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Potential Use Of Structural Layout Optimization At The Conceptual Design Stage

Peter Park, Matthew Gilbert, Andy Tyas and Olga Popovic-Larsen

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Abstract

Despite recent developments in computer-aided design in architecture, both in terms of form generation techniques and performance-based design tools, there still appears to be polarization between the 'visual' and the 'technical' elements of design. Two causes of this are discussed: long-standing tradition within the discipline and perception of design as primarily a visual exercise. Structural layout optimization is a technique which enables automatic identification of optimal arrangements of structural elements in frames. As the technique appears to have the potential to help reduce the polarization between the visual and the technical elements of design, it can be considered as an 'integrative' form generation tool. Applications of the technique are considered via three design examples, demonstrating both its potential and areas where refinement is required before it is suitable for application in practice.

1. INTRODUCTION

Distinct paradigms in architectural design history can be identified according to distinct eras of design and construction tools and materials as much as by the design theories and ideas of those eras [1]. Typically 'technical' and 'visual' aspects are considered largely in isolation, and the limited number of publications in the literature on the interdisciplinary nature of architecture and engineering suggests that there is a systematic, and to some extent institutionalized, divide between the 'visual' and 'technical' aspects of design. This situation is thought to be largely due to the sheer scale, nature and complexity of modern building projects, with the divide apparently concretized by the difference in the nature of the design tools used by architects and engineers*.

In this light, computer-based tools and techniques (e.g. traditional CAD software) developed in recent years should ideally be 'integrative', in the sense that they should help to narrow the divide between the 'visual' and 'technical' aspects of design. Unfortunately there is little evidence that this is occurring.

However, while the application of computer-based technology has made numerous large-scale projects possible, it is also true to say that it has led to specialist silos of knowledge within the building design sphere. Thus on the one hand there are groups working with highly advanced visual techniques (e.g. form finding and form generation techniques), and on the other hand groups working with highly advanced physical modelling tools (e.g. finite element based tools and other performance-based design approaches). This has arguably resulted in an over-emphasis on certain aspects of the design process, depending on whether the design process has been initiated as a visual or performance-based exercise.

This apparent polarization, which began from intentional division for design convenience, may now be counterproductive and, given the level of development of the seemingly disparate technologies being applied, it seems anomalous that few steps appear currently being taken to provide a bridge between them. This paper seeks to address this by examining the role of structure in architectural form conceptualization, and considering the potential role for the *structural layout optimization* technique to be used early in the design process (referred to here as the 'conceptual design stage'), and the extent to which this can help bridge this apparent divide.

2. FORM GENERATION METHODS IN CONTEXT

2.1. Background

Form generation involves definition or conception of the external shape of an object or arrangement of its constituent elements. In recent years various methods have been applied to generate highly irregular and/or curvilinear forms (cf. buildings based on simple geometrical shapes, which

*Notwithstanding the apparent 'integrative' nature of for example modern 'Building Information Models' (BIM).

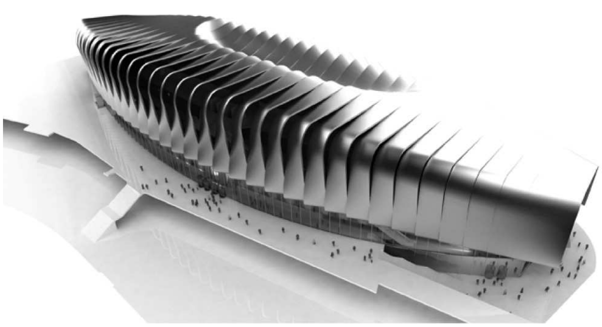
were common prior to the ubiquitous use of CAD). Evidence of this can be obtained from the portfolios of prominent architecture studios (e.g. Gehry & Partners; Future Systems; Foster & Partners), and in the entries to influential architecture competitions (e.g. RIBA Stirling Prize; Emporis Skyscrapers Award; AIA Progressive Architecture Award).

Though form may be considered as just one facet of architectural design, it is undeniably a highly important one. Entirely manual form generation techniques can be applied, though computational form generation techniques are likely to find increased use in the future, including [2]:

1. Parametric modelling techniques (using non-Euclidean geometries, NURBS etc.)
2. Metamorphosis & evolutionary architecture techniques
3. Performance-based methods (e.g. based on mathematical layout & topology optimization techniques – see Section 4 for a more in-depth consideration)

Alternatively, irrespective of whether manual or computational methods are being applied, a number of categories of form generation can be identified: imitation; controlled randomization; repetition (including mirroring, alignment and segmentation); variation (including misalignment); geometricism (use of simple geometric shapes as primary elements); use of relevant physical principles (other than purely geometric or visual principles); 2D-to-3D extrusion. All the aforementioned can be influenced by internal usage or arrangement requirements, and also by inherent limitations of the available tools and/or structural principles.

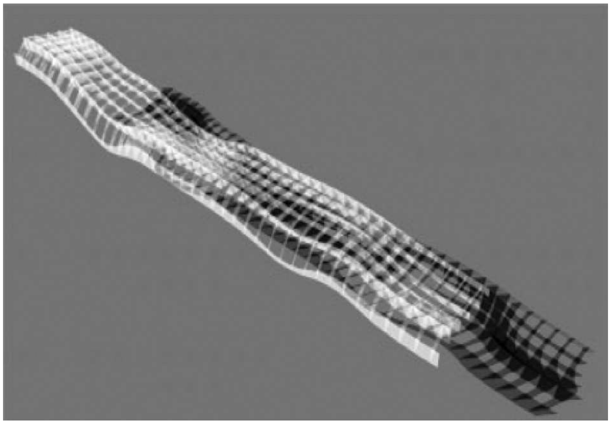
Among contemporary design projects (e.g. see Figure 1, Figure 2; References [3],[4]), the form generation methods used in practice typically assume that the structure functions purely according to some visual or 'geometric' principles. i.e. a visual representation of 'form' is prescribed, with spatial and aesthetic considerations taken into account, but with physical principles largely ignored+. These physical principles, together with other primarily 'functional' or 'technical' subsidiary aspects, are usually only accounted for at the subsequent detailed design stage, thus finally allowing the form to be realized 'off the computer screen'.



◀ Figure 1 : Garibaldi Exhibition Centre, Grimshaw-Architects, Milan 2006.

+For example, in the case of experimental folding forms (e.g. Figure 1) initial concept models may be constructed using a material very different to that which can feasibly be used in practice, frequently leading to forms which are in practice unrealisable.

► Figure 2 : Brian Boyer – non-structurally initiated.



► Figure 3 : Guggenheim Museum, Gehry & Partners.

2.2. Division between the 'visual' and 'technical' elements of design

It is sometimes suggested that the division between the 'visual' and 'technical' elements of design is a necessary consequence of the dramatic increase in the scale and complexity of modern projects. However, the division can also be partly attributed to long-standing tradition, with historical roots dating back to the time of Bacon [5]. It is also generally accepted that in the 1800s a clear division arose between proponents of the Enlightenment ideology (viewing science as 'the truth') and proponents of the then emerging Romanticism ideology (viewing science as dehumanising, and leading to the destruction of beauty) [6], [7]. These struggles were later reflected in theories of architecture, and subsequently in buildings [8], and continue to influence design practice to the present day.

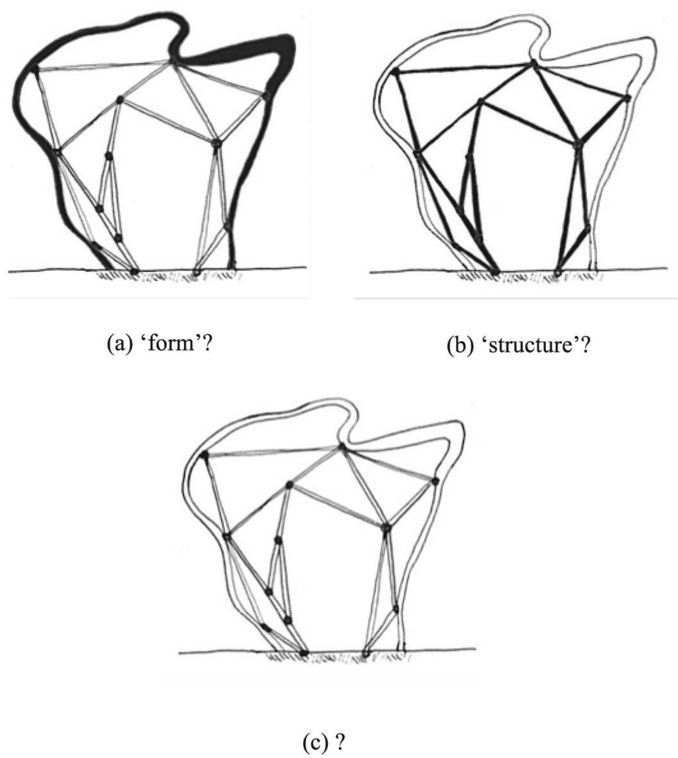
However it is relevant to consider the following questions: why did a functional, efficient and rational design like the Eiffel Tower not become a definitive aesthetic, considering its harmonious combination of structural efficiency and aesthetic elegance? Why is structural efficiency not viewed as synonymous with beauty? [9] And why are such harmonious combinations not more often the norm in modern-day buildings?

Many prominent contemporary large-scale building designs are 'form-oriented' or 'form-led' (that is to say the iconoclastic external envelope or overall form is sought with high priority, often at the expense of other aspects of building). Inevitably, this begs the question: what is 'form' in architecture, and why does form so often seem to govern the design of a building? Loosely 'form' can be defined as 'a visually perceivable pattern or structure with spatial attributes', and for an object to really take 'form', it has to physically exist (i.e. to be 'realisable' in practice).

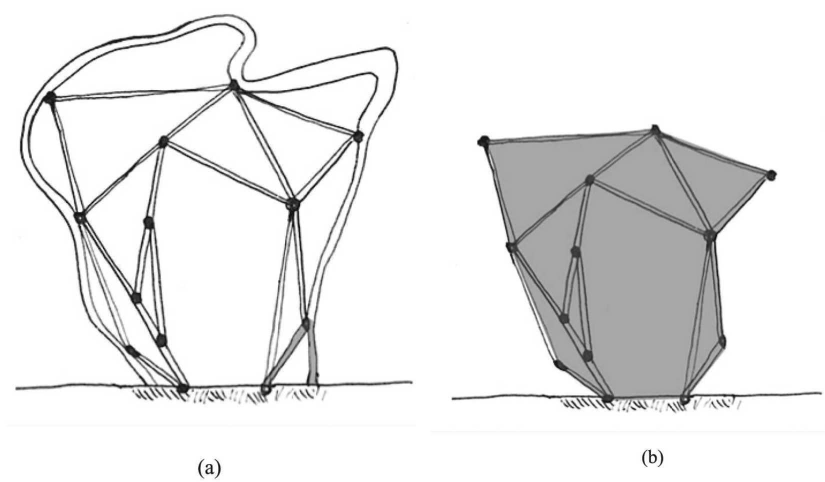
2.3. Inter-relationship between form and structure

It is of interest to establish the nature of the relationship between 'form' and 'structure'. Figure 3 shows The Guggenheim Museum, Bilbao, designed by Gehry & Partners, with truss structures covered in a mesh-type envelope. Figure 4 shows a section through a generic free-form building of a similar type. Figure 4(a) highlights elements that are conventionally perceived to constitute 'form' (i.e. surface), whereas Figure 4(b) highlights those that are conventionally perceived to constitute 'structure'. Figure 4 (c) shows both sets of elements. The drawings in Figure 5 clearly highlight ambiguity in the conventional design definitions of 'form' and 'structure'.

Figure 5 (a) highlights a part of the building that can be considered to define both 'form' and 'structure', while Figure 5 (b) highlights the structural skeleton taking what appears to be a 'form'. Unsurprisingly this ambiguity leads us to question the clear-cut division between the two aspects of a building, devised originally for the convenience of designers, fabricators etc. Indeed, the illustrated ambiguity highlights the inevitable interaction between 'form' and 'structure'. Nevertheless, this separation, which was initially developed for practical convenience, is still widely accepted in standard design practice, and inevitably influences the way many designers think and work. How did this happen? Is it because we ultimately perceive design as a primarily visual exercise, with modern computer software applications only serving to reinforce this perception?



► Figure 4 : Ambiguity in definitions of ‘form’ vs. ‘structure’.



► Figure 5 : Ambiguity between ‘form’ and ‘structure’: (a) a triangular structural element that can be considered to define both ‘form’ and ‘structure’ is highlighted; (b) the envelope of elements considered as constituting ‘structure’ is highlighted, showing that this also defines ‘form’.

2.4. A critical appraisal of the role of computers in the design process

The numerous methods in which a 'form' can be generated have been briefly outlined in the previous section. However, given its ever-increasing role in the design process, it is useful in particular to critically appraise the role played by the computer.

Firstly it can be observed that most common computer interface components are actually unidirectional (there are exceptions – e.g. touch screens used in 3D sketching [10]), which limits the degree of interaction between designer and computer. The most effective machine-to-human interface is currently the computer monitor, where all inputs and outputs are visualized. With the contemporary method of 'monitor and mouse' almost all information has to be visually communicated between the media and the designer. This is in common with the conventional method of 'pen and paper', indicating that visual communication of information obviously predates computer aided design processes. However, the particular mode of operation to which many designers have become committed when using a computer can be highly restrictive.

Of course if ideas to be communicated are essentially of a visual nature this does not pose a problem. However, upon analysing any building, not even the simplest of objects (e.g. a humble doorknob) is in reality a mere visual entity. For example, surface texture, weight, structure and temperature are other aspects which are essentially filtered out through a simple visual representation. It should also be pointed out that a visual 'form' is still an idea, and visual 'existence' of a design object is virtual; a design object has to be more fully justified in order to physically exist. Transformation of ideas into reality is (or should be) at the heart of the architectural design process.

Nevertheless, many designers continue to consider visual representation as the primary means of communication, symptomatic of the current 'ocular-centric' culture in which we live, and which appears to extend into the sphere of the computer-aided design process in architecture [11]. This means that simulations, or other means of supporting more abstract ideas or principles (e.g. level of comfort or physical stability), tend to be filtered out through the use of computer visualization, and collation of other information is still separately required. This appears to be a significant missed opportunity, and an aim of the present study is to evaluate the potential for such simulations to be placed closer to the heart of the conceptual design process.

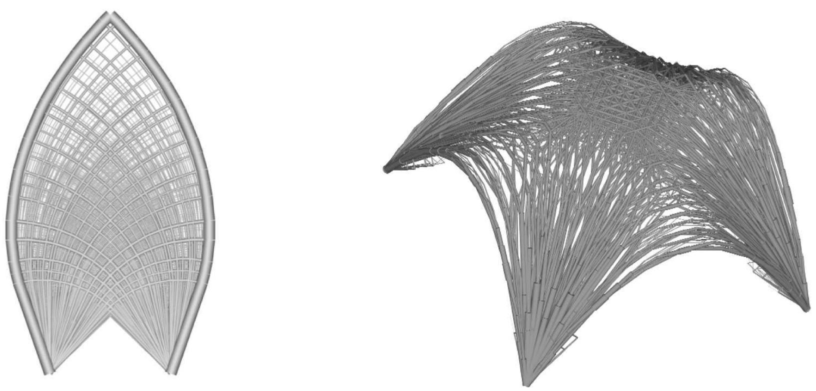
2.5. The computer as a genuinely integrative design tool

Although the computer can never model a building in its complete entirety, it is clearly capable of modelling much more than purely visual aspects. It can be argued that the proliferation of visualization techniques has not necessarily 'improved' or expanded the boundaries of what architecture can be; indeed over-emphasis on the visual can reduce architecture to a mere visual sensation or, in more practical terms, can simply waste time. Although other performance-related aspects of a building can be used to initiate the concept design phase, and some enlightened designers adopt this strategy, this appears not to be commonplace in standard design practice. This seems regrettable as ideally designers should have the freedom to explore other, non 'visual', aspects of design, be it to identify a physical solution to a social problem or to synthesize a functional sculpture.

Indeed, given the immense capabilities of a modern desktop computer, it should be feasible to ensure that the physical behaviour of any form being designed can be taken into account at the initial conceptual design stage. Incorporation of structural considerations via the use of mathematical optimization techniques is potentially one small step towards achieving this. With this in mind, a software application originally developed for use by structural engineers to identify the optimal arrangement of structural members in frameworks has recently been re-evaluated by the present authors with a view to using it in the architectural design process. The software is based on the structural layout optimization technique which will now be briefly described.

The structural layout optimization technique was first developed in the 1960's in order to automatically identify the optimal arrangement of members in either 2D or 3D frameworks, satisfying predefined constraints and a predefined optimality criteria [12]. Recent advances have meant that very large scale design problems can now be tackled [13]. An advantage of using the original layout optimization formulation, in which all members in the optimal minimum volume (weight) structure are 'fully stressed', is that highly-developed mathematical optimization solvers can be used to identify optimal solutions in a short space of time (see Appendix for details of the basic formulation).

Sample 2D and 3D output is shown in Figure 6; note that in order to generate structurally sound concept designs boundary conditions and details of the applied loading must initially be specified.



► Figure 6 : Sample structures generated using layout optimization (after Gilbert et al. [14]): (a) 2D 'Michell structure' (design constraints: two pinned supports at base and a horizontal point load at the top of the domain); 3D roof structure (design constraints: 4 pinned supports at base and uniformly distributed vertical 'transmissible' loading).

Although obtained by specifying relatively simple loading and support conditions, these optimal forms might be considered to exhibit aesthetic characteristics reminiscent of 'emergent' forms (the term 'emergent' refers to *"the spontaneous occurrence of an organization or a behaviour that is greater than the sum of its parts"* [15]). This type of optimization tool would therefore appear to have the potential to appeal to a wide range of users, including architects and mega-sculptors.

Considering a potential use of the structural layout optimization technique in an architectural design environment, various questions arise. For example, why should the least volume (weight) structure be sought, unless the weight is of critical importance, which in most cases is not? Furthermore, surely such a technique has the potential to adversely impinge on the creative process at the form conceptualization stage?[**] Whilst both these questions, and no doubt many others, deserve answers in due course, this paper seeks instead to address a rather simpler question, namely is there potentially a place for structural layout optimization techniques in the architectural design process, at the conceptual design stage?

It is also worth pointing out that we are in a sense here more interested in the nature of the forms generated (together with the process of identifying them, and how this process can be changed to manipulate the forms generated), than whether or not the forms are structurally 'optimal'. This clearly brings us outside the traditional domain of engineering, where the goal is generally to single-mindedly seek out the most efficient (cost-effective) structural solutions.

Finally, it should be noted that application of optimization technology in design is by no means new; it is widely used in the automotive and aerospace industries, though has to date found comparatively little application in the construction industry. However, recently some buildings have been designed with the help of this technology (e.g. see [16]), and tools such as the 'EifForm' design software developed by Shea [17] have attracted significant interest.

3. DESIGN EXAMPLES

It is now useful to consider a number of examples which illustrate how structural layout optimization technology might be applied in reality. These examples offer a range of opportunities for the software to identify possible solutions, ranging from an initial relatively unconstrained example ('Thinking pods') to a much more highly constrained multi-storey building example with supports prescribed to coincide with building frame locations, and with realistic loading conditions.

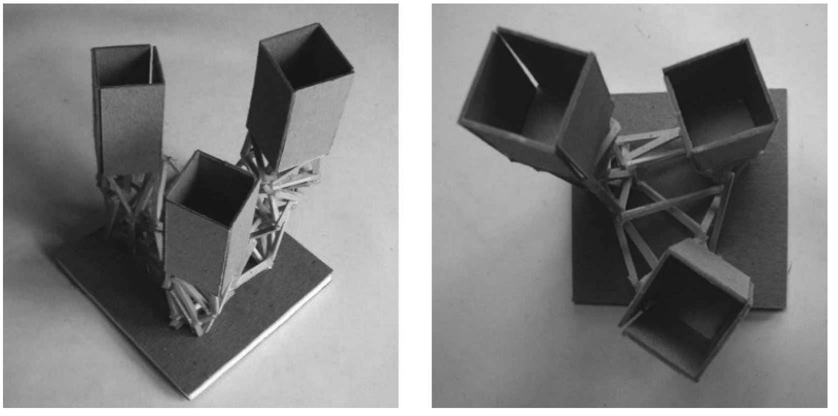
[**] Issues of form conception in various branches of architecture-related disciplines have been the subject of much debate, e.g. [2], [4] and [16].

3.1. ‘Thinking Pods’

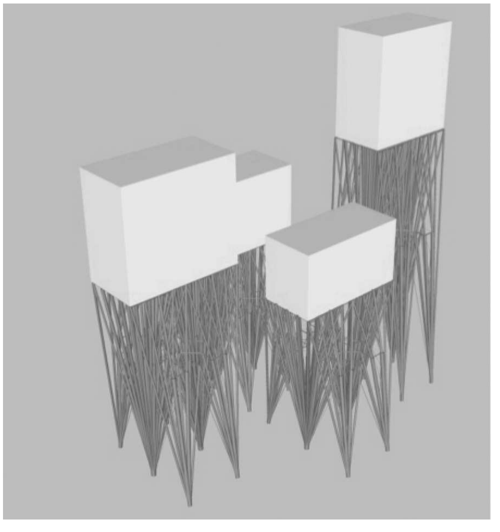
Here the brief was to design a multitude of elevated spaces for relaxation and cogitation, supported high above the ground on stilts in a wooded area of a North American University campus. Initial concept design was carried out in collaboration with a student of architecture studying on the campus, with the most promising manually identified design concept shown in Figure 7. It shows individual cuboid-chambers (typical size: $4 \times 5 \times 5$ m) supported on a web of interconnected space trusses. Though the geometries of these space trusses were not explicitly defined, the plan areas and overall elevations of the chambers were fixed.

The same overall design constraints were then fed into the structural layout optimization tool; the solution obtained is shown in Figure 8 (using simultaneous vertical and horizontal loading as the design load case).

► Figure 7 : Conceptual design of ‘thinking pods’: manually derived.



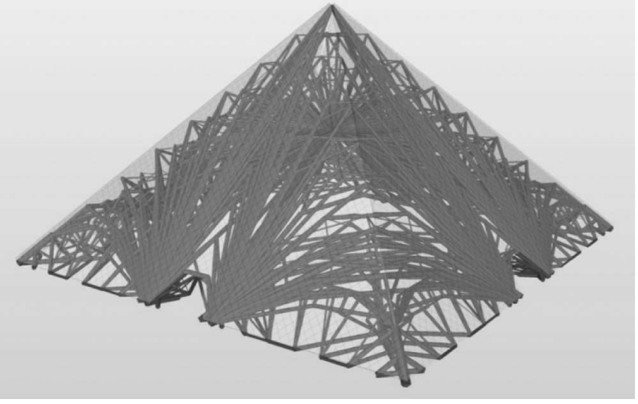
► Figure 8 : Conceptual design of ‘thinking pods’: obtained using structural layout optimization.



While the manually derived concept model featured pods supported by trusses which were for reasons of efficiency interlinked (e.g. see Figure 7 (b)), the design obtained using structural layout optimization techniques, shown on Figure 8, did not. This was to ensure a rapid run time, rather than due to an intrinsic shortcoming of the layout optimization technique itself. Additionally, for sake of simplicity, it was assumed that the floor of a given pod had negligible bending strength, with a consequence that loads were applied to closely spaced nodes distributed across the base of the pod. Since every load must be carried by a structural element, this leads to a somewhat impractical proliferation of members, which then converge on the supports at ground level (see Figure 8). This serves to highlight the need to incorporate adequate user-controls when developing a practical software tool based on this approach. Additionally, in a real design setting, the ability to specify limits on the number of members converging on a joint, or the positions of individual joints (nodes) would often be highly desirable. To incorporate certain practical constraints use of a more complex mathematical formulation than the linear formulation used here would be required (e.g. see Appendix A3 for brief details of a potential MILP-based approach).

3.2. Pharaonic Village Project

Here the brief was to design a children's games area, covering an area of 20m by 20m located on an Egyptian-themed restaurant complex in the Middle East.



◀ Figure 9 : Conceptual design of glass pyramid obtained using structural layout optimization.

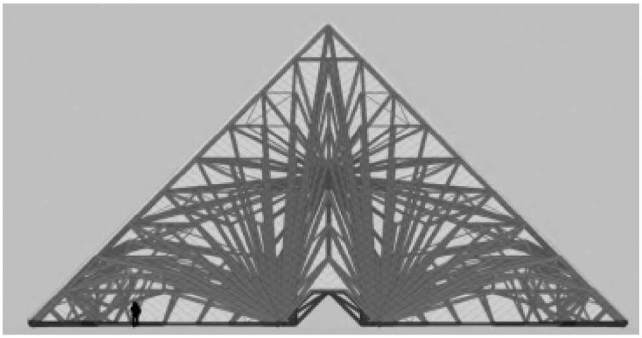
To be in keeping with the Egyptian theme, the requirement was to create a glass-clad pyramid, with sides inclined at 45-degrees. i.e. the external envelope was prescribed at the start of the design process, but the means of supporting this was left unspecified. It was also required that each face of the pyramid should have a central opening at ground level. To identify a suitable supporting structure the structural layout optimization tool was used. Assuming symmetrical loading (vertical and horizontal) and geometry, only one eighth of the structure needed to be modelled. Point supports were specified at the corners of the pyramid and to each side of the ground level openings. The permissible design domain was limited to ensure that structural members would not intrude excessively into the internal usable space. Initial results are shown in Figures 9 and 10.

It appears that the structural solution obtained using the layout optimization tool is rather complex, especially considering that at the initial stage it is basic design concepts that are usually being sought. However, essential features of the solution can be extracted for use in later stages of the design process. For example, Figure 11 show a simplified version of the same basic design (simplification was achieved via a semi-automatic procedure which involved firstly filtering out very small members, then manually removing selected members in congested areas). The simplified version clearly reveals the essential, and apparently novel, structural principle at work (i.e. an elevated central 'node' from which many members radiate), and provides a solution with uninterrupted floor space as required.

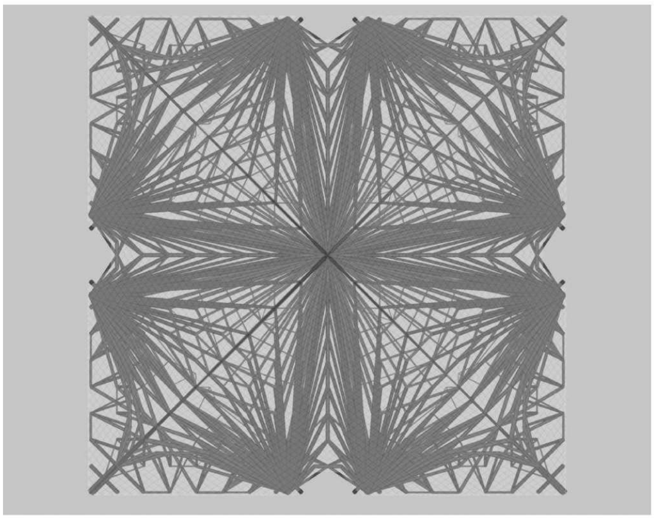
3.3. Canopy for roof terrace in multi-storey building

The third example involves the design of a sloping canopy roof for a multi-storey office building to be constructed in central London (size: 35 $\times$ 40 m in plan). In this example real design load data was used, thereby in principle allowing the solution generated to be compared against the more conventional beam-grid design which was used in practice.

The design solution is shown in Figure 12, Figure 13 and Figure 14. The solution has significantly more visual interest than a conventional beam-grid design, provides an uninterrupted floor space and also appears much more structurally efficient (<10% of the weight of the adopted beam-grid design, though care must be taken in comparing the result from a relatively simplistic optimization with the real design which will inevitably have rationalised the number of different members and required them to fit a more regular grid to simplify glazing details, and in which all members will have been designed to meet the requirements of building codes with adequate factors of safety. Nevertheless, the very large potential weight saving is noteworthy and perhaps gives an indication of how economy of material use is currently highly subordinate to simplicity of construction).



(a)

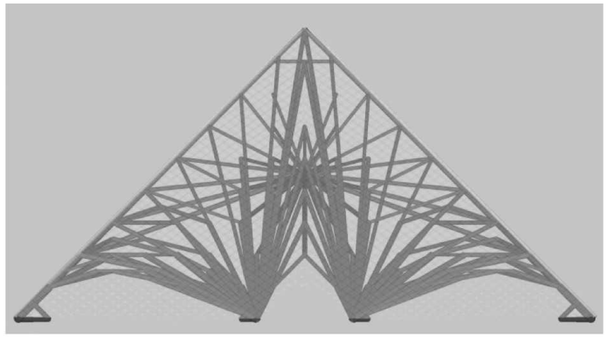


(b)

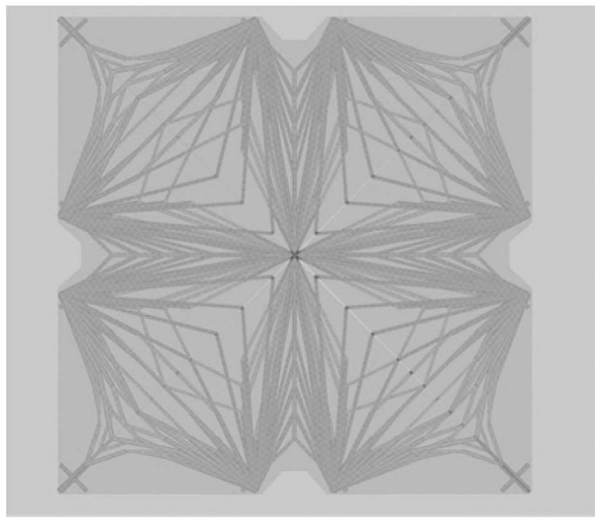
(c)

◀ Figure 10 : Conceptual design of glass pyramid obtained using structural layout optimization - initial detailed solution: (a) side view; (b) plan view; (c) internal view.

► Figure 11 : Conceptual design of glass pyramid obtained using structural layout optimization - subsequent simplified solution: (a) side view; (b) plan view; (c) internal view..

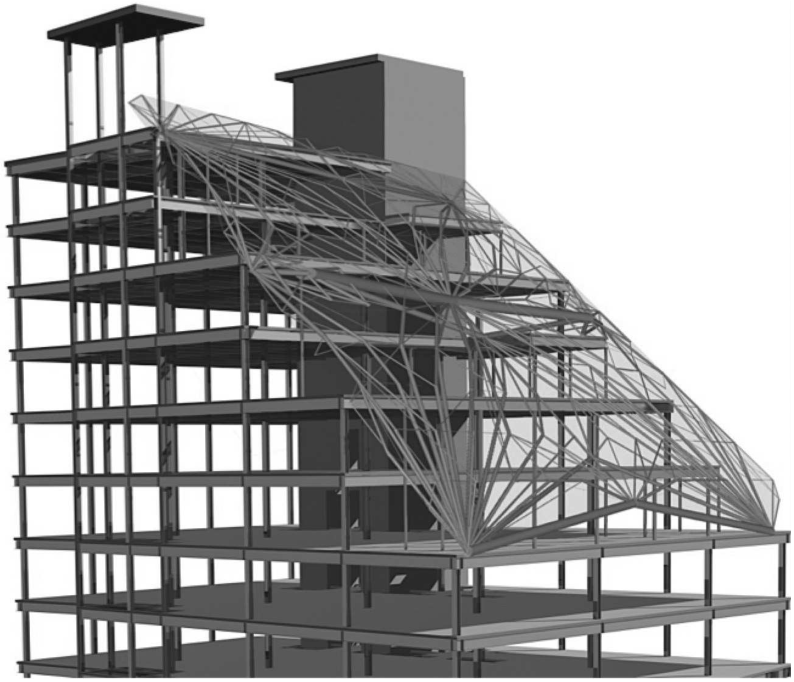


(a)

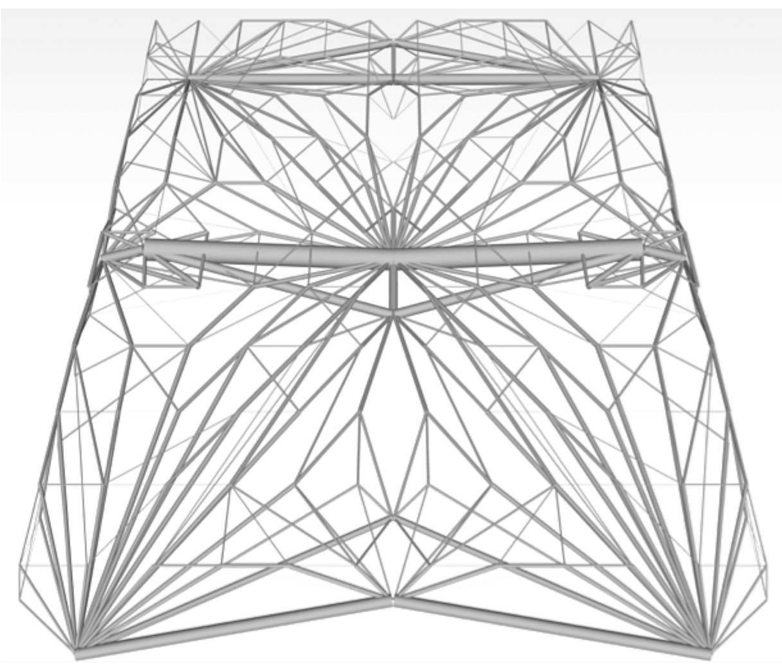


(b)

(c)



◀ Figure 12 : New canopy for roof terrace: in context of the main building framing elements.



◀ Figure 13 : New canopy for roof terrace: front view.

► Figure 14 : New canopy for roof terrace: side view.

3.4. Commentary

The design examples considered have revealed at least two potential usage patterns. The structural layout optimization tool described could for example be used in the following modes:

- 'Full Automatic': In this mode the user specifies the bare minimum of design constraints prior to carrying out an optimization. This has the potential to yield interesting, possibly 'emergent', forms.
- 'Optimization with Prescribed Outer Geometry': In this mode the geometry of the outer envelope is fully or partially prescribed by the user prior to carrying out an optimization. A possible application includes rationalization of the layout and sizes of internal structural members in a design solution where the outer geometry has already been finalized. Areas where structural members cannot be positioned can also be specified if required.

It should be noted that the design study described in section 3.1 ('Thinking Pods') is essentially an example of the 'Full Automatic' method whereas that described in section 3.2 ('Pharaonic Village Project') is an example of 'Optimization with Prescribed Outer Geometry'. The final design study described in section 3.3 ('Canopy for roof terrace in multi-storey building') is also an example of 'Optimization with Prescribed Outer Geometry', although the original geometry was adjusted slightly in order to achieve the desired effect.

It should also be noted that the forms generated using layout optimization are a function of the input parameters (e.g. permissible extent of design domain, potential locations of supports etc.), some of which are uncertain at the conceptual design stage. Thus in practice it will be useful to generate a range of forms by varying the input parameters, reserving for further scrutiny those which appear promising. Finally, there are of course numerous issues which influence what makes a 'good design' and, although the focus of the present paper, structural considerations are just one of them.

4. CONCLUSIONS

1. The longstanding divide between the 'technical' and 'visual' aspects of architecture does not appear to being bridged by current and emerging computer-based design tools, notwithstanding the apparent integrative nature of some tools (e.g. Building Information Models). Furthermore, because of the 'visual' nature of the interaction between a computer and user, it may be argued that such computer-based tools are even widening this divide.
2. Structural layout optimization is a technology which has the potential to provide the architect with the ability to rapidly identify concept designs which are intrinsically structurally sound. The technology therefore has the potential to play a small part in bridging the divide between the 'technical' and 'visual' aspects of architecture.
3. In this paper the structural layout optimization technique has been applied to a number of conceptual design problems, allowing several potential usage patterns to be identified. For example, when minimal design constraints are imposed the technique can yield interesting 'emergent' forms; when the technology is applied to design problems where the outer envelope has already been fixed, the technique can be used to identify efficient locations for supporting framing elements.
4. Further work is required to increase the power and flexibility of the structural layout optimization tool used in this study, and also to make it fully interactive.

Appendix: STRUCTURAL LAYOUT OPTIMIZATION

A1. Description of structural layout optimization process

The structural layout optimization process involves several steps: (i) the designer defines the extent of the design domain, and also the support and load conditions; (ii) the design domain is populated with *n* nodes, typically uniformly spaced, which represent the potential end-points of structural members; (iii) the *n* nodes are inter-connected with *m* potential structural members, forming a so-called 'ground structure'; (iv) optimization techniques (e.g. linear programming, LP [12]) are used to identify the subset of members present in the structure that best fulfils the required design criteria (e.g. to find the structure which uses the minimum volume of material).

A2. Linear programming (LP) structural layout optimization formulation

The equilibrium LP plastic design formulation for a 2D ground structure subjected to a single load case and containing *m* members and *n* nodes where the design objective is to find the minimum structural volume can be stated as follows[13]:

minimize

$$V = \mathbf{q}^{\mathrm{T}}\mathbf{c} \tag{A1}$$

subject to:

$$\mathbf{Bq} = \mathbf{f} \tag{A2}$$

$$q_i^+, q_i^- \geq 0, i = 1,...,m \tag{A3}$$

where V is the total volume of the structure, $\mathbf{q}^{\mathrm{T}} = \{q_1^+, -q_l^-, q_2^+, -q_2^-, ..., q_m^+, -q_m^-\}$, $\mathbf{c}^{\mathrm{T}} = \{l_1/\sigma_1^+, -l_1/\sigma_l^-, l_2/\sigma_2^+, -l_2/\sigma_2^-, ..., l_m/\sigma_m^+, -l_m/\sigma_m^-\}$ $\mathbf{B}$ is a suitable $(2n\times 2m)$equilibrium matrix, $\mathbf{f}^{\mathrm{T}} = \{f_1^x, f_1^y, f_2^x, f_2^y, ..., f_n^x, f_n^y\}$ and where $l_i, q_i^+, q_i^-, \sigma_i^+, \sigma_i^-$ represent the length and tensile and compressive member forces and stresses in member *i* respectively. Finally, f_j^x, f_j^y are the *x* and *y* direction live load components applied to node *j*. The LP variables are the tensile and compressive member forces in $\mathbf{q}$.

A3. Mixed integer linear programming (MILP) structural layout optimization formulation

As a variation on the formulation given in Eqns (A1 – A3), it is possible to introduce additional binary and integer variables to indicate for example whether a given member is 'on' (present) or 'off' (absent) in the final structural solution, giving rise in mathematical terms to a 'mixed integer linear programming' (MILP) formulation. Such variables make it possible to for example specify the maximum number of members converging on a given joint, increasing the power of layout optimization as far as the designer is concerned, albeit at the expense of computational efficiency. It is also possible to develop MILP formulations which allow more accurate modelling of the behaviour of compression members, which will in reality buckle if overly slender. The usefulness of various MILP formulations are currently being investigated by the authors.

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