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Time scaling of the electron flux increase at GEO: The local energy diffusion model vs observations

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Abstract. The characteristic time scaling of the electron flux evolution at geosynchronous orbit (GEO), resulting from the quasilinear wave-particle interaction, is investigated. The upper limit of the electron flux increase rate, due to the interaction with waves, is deduced from the energy diffusion equation (EDE). Such a time scaling allows for a comparison with experimentally measured fluxes of energetic electrons at GEO. It is shown that the analytically deduced time scaling is too slow to explain the observed increase in fluxes. It is concluded that radial diffusion plays the most significant role in the build up of the energetic electrons population at GEO. However, this conclusion is only justified if the seed population energies are very low.

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1. Introduction

The radiation belts were among the very few regions discovered in the terrestrial magnetosphere by the first spacecraft measurements *Van Allen* [1959]. However, in spite of being discovered so early, the physical processes related to the evolution of radiation belts are not yet understood. The main puzzle is the evolution of the high energy electron fluxes at the outer radiation belt. These fluxes are very dynamic and can change by orders of magnitude on a short time scale. High fluxes of the relativistic electrons represent a serious hazard to the spacecraft. As the outer radiation belt usually encompasses the Geosynchronous Orbit (GEO), it is one of the key orbits for many modern technological systems. The understanding of the evolution of the relativistic electron fluxes can reduce the vulnerability of spacecraft operating at GEO. A number of studies have focused on the development of forecasting models deduced directly from data [*Baker et al.*, 1990; *Li et al.*, 2001; *Wei et al.*, 2011]. While such data deduced tools have been very successful in many fields, their forecasts of the radiation belts still lack the required reliability and accuracy.

Currently, a number of models have been proposed to explain acceleration of the electrons in the outer radiation belt. The most promising are based on either radial diffusion [*Falthammar*, 1968; *Schulz and Lanzerotti*, 1974] or the interaction with waves within the outer radiation belt itself [*Temerin et al.*, 1994; *Reeves et al.*, 2009]. According to the models based on radial diffusion, the energization of electrons takes place due to the earthward diffusion of the initial seed population and the conservation of the first and the second adiabatic invariants. It was shown in a number of studies that ULF waves at the boundaries of the magnetosphere are able to enhance the radial diffusion [*Elkington et al.*, 1999; *Hudson et al.*, 1999, 2000, 2001]. Models that are based on the interaction of the electron population with waves [*Shklyar*, 2011; *Shklyar and Matsumoto*, 2009; *Shprits et al.*, 2008; *Omura et al.*, 2007; *Horne et al.*, 2005; *Summers and Thorne*, 2003; *Summers et al.*, 1998, 2002, 2004; *Albert*, 2003, 2005] assume the local diffusion in the pitch angle and in the energy space. This results in the acceleration of one part of electron population and the filling of the loss cone by the subsequent precipitation of another part.

Other proposed models, which are considered less promising, involve a variety of physical processes to explain electron acceleration. These include exotic models such as those based on the Jovian origin of electrons [*Baker et al.*, 1979]. A detailed review of the acceleration mechanisms, proposed in addition to the radial and local diffusion, are given in *Friedel et al.* [2002].

The relationship between the solar wind parameters and the fluxes of energetic electrons in the outer radiation belt have been investigated by *Paulikas and Blake* [1979]. The main conclusion of *Paulikas and Blake* [1979] was that it is the solar wind velocity that controls the radiation belt fluxes. Recently, *Reeves et al.* [2011] revisited *Paulikas and Blake* [1979] results and found that the relationship between the velocity and electron flux is far more complex than the roughly linear correlation suggested by *Paulikas and Blake* [1979]. *Balikhin et al.* [2011] and *Boynton et al.* [2012] employed the Error Reduction Ratio (ERR) to assess the significance of the influence on the outer radiation belt electron fluxes of various solar wind parameters. In these studies, daily averaged measurements of the electron fluxes at GEO have been subjected to the ERR analysis. The data were obtained by the LANL Synchronous Orbit Particle Analyzer (SOPA) instrument and are available at ftp:/ftp.agu.org/apend/ja/2010ja015735 as auxiliary materials to [*Reeves et al.*, 2011]. Even though both *Balikhin et al.* [2011] and *Boynton et al.* [2012] stated the importance of the solar wind density for higher energies, the ERR results confirmed that the fluxes of the electrons with energies below 1 MeV are indeed controlled by the solar wind velocity. The other conclusion of the ERR analysis was that while the fluxes of the low energy electrons (e.g. 24.1 keV) are affected by the solar wind velocity on the same day, as the energy increases the previous days solar wind velocity becomes more and more important. The previous days solar wind velocity becomes the most effective parameter as energy reaches 172.5 keV. A further increase in the energy leads to the effect of the solar wind velocity, measured two days prior, becoming more and more important and so on.

The plot showing the energy of the electron flux against the effective time delay of the solar wind velocity calculated from the ERR analysis by *Boynton et al.* [2012] is shown in Figure 1 of *Boynton et al.* [2012]. While the ERR approach has a rigorous mathematical foundation and is appropriate for nonlinear systems, it is always useful when it is possible to support the results by simpler means, which don't require complicated mathematics. The time delay as a function of energy, for the same data used by *Boynton et al.* [2012] but based on correlation function, is shown in Figure 1. The plot, displayed in Figure 1, is almost identical to the one obtained by the ERR analysis in *Boynton et al.* [2012].

The dependence of the time delay between the change of the solar wind velocity and the effect on the electron fluxes, t_d, upon the energy has been detected before [*Li*, 2004; *Li et al.*, 2005]. As it was explained in *Li* [2004] and *Li et al.* [2005], such a time delay has an obvious explanation in the frame of both the radial diffusion model and the local interaction with waves model. In the case of the radial diffusion, it can be explained if it takes longer for the higher energy electrons to reach the GEO orbit. In the case of wave-particle interactions and so called energy diffusion processes, it should take more time for a particle of an initial seed population to achieve higher energies. The main aim of the present paper is to estimate the rate of the electron fluxes energy increase, in the frame of the local diffusion, due to the interaction with waves and compare this with the dependences displayed in Figure 1 and similar results obtained in previous studies.

The relationship displayed in Figure 1 is much steeper than it would be expected from the local diffusion type processes, where fluxes with higher energies are resulting from the local energy diffusion of the same seed population. The time scale of the widening of the distribution function, according to the standard diffusion equation, should be similar to the square root of time, $\sqrt{t}$. In such a case, if it requires about one day to accelerate a seed population to energies of about 172.5 keV it should take about 25 days to reach 900 keV in the case of the same low energy seed population. Figure 1 shows that 900 keV are reached about 10 times faster. This huge difference is the main motivation for the present study, since a very rough upper limit on the local diffusion time scale can still be considerably slower than dependence in Figure 1. The energy diffusion equation obtained in [*Horne et al.*, 2005] is the basis of the present analytical estimate. It is worth noting that the above speculation implicitly assumes that the initial seed population has an energy range below 172 keV. In the case when the seed population has a wide distribution and reaches energies of say 900 keV, time scaling will not only reflect the diffusion equation but also the initial seed particle energy distribution. In this case, the fast increase of fluxes at 900 keV can be explained by the initial seed population distribution.

2. The Energy Diffusion Equation: Simplifications and solutions

The time scales of the modification of the electron distribution function, F, have been numerically studied in [*Horne et al.*, 2005]. The energy diffusion equation used by [*Horne et al.*, 2005] is the following:

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial E}\left[A(E)D\frac{\partial}{\partial E}\left[\frac{F}{A(E)}\right]\right] - \frac{F}{\tau_L} \quad (1)$$

Where A is:

$$A = (E + E_0)(E + 2E_0)^{\frac{1}{2}}E^{\frac{1}{2}}$$

E is the kinetic energy, D is the bounce-averaged energy diffusion coefficient, τ_L is the effective timescale for losses to the atmosphere and $E_0 = mc^2$ is the rest energy of the electron. In the derivation of the above relationship, [*Horne et al.*, 2005] neglected the mixed pitch angle-energy diffusion coefficients and used the rate of pitch angle diffusion near the loss cone to calculate the timescale for the losses to the atmosphere. In addition, the pitch angle isotropy has been assumed and bounce averaging has been implemented (see [*Horne et al.*, 2005] for details).

As it is shown in *Horne et al.* [2005] the distribution function $F(E, \alpha_{eq})$, which depends upon energy and the equatorial pitch angle α_{eq}, is related to the fluxes $J(E, \alpha_{eq})$ by:

$$F(E, \alpha_{eq}) = \frac{E + E_0}{c(E + 2E_0)^{\frac{1}{2}}E^{\frac{1}{2}}}J(E, \alpha_{eq}) \quad (2)$$

These equations, (1) and (2), are used in the present paper to estimate the time scale of the fluxes increase as a function of energy.

For $A = \text{const}$, $D = \text{const}$, and no losses $\tau_L \to \infty$, the solution of (1), corresponding to the well known diffusive broadening of an initially cold distribution, reads

$$F = Kt^{-1/2}\exp(-E^2/4Dt) \quad (3)$$

where K is a constant. If the loss time is constant $\tau_L = \text{const}$, the solution will have the form

$$F = Kt^{-1/2} \exp(-E^2/4Dt) \exp(-t/\tau_L) \quad (4)$$

It is not possible to analytically solve (1) in the general case for an arbitrary dependence of A, D, and τ_L on energy. The dependence of A upon the energy is rather cumbersome and does not allow for a compact analytical solution. However, this relationship can be significantly simplified if A is considered for three different ranges: 1) Sub-relativistic $E \ll E_0$, 2) $E \approx E_0; E - E_0 \ll E_0$ and 3) $E \gg E_0$. In all three cases $A \propto E^{\beta}$, where β is equal to $\frac{1}{2}$, 0 and 2 correspondingly. In the second case, if $\beta = 0$ then equation (4) becomes a standard diffusion equation with constant coefficients and a known time scaling of $\sqrt{t}$, therefore, below only the first and the third limit will be considered. In the case of decreasing D with energy, replacing it with a constant will only lead to the overestimation of the speed of higher energy flux increases. The numerical estimates of the bounce-averaged diffusion rates in the case of whistler mode chorus waves, for a number of energies and Magnetic Local Time (MLT) ranges, have been calculated in *Horne et al.* [2005]. For example, the bounce-averaged pitch angle and energy diffusion rates of whistler mode chorus waves for the night, pre-noon, and afternoon models, weighted according to the MLT occurrence of the waves, are displayed in Figure 6 by *Horne et al.* [2005]. It can be seen from this figure that there is no increase with energy for the diffusion coefficients in the ranges of $100 - 1000$ keV. This figure shows an absence of any strong dependence of D on energy in the range $100 - 1000$ keV. It is worth noting that the diffusion rates displayed in this figure by *Horne et al.* [2005] are equal to the diffusion coefficients normalised by E^2. In the other example by *Shklyar and Kliem* [2006], the energy diffusion rate has been calculated in the case of electrostatic upper hybrid waves. It was shown that the rates are proportional to the electron γ factor. So they should be almost independent upon energy for sub-relativistic electrons and exhibit a weak dependence when the kinetic energy is of the order of the rest energy E_0. Therefore, for our approximation it is sufficient to treat the diffusion coefficient as constant.

To summarize, we are seeking a solution for the equation

$$\frac{\partial F}{\partial t} = D\frac{\partial}{\partial E}\left[E^\beta \frac{\partial}{\partial E}\left[\frac{F}{E^\beta}\right]\right], \tag{5}$$

which would describe the broadening of the initially cold distribution in the case of an arbitrary β. Here, for brevity and convenience we switched to the dimensionless energy and time as follows $E/E_0 \rightarrow E$, $Dt/E_0^2 \rightarrow t$. Introducing a new variable f

$$F = fE^{\frac{\beta+1}{2}}$$

leads to the replacing of (5) by

$$\frac{\partial f}{\partial t} = \frac{\partial^2}{\partial E^2}f + \frac{1}{E}\frac{\partial f}{\partial E} - \frac{b^2}{E^2}f, \tag{6}$$

where $b = \frac{\|\beta-1\|}{2}$. The eigenfunctions of the linear equation (6) can be found using separation of variables:

$$f = T(t)Y(E)$$

This leads to a simple equation for T

$$\frac{1}{T}\frac{dT}{dt} = -k^2.$$

Therefore

$$T(t) = T_0 \exp(-k^2 t)$$

And to the equation of the Bessel type for Y

$$\frac{d^2Y}{dE^2} + \frac{1}{E}\frac{dY}{dE} + (k^2 - \frac{b^2}{E^2})Y = 0 \tag{7}$$

The solution of this equation is a combination of the Bessel J_b and the Neumann N_b with coefficients that in general are arbitrary functions of k:

$$Y(E) = c_1(k)J_b(kE) + c_2(k)N_b(kE)$$

For physical solutions the Neumann function part should be equal to zero, since N_b diverges when $E \to 0$. Therefore the solution of (6) is

$$F = \int_0^\infty dk \exp{(-k^2Dt)}C(k)J_b(kE)E^{\frac{\beta+1}{2}} \tag{8}$$

The initial condition at $t_0 = 0$, $F(E, t_0) = F_0(E)$, should be used to find $C(k)$:

$$F_0(E)E^{-\frac{\beta}{2}} = \int_0^\infty dk\sqrt{kE}C(k)J_b(kE) \tag{9}$$

This equation shows that the function $F_0(E)E^{-\frac{\beta}{2}}$ is a Bessel transform image of $C(k)$. So $C(k)$ can be found using the inverse Bessel transform of the initial distribution:

$$C(k) = \int_0^\infty dE\sqrt{kE}F_0(E)E^{-\frac{\beta}{2}}J_b(kE) \tag{10}$$

In general, the choice of the initial distribution will not have a strong effect on the the rough estimate of the time scaling. Choosing the initial distribution with a free parameter s:

$$F_0(E)E^{-\frac{\beta}{2}} = \frac{E^{b+1/2}}{(2s)^{b+1}}\exp(-\frac{E^2}{4s}), \tag{11}$$

leads to the following $C(k)$:

$$C(k) = k^{b+1/2}\exp{(-sk^2)} \tag{12}$$

Such a solution can also be found by a less cumbersome way. In analogy with the diffusion equation with constant coefficients, we shall seek for the solution in the form

$$F = K(t + t_0)^{\beta} f(E^2/(t + t_0)) \tag{13}$$

for $t > 0$. The initial distribution is then

$$F = Kt_0^{\beta} f(E^2/t_0), \tag{14}$$

and is a generalisation of (11). Direct substitution gives:

$$F(E,t) = KE^{(\beta+1)/2+|\beta-1|/2}(D(t+t_0))^{-|\beta-1|/2-1} \exp\left(-\frac{E^2}{4DE_0^2(t+t_0)}\right) \tag{15}$$

In the sub-relativistic limit (case 1), $\beta = \frac{1}{2}$ and

$$F = KE(t+t_0)^{-5/4} \exp\left(-\frac{E^2}{4DE_0^2(t+t_0))}\right) \tag{16}$$

In the highly relativistic case $E \gg E_0$ (case 3), $\beta = 2$ and

$$F = KE^2(t+t_0)^{-3/2} \exp\left(-\frac{E^2}{4DE_0^2(t+t_0)}\right) \tag{17}$$

Both solutions (16) and (17) correspond to $s = 4DE_0^2 t_0$. In what follows we considered the asymptotic solutions with $t \gg t_0$. The limit $t_0 \to 0$ corresponds to the transition of the limit $s \to 0$ and describes the distribution which has zero temperature at $t = 0$. For this initial distribution, $F(E \neq 0) = 0$ while $F(E = 0) \to \infty$.It is worth noting that in contrast with the global solution of the diffusion equation with constant coefficients, these partial solutions for the different energy ranges do not have to conserve the particle numbers separately in each energy region. Only the conservation of the total particle number is required.

3. Time scales of the solutions

Equation (2) can be used to translate the change of the distribution function into the change of fluxes. The shape of the fluxes that correspond to the solutions (16) and (17) are displayed in Figures 2 and 3 respectively. The fluxes are displayed for 3 values of normalised energy; 0.1 (solid blue), 0.3 (solid red) and 0.5 (solid black) in Figure 2 and 3 (solid blue), 4 (solid red) and 5 (solid black) in Figure 3. In both figures all fluxes initially exhibit a slow change that is replaced by a steep increase and exponential decay after reaching a maximum. Since the function $(Dt)^{-n}\exp(-E^2/4Dt)$ reaches maximum at $Dt = E^2/4n$, the time taken to reach the maximum for the different energies obey the $t \propto E^2$ time scaling, independently of n.

Experimental data shows that after the increase of the solar wind velocity, the increase of low energy fluxes occurs in less than a day, higher energy fluxes on the next day and so on. So it is the initiation of the steep increase of fluxes that should be studied. The time moment when the flux reached 10 percent of maximum have been chosen for a steep flux increase time. The corresponding times for the three values of normalised energy are $t_{E=0.1} = 4.60 \times 10^{-4}$, $t_{E=0.2} = 1.89 \times 10^{-3}$ and $t_{E=0.3} = 4.19 \times 10^{-3}$ for sub-reativistic (case 1, Figure 2) and $t_{E=3} = 0.386$, $t_{E=4} = 0.685$ and $t_{E=5} = 1.067$ for the highly relativistic (case 3, Figure 3). In the sub-relativistic case $t_{E=0.1} : t_{E=0.2} : t_{E=0.3} = 1 : 4.10 : 9.10$, In the highly relativistic case $t_{E=3} : t_{E=4} : t_{E=5} = 1 : 1.78 : 2.77$. As it was expected, the time scaling is close to the $\sqrt{t}$ from the diffusion equation with constant coefficients.

A much faster increase of the energetic electron fluxes is observed in the experimental data at GEO. It is possible to conclude that the observed timing of the flux increase at GEO is much faster than expected from the local energy diffusion due to the interactions with observed waves. However, it can be explained by radial diffusion if the mobility of ions in the process is decreasing with the energy. The comparison of time scales cannot be used as an argument to rule out the important effects of energy diffusion on the population of energetic electrons at GEO. It is possible that the acceleration takes place due to the interaction with waves somewhere deeper in the magnetosphere at an L parameter in the range of $4-5$. Such a process will create a maximum in the space phase in the region of the acceleration and initiate outward diffusion that will bring high energy electrons to GEO.

The square root of time scaling should be valid for an arbitrary β as can seen from the way solutions (16) and (17) are obtained. This can be used to argue that the time scaling should not drastically differ to the original relationship for A when it is approximated in different energy ranges. However, as it is hard to prove analytically, it can be checked numerically. A decrease of D with energy can only slow down the build up of higher energy fluxes. As it was mentioned above, the plotted values of D in [*Horne et al.*, 2005] indicate that it does not undergo a significant change in the range of $100-1000$ keV. However, if a week dependence of D upon the energy exists, it should also not lead to a such a drastic change of the time scaling, which correspond to observations. Again, only numerical calculations with experimentally deduced values of D are able to prove it.

The main conclusion of the present paper is that the time scaling of the energy diffusion equation is too slow to explain the increase of fluxes and GEO and therefore, radial diffusion should play the key role in the evolution of high energy distributions at GEO. It must be noted that for this conclusion to be valid, the seed population energies must be low enough. It can be seen in Figure 1 that the slope between two highest energies, 925 keV and 1.3 MeV, is slightly steeper in comparison to the mean slope, from 170 keV to 1.3 MeV. Therefore, the above conclusion is valid if the upper limit for the energy of seed population is below 925 keV.

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Figure 1. A log-log plot showing the energy of the electron flux E, against the effective time delay of the solar wind velocity t calculated from the correlation function results.

Figure 2. The sub-relativistic case, calculated from (16), showing the evolution of fluxes in time for the normalised energies of 0.1 (solid blue), 0.3 (solid red) and 0.5 (solid black).

Figure 3. The highly relativistic case, calculated from (17), showing the evolution of fluxes in time for the normalised energies of 3 (solid blue), 4 (solid red) and 5 (solid black)

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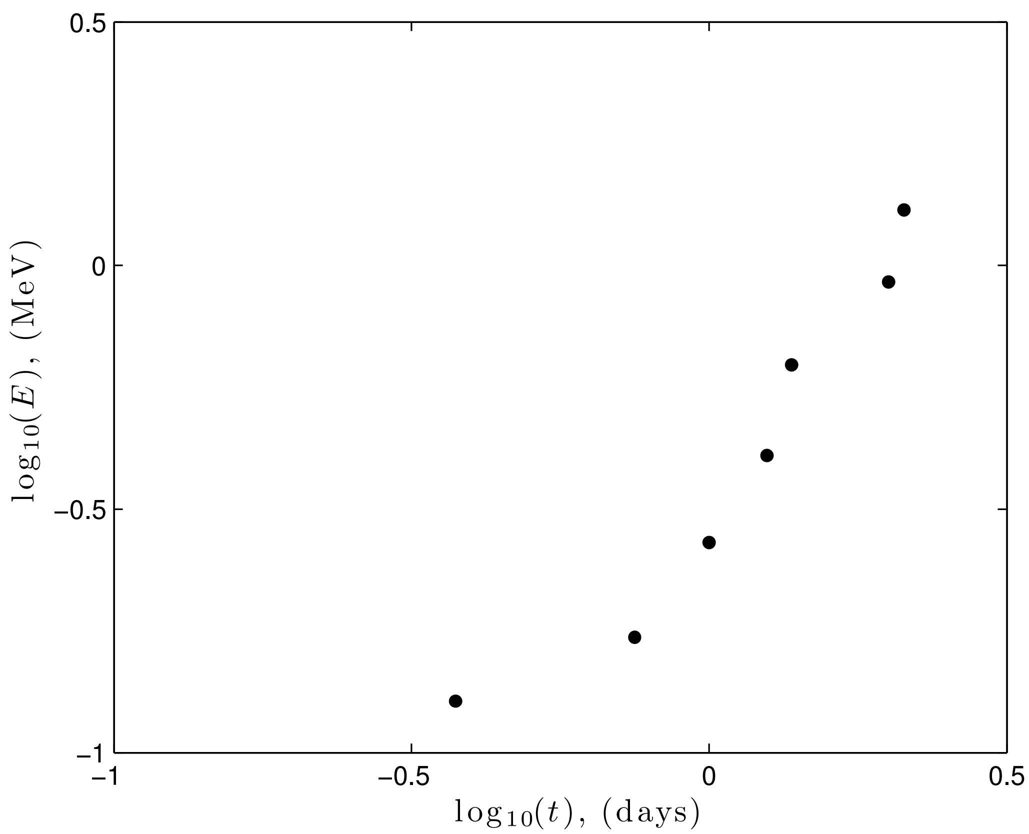
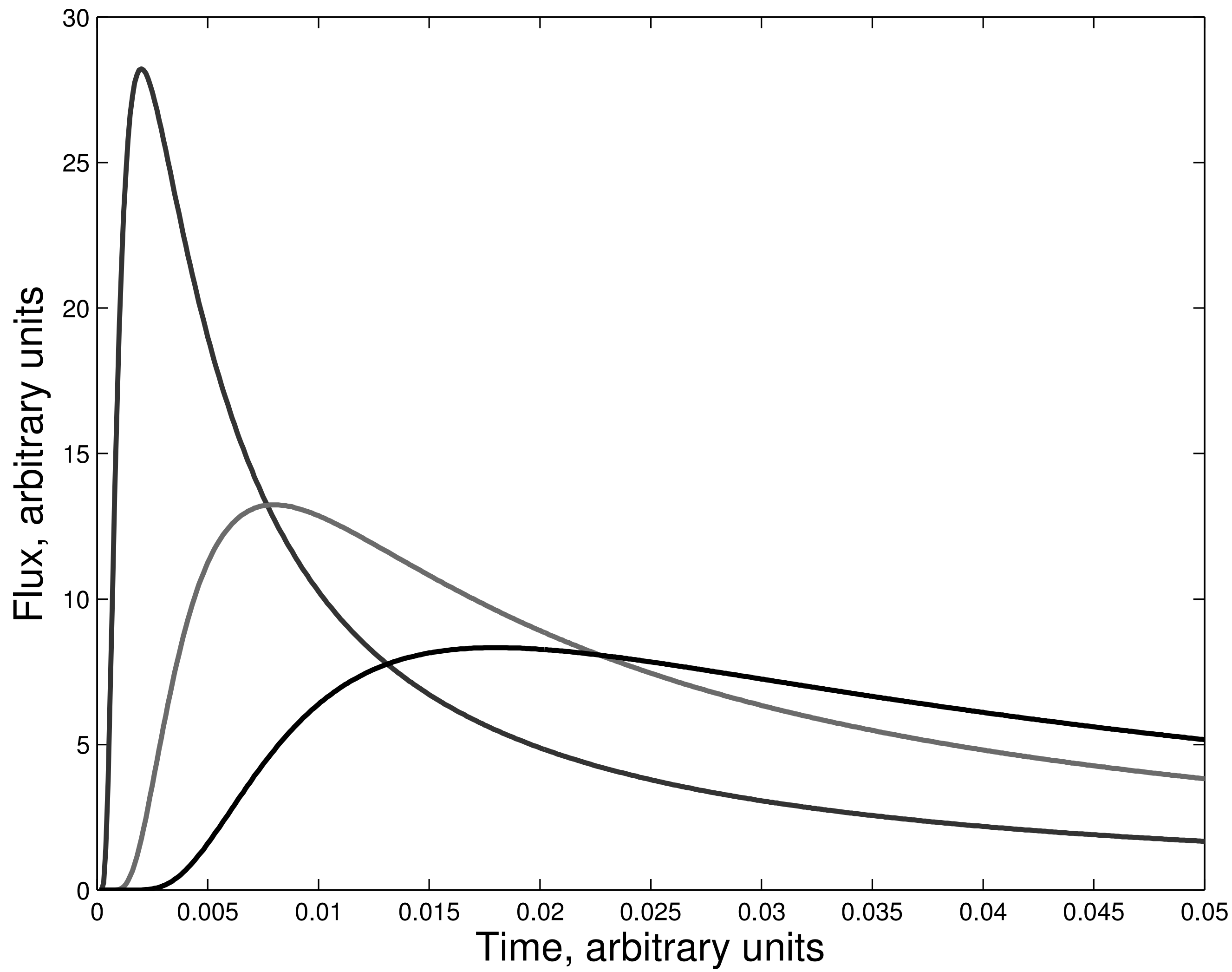
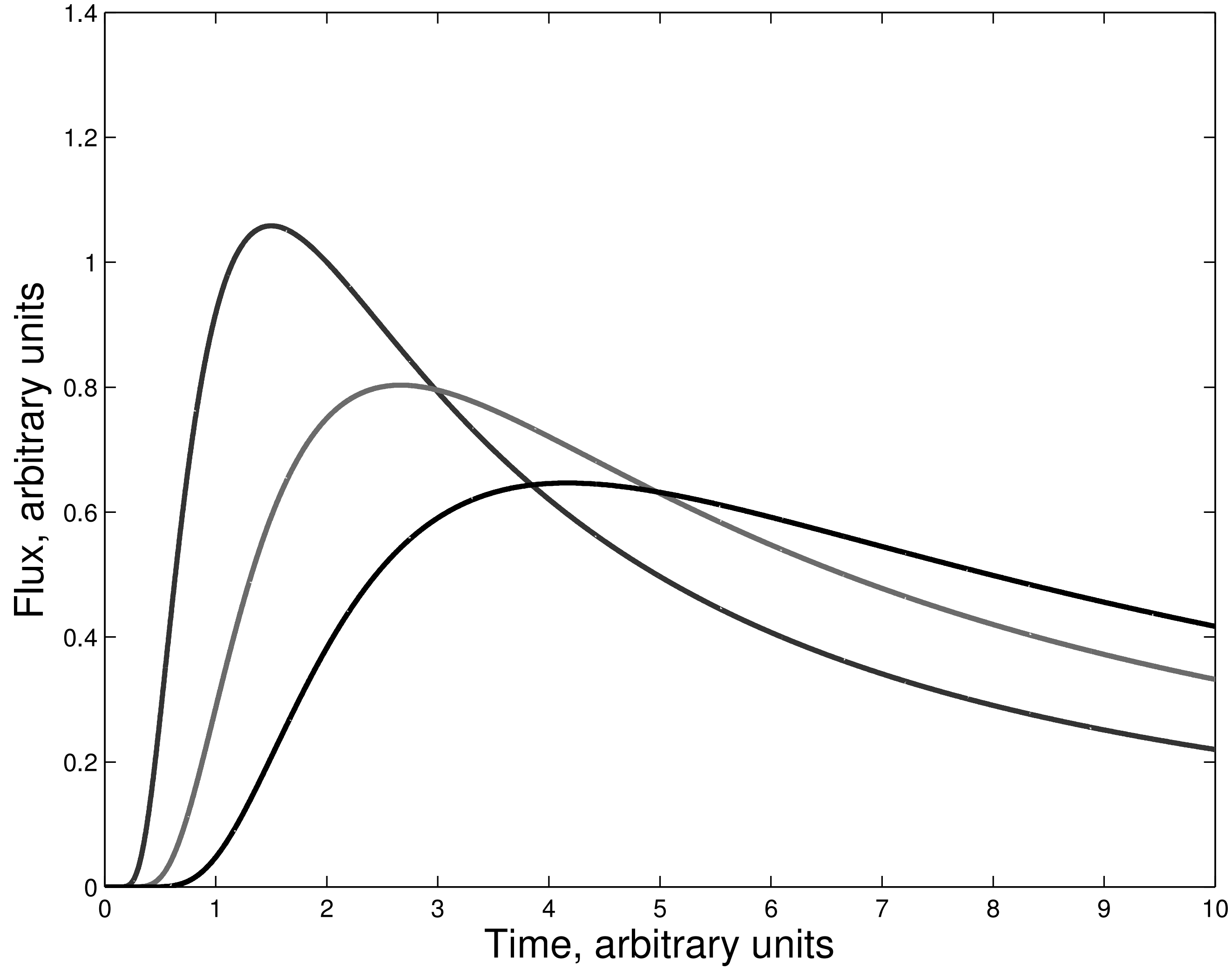

0.5
0
−0.5
−1
−1
−0.5
0
0.5
$\log_{10}(E)$, (MeV)
$\log_{10}(t)$, (days)



Flux, arbitrary units
Time, arbitrary units
0
5
10
15
20
25
30
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05



Flux, arbitrary units
Time, arbitrary units
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
1
2
3
4
5
6
7
8
9
10