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TRACKING THE FEATURES OF A SPATIALLY DISTRIBUTED CONTINUOUS FIELD

(the Idealised 2D, Deterministic Case)

by

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(THE IDEALISED 2D, DETERMINISTIC CASE)

Abstract

This is the first of several reports on research that aims to utilise a model of the surrounding field together with measured data therefrom to locate and orientate an observer. A single valued solution is developed to calculate the position of a vehicle in a continuous field environment described by a known polynomial from a minimum number of equispaced field measurements taken at that location and known coefficients of the polynomial. Where insufficient measurements from a single scan of the field are available, a formula for utilising extra data from several scans has been produced. The formulas have been demonstrated in many example fields, two of which are presented here.

The investigation and test results to date have been confined to the purely deterministic case and future reports in this series will address the effect of noise.

1. Introduction

Over the past decade or so, great interest has been shown in achieving autonomy of moving vehicles [1]. Detection and avoidance of obstacles within the traversed domain is often a key criterion in choosing an acceptable path for the mobile through the domain between prespecified start and destination points. Obstacles might

(a) be objects insurmountable by the vehicle or
(b) represent merely "difficult terrain" features.

In case (a), the obstacle must be avoided totally at all costs. In case (b), the feature should be avoided only to the extent that some overall cost function (involving perhaps journey time, energy utilisation, overheating etc.) is minimised.

Generally, there will exist some form of domain map providing à priori information of obstacle and feature location to the on-line vehicle guidance controller. This map may be a shop-floor layout plan [2] for a factory mobile or, perhaps, a satellite image of iceberg distribution for navigating Arctic waters. In addition, the mobile will carry onboard obstacle detector systems (based on vision, ultrasonic and/or tactile sensors), together with some form of position-reckoning system (compass based, inertial, scanning laser triangulation etc.). These, used together, should permit correlation of obstacles and features observed by the vehicle (i.e. within its field of view) with members of the set of obstacles prestored on the map. Indeed, such correlation may be essential for recalibration of the position-reckoning system from time to time or continuously. This is because the position-reckoning system can be prone to drift and the resolution of the domain map may be insufficiently precise for accurate navigation with respect to the real life obstacles in real time.

The actual path traversed by the mobile through the spatial domain will determine the features and obstacles actually encountered therein on a particular journey, and hence the particular images that are picked up by the onboard scanning system. Of course, the actual path will differ continuously from the planned path (to only within acceptable limits hopefully). A task of the feature- and obstacle- detection systems is to measure such deviations and either initiate the necessary mid course corrections automatically or, at least, correct the readings of the position-reckoning system in the light of the real world observations.

1.1. Tracking within a Continuous Field

The foregoing discussion concerned essentially discrete features within a spatial domain although, in the case of ice-flows for instance, these can merge into semi-continuous features. Many surface geological formations viewed from altitude can also present remarkably regular continuous patterns over long distances and these can provide a useful aid to aerial navigation. The same is true of large scale underground geological structures [3], the features of which can assist mining and tunnelling machine guidance [4]. Such geological fields can be modelled as multi layer sandwiches of ingredients that differ significantly in their physical properties such as their optical reflectivity, natural radiation emission, density and mechanical hardness. The layering can be remarkably consistent over thousands of square metres. The layering is obvious from laboratory testing of core samples, and from human observation of rock faces exposed in the course of mining operations. Moreover, scanning optical [5] and tactile sensor systems [6, 7] can reproduce an electronic replica signature of the layering, the latter on line and the former at safe distance behind the mining machine or between cutting cycles.

During mining operations it is required that the machine follow the geological structure so as to extract a band of preselected consecutive layers from the sandwich,

(a) to avoid dangerously hard layers (i.e. continuous obstacles) above or below and
(b) to extract those layers that are richest in the desired mineral.

In general, the layers are remarkably constant in thickness but, unfortunately, the sandwich structure as a whole is rarely level or even flat. This is because of large scale earth movements subsequent to the epoch of layer formation. The gentle bending and twisting of the geological world in which the mining machine operates is one reason for automatic guidance with respect to the features of the formation. Merely following a straight line, (as defined by a pre directed laser beam for instance) is rarely adequate for a mining- (as opposed to tunnelling-) machine. The other reason for automatic guidance is that such a machine, even if correctly launched within a flat geological formation, would soon drift off course, just like any other vehicle, ship or aircraft released with its controls locked with respect to the vehicle body: chassis, hull or fuselage.

In the mining application, research to date has focused on using data from only a single scan of the field to locate the machine at any particular instance, or has merely used moving average techniques over recent scans to attenuate the effect of field measurement noise on the positional estimate. No serious attempt to incorporate spatial model of the field within the estimator and its memory has been made. The fusion of optimal positional estimation and geostatistical models, although attractive as a vague idea, has proved difficult to formulate so far. There is a need to get back to basic concepts and in this report we begin by posing and solving the simplest possible problem of the genre. It is intended to add details later.

2. Idealising the Continuous Field Tracking Problem

2.1. Modelling of the Field

Let H be a measure of the physical property (hardness, reflectivity, etc.) of the material in the field, the distribution of which is to be scanned, sensed and tracked.

In this two dimensional version of what, in general, would be a three dimensional field, H would be a function of distance x, measured along the contours of the formation and y, the distance measured orthogonally to x from a boundary (say the upper ore, as shown in Fig. 1) of the field. Thus

$$H = H(x, y) \qquad (1)$$

in the 2D situation. However, we shall confine attention to perfectly isotropic situations as regards the x direction so that the field description reduces to simply

$$H = H(y) \qquad (2)$$

$$0 \leq y \leq y_f \qquad (3)$$

where y_f is the constant depth of the total formation.

Although discussion in earlier Sections has suggested discrete layering of the formation we shall assume here that the field characteristics can be described adequately by a function $H(y)$ that is continuous in y, subject to limits (3) of course. In particular we here assume a polynomial representation

$$H(y) = a_1 y + a_2 y^2 + \ldots\ldots\ldots + a_n y^n \qquad (4)$$

of order n sufficient, with appropriate choice of constant parameter values $a_1 \ldots a_n$ to represent all the features of interest within the field i.e. to produce a map of $H(y)$ with adequate relief.

Now the machine will sample the function $H(y)$ only at specific values of y that will depend on the scan number j and the sample number i within the scan. Thus we must confine attention to only the locations

$$y = y(i, j) \qquad (5)$$

$$1 \leq i \leq m \qquad (6)$$

where m = the number of samples per scan. Here the samples in each scan are assumed to be equispaced at intervals Δy as shown in Fig. 1 so that

$$y(i, j) = y_0(j) + (i-1)\Delta y \qquad (7)$$

For all scan numbers j therefore, the scan width is a constant given by

$$y(m, j) - y_0(j) = (m-1)\Delta y \qquad (8)$$

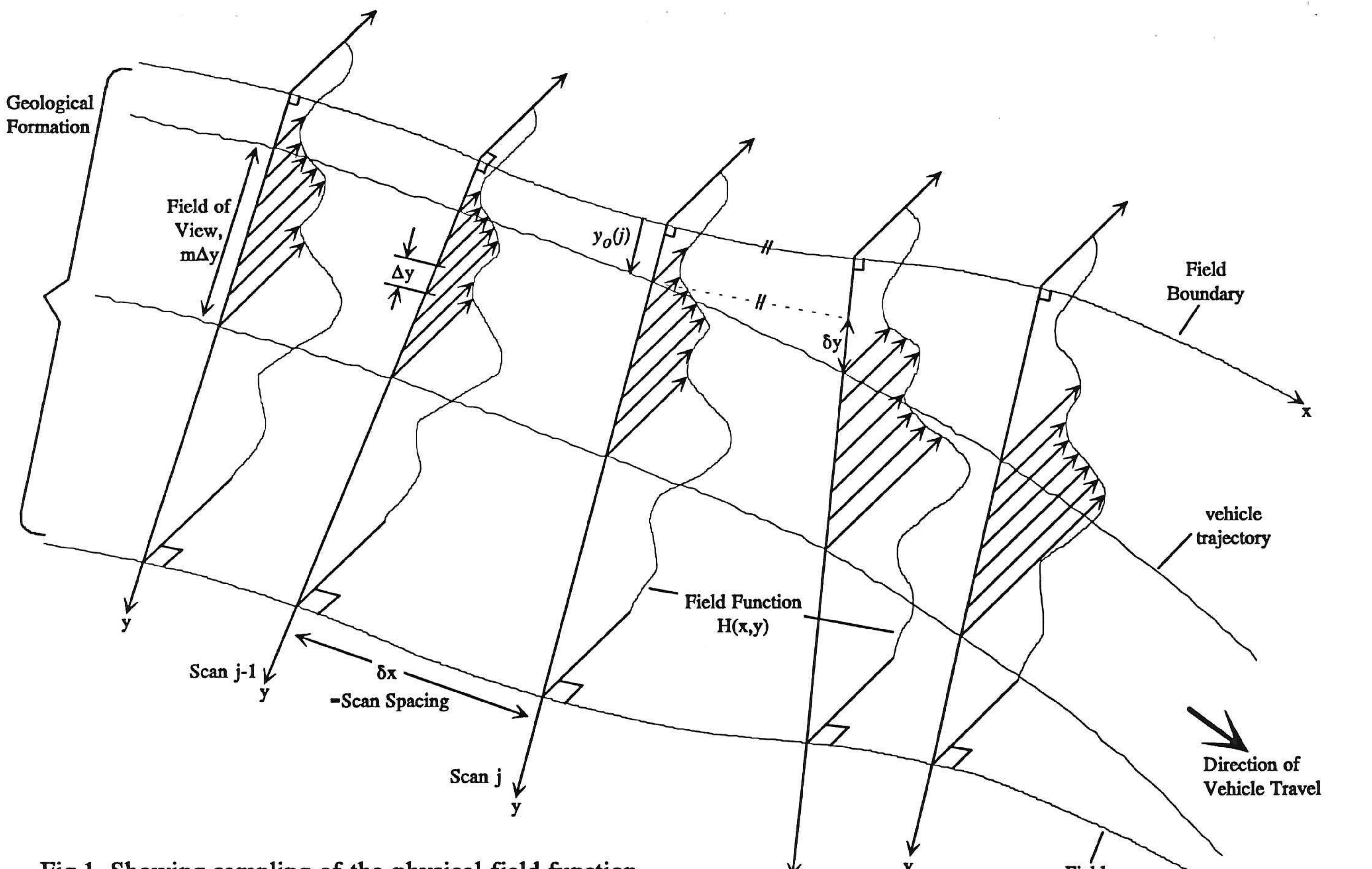

Fig 1. Showing sampling of the physical field function by a travelling observer vehicle

but each scan j is subject to floating limits specified by:

$$y_0(j) \le y(i,j) \le (m-1)\Delta y + y_0(j) \qquad (9)$$

2.2. Modelling the Machine Trajectory Through the Field

The object of the tracking exercise is to find successive $y_0(j)$ values (from the measured values of $H(y)$, knowing equation (4) and some initial value, say $y_0(1)$) as the machine moves through the field.

Now if $m<n$, there will be insufficient data from a single scan j to allow calculation of $y_0(j)$ in general. Information from previous scans will be required together with a knowledge of the form (though not necessarily the parameter values) of the equation of the machine trajectory with respect to chosen boundary of the formation which forms the x-axis. Since scans take place only at discrete x-locations, here assumed nearly equispaced at δx,

$$x = j\delta x \qquad (10)$$

the above mentioned trajectory can be expressed in the discrete form:

$$y_0(j) = y_0(j-1) + g(j)\delta x \qquad (11)$$

In general $g(j)$ will be x-dependent and therefore vary from scan to scan but, for this preliminary report, we shall assume that $g(j)$ is constant, i.e.

$$g(j) = g = \text{a constant} \qquad (12)$$

This would apply exactly for example to a flat, though not necessarily level, geological formation and to a machine trajectory that is straight (in a Euclidean sense) but not necessarily parallel to the field. It would apply exactly also to a curved field provided the curvature of the machine satisfied equation (12). As an approximation, the assumption (12) would be allowable also provided the variation of $g(j)$ were small over the number of scans r necessary in the memory of the tracking algorithm (yet to be designed). Based on assumption (12), it follows that the offset δy between successive scans is given by

$$\delta y = y_0(j) - y_0(j-1) = g\delta x \qquad (13)$$

$$\frac{\delta x}{\delta y} = g = \text{a constant} \qquad (14)$$

2.3. Representing the Field after Scanning and Sampling

Now associated with scan j (taken at station $x=j\delta x$ from $x=0$) will be produced an m-valued discrete sampled function of $H(x,y)=H\{j\delta x, y(i,j)\}$. Hence restating equation (7)

$$y(i,j) = y_0(j) + (i-1)\Delta y, \quad 1 \le i \le m \qquad (15)$$

where $y_0(j)$ is an unknown variable (the value of which is the object of the tracking exercise). Now, although for a field that is isotropic in x-direction (as assumed here), field H needs only one spatial coordinate or argument, (i.e. $H(x,y)$ can be denoted simply as $H(y)$), the sampled function needs two arguments. This is not because x_i is unknown but because y is unknown and x-dependent (since the initial value of $y(i,j)$, i.e. $y_0(j)$ is unknown and j-dependent). We therefore denote the jth discretely-sampled H function, produced at location $y(i,j)$, by the symbol $H_j(i)$, $1 \leq i \leq m$.

2.4. Statement of the Overall Tracking Problem

In terms of the notation introduced in Sections 2.1 to 2.3, the idealised tracking problem here can be formally stated as follows:

Given $H_j(i)$, $1 \leq j \leq r$, $1 \leq i \leq m$, i.e. given m samples, taken in the y direction at <u>*known*</u> *constant intervals Δy starting at* <u>*unknown*</u> *$y_0(j)$, from each of r scans separated in the x direction at* <u>*known*</u> *constant intervals δx, of field $H(x,y)$ where the constant coefficients $a_1, a_2, ..., a_n$ of its polynomial representation*

$$H(x,y) = H(y) = a_1 y + a_2 y^2 ....a_n y^n \qquad (16)$$

are also known and, given that the path of the machine is linear in x, y space so that:

$$\frac{dy_0}{dx} = \underline{\text{unknown}}\ \text{constant}\ g \qquad (17)$$

find g, and hence $y_0(j)$, $1 \leq j \leq r$, given launching position $y_0(1)$.

Clearly it is computationally desirable to keep integer r to a minimum for a given integer m. Fig. 2 illustrates the problem very simply. Effectively, we are given r batches, equispaced at δy, each of m samples of $H(y)$ equispaced at Δy, δy being given by

$$\delta y = g \delta x \qquad (18)$$

where δx is known but constant g is not. The initial position $y_0(1)$ is given and δy is to be found. The solution of the problem is considered in Section 3.

3. Method of Solution

3.1. Special Case (Single Scan) Problem

By way of introduction we first consider the special case of

$$\delta y = m \Delta y \text{ and} \qquad (19)$$

$$mr = n \qquad (20)$$

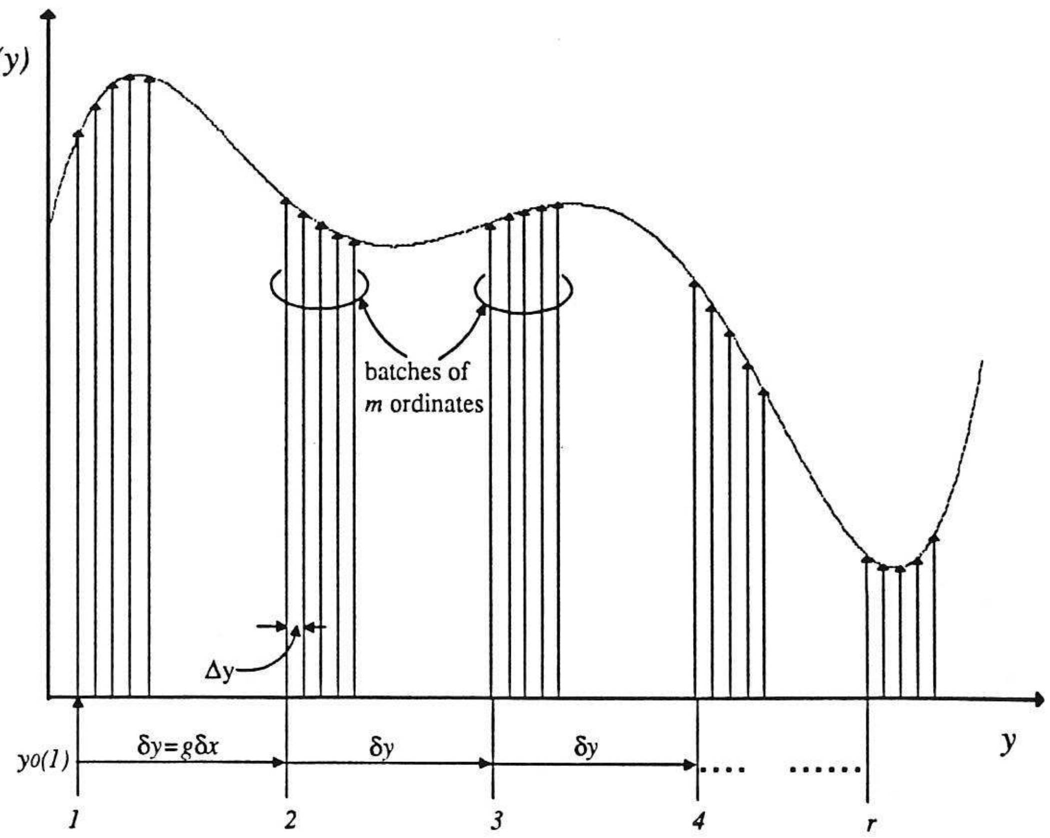

Fig. 2. *r* batches of *m* ordinates of a polynomial field function

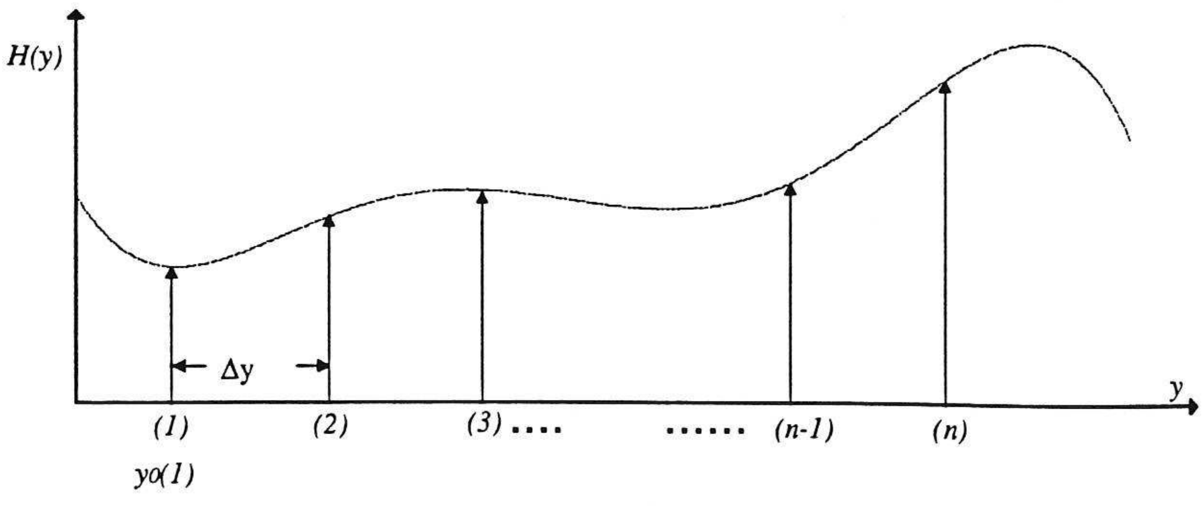

Fig. 3. The single scan solution uses *n* sampled values from a single scan to track a field of order *n*.

so that effectively we are given n consecutive samples of $H(y)$ equispaced at known interval Δy as shown in Figure 3. We shall here consider initial value $y_0(1)$ to be unknown since δy is known in this special case (via equation (19)) if Δy is known. The problem therefore reduces to finding $y_0(1)$ from the n given samples of $H(y)$ and a knowledge of Δy. Since the number of given samples = order of the polynomial describing $H(y)$ in this case the problem is clearly tractable. The method proposed is based on the following analysis.

This special case can be considered as a single scan problem producing the n equispaced samples needed to determine $y_0(1)$. i.e. we can effectively set

$$m = n \text{ and} \tag{21}$$
$$r = 1 \tag{22}$$

and the n samples of $H(y)$ can be denoted by $H_1(i)$, $1 \leq i \leq n$. Now, if we denote first, second.....n-1 th order finite differences between the samples as follows

$$\Delta H_1(i) = H_1(i+1) - H_1(i), \quad 1 \leq i \leq n-1 \tag{23}$$
$$\Delta^2 H_1(i) = \Delta H_1(i+1) - \Delta H_1(i), \quad 1 \leq i \leq n-2 \tag{24}$$
$$\vdots$$
$$\Delta^q H_1(i) = \Delta^{q-1} H_1(i+1) - \Delta^{q-1} H_1(i), \quad 1 \leq i \leq n-q \tag{25}$$
$$\vdots$$
$$\Delta^{n-1} H_1(i) = \Delta^{n-2} H_1(i+1) - \Delta^{n-2} H_1(i), \quad 1 \leq i \leq n-(n-1), \text{i.e. } i = 1 \tag{26}$$

It is obvious from (23) to (26) that with increasing order of the finite differences, the number of values from the given n ordinates of $H_1(i)$ reduces progressively until the differences of order n-1 yields only a single unique value, $\Delta^{n-1}H_1(1)$. Furthermore, this difference is evaluated at the unknown value of $y = y_0(1)$. For simplicity of notation we may therefore write

$$\Delta^{n-1} H_1(1) = \Delta^{n-1} H\{y_0(1)\} \tag{27}$$

or, if it is understood that $\Delta^{n-1}H$ is evaluated only at $y_0(1)$ we may write more simply

$$\Delta^{n-1} H_1(1) = \Delta^{n-1} H \tag{28}$$

Now as shown in Appendix A, the following relations apply between the n-1 th finite difference and the n-1 th and the n th derivatives of H w.r.t. y, all evaluated at the same value of y (= $y_0(1)$ here) i.e.

$$\frac{\Delta^{n-1} H}{\Delta y^{n-1}} = D^{n-1} H + \frac{(n-1)}{2} D^n H \Delta y \tag{29}$$

where $D^{n-1}H$ is merely a linear function of y, i.e.

$$D^{n-1} H = (n-1)!\, a_{n-1} + n!\, a_n y \tag{30}$$

and D^nH is a constant i.e.

$$D^n H = n!a_n \tag{31}$$

Thus, since Δy, n and the coefficients a_n and a_{n-1} are known à priori, and since $\Delta^{n-1}H$ may be computed by successive differencing of the n equispaced samples of H measured, the value of y can be determined directly from (29), which is merely a linear algebraic equation in y. The resulting value is of course $y_0(1)$, the y-location of the first sample. Successive values of y across the scan are readily found of course by successive addition of known increment Δy.

3.1.1. Finding the Sample Spacing

An interesting variant on the above problem is finding Δy if this is unknown à priori, but $y_0(1)$ is given instead. An additional sample $H_1(n+1)$ is needed thus allowing computation of

$$\Delta^n H_1(1) = \Delta^{n-1} H_1(2) - \Delta^{n-1} H_1(1) \tag{32}$$

since two n-1 th order differences are now calculable. Appendix A readily shows that, if we denote $\Delta^n H_1(1)$ simply as $\Delta^n H$ then

$$\frac{\Delta^n H}{\Delta y^n} = D^n H = \text{constant} = n!a_n \tag{33}$$

where difference and derivative again being evaluated at the same value of y (here $= y_0(1)$) and, from (29) and (33) we now deduce

$$\Delta y = \frac{D^{n-1}H}{D^n H\left(\Delta^{n-1}H / \Delta^n H - (n-1)/2\right)} \tag{34}$$

i.e. a simple linear equation relating Δy to $y_0(1)$ in terms of known finite differences and known polynomial parameters. An example of the use of equation (29) is given in Section 4.

3.2. Solution to the Multiscan Tracking Problem

We now return to the general case of $m < n$, where $m+1$ = number of samples per scan, so that more than a single scan is needed to provide sufficient samples of $H(x,y)$ to permit the calculation of the constant but unknown machine movement, δy, between successive scans (see Figs. 1 and 2).

Generalising the notation of equations (23) to (26) to allow for variable scan number j we may write

$$\Delta H_j(i) = H_j(i+1) - H_j(i), \quad 1 \le i \le m, 1 \le j \le r \tag{35}$$

$$\Delta^2 H_j(i) = \Delta H_j(i+1) - \Delta H_j(i), \quad 1 \le i \le m-1, 1 \le j \le r \tag{36}$$

$$\vdots$$
$$\Delta^q H_j(i) = \Delta^{q-1} H_j(i+1) - \Delta^{q-1} H_j(i), \quad 1 \le i \le m+1-q, 1 \le j \le r \tag{37}$$
$$\vdots$$
$$\Delta^m H_j(i) = \Delta^{m-1} H_j(i+1) - \Delta^{m-1} H_j(i), \quad 1 \le i \le m+1-m, 1 \le j \le r, \quad \text{i.e. } i = 1 \tag{38}$$

The above equations define a notation for different orders (1 to m) of finite difference between consecutive samples within general scan number j. Of particular interest here is $\Delta^m H_j(i)$ which is single valued (given only $m+1$ H-samples) for any chosen scan j since only the value for $i = 1$ is applicable. We may therefore simplify the notation for this m'th order finite difference as follows

$$\Delta^m H_j(i) = \Delta^m H_j(1) \equiv \Delta^m H_j \tag{39}$$

Similarly we can define notation for different orders of finite difference between H samples taken for a given value of interscan sample number i, thus

$$\delta H_j(i) = H_{j+1}(i) - H_j(i), \quad 1 \le i \le m+1, 1 \le j \le r \tag{40}$$
$$\delta^2 H_j(i) = \delta H_j(i+1) - \delta H_j(i), \quad 1 \le i \le m+1, 1 \le j \le r-1 \tag{41}$$
$$\vdots$$
$$\delta^p H_j(i) = \delta^{p-1} H_{j+1}(i) - \delta^{p-1} H_j(i), \quad 1 \le i \le m+1, 1 \le j \le r-p+1 \tag{42}$$
$$\vdots$$
$$\delta^r H_j(i) = \Delta^{r-1} H_{j+1}(i) - \delta^{r-1} H_j(i), \quad 1 \le i \le m+1, 1 \le j \le r+1-r, \quad \text{i.e. } j = 1 \tag{43}$$

The r'th order finite difference will be of particular interest and, being single valued for a given i, can be defined by the simple notation

$$\delta^r H_j(i) \equiv \delta^r H_1(i) \tag{44}$$

Operator δ^p, $1 \le p \le r$ can be applied not only to $H_j(i)$ but also to its orthogonal finite difference functions $\Delta^q H_j(i)$, $1 \le q \le m$, so that

$$\delta^p \{\Delta^q H_j(i)\} = \delta^{p-1} \{\Delta^q H_{j+1}(i)\} - \delta^{p-1} \{\Delta^q H_j(i)\} \tag{45}$$

In the particular case of $p = r$ and $q = m$, we note that $\delta^r \{\Delta^m H_j(i)\}$ will be single valued since only the integer values $j = 1$, $i = 1$, are applicable for $r+1$ scans of $m+1$ samples each. Thus the notation may be simplified to

$$\delta^r \{\Delta^m H_j(i)\} = \delta^r \{\Delta^m H_1(1)\} \equiv \delta^r \{\Delta^m H\} \tag{46}$$

The r-1 th order difference across the scans, i.e. $\delta^{r-1} \{\Delta^m H_j(i)\}$ can take two values depending on whether j is set to 1 or 2. Here we shall always choose $j = 1$ so that we may write, again for notational simplicity:

$$\delta^{r-1}\{\Delta^m H_1(i)\} = \delta^{r-1}\{\Delta^m H_1(1)\} \equiv \delta^{r-1}\{\Delta^m H\} \quad (47)$$

Appendix B proves that, having calculated $\delta^{r-1}\{\Delta^m H\}$ and $\delta^r\{\Delta^m H\}$ from the r+1 scans spaced at δy, each of m+1 ordinates spaced at Δy then the desired unknown machine shift, δy, per scan can be calculated simply from the formula

$$\delta y = \frac{D^{n-1}H / D^n H + m\Delta y / 2}{\delta^{r-1}(\Delta^m H) / \delta^r(\Delta^m H) - (r-1)/2} \quad (48)$$

provided the product of integers r and m equal the order n of the polynomial $H(y)$, i.e. provided

$$m + r = n \quad (49)$$

D^nH is again given by

$$D^n H = n!\,a_n \quad (50)$$

and $D^{n-1}H$ by

$$D^{n-1}H = (n-1)!\,a_{n-1} + n!\,a_n y \quad (51)$$

and, for calculating the terms D^nH, $\delta^{r-1}(\Delta^m H)$ and $\delta^r(\Delta^m H)$ we set

$$y = y_0(1) \quad (52)$$

where launch position $y_0(1)$ is known à priori. An examples of the use of the formula is given in Section 4.

4. Examples

4.1 The Single Scan Case

Consider the polynomial

$$H(y) = \sum_{i=1}^{7} a_i y^i$$

where $a_1 = 0.29159$
$a_2 = -0.5379$
$a_3 = 0.40998$
$a_4 = -0.1484$
$a_5 = 2.7089$
$a_6 = -0.2405$
$a_7 = 0.00825$

A plot of this polynomial in the range [0,6] is shown in Fig. 4

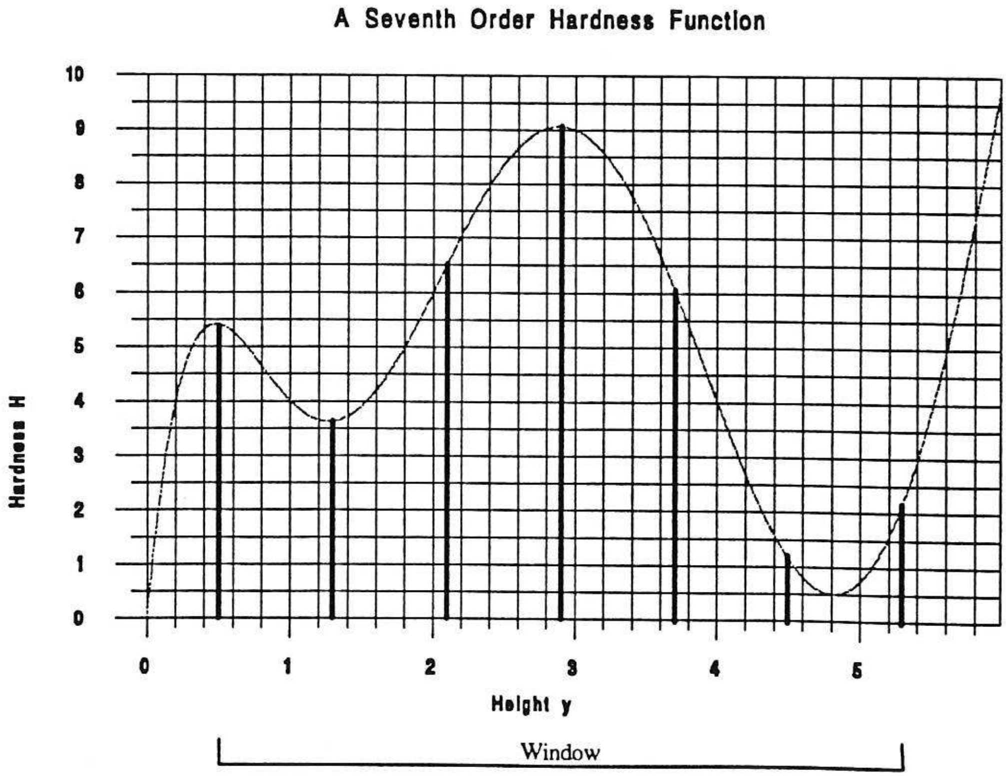

Fig. 4. A 7th order polynomial field function

Now the polynomial function is sampled at points y_i ,$i=1,..,7$ with $\Delta y=0.8$ and $y_1=0.5$ (yet to be determined using our algorithm). Seven sampled values thus obtained are

$$H(y_1) = 5.4099$$
$$H(y_2) = 3.6288$$
$$H(y_3) = 6.5245$$
$$H(y_4) = 9.0635$$
$$H(y_5) = 6.0441$$
$$H(y_6) = 1.1466$$
$$H(y_7) = 2.1986$$

Solving equations (29) to (31) for y yields

$$y_1 = \frac{1}{n!\,a_n}\frac{\Delta^{n-1}H}{\Delta y^{n-1}} - \frac{a_{n-1}}{na_n} - \frac{n-1}{2}\Delta y \qquad (53)$$

For $n=7$ and $\Delta y=0.8$ this becomes

$$y_1 = 1.7646 + 0.0917\ \Delta^{n-1}H \qquad (54)$$

For the this purely deterministic case the above sampled values yield

$$\Delta^{n-1}H = -13.7853 \qquad (55)$$

Hence equations (54) and (55) implies

$y_1 = 0.5$ *(as expected)*

4.2 The Multiscan Case

An example of multiscan tracking is given here. Fourth order polynomial function used here is

$$H(y) = \sum_{i=1}^{4} a_i y^i \qquad (56)$$

where
$$a_1 = -2.624$$
$$a_2 = 2.8790$$
$$a_3 = -0.605 \qquad (57)$$
$$a_4 = 0.0357$$

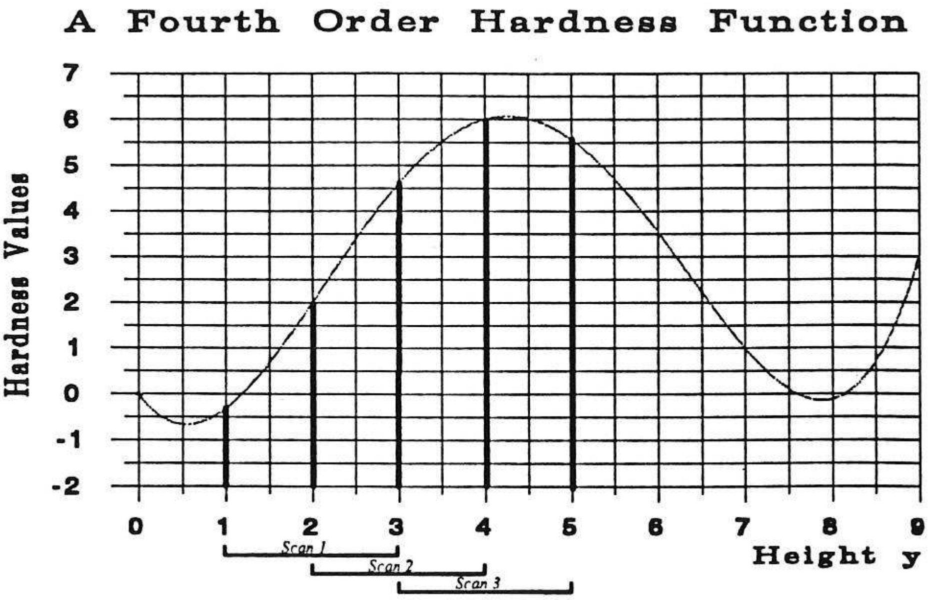

Fig. 5. Sampling field function in 3 scan

A plot of this curve is shown in Fig. 5.

Degree of polynomial	n	= 4
Initial Sampling Position	y	= 1
Sampling Interval	Δy	= 1
Number of Scans	$r + 1$	= 3
Samples per Scam	$m + 1$	= 3
Change of Height per scan	δy	= 1 *(To be determined)*

Result of this sampling is shown in the table below.

Sample No	Scan Number 1	Scan Number 2	Scan Number 3
1	-0.314	2.0	4.6
2	2.0	4.6	6.0
3	4.6	6.0	5.57

Table 1. Sampled Values of the Polynomial Field

With the above parameters and using the polynomial coefficients of equation (57), equations (48), (50) and (51) yields

$$\delta y = \frac{-2.333}{\delta^{r-1}(\Delta^m H)/\delta^r(\Delta^m H)-1} \qquad (58)$$

Where two double differences are calculated using sampled values of the Table 1 as

$$\delta^{r-1}(\Delta^m H) = 0.857 \quad (59)$$

$$\delta^{r}(\Delta^m H) = -1.486 \quad (60)$$

Putting these values in equation (33) implies

$\delta y=1$ *(As expected)*

5. Conclusions

A single valued solution has been developed to calculate the position of a vehicle in a continuous field environment described by a known polynomial from a minimum number of equispaced field measurements taken at that location and known coefficients of the polynomial. Where insufficient measurements from a single scan of the field are available, a formula for utilising extra data from several scans has been produced. The formulas have been demonstrated in many example fields, two of which are presented here.

The investigation and test results to date have been confined to the purely deterministic case and future reports in this series will address the effect of noise.

6. Appendices

6.1 Appendix A

From Taylor's Theorem

$$\frac{\Delta H}{\Delta y} = DH + \frac{D^2}{2!}\Delta y + \frac{D^3}{3!}\Delta y^2 + \frac{D4}{4!}\Delta y^3 + ... + \frac{D^{n-1}}{n-1!}\Delta y^{n-2} + \frac{D^n}{n!}\Delta y^{n-1}$$

now

$$\Delta^2 H = H(y+\Delta 2y) - 2H(y+\Delta y) + H(y)$$

$$= H + DH(2\Delta y) + \frac{D^2}{2!}(2\Delta y)^2 + \frac{D^3}{3!}(2\Delta y)^3 + \frac{D4}{4!}(2\Delta y)^4 + ... + \frac{D^{n-1}}{n-1!}(2\Delta y)^{n-1} + \frac{D^n}{n!}(2\Delta y)^n$$

$$-2\left(H + DH\Delta y + \frac{D^2}{2!}\Delta y^2 + \frac{D^3}{3!}\Delta y^3 + \frac{D4}{4!}\Delta y^4 + ... + \frac{D^{n-1}}{n-1!}\Delta y^{n-1} + \frac{D^n}{n!}\Delta y^n\right) + H$$

$$\therefore \frac{\Delta^2 H}{\Delta y^2} = D^2 H + \frac{2D^3 H}{2!}\Delta y + \text{Higher order terms in } D \text{ and } \Delta y$$

$$\Delta^3 H = H(y+\Delta 3y) - 3H(y+2\Delta y) + 3H(y+\Delta y) - H(y)$$

$$= H + DH(3\Delta y) + \frac{D^2}{2!}(3\Delta y)^2 + \frac{D^3}{3!}(3\Delta y)^3 + \frac{D^4}{4!}(3\Delta y)^4 + ... + \frac{D^{n-1}}{n-1!}(3\Delta y)^{n-1} + \frac{D^n}{n!}(3\Delta y)^n$$

$$-3\left(H + DH(2\Delta y) + \frac{D^2}{2!}(2\Delta y)^2 + \frac{D^3}{3!}(2\Delta y)^3 + \frac{D^4}{4!}(2\Delta y)^4 + ... + \frac{D^{n-1}}{n-1!}(2\Delta y)^{n-1} + \frac{D^n}{n!}(2\Delta y)^n\right)$$

$$+3\left(H + DH\Delta y + \frac{D^2}{2!}\Delta y^2 + \frac{D^3}{3!}\Delta y^3 + \frac{D^4}{4!}\Delta y^4 + ... + \frac{D^{n-1}}{n-1!}\Delta y^{n-1} + \frac{D^n}{n!}\Delta y^n\right) + H$$

$$\therefore \frac{\Delta^3 H}{\Delta y^3} = D^3 H + \frac{3D^3 H}{2!}\Delta y + \text{Higher order terms in } D \text{ and } \Delta y$$

Thus, finally we get

$$\frac{\Delta^{n-1} H}{\Delta y^{n-1}} = D^{n-1} H + \frac{n-1}{2!} D^n H \Delta y + (\text{no. higher order terms})$$

Therefore

$$\boxed{\frac{\Delta^{n-1} H}{\Delta y^{n-1}} = D^{n-1} H + \frac{n-1}{2!} D^n H \Delta y} \qquad (61)$$

6.2 Appendix B

For an *n*th order polynomial (see Appendix A) we can write

$$\frac{\Delta^{n-1}H}{\Delta y^{n-1}} = D^{n-1}H + \frac{n-1}{2!}D^{n}H\Delta y \tag{62}$$

Let $n=m+r$ and if $F(y) = \Delta^{m}H$ is an nth order polynomial in *y* and

$$\frac{\delta^{r-1}H}{\delta y^{r-1}} = D^{r-1}F + \frac{n-1}{2!}D^{r}F\delta y \tag{63}$$

then

$$\frac{\Delta^{m}H}{\Delta y^{m}} = \frac{F(y)}{\Delta y^{m}} = D^{m}H + \frac{m}{2}D^{m+1}H\Delta y + higher\ order\ terms$$

therefore *F(y)* can be written as

$$F(y) = \Delta y^{m}\left[D^{m}H + \frac{m}{2}D^{m+1}H\Delta y\right] + higher\ order\ terms$$

$\therefore$

$$D^{r-1}F(y) = \Delta y^{m}\left[D^{r-1}D^{m}H + \frac{m}{2}D^{r-1}D^{m+1}H\Delta y\right] + D^{r-1}[higher\ order\ terms]$$

$\therefore$

$$D^{r-1}F(y) = \Delta y^{m}\left[D^{r-1}D^{m}H + \frac{m}{2}D^{r-1}D^{m+1}H\Delta y\right] + D^{r-1}[higher\ order\ terms]$$

$\therefore$

$$D^{r-1}F(y) = \Delta y^{m}\left[D^{n-1}H + \frac{m}{2}D^{n}H\Delta y\right] + [no\ higher\ order\ terms]$$

Hence $D^{r-1}F$ can be written as

$$D^{r-1}F = \left[D^{n-1}H + \frac{m}{2}D^{n}H\Delta y\right]\Delta y^{m} \tag{64}$$

Differentiating again

$$D^{r}F = D^{n}H\Delta y^{m} \tag{65}$$

Now using equations (63), (64) and (65) implies that

$$\frac{\delta^{r-1}\Delta^{m}H}{\delta y^{r-1}} = \left(D^{n-1}H + \frac{m}{2}D^{n}H\Delta y\right)\Delta y^{m} + \frac{r-1}{2}D^{n}H\Delta y^{m}\delta y \tag{66}$$

$\therefore$

$$\frac{\delta^{r-1}\Delta^m H}{\delta y^{r-1}\Delta y^m} = D^{n-1}H + \left\{\frac{m}{2}\Delta y + \frac{r-1}{2}\delta y\right\} D^n H \tag{67}$$

Similarly

$$\frac{\delta^r \Delta^m H}{\delta y^r \Delta y^m} = D^n H \qquad (\because D^{n+1}H = 0) \tag{68}$$

Hence

$$\frac{\delta^{r-1}\Delta^m H}{\Delta y^m} = \left(D^{n-1}H + \frac{m}{2}D^n H \Delta y\right)\delta y^{r-1} + \frac{r-1}{2}D^n H \delta y^r \tag{69}$$

Now equation (68) implies that

$$\delta y^{r-1} = \frac{\delta^r(\Delta^m H)}{\delta y \Delta y^m D^n H}$$

Therefore equation (69) becomes

$$\frac{\delta^{r-1}\Delta^m H}{\Delta y^m} = \left(D^{n-1}H + \frac{m}{2}D^n H \Delta y\right)\frac{\delta^r(\Delta^m H)}{\delta y \Delta y^m D^n H} + \frac{r-1}{2}D^n H \frac{\delta^r(\Delta^m H)}{\Delta y^m D^n H}$$

$\therefore$

$$\delta^{r-1}\Delta^m H = \left(D^{n-1}H + \frac{m}{2}D^n H \Delta y\right)\frac{\delta^r(\Delta^m H)}{\delta y D^n H} + \frac{r-1}{2}\delta^r(\Delta^m H)$$

$\therefore$

$$\frac{\delta^r \Delta^m H\left(D^{n-1}H + \frac{m}{2}D^n H \Delta y\right)}{\delta y D^n H} = \delta^{r-1}(\Delta^m H) - \frac{r-1}{2}\delta^r(\Delta^m H)$$

$\therefore$

$$\frac{\delta y}{\delta^r \Delta^m H\left(\frac{D^{n-1}H}{D^n H} + \frac{m}{2}\Delta y\right)} = \frac{1}{\delta^{r-1}(\Delta^m H) - \frac{r-1}{2}\delta^r(\Delta^m H)}$$

Hence shift per scan δy is given by

$$\boxed{\delta y = \frac{D^{n-1}H / D^n H + m\Delta y / 2}{\delta^{r-1}(\Delta^m H) / \delta^r(\Delta^m H) - (r-1)/2}} \tag{70}$$

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