 1957). The phase response of an elliptic filter is more nonlinear in the passband than a comparable Butterworth filter, especially near the band edge.

To study the system performance with filtered torque input a low-pass filtered bang-bang input torque is used in the experimental set-up of Figure 6 and the system response is measured in the form of hub angle and end-point acceleration. A plot of low-pass filtered bang-bang input torque using a Butterworth filter is shown in Figure 11. It appears from

Figure 11 that the spectral energy in the first resonance frequency of the system is reduced significantly. The corresponding hub angle and end-point acceleration with this input torque are shown in Figures 12 and 13 respectively. The spectral density of residual end-point accelerometer signal is shown in Figure 14. It follows from this diagram that all the higher modes are attenuated significantly. Moreover, comparing these with the response using bang-bang torque (Figures 8, 9 and 10) shows significant improvement.

Figure 15 shows the filtered bang-bang input torque using an elliptic low-pass filter. Comparing the spectral density of the input torque in Figure 15 with that using a Butterworth filter (Figure 11) reveals that the energy level in the stop band is much higher for the elliptic filter. Figures 16 and 17 show the flexible manipulator's hub angle and end-point acceleration respectively using the elliptic filtered input torque. The spectral density of the corresponding residual fluctuations (vibrations) at the end-point of the flexible manipulator is shown in Figure 18. It follows from these diagrams that the flexible modes of the system are totally suppressed.

It follows from the performance of the system with the two types of low-pass filtered input torque that better performance is achieved with the Butterworth filter. Comparing the results using low-pass filtered and gaussian shaped input torques reveals that the response time is not cost effective with the low-pass filtered input torque. As an alternative to digital filters, analogue low-pass filters can be utilised. However, with analogue filters the system will require considerably longer time to complete the move. The results above demonstrate that filtered input torque will be favoured specially when the natural frequency of the system is relatively high, permitting a reasonably wide bandwidth for the filtered input. However, for systems with lower natural frequencies the filtered signal bandwidth must be reduced considerably. This will lead to a further increase in the response time. Nevertheless, it has been demonstrated above that satisfactory system performance is achieved with the low-pass filtered input torque at the expense of higher response time. The response time for a given angle can be significantly reduced if some excitation energy is permitted in the input function above the lowest system natural frequency. This can be done by introducing excitation energy above the lowest natural frequency by notching out only the frequencies in the square wave frequency spectrum that correspond to system natural frequencies.

3 Closed-loop control

In this section, closed-loop control strategies that use both the collocated and non-collocated feedback are proposed and developed. A collocated PD control, incorporating hub angle and hub velocity feedback, is initially developed. This is then extended to, additionally, incorporate end-point acceleration feedback through a PID configuration.

3.1 Joint based collocated control

A common strategy in the control of manipulator systems involves the utilisation of PD feedback of collocated sensor signals. Such a strategy is adopted at this stage of the investigation here. A block diagram of the PD controller is shown in Figure 19, where K_p and K_v are the proportional and derivative gains respectively, θ, $\dot{\theta}$ and α represent hub angle, hub velocity and end-point acceleration respectively, R_f is the reference hub angle and A_c is the gain of the motor amplifier. Here the motor/amplifier set is considered as a linear gain A_c, as the set is found to function linearly in the frequency range of interest. To design the PD controller a linear state-space model of the flexible manipulator was obtained by linearizing the system equations of motion of the system. The first two flexible modes of the manipulator were assumed to be dominantly significant. The control signal $u(s)$ in Figure 19 can thus be written as

$$u(s) = A_c\left[K_p\left(R_f(s) - \theta(s)\right) - K_v\dot{\theta}\right]$$

where, s is the Laplace variable. The closed-loop transfer function is, therefore, obtained as

$$\frac{\theta(s)}{R_f(s)} = \frac{K_p H(s) A_c}{1 + A_c K_v \left(s + K_p / K_v\right) H(s)}$$

where, $H(s)$ is the open-loop transfer function from the input torque to hub angle, given by

$$H(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \qquad (5)$$

where, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are the characteristic matrix, input matrix and output matrix of the system respectively and $\mathbf{I}$ is the identity matrix. The closed-loop poles of the system are, thus, given by the closed-loop characteristic equation as

$$1 + K_v (s + Z) H(s) A_c = 0$$

where, $Z = K_p / K_v$ represents the compensator zero which determines the control performance and characterises the shape of root-locus of the closed-loop system. It is well known that theoretically any choice of the gains K_p and K_v assures the stability of the system (Gravarter, 1970). However, in practice this does not hold. This is due to the uncontrolled dynamics of the flexible manipulator, actuator and sensor as well as delays caused by measuring and sampling of feedback signals. The root-locus plots of the closed-loop system for $Z = 1$, 2, 3, 4 and 5 are shown in Figures 20 - 24 for the derivative gain K_v varying from 0 to 1.2. In this process the value of K_p will also vary, to keep Z constant. It follows from these root-locus plots that $Z = 2$ gives the resulting dominant roots with maximum negative real parts.

To study the performance in closed-loop control, the response with the PD controller is compared with the open-loop response using a bang-bang torque input. A step reference input (hub angle) is provided to the system with PD feedback control. The open-loop bang-bang input torque utilised and the corresponding torque input obtained for the closed-loop system achieving the same steady-state angular displacement as the open-loop system are shown in Figure 25. It is noted that the initial torque input with PD control is higher but the total energy input for the specified movement is lower than the open-loop system. Figure 26 shows plots of the hub angle as function of time for the open-loop and closed-loop systems. It is noted that the open-loop system reaches the desired angle in 0.8 seconds. However, oscillations persist even after 2 seconds. Moreover, the overall movement is not smooth. With the PD control the motion is smooth and the desired angle is reached within 1.8 seconds. The hub velocity corresponding to the open-loop and PD controlled systems is shown in Figure 27. For the open-loop system, the hub velocity appears to remain oscillatory throughout the movement to about 4 seconds. With the PD control, however, apart from slight oscillations at the beginning of the motion, the hub velocity settles smoothly at zero in 1.9 seconds. A comparison between the end-point acceleration and the corresponding end-point elastic deflection for the open-loop and PD-controlled systems are shown in Figures 28 and 29. It is noted that with the PD control, oscillations disappear quickly and the system smoothly comes to rest at about 1.8 seconds, whereas, for the open-loop system the response remains oscillatory to about 4 seconds. The energy input to the system, as shown in Figure 30, appears to be much smoother for the PD controlled system as compared to the open-loop system. This is important in such an application. The results presented above demonstrate the significant improvement in system performance with PD control, using hub angle and hub velocity feedback, as compared to the open-loop system.

3.2 Hybrid collocated and non-collocated control

A block diagram of the control structure incorporating a combined collocated and non-collocated controller is shown in Figure 31. The controller design utilises end-point acceleration feedback through a PID control scheme. Moreover, the hub angle and hub velocity feedback are also used in a PD configuration for control of the rigid body motion of the manipulator. The control structure utilised thus comprises of two feedback loops: one using the filtered end-point acceleration as input to a control law, and the other using the filtered hub angle and hub velocity as input to a separate control law. These two loops are then summed to give a command motor input voltage to produce a torque.

Consider first the rigid body control loop in which the hub angle θ and hub velocity $\dot{\theta}$ are the output variables. The open-loop transfer function is obtained using equation (5). To design the controller in this loop a low-pass filter is required for both θ and $\dot{\theta}$ so that the flexible modes are attenuated before reaching the controller input. The appropriate proportional and derivative gains are determined from a root-locus analysis, producing 40dB gain margin and ample phase margin.

The flexible motion of the manipulator is controlled using the end-point acceleration feedback through a PID controller. The transfer function of the flexible manipulator with end-point acceleration as output is obtained using equation (5). The end-point acceleration is fed back through a low-pass filter with a cutoff frequency of 40Hz. The values of proportional, derivative and integral gains are adjusted using the Ziegler-Nichols procedure (Warwick, 1989).

Figure 32 shows the hub angle of the manipulator using the hybrid collocated and non-collocated controller. It is noted that this reaches a steady-state value within 0.5 seconds, i.e. in one thirds of the time with the PD controlled system (Figure 26). The hub velocity with the hybrid controller is shown in Figure 33, where it is noted that the magnitude of the velocity is higher than that with the PD controlled system (Figure 27) but reaches a steady-state in a shorter period of time. In a similar manner as the hub velocity, the magnitude of end-point acceleration is higher with the hybrid controller (Figure 34) and reaches a steady level earlier than the PD controlled system (Figure 28). Comparing Figures 35 and 29 reveals that, similar to the end-point acceleration, the elastic deflection settles down much quicker in the case of the hybrid controller. However, the magnitude of the elastic deflection is relatively higher in this case. As shown in Figures 36 and 25, the control effort, i.e. the torque at the system input, is much higher in the case of the hybrid controller. This can be justified by its quick settling time. Similar to the control effort, the peak energy input to the system, as shown in Figure 37, is also higher with the hybrid controller as compared to the PD controlled system (Figure 30), but settles down quickly.

The use of acceleration feedback for rigid or flexible manipulator control has intuitive appeal from an engineering design viewpoint. Primary advantages include the fact that sensing for control implementations is done with structure mounted devices so that, e.g. camera position or field of view are not issues of concern, as attempted by other researchers, and from a practical implementation viewpoint it is easy and of low cost. Moreover, applications to multi-link flexible manipulators will probably demand use of these advantages to a greater extent. A problem associated with the hybrid collocated and non-collocated controller designed for flexible manipulator systems, however, as noted in Figure 36 is that the control effort at the manipulator input produces a spike at the beginning of the move. This may cause damage to the actuator and/or to the flexible manipulator system itself. This behaviour, therefore, must be taken into account during the implementation of the controller.

The control strategies developed above have been demonstrated to perform well and to a satisfactory level under situations where the characteristics of the system do not change. In a time-varying environment, for example when a manipulator is handling varying payloads, the characteristics of the controller will be required to be updated according to the changes in system characteristics. Such a strategy can be adopted by devising an adaptive control mechanism within the system using the fixed control strategies developed above to incorporate on-line estimation of the system model, controller design and implementation.

4 Conclusion

The development of open-loop and closed-loop control strategies for flexible manipulator systems has been presented and verified within a single-link flexible manipulator system. Open-loop control methods involve the development of the control input by considering the physical and vibrational properties of the flexible manipulator system. The control input is minimise the energy input at system resonances so that system vibrations are reduced. Gaussian shaped and lowpass filtered input torque functions have been developed and investigated in an open-loop control configuration. Remarkable improvement in system response with these control functions has been achieved as compared to a bang-bang input torque. It has also been revealed that, as far as the total response time is concerned, the low-pass filtered input torque technique is not cost effective as compared to the gaussian shaped input torque method.

Two controller design strategies in closed-loop configuration have been presented. A hub angle and hub velocity based collocated controller has been developed and its performance has been assessed in comparison to a bang-bang input torque for similar scale of manipulator movement. The results show that considerable improvement in system performance is achieved with the PD controlled system. A hybrid collocated and non-collocated control strategy based on hub angle and hub velocity feedback for rigid body motion control and the end-point acceleration feedback for flexible motion control has also been developed. Results of such a strategy demonstrate that the system performance improves significantly. The use of acceleration feedback offers several advantages, namely, ease of implementation, raggedness, relatively low cost, and advantages of structure mounted sensing. This latter point is extremely important for extensions of this work to multi-link systems where the use of, for example, cameras may be impractical.

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Table 1: Physical dimensions and characteristics of the flexible manipulator.

Length	$960mm$
Thickness	$3.2004mm$
Width	$19.008mm$
Mass density per volume	$2710\,kg/m^3$
Young's Modulus	$7.11\times10^{10}\,N/m^2$
Area moment of inertia	$5.1924\times10^{-11}\,m^4$
Hub inertia	$5.86\times10^{-4}\,kgm^2$
Manipulator moment of inertia	$0.0495kgm^2$

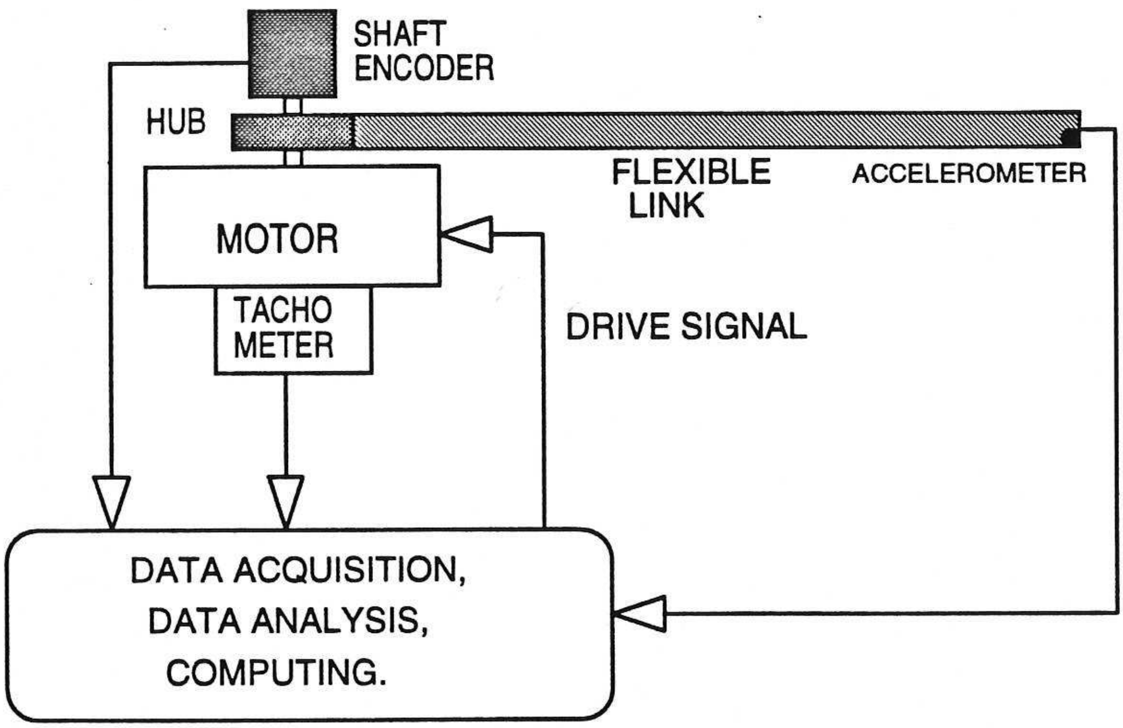


Figure 1: Schematic diagram of the single-link manipulator system.

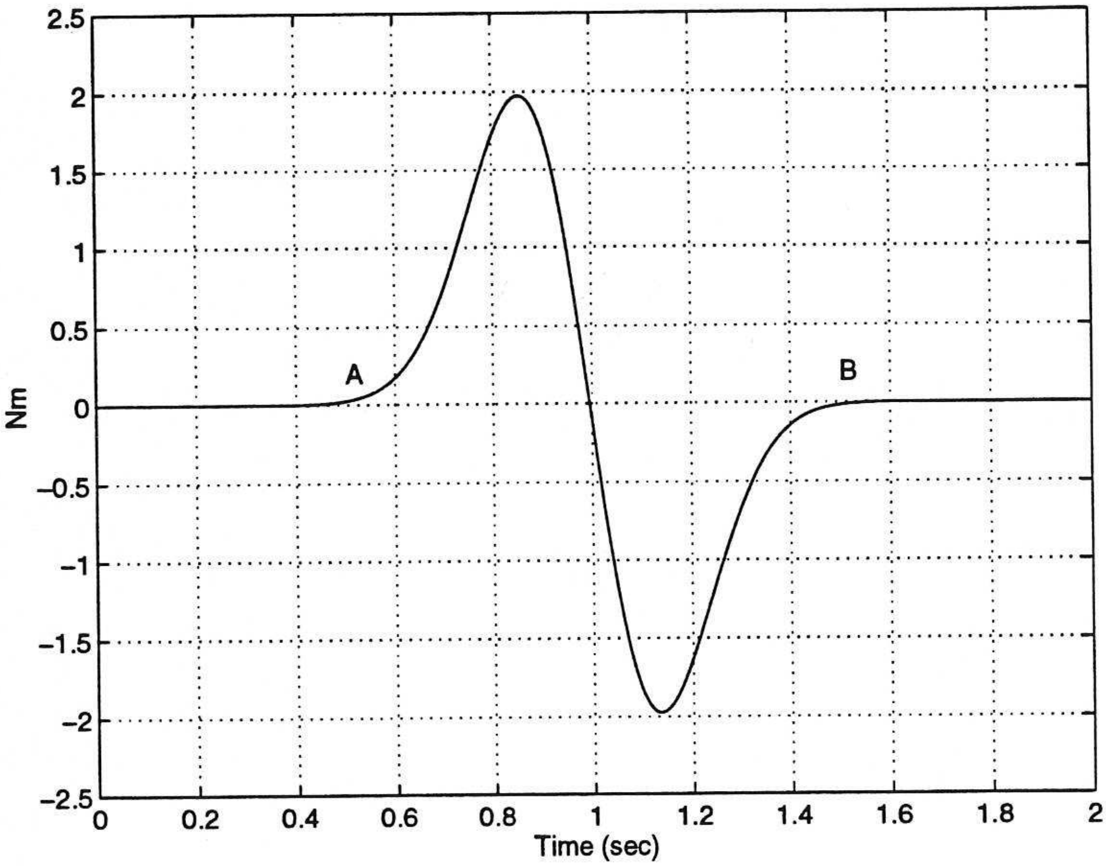


Figure 2: The gaussian shaped torque input.

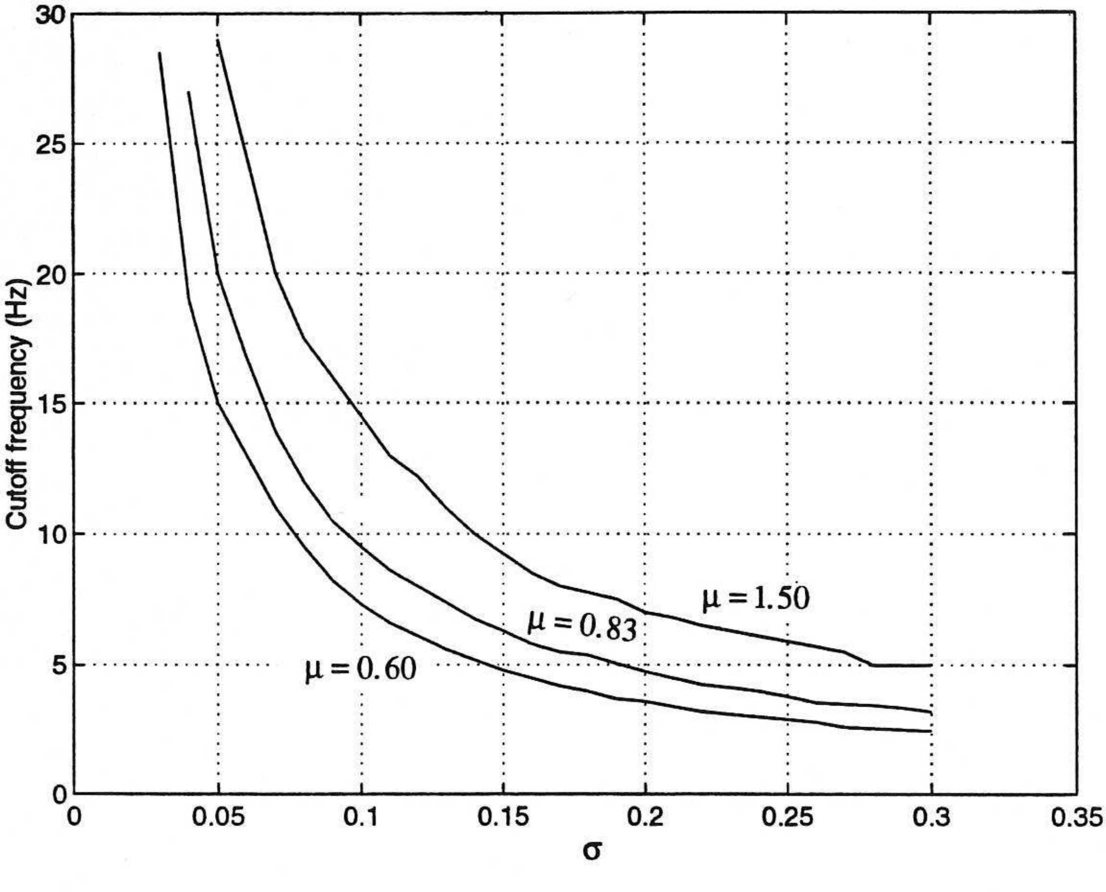


Figure 3: The cutoff frequency of the gaussian shaped torque input as a function of σ.

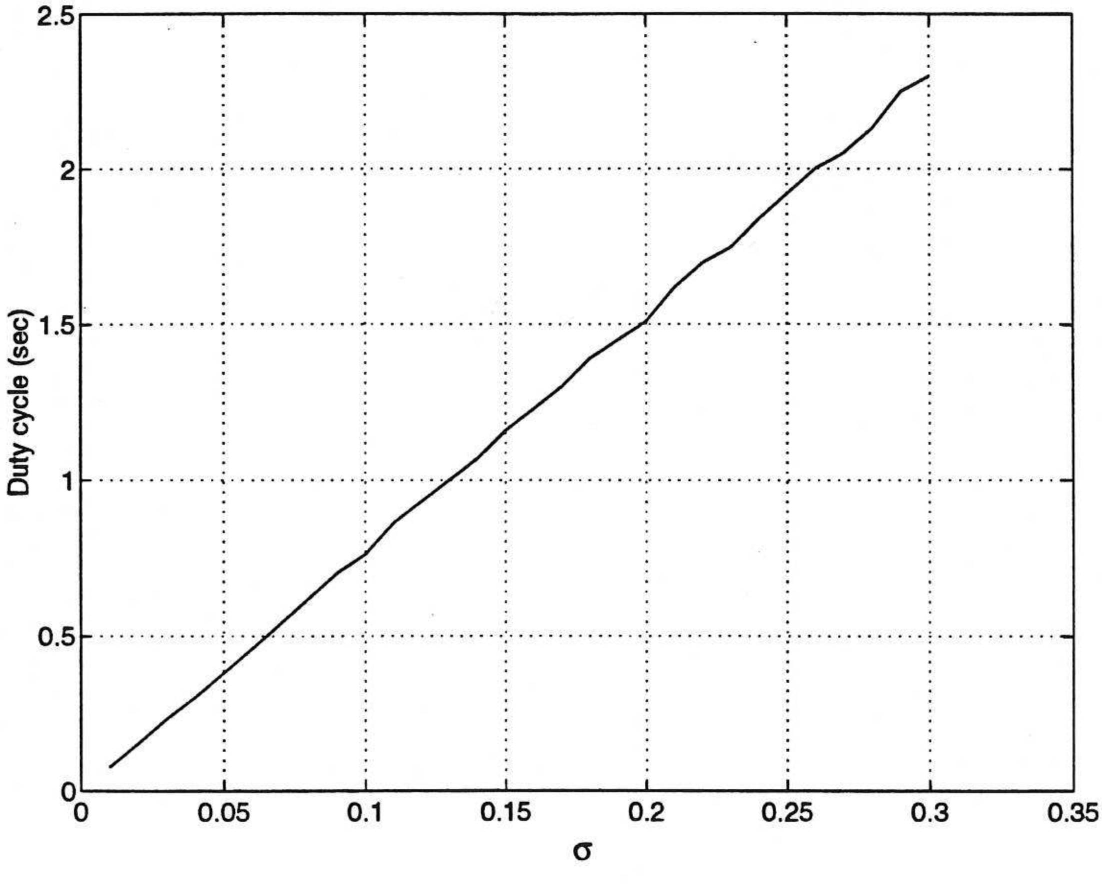


Figure 4: The duty cycle of the gaussian shaped torque input as a function of σ.

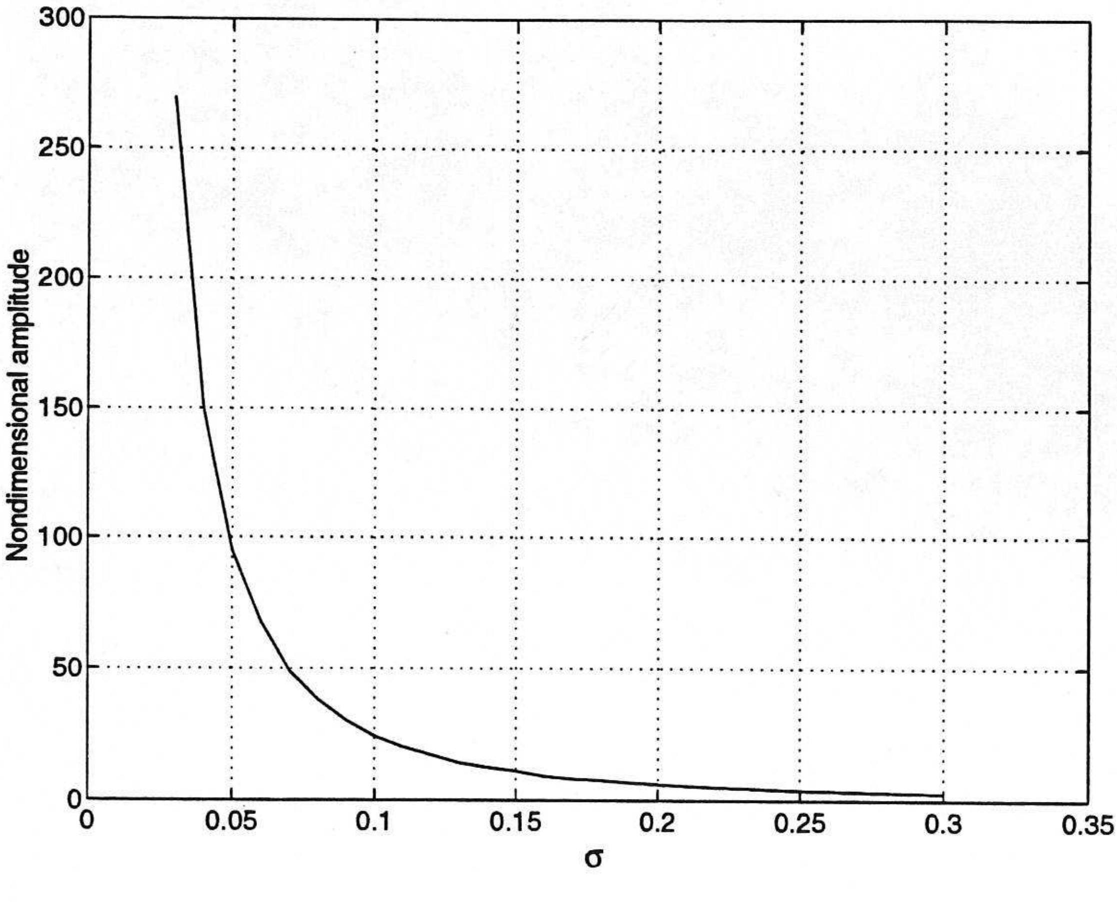


Figure 5: Amplitude of the gaussian shaped input torque as a function of σ.

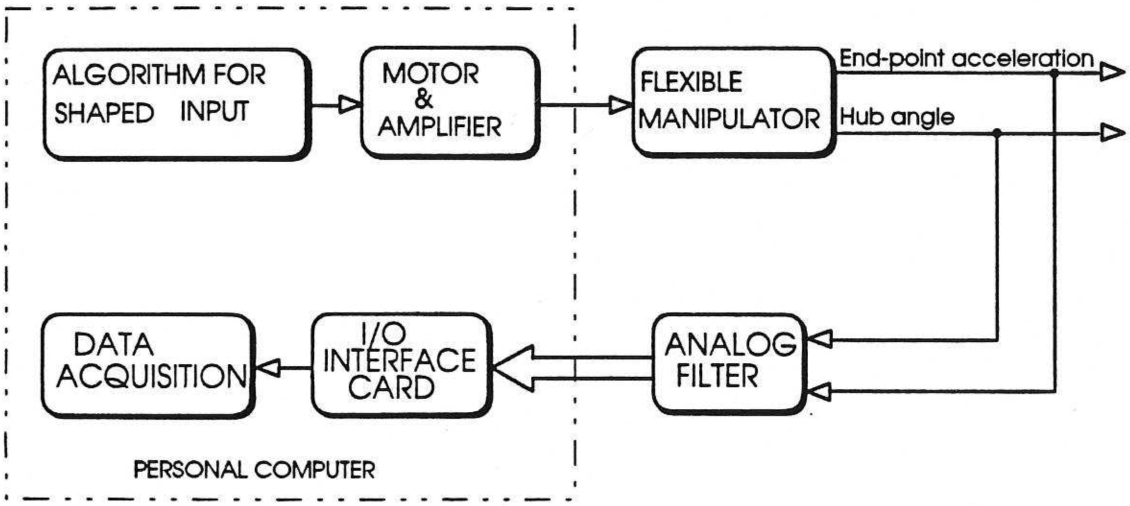
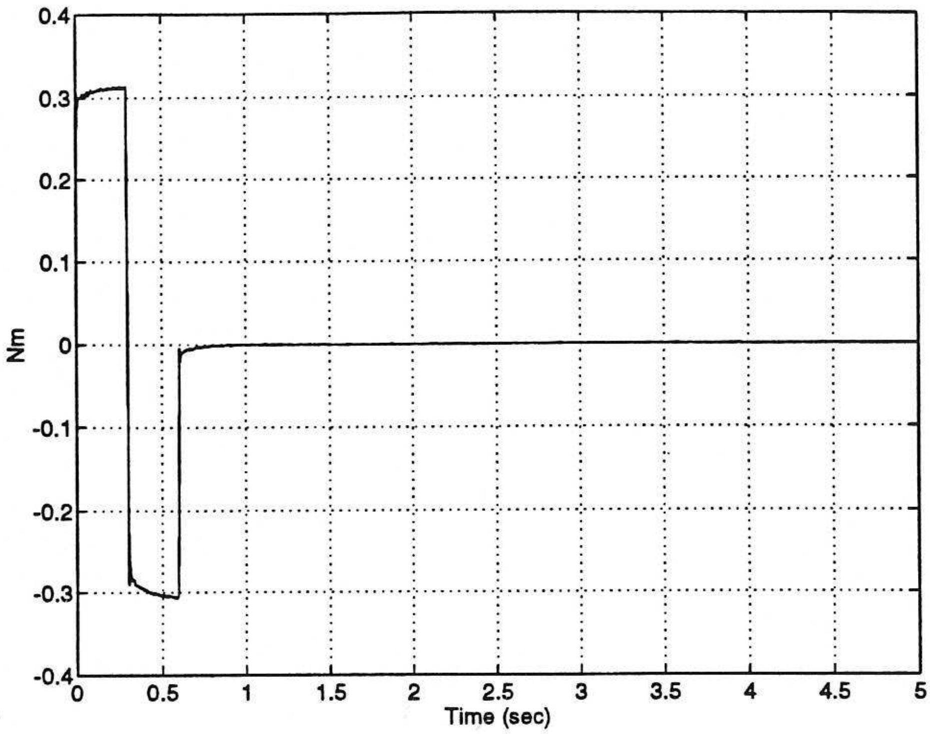
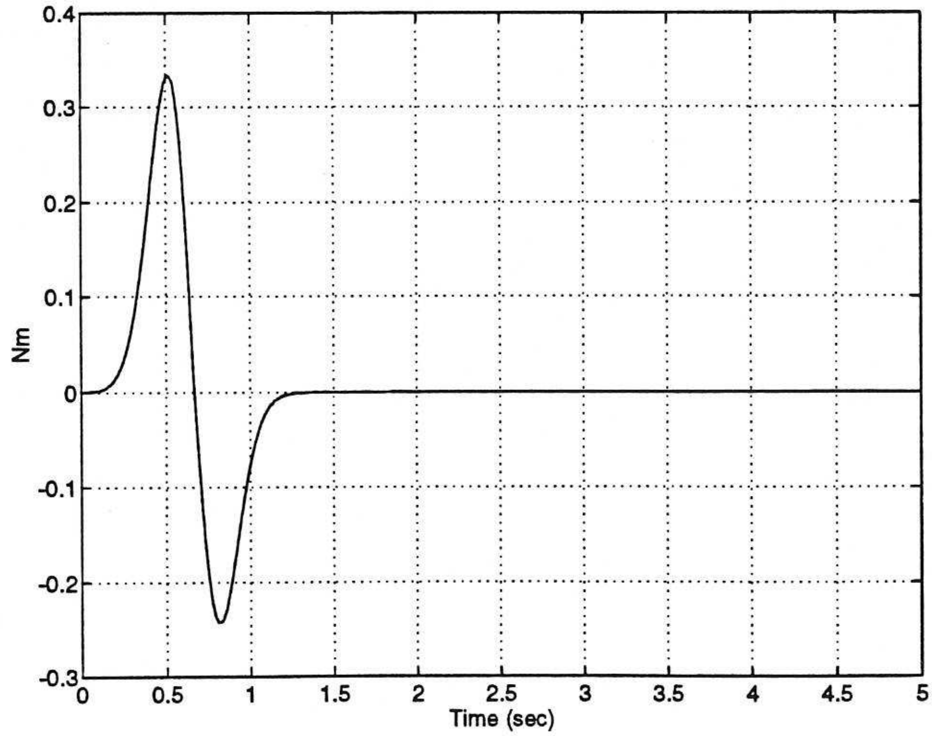


Figure 6: Experimental set-up for open-loop system excitation with shaped input.

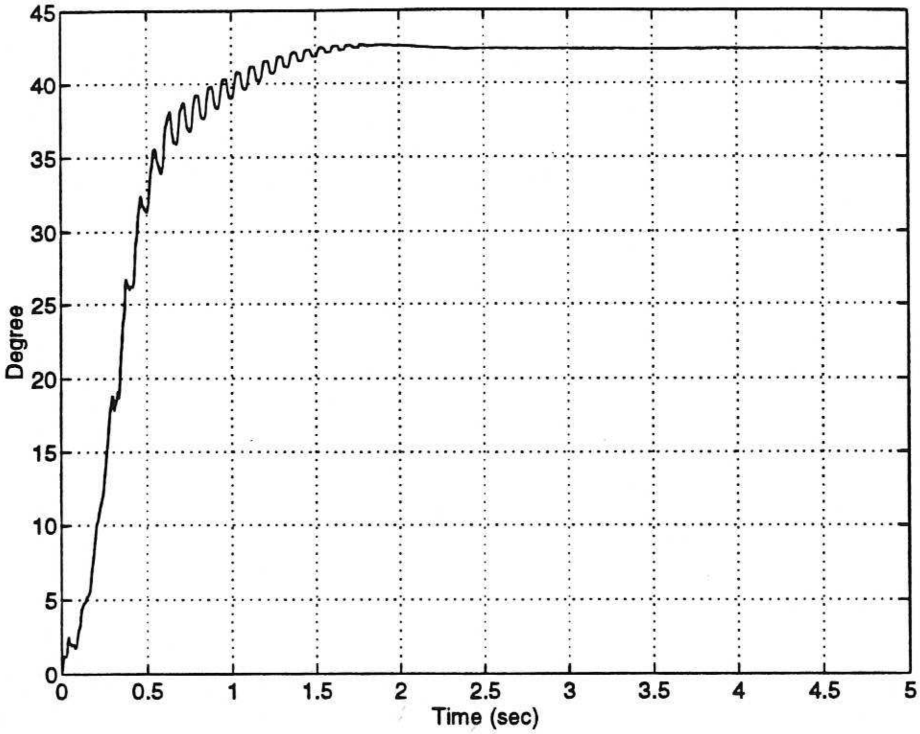


(a)

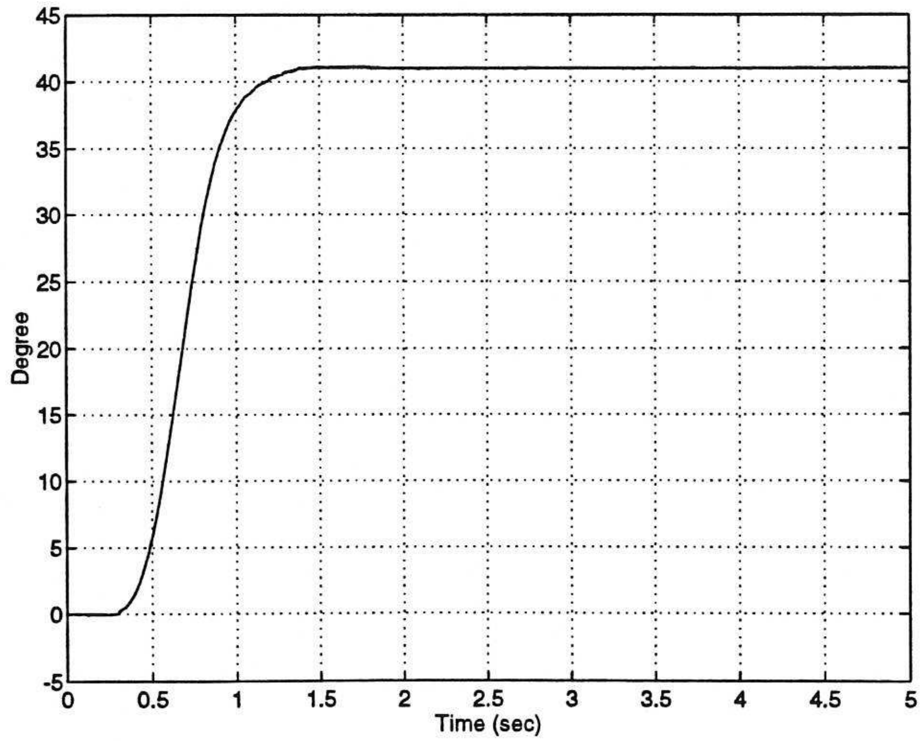


(b)

Figure 7: Input torque profiles.
(a) Bang-bang.
(b) Gaussian shaped.

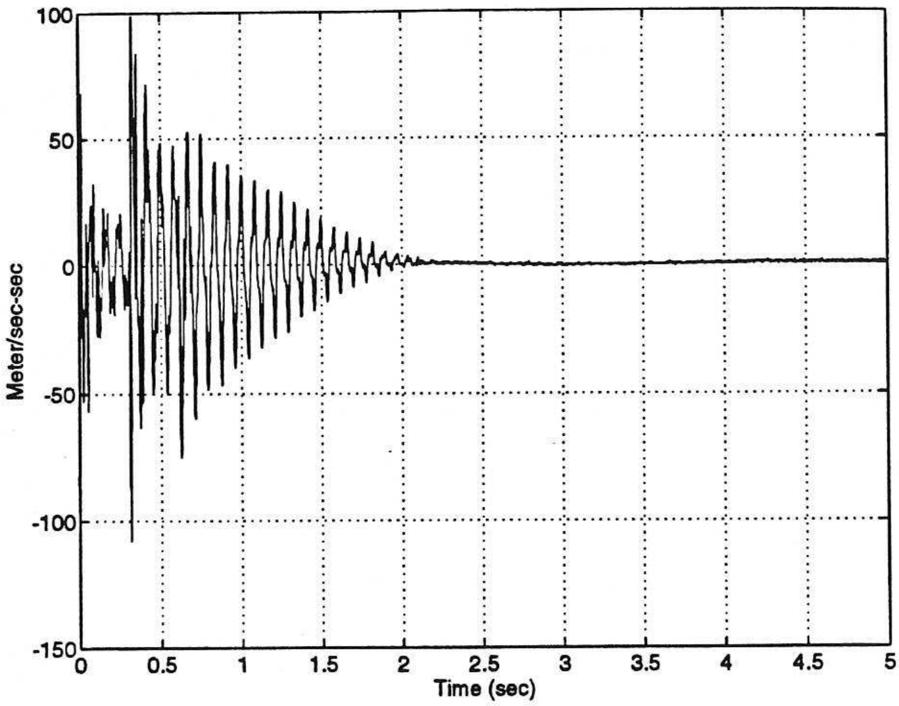


(a)

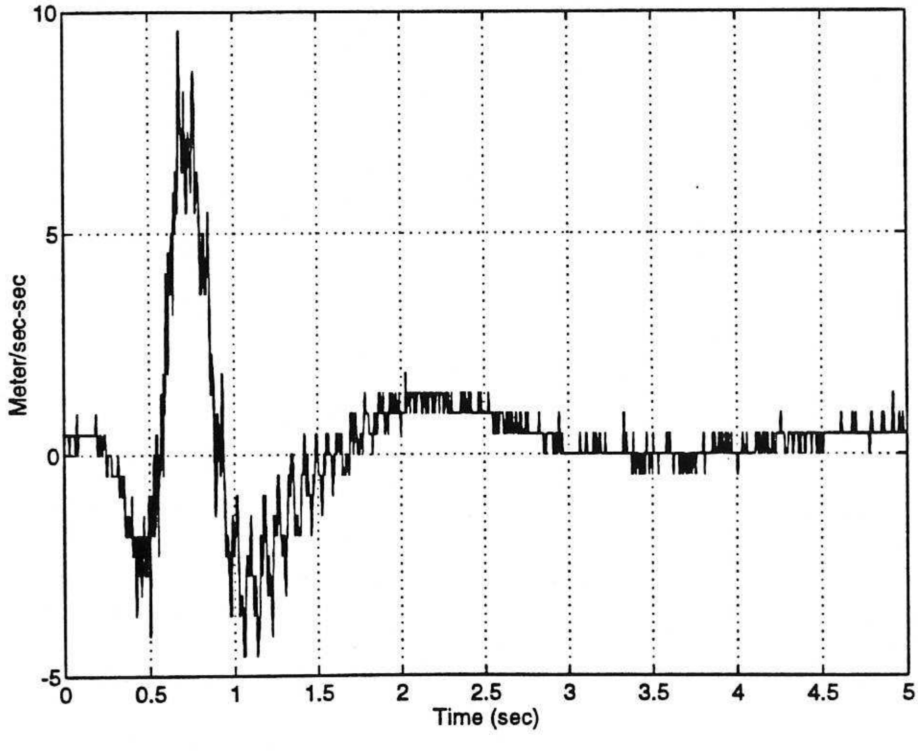


(b)

Figure 8: Hub angular displacement of the flexible manipulator.
(a) With the bang-bang input torque.
(b) With the gaussian shaped input torque.

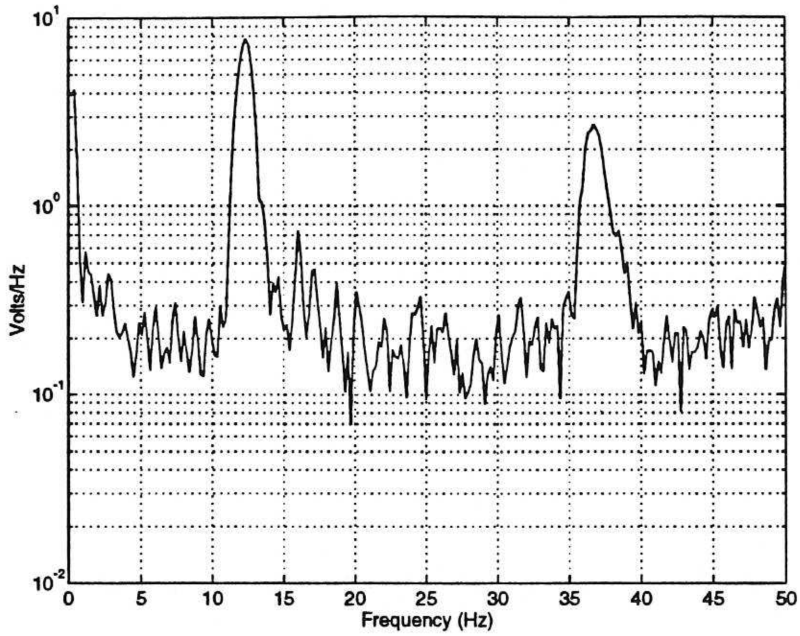


(a)

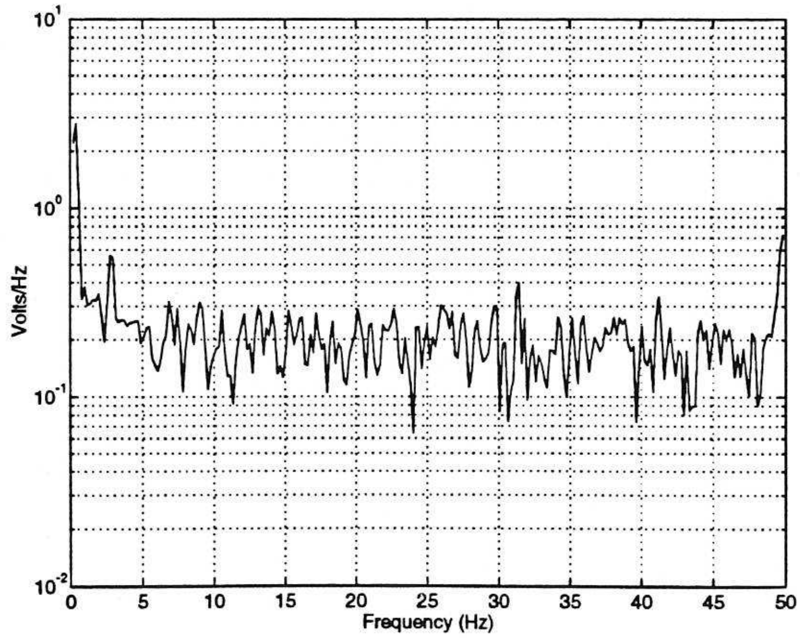


(b)

Figure 9: End-point acceleration of the flexible manipulator.
(a) With the bang-bang input torque.
(b) With the gaussian shaped input torque.

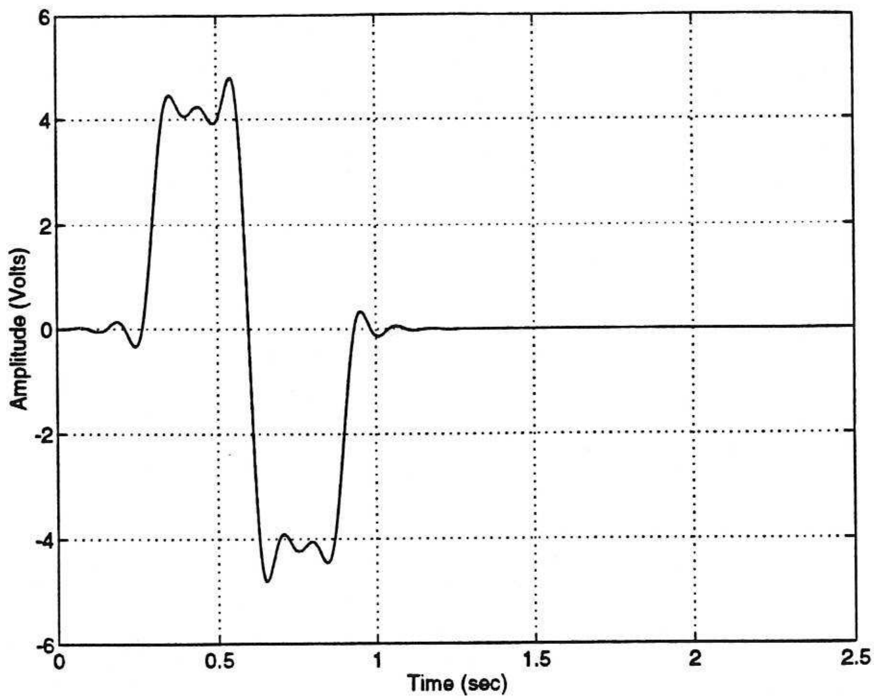


(a)

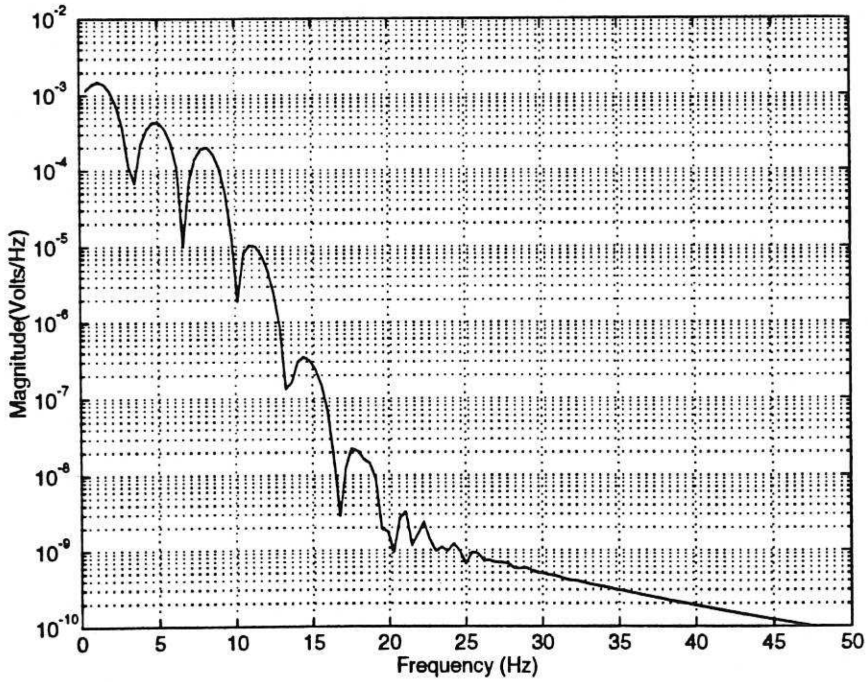


(b)

Figure 10: Autopower spectrum of the residual acceleration output.
(a) With the bang-bang input torque.
(b) With the gaussian shaped input torque.



(b)



(b)

Figure 11: Butterworth low-pass filtered input torque;
(a) Time-domain.
(b) Spectral density.

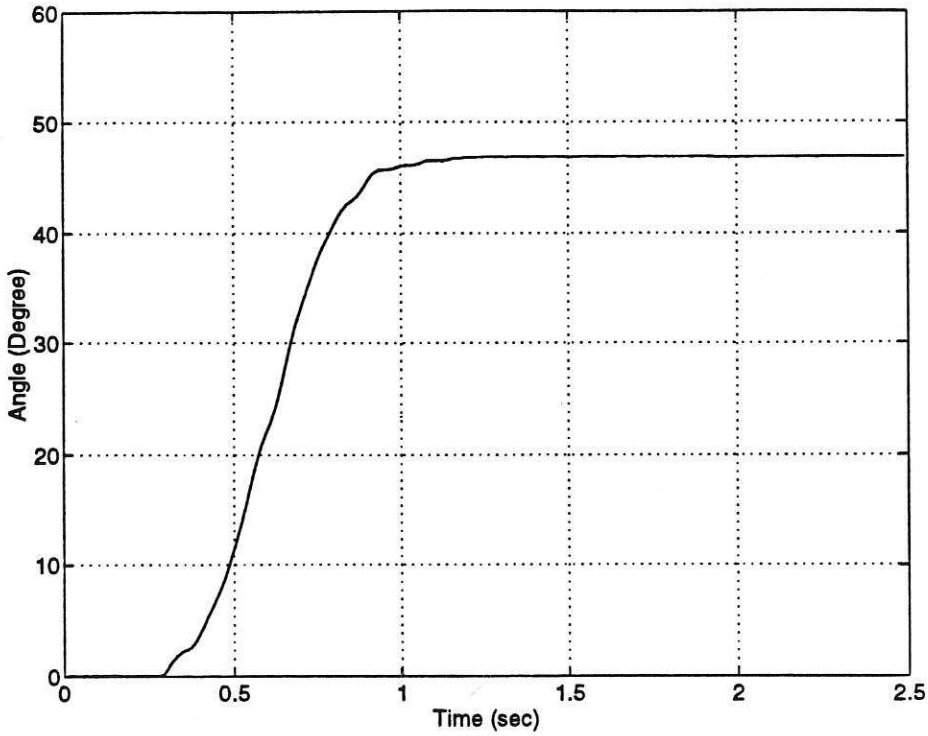


Figure 12: Hub angle with Butterworth filtered input torque.

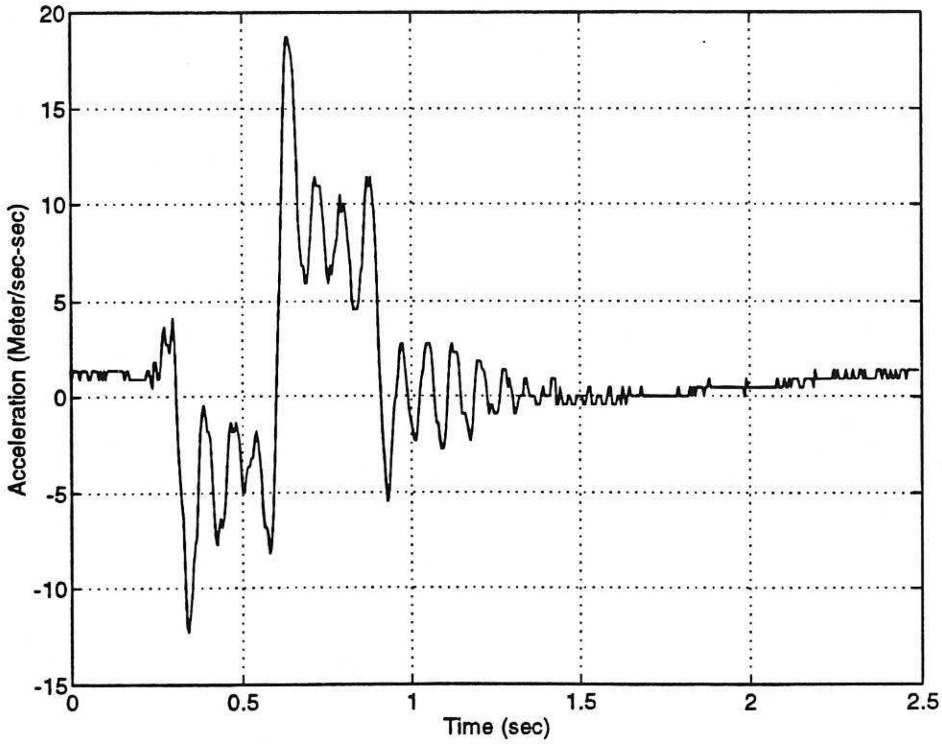


Figure 13: End-point acceleration with Butterworth filtered input torque.

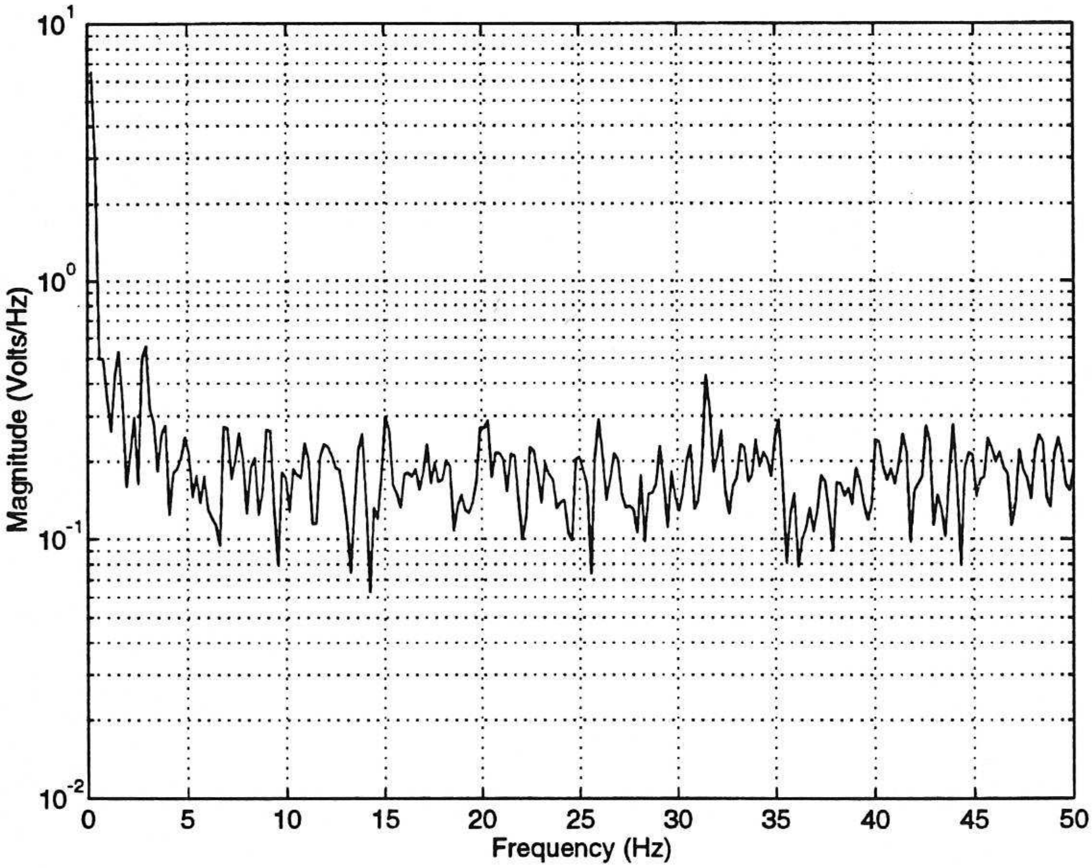
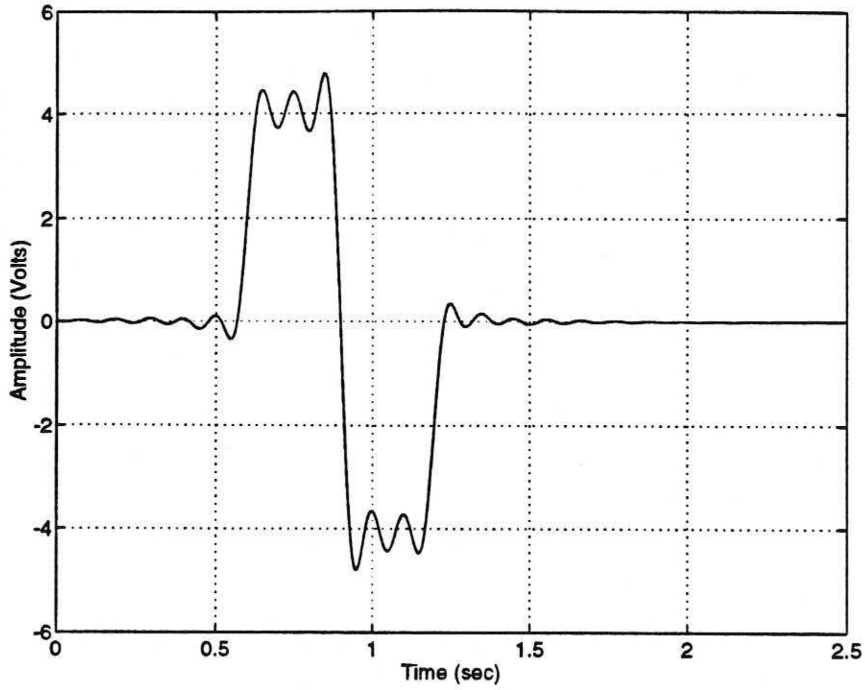
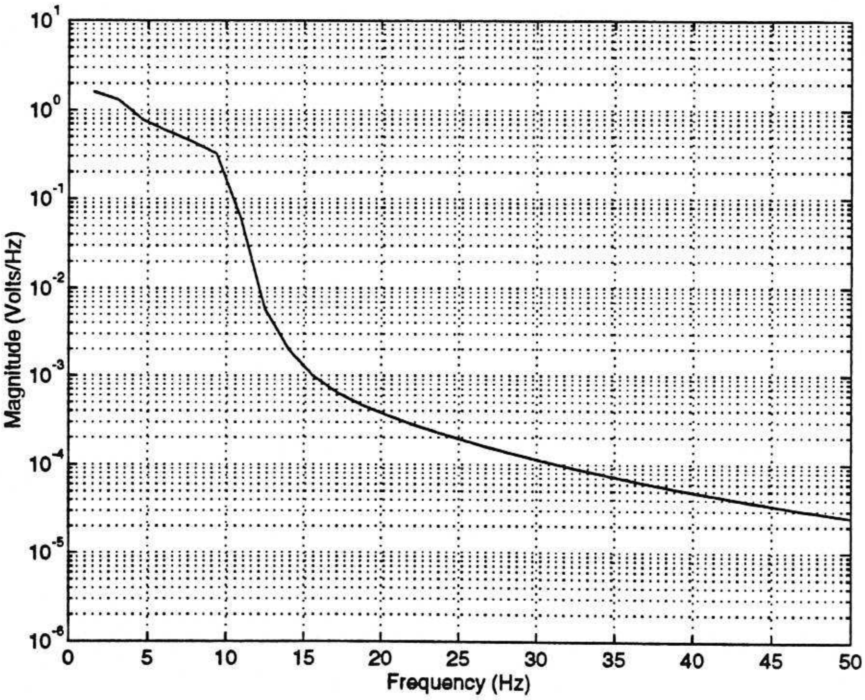


Figure 14: Spectral density of the residual end-point acceleration signal.



(a)



(b)

Figure 15: Elliptic low-pass filtered input torque;
(a) Time-domain.
(b) Spectral density.

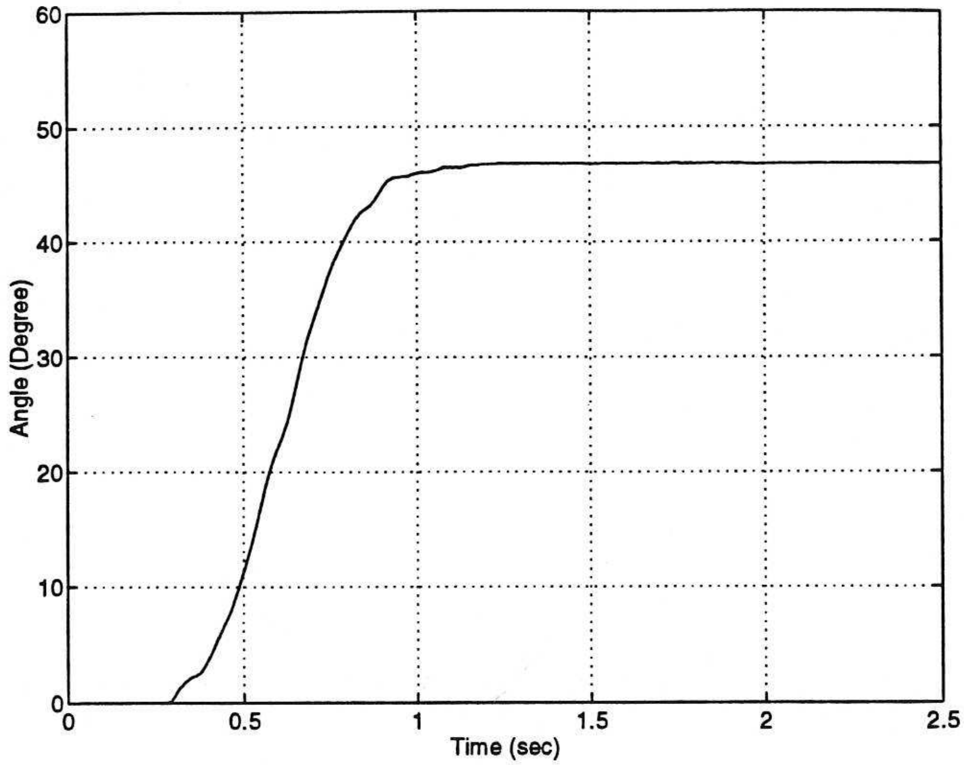


Figure 16: Hub angle with Elliptic filtered input torque.

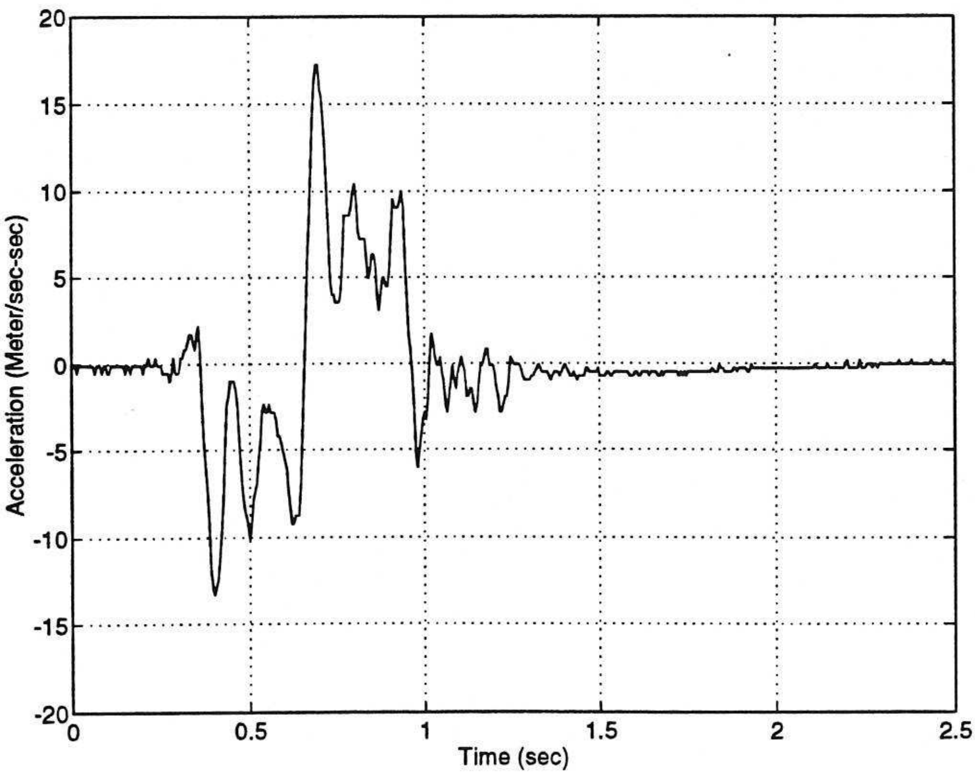


Figure 17: End-point acceleration with Elliptic filtered input torque.

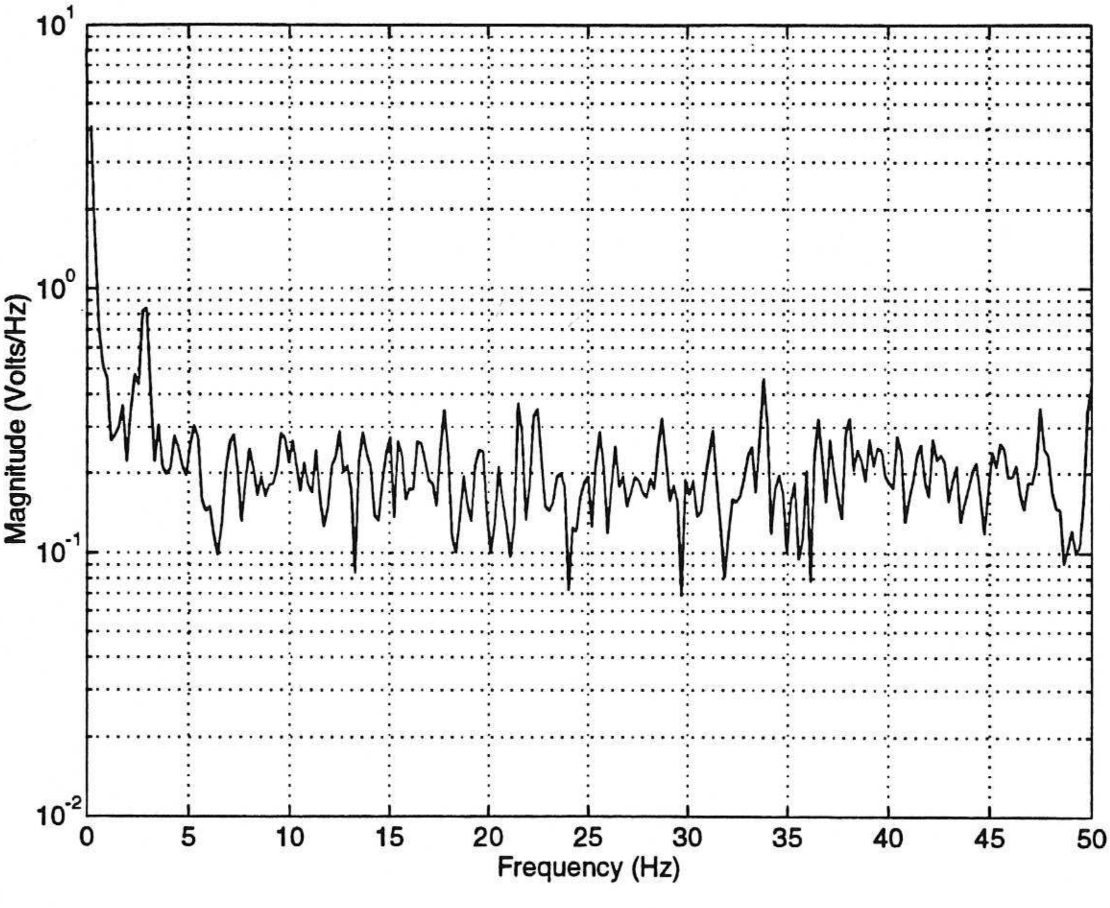


Figure 18: Spectral of the residual end-point acceleration signal.

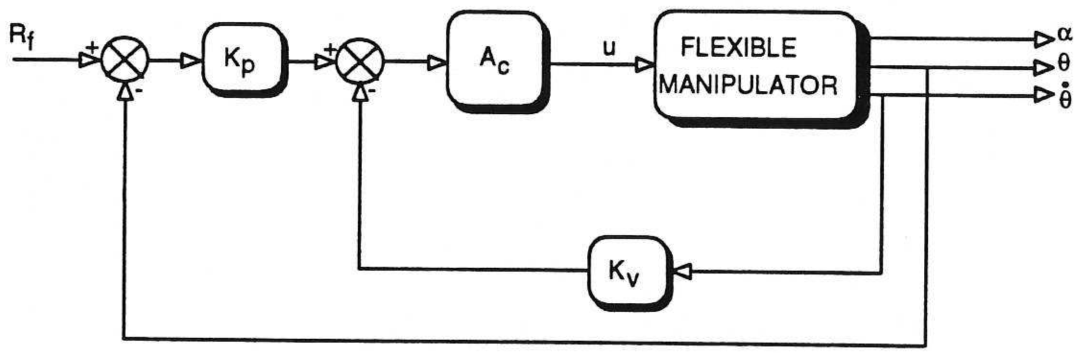


Figure 19: Flexible manipulator system with joint based collocated controller.

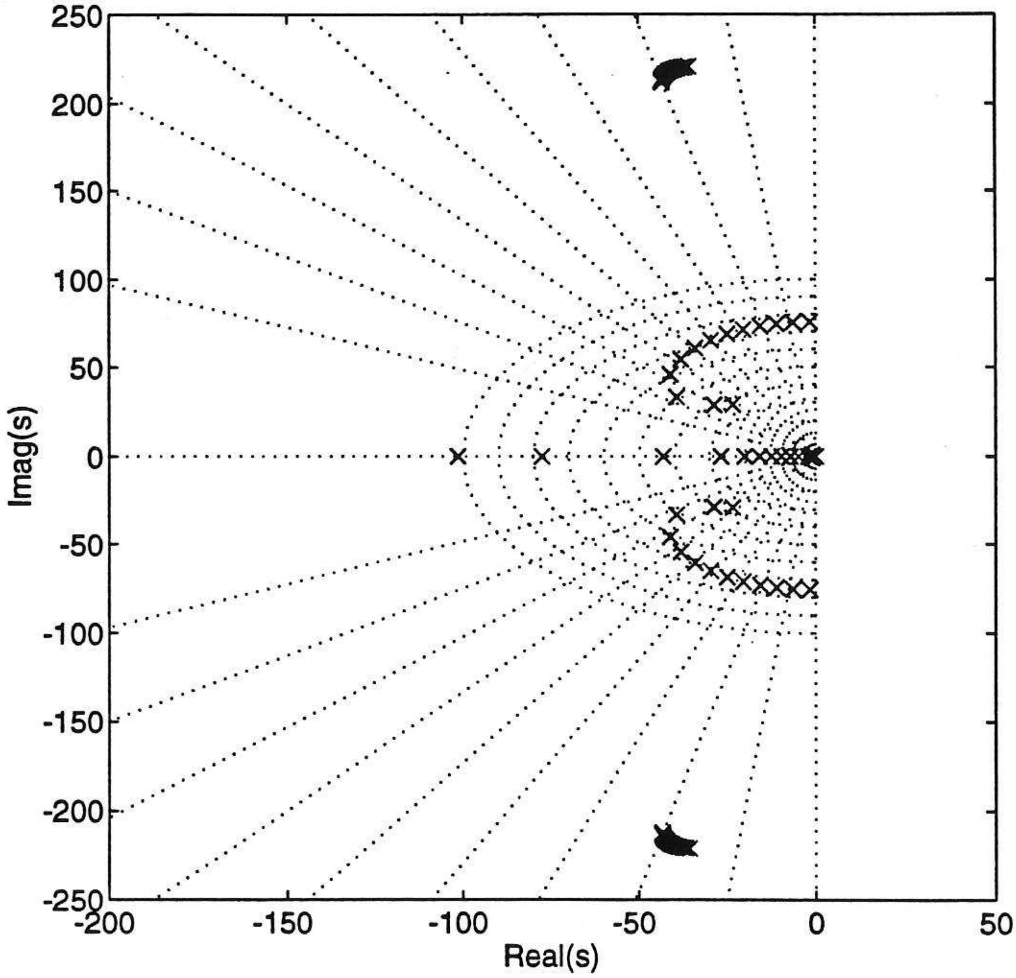


Figure 20: Closed-loop root locus with Z=1.

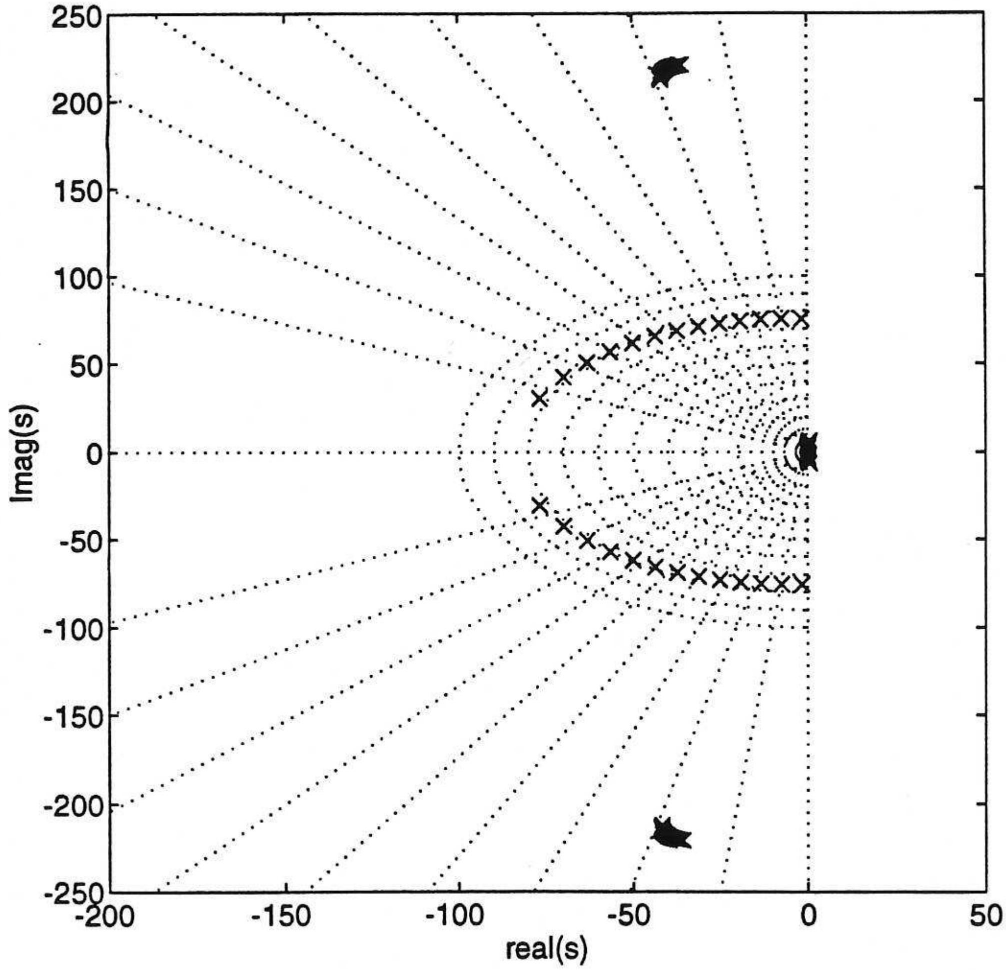


Figure 21: Closed-loop root locus with Z=2.

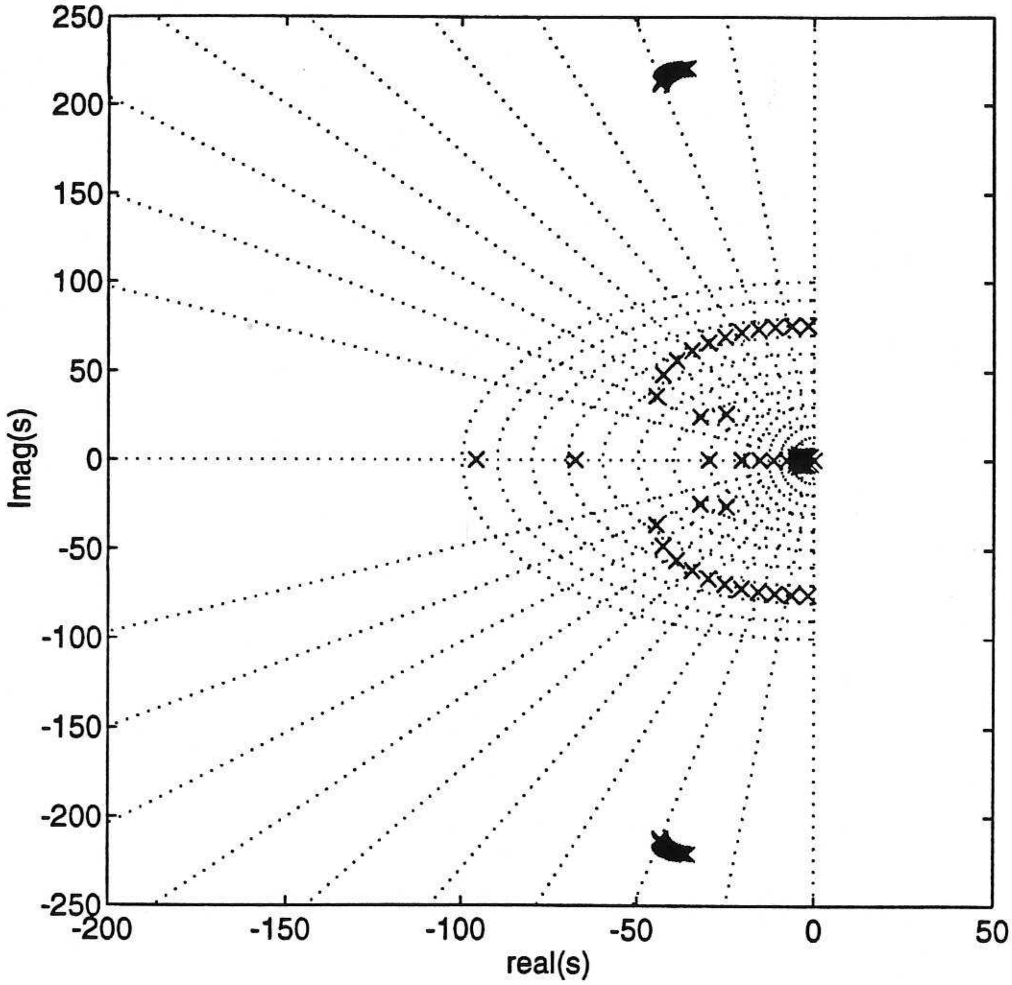


Figure 22: Closed-loop root locus with Z=3.

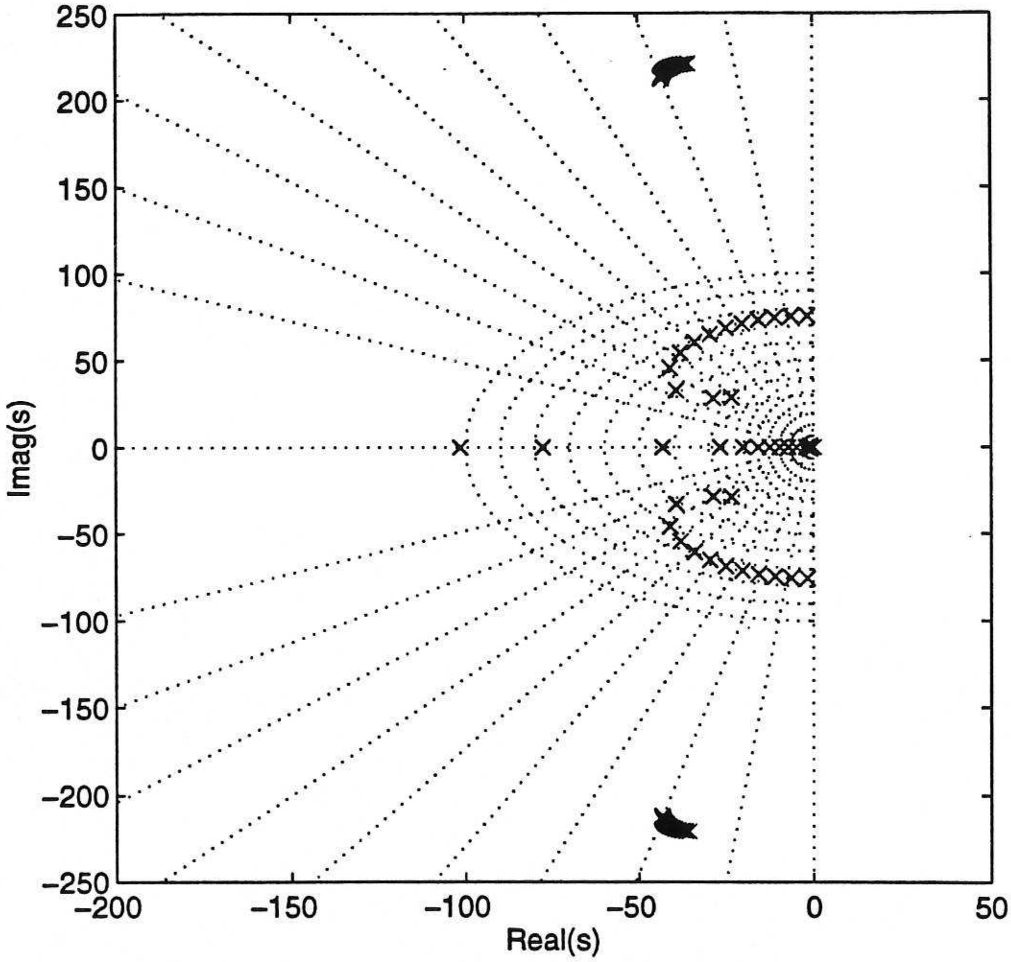


Figure 23: Closed-loop root locus with Z=4.

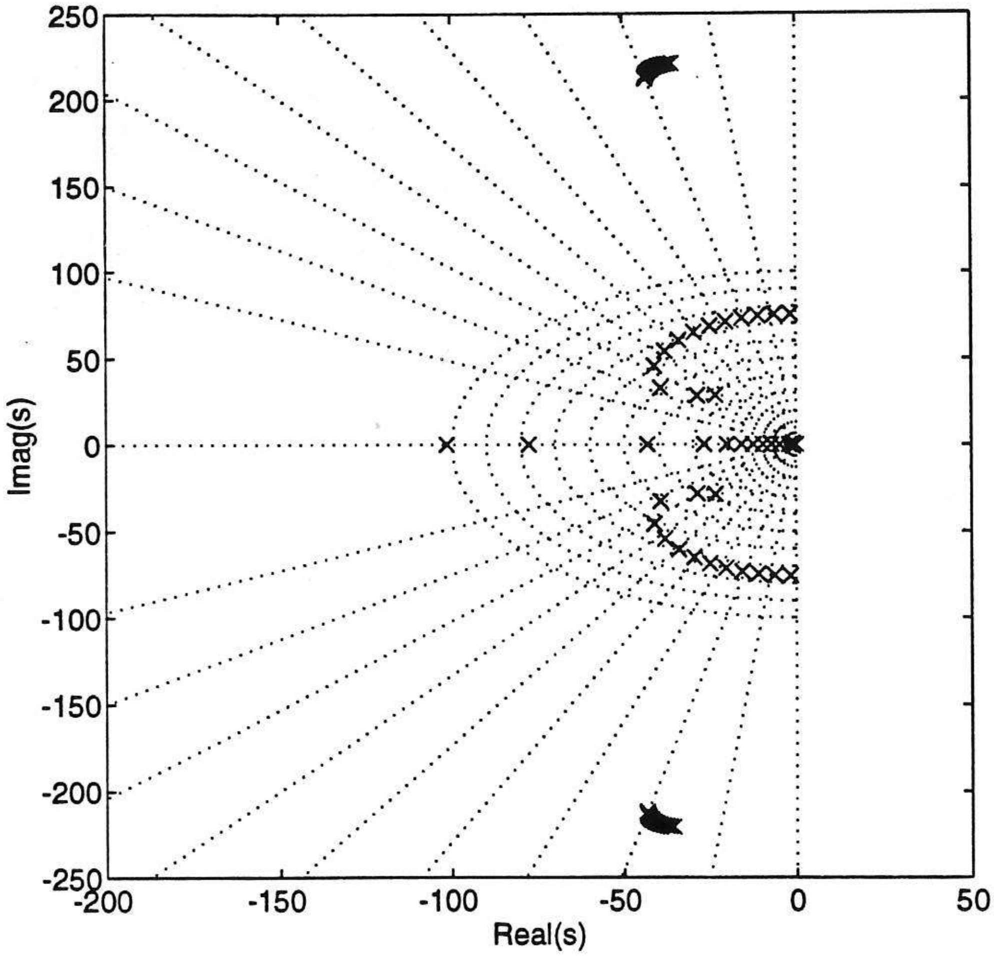
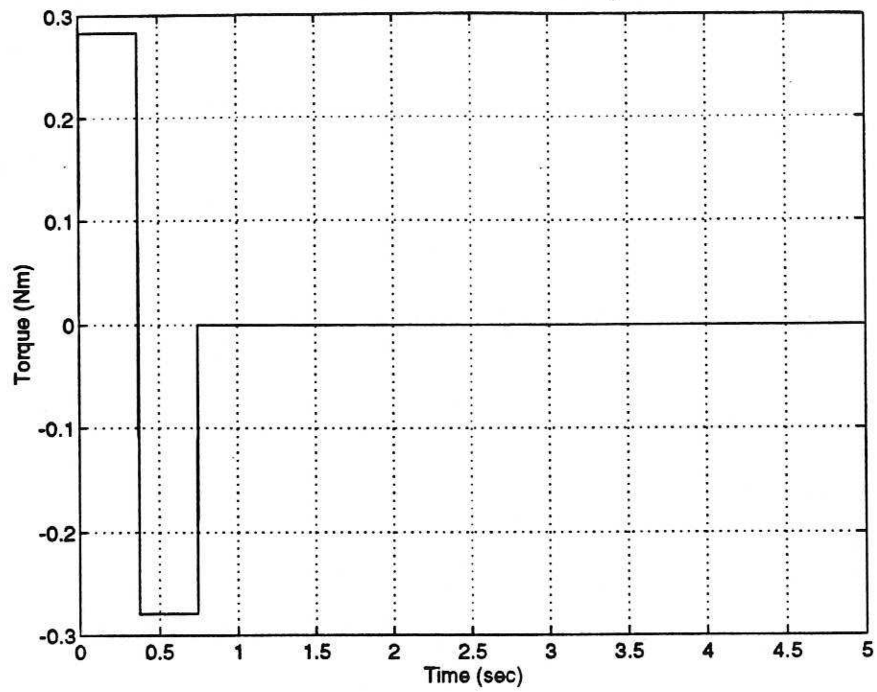
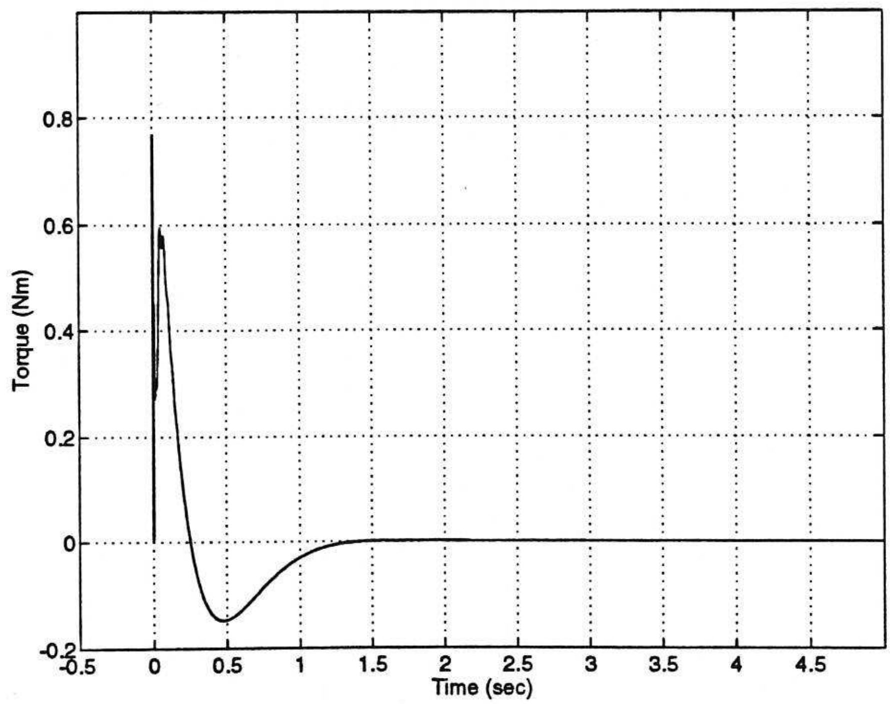


Figure 24: Closed-loop root locus with Z=5.

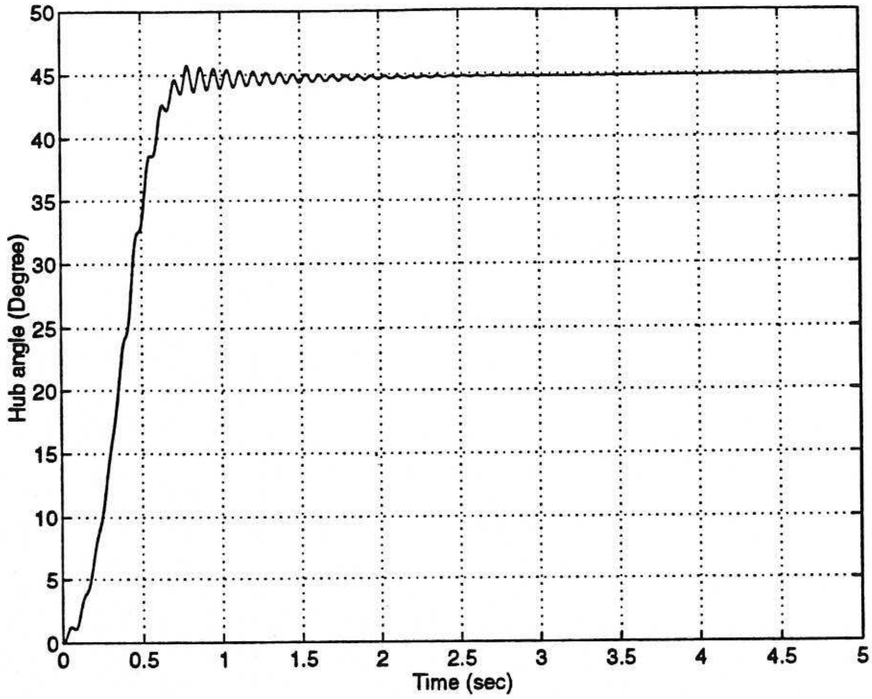


(a)

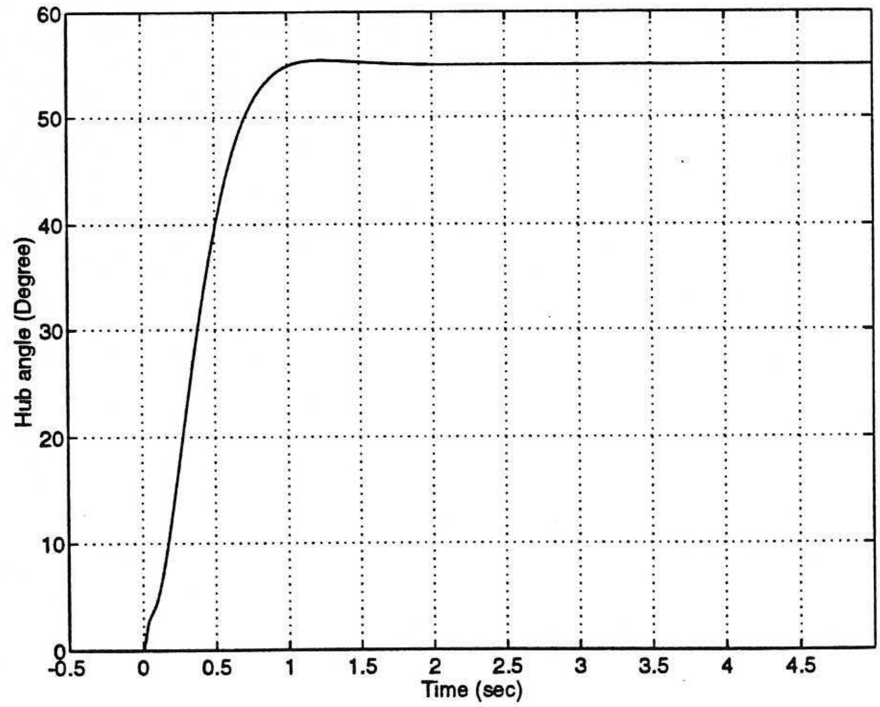


(b)

Figure 25: Torque input at the hub of the flexible manipulator;
(a) Open-loop bang-bang.
(b) PD controlled.

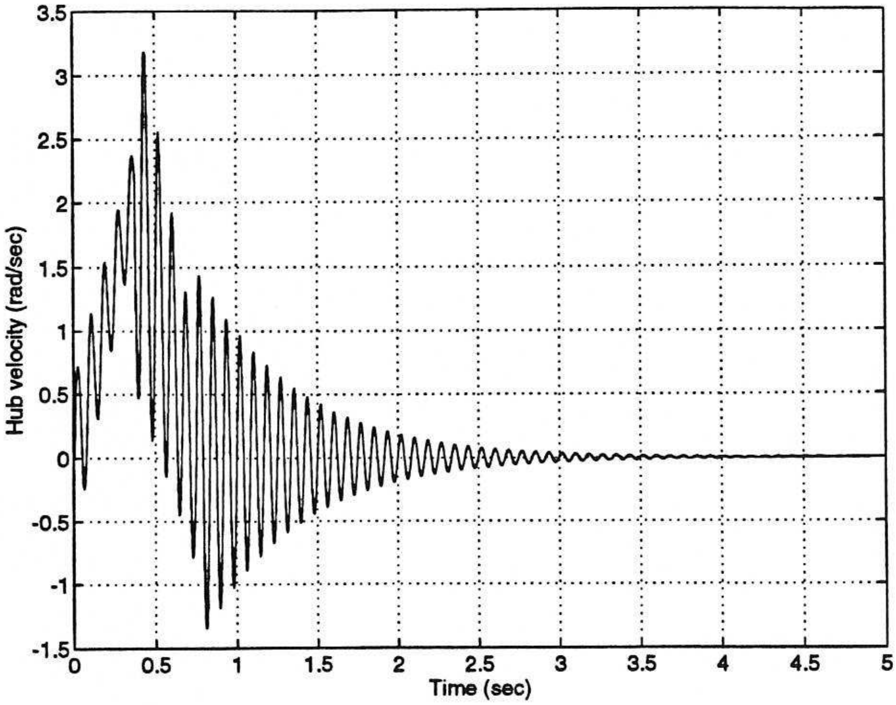


(a)

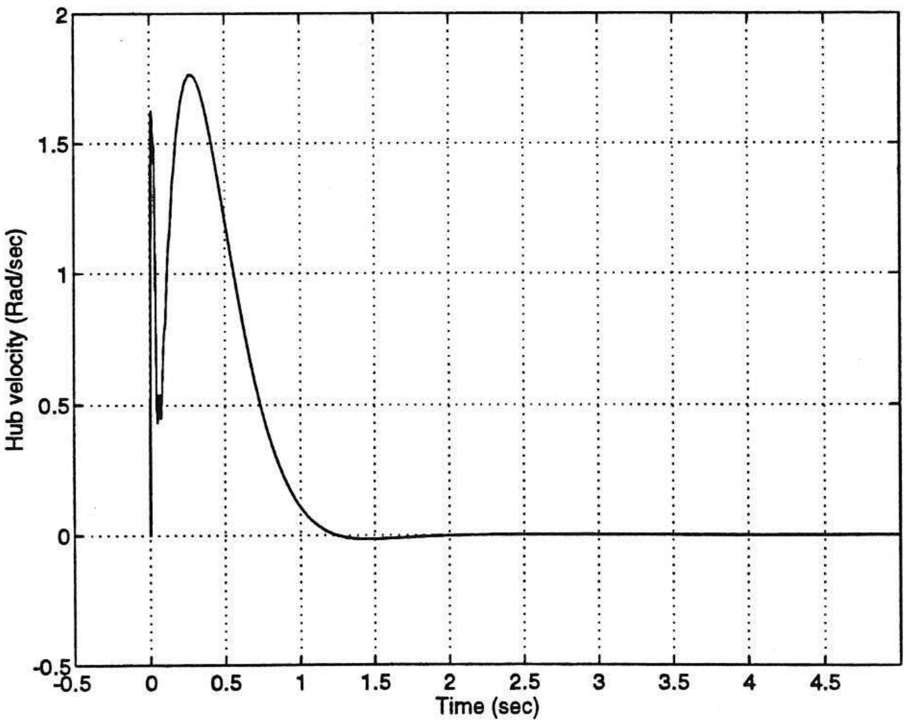


(b)

Figure 26: Hub angle of the flexible manipulator;
(a) Open-loop bang-bang.
(b) PD controlled.

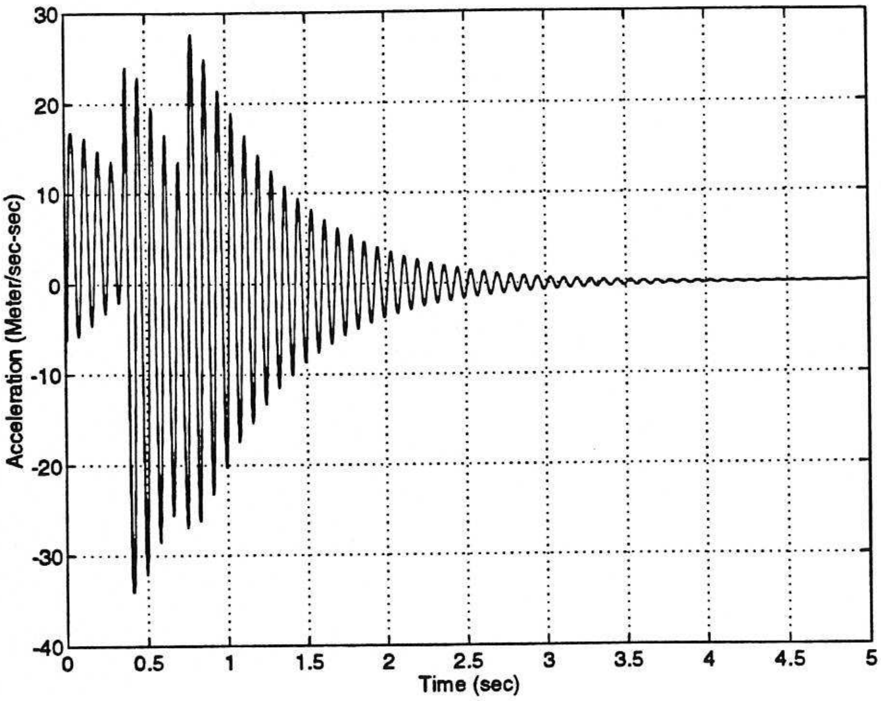


(a)

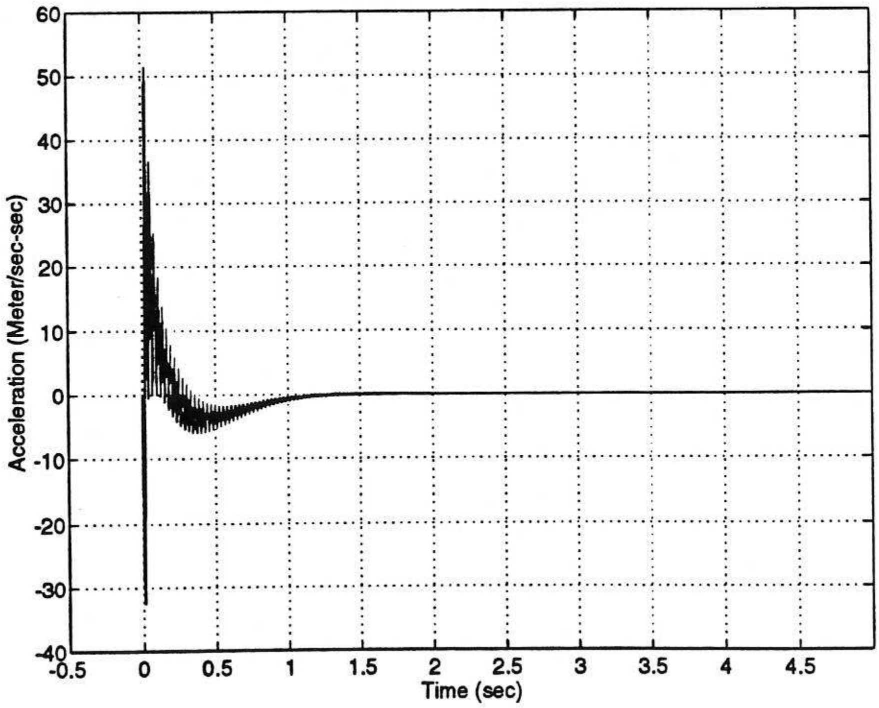


(b)

Figure 27: Hub velocity of the flexible manipulator;
(a) Open-loop bang-bang.
(b) PD controlled.

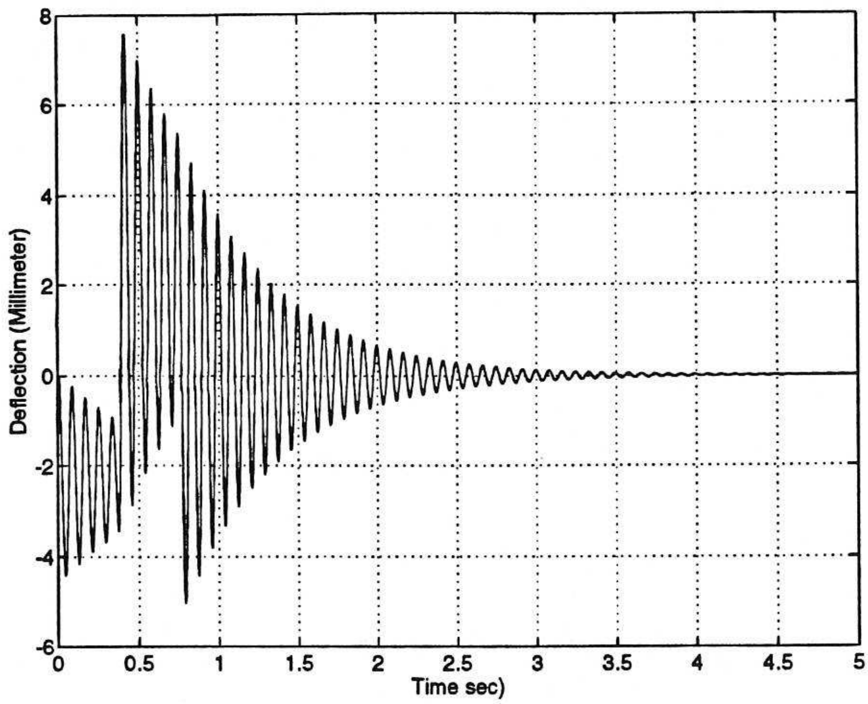


(a)

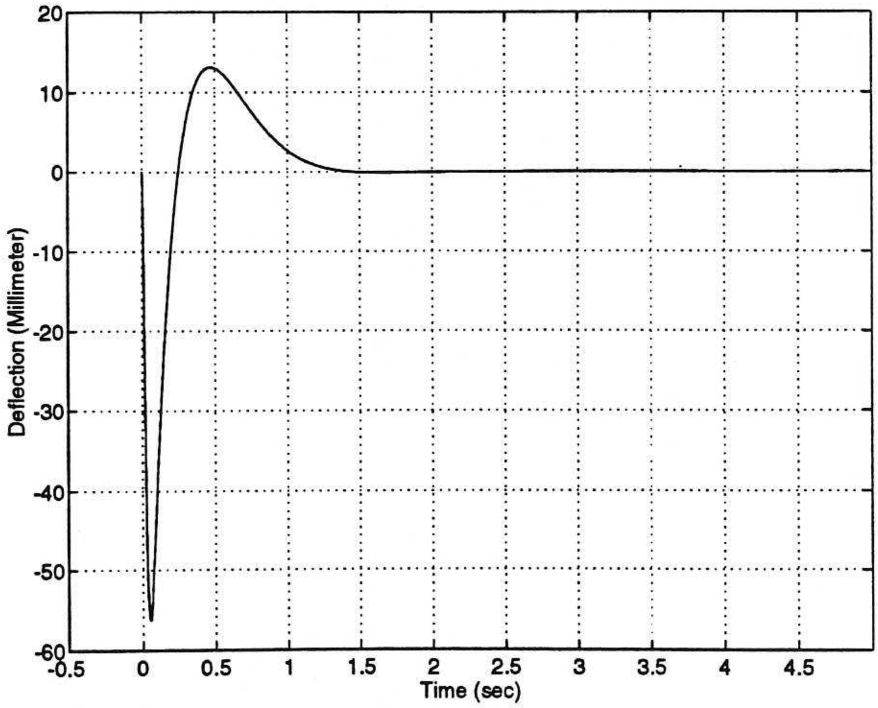


(b)

Figure 28: End-point acceleration of the flexible manipulator;
(a) Open-loop bang-bang.
(b) PD controlled.

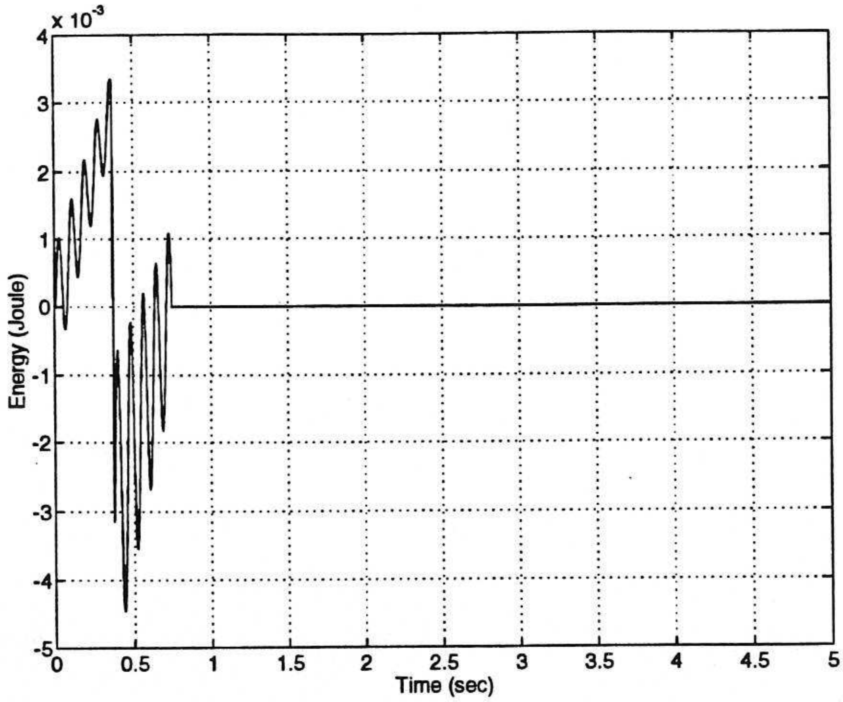


(a)

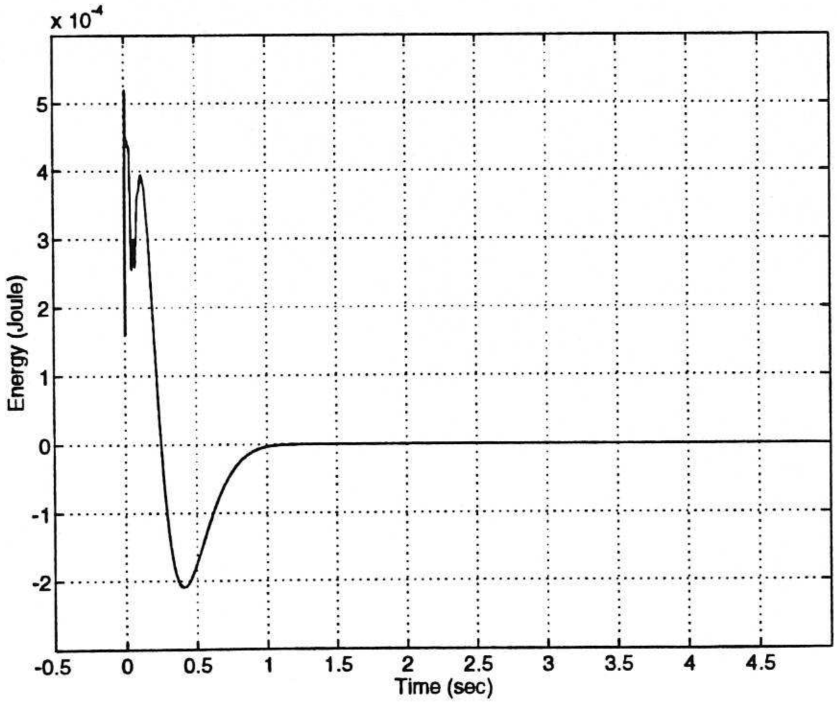


(b)

Figure 29: End-point elastic movement of the flexible manipulator;
(a) Open-loop bang-bang.
(b) PD controlled.



(a)



(b)

Figure 30: Input energy to the flexible manipulator;
(a) Open-loop bang-bang.
(b) PD controlled.

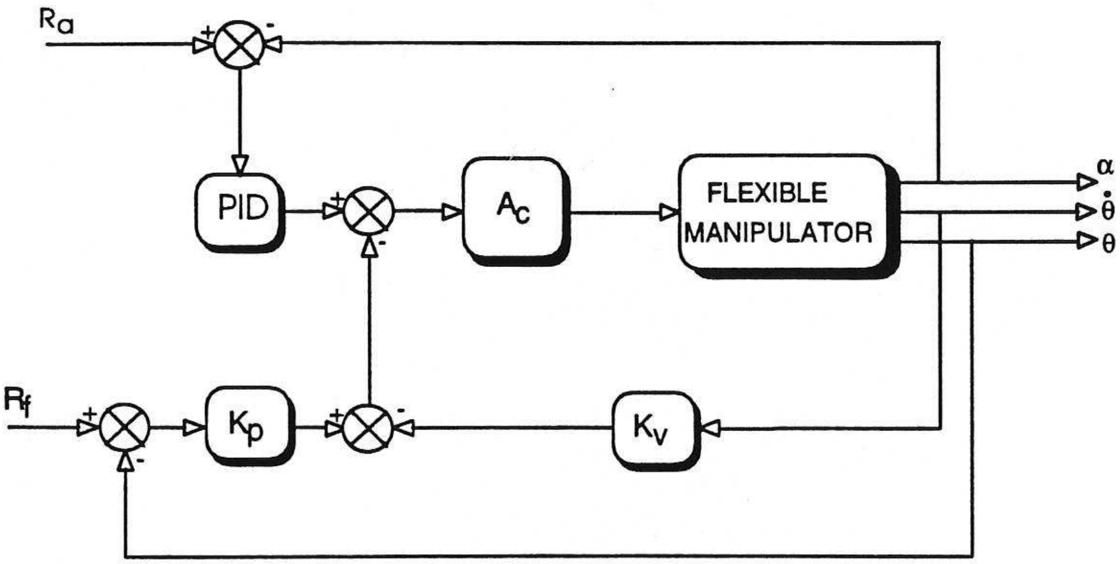


Figure 31: Block diagram of the hybrid collocated and non-collocated controller.

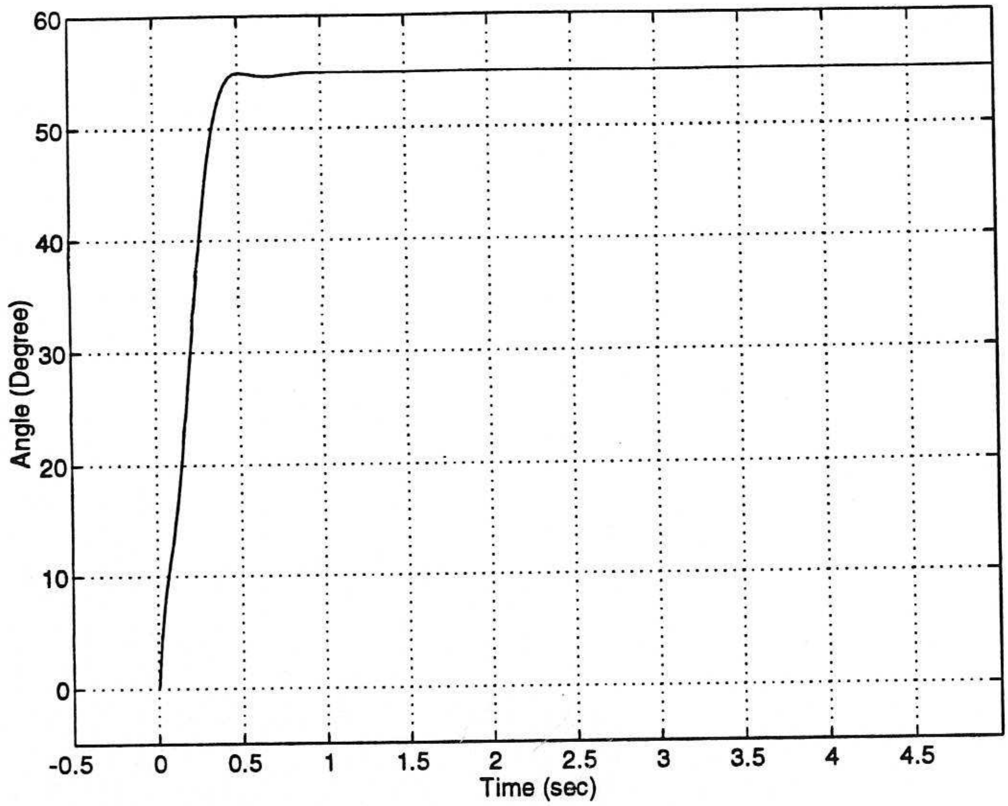


Figure 32: Hub angle of the flexible manipulator with the hybrid controller.

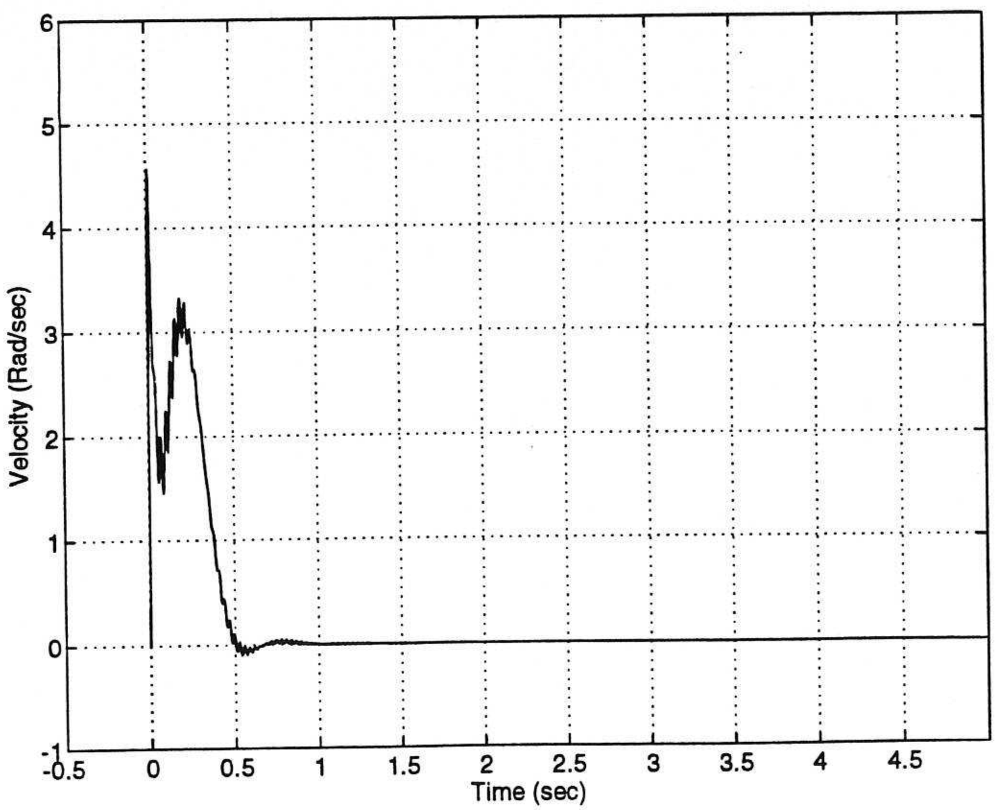


Figure 33: Hub velocity of the flexible manipulator with the hybrid controller.

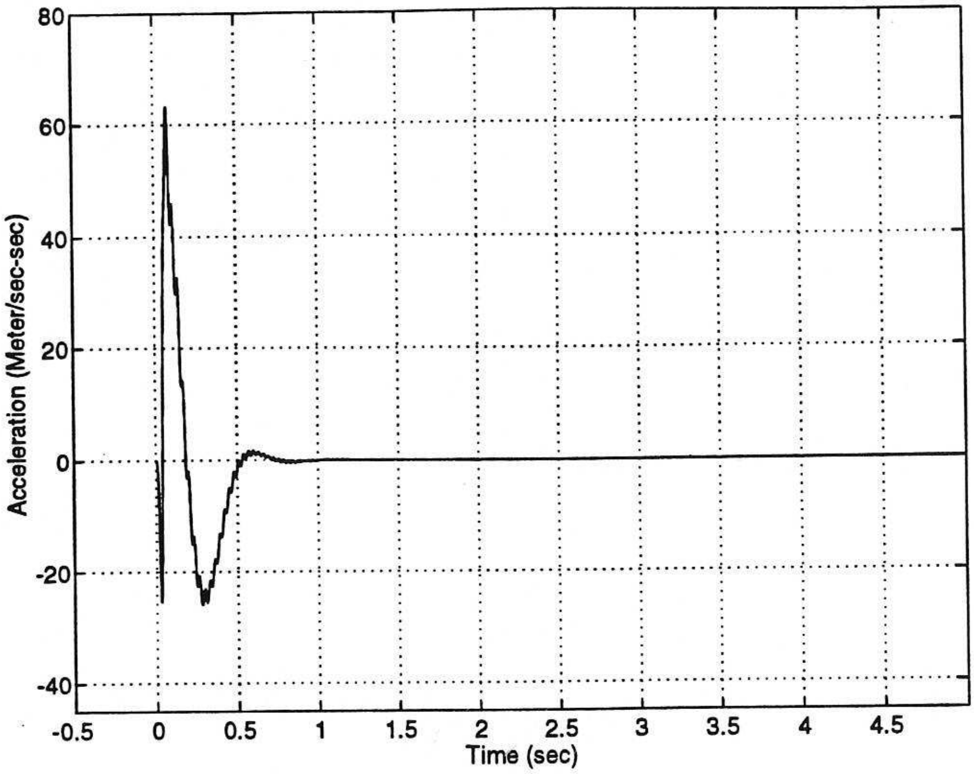


Figure 34: End-point acceleration of the flexible manipulator with the hybrid controller.

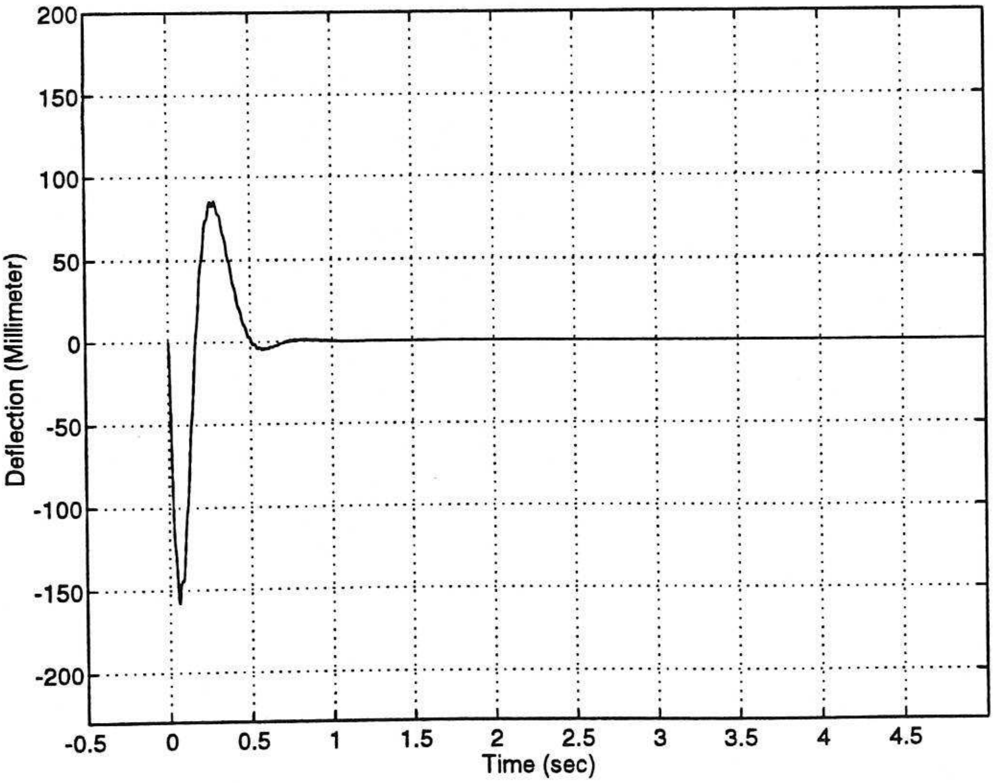


Figure 35: End-point elastic movement of the flexible manipulator with the hybrid controller.

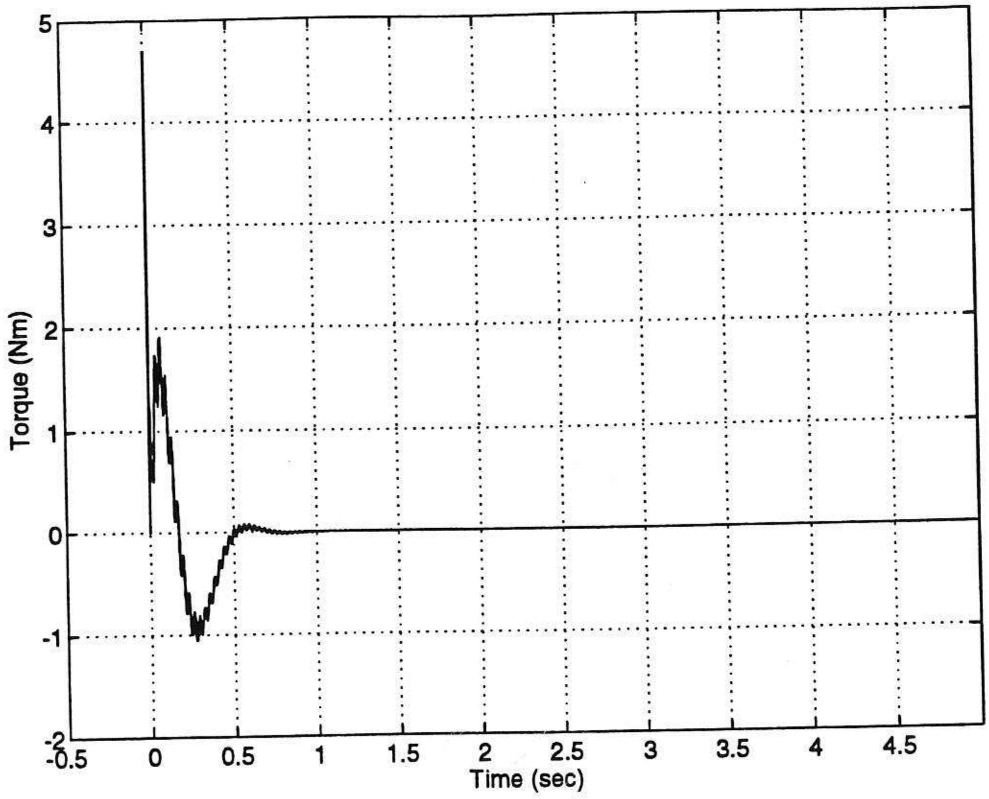


Figure 36: Control effort at the hub of the flexible manipulator with the hybrid controller.

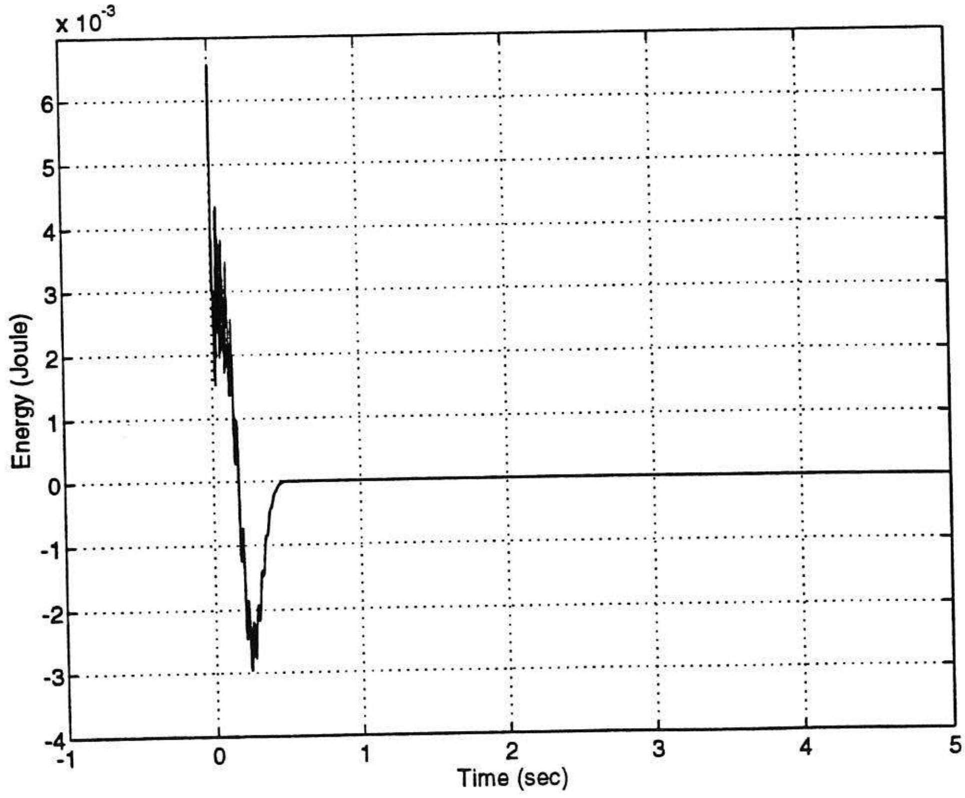


Figure 37: Input energy to the flexible manipulator system with the hybrid controller.