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THE NONLINEAR IDENTIFICATION OF A HEAT EXCHANGER

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Abstract

The practical application of a recently introduced orthogonal parameter estimation algorithm is investigated by identifying a nonlinear model of a heat exchanger based on input-output records. A new criterion called ERR_i (Error Reduction Ratio) is employed to select significant terms in the NARMAX model expansion to yield a parsimonious model. The fitted model is then compared with a model previously obtained using a prediction error algorithm coupled with stepwise regression, and validated by computing various correlation tests and plotting predicted outputs.

1. Introduction

The recent introduction of the NARMAX model [Leontaritis and Billings 1985 a,b] (Nonlinear AutoRegressive Moving Average model with eXogenous inputs) into the identification of nonlinear systems provides a system representation which can be characterized by a small number of parameters. This is due to the fact that the NARMAX model maps both past inputs and past outputs into the present system output which considerably reduces the computational burden and excessive parameter set associated with the traditional functional series [Schetzen 1980, Marmarelis and Marmarelis 1978] expansions which map only past inputs into the present output.

The main difficulty in nonlinear identification using the NARMAX model is the model structure determination. Once the exact model structure is determined, the unknown parameters can be estimated using standard routines [Billings and Voon 1984, 1986a, Korenberg, Billings and Liu 1987]. In practice, it may be either impossible or very difficult to derive an analytical model and the system under consideration is often treated as a black or grey box. The successful application of the NARMAX model in nonlinear identification is therefore dependent upon detecting which terms in the polynomial expansion should be included in the final model. Several well known methods adopted for this purpose including a stepwise regression based algorithm and a log determinant ratio test have been documented elsewhere [Billings and Voon 1986a, Leontaritis and Billings 1987].

In this paper, a new criterion called ERR_i is used to detect or select significant terms for a nonlinear heat exchanger system. The resultant model is estimated using an orthogonal parameter estimation algorithm and validated by computing various correlation tests and plotting predicted outputs. A comparison between this model and a model estimated using a prediction error/stepwise regression routine [Billings and Fadzil 1985] is examined.

2. Problem Formulation

The NARMAX model is used as a basis to develop a nonlinear representation of a heat exchanger. It has been shown that this model can represent a wide class of nonlinear systems and has the general form

$$y(t)=dc+F^l[y(t-1),\cdots,y(t-n_y),u(t-d),\cdots,u(t-d-n_u+1),\xi(t-1),\cdots,\xi(t-n_\xi)]+\xi(t) \quad (1)$$

where $u(t)$ and $y(t)$ represent the measured input and output respectively, $\xi(t)$ is the prediction error defined as

$$\xi(t) = y(t) - \hat{y}(t) \quad (2)$$

and n_y, n_u and n_ξ represent the dynamic order of output $y(t)$, input $u(t)$ and prediction error $\xi(t)$ respectively and $F^l[\cdot]$ is a nonlinear function of l degree nonlinearity, d is the time delay in the input and dc is a constant term. Inclusion of the constant term is justified since removing mean levels from input-output records when estimating nonlinear models will almost always induce input sensitivity and consequently change the structure of the model [Billings and Voon 1984].

Expanding equation (1) as a polynomial yields

$$y(t) = \sum_{i=0}^{M} \theta_i P_i(t) + \xi(t) \qquad (3)$$

where

$$P_m(t)=y(t-n_{y1}) \cdots y(t-n_{yk})u(t-d+1-n_{u1}) \cdots$$
$$\cdot u(t-d+1-n_{uj})\xi(t-n_{\xi 1}) \cdots \xi(t-n_{\xi q}) \qquad (4)$$

for $m=0, 1, 2, \cdots, M$

$k \geq 0, \quad j \geq 0, \quad q \geq 0$

$1 \leq n_{y1} \leq n_y, \cdots, 1 \leq n_{yk} \leq n_y$

$0 \leq n_{u1} \leq n_u, \cdots, 0 \leq n_{uj} \leq n_u$

$1 \leq n_{\xi 1} \leq n_\xi, \cdots, 1 \leq n_{\xi q} \leq n_\xi$

and

$k=0$ indicates that $P_m(t)$ contains no $y(.)$ terms,

$j=0$ indicates that $P_m(t)$ contains no $u(.)$ terms,

$q=0$ indicates that $P_m(t)$ contains no $\xi(.)$ terms,

If $k=j=q=0$, the corresponding term $P_m(t)$ is set to 1.0 so that a constant term $\theta_m=dc$ may be accommodated in the model.

For the convenience of later analysis, we refer to those $P_m(.)$ that contain no $\xi(.)$ as deterministic terms ($q=0$), the rest as noise terms ($q \neq 0$).

The maximum number of terms is $M+1$ where M can be computed from

$$M = \sum_{i=1}^{l} n_i = n_1+n_2+ \cdots\cdots +n_l \qquad (5)$$

$$n_i = [n_{i-1}(n_y+n_u+n_\xi+i-1)]/i \quad \textit{where} \quad n_0=1$$

For example, a first order system ($n_y=n_u=n_\xi=1$) with second degree nonlinearity is given by

$$y(t)=dc+\theta_1 y(t-1)+\theta_2 u(t-1)+\theta_3 y^2(t-1)$$
$$+\theta_4 y(t-1)u(t-1)+\theta_5 u^2(t-1)+\theta_6 \xi(t-1)$$
$$+\theta_7 y(t-1)\xi(t-1)+\theta_8 u(t-1)\xi(t-1)$$
$$+\theta_9 \xi^2(t-1)+\xi(t) \qquad (6)$$

by defining $P_0(t)=1.0$, $P_1(t)=y(t-1)$, $P_2(t)=u(t-1)$, $P_3(t)=y^2(t-1)$, $P_4(t)=y(t-1)u(t-1)$, $P_5(t)=u^2(t-1)$, $P_6(t)=\xi(t-1)$, $P_7(t)=y(t-1)\xi(t-1)$, $P_8(t)=u(t-1)\xi(t-1)$, $P_9(t)=\xi^2(t-1)$ and $\theta_0=dc$, eqn (6) can be written in the form of eqn (3). There are ten coefficients in the above model, but for a second order system ($n_y=n_u=n_\xi=2$) with a cubic polynomial expansion ($l=3$) there will be eighty four terms in the model and the excessive number of parameters becomes evident. The complete NARMAX model representation of a nonlinear system may therefore cause severe computational demands on the parameter estimation algorithms. Even if unlimited computer facilities were available the problem of overparameterization would probably cause numerical ill-conditioning.

Fortunately, experience has shown that provided the significant terms in the NARMAX model can be detected, typically only around ten such terms are adequate to describe highly nonlinear dynamics and the remainder can be deleted with little deterioration in prediction accuracy [Billings and Fadzil 1985, Billings and Voon 1984]. Hence, the detection of significant terms or the selection of which terms to include in the model becomes critically important in nonlinear identification using the NARMAX representation.

3. The Orthogonal Algorithm

The recently introduced orthogonal estimation algorithm for input-output records corrupted by noise provides a simple and efficient way of obtaining unbiased parameter estimates for the NARMAX model. The ERR_i test which is a by-product of the algorithm can provide information regarding the significance of terms in the polynomial expansion in eqn (3). For a detailed derivation of the orthogonal algorithm the reader is referred to Korenberg, Billings and Liu (1987). Only the main results associated with ERR_i are presented here.

In order to estimate the coefficients θ_i in eqn (3), an auxiliary model [Korenberg 1985]

$$y(t)=\sum_{m=0}^{M} g_m W_m(t)+\xi(t) \qquad (7)$$

is introduced such that $W_i(t)$ satisfies the orthogonality condition over the data record of length N

$$\sum_{t=1}^{N} W_j(t)W_{k+1}(t)=0 \quad j=0,1, \cdots, k \qquad (8)$$

This can be achieved by setting

$$W_0(t)=P_0(t)$$

$$W_m(t)=P_m(t)-\sum_{r=0}^{m-1} \alpha_{rm} W_r(t) \qquad (9)$$

and

$$\alpha_{rm}=\frac{\sum_{t=1}^{N} P_m(t)W_r(t)}{\sum_{t=1}^{N} W_r^2(t)}$$

Then using eqn (7) yields

$$\hat{g}_m=\frac{\sum_{t=1}^{N}y(t)W_m(t)}{\sum_{t=1}^{N}W_m^2(t)} \qquad (10)$$

Now the coefficients in the original model (3) can be computed from

$$\hat{\theta}_M=\hat{g}_M \qquad (11a)$$

$$\hat{\theta}_m=\hat{g}_m-\sum_{j=m+1}^{M}\alpha_{mj}\hat{\theta}_j \quad m=M-1, M-2, \cdots, 1,0 \qquad (11b)$$

which can be easily established using the identity

$$\sum_{m=0}^{M}\hat{\theta}_m P_m(t)=\sum_{m=0}^{M}\hat{g}_m W_m(t)$$

By choosing $W_0(t)=P_0(t)=1.0$, it can be proved from the orthogonality property of $W_i(.)$ and eqn (7) that the ERR_i is given by

$$ERR_i=\frac{g_i^2\sum_{t=1}^{N}W_i^2(t)}{\sum_{t=1}^{N}y^2(t)-\frac{1}{N}\left[\sum_{t=1}^{N}y(t)\right]^2}\cdot 100 \qquad (12)$$

for i=1,2, $\cdots$, M. The ERR_i gives the percentage reduction, by including the i'th term in the model, from the maximum mean squared error with the effects of the constant terms eliminated. The ratio can also be interpreted as the percentage reduction in the total sum of the squared errors due to the inclusion of the i'th term, with the effects of the constant removed. This ratio can therefore be used to provide an indication of the significance of terms in the model.

In the implementation of the ERR_i test, two kinds of terms should be treated separately if both unbiasedness of the estimates and parsimony of the model are desired. For deterministic terms, typical threshold values of ERR_i (called C_d) are taken to be 0.05 to 0.5 and for noise terms lower threshold values (called $C_{d\varepsilon}$) are taken to be 0.005 to 0.1.

The prediction error $\xi(.)$ is not known a priori and can be computed from

$$\xi(t)=y(t)-\sum\hat{g}_jW_j(t) \qquad (13)$$

where the summation will be specified later. The orthogonal estimation routine incorporating the ERR_i test is then given by the following algorithm

(i) Select values for n_y, n_u, n_ξ, d and l in eqn (1) and set $\xi(t)$=0.0 for t=1, $\cdots$,N. Select C_d and $C_{d\varepsilon}$.

(ii) Estimate all the auxiliary parameters that correspond to the candidate deterministic terms using eqn's (9) and (10).

(iii) If $\xi(t)$=0.0, t=1,2, $\cdots$, N goto (iv) otherwise estimate all the auxiliary parameters that correspond to the candidate noise terms using eqn's (9) and (10)

(iv) Compute ERR_i's using eqn (11), test against the thresholds C_d, $C_{d\varepsilon}$ and remove insignificant terms

(v) Estimate the prediction errors using eqn (13) where the summation is over all selected terms.

(vi) If any deterministic terms were deleted in step (iv) then take the remaining deterministic terms as candidates and go to step (ii), otherwise take the remaining noise terms as candidates and go to step (iii), and repeat until convergence.

(vii) Estimate the NARMAX model coefficients using eqn (11)

Numerous simulations have been done using the above algorithm [Korenberg, Billings and Liu 1987], here we apply it to the identification of a heat exchanger.

4. The Data Set

The heater exchanger system (Fig.1) consists of a heater, pump, fan and radiator. Heated water is pumped through the radiator around a closed loop and the fan blows air across the radiator. The temperature drop across the radiator and the air flow rate can be controlled by adjusting the inputs to the heater and the fan. Fig.2 gives a block diagram of the system. It has been shown that Loop L_{12} and Loop L_{22} are linear while Loop N_{11} is nonlinear.

A detailed description of the heat exchanger system and experiment design can be found in the literature [Billings and Fadzil 1985]. For the purpose of this paper, only the nonlinear loop N_{11} will be studied. The input was a Gaussian white excitation with a mean of -0.1177 and a bandwidth of 0.5Hz for the first five hundred input-output records and a bandwidth of 0.05Hz for the second five hundred records. In the following analysis, the first 500 points in the data set will be used as the estimation set, and the second 500 points as the testing set. For comparison with previous results a linear model is identified initially and then extended to include nonlinear terms.

5. Linear Analysis

A linear analysis was based on the modified data set by removing the mean level from the raw input-output records. The specification of a second order process model ($n_y=n_u=2$, $d=1$) and a second order noise model ($n_\xi=2$) produced a model with linear model validity tests illustrated in Fig.3. Inspection of Fig.3 shows that $\Phi_{u\xi}(\tau)$ is slightly outside the 95% confidence bands and $\Phi_{\xi\xi}(\tau)$ is well outside the 95% confidence bands. This is almost identical to the results of Billings and Fadzil [1985] where it was argued that this indicated a deficiency in the noise model.

The noise model order was therefore increased to four and the model re-estimated to yield

$$y'(t)=0.03177+0.9245y'(t-1)-0.2022y'(t-2)$$
$$+0.2669u'(t-1)-0.05577u'(t-2)$$
$$-0.1837\xi(t-1)+0.03405\xi(t-2)+0.2458\xi(t-3)$$
$$0.02721\xi(t-4)+\xi(t) \qquad (14)$$

which appeared to be the best fit according to the linear model validity tests. However, since the process is nonlinear it also must be tested by nonlinear model validity tests to see if this is an acceptable approximation. The correlation validity tests introduced below can serve for this purpose. It has been shown that [Billings and Voon 1986b] when the system is nonlinear the residuals $\xi(t)$ should be unpredictable from all linear and nonlinear combinations of past inputs and outputs and this will hold iff

$$\Phi_{\xi\xi}(\tau)=\delta(\tau)$$
$$\Phi_{u\xi}(\tau)=0 \quad \text{for } all\ \tau \qquad (15a)$$
$$\Phi_{\xi\xi u}(\tau)=0 \quad \text{for } \tau\geq 0$$

Notice the fact that the conventional model validity tests $\Phi_{\xi\xi}(\tau)$ and $\Phi_{u\xi}(\tau)$ are not sufficient when the system under consideration is nonlinear. When instrumental variables or suboptimal least squares are used the residuals may be coloured. In this case the process model will be an unbiased adequate fit iff

$$\Phi_{u\xi}(\tau)=0 \quad \text{for } all\ \tau$$
$$\Phi_{u^{2'}\xi}(\tau)=0 \quad \text{for } all\ \tau \qquad (15b)$$
$$\Phi_{u^{2'}\xi^2}(\tau)=0 \quad \text{for } all\ \tau$$

where $\xi(t)$ represent estimates of the prediction error sequence and the $'$ indicates that the mean has been removed from a signal.

As is the case with the prediction error algorithm, experience has also shown that when using the orthogonal algorithm the tests in both equation (15a) and (15b) can often provide an experimenter with valuable information regarding the deficiencies in the resultant model, and can indicate which terms, if any, are missing from the model. These terms can then be forced in and the new model can be estimated and tested again.

Computing the correlation tests gave the results illustrated in Fig.4, which show that although the linear covariance tests $\Phi_{\xi\xi}(\tau)$, $\Phi_{u\xi}(\tau)$ now indicate that the model is adequate, $\Phi_{u^{2'}\xi}(\tau)$ and $\Phi_{u^{2'}\xi^2}(\tau)$ are well outside the confidance bands indicating that nonlinear terms have been omitted from the model [Billings and Voon 1986b] . Notice that (Fig.5) whilst the predicted output gives an acceptable approximation over the estimation set, the experimental condition for which the model was estimated, the deficiency is apparent over the testing set(last 500 points), which were collected for a different experimental condition. We can therefore conclude that the linear covariance tests are not sufficient and can be misleading when the system under consideration is nonlinear. In the following section, estimation of a nonlinear model is investigated.

6. Nonlinear Identification

In nonlinear identification the raw data set was used, because using normalised data may result in an input sensitive model [Billings and Voon 1984]. As before, the first 500 points in the data set were used as the estimation set and the second 500 as the testing set.

With an initial specification of $n_y=n_u=n_\xi=2$, $d=1$, $l=3$, there will be eighty four candidate terms in the NARMAX model expansion. With a significance threshold of $C_d=0.2$ and $C_{de}=0.05$, the estimator produced the following model

$$y(t)=2.140+1.293y(t-1)-0.3925y(t-2)$$
$$+0.4809u(t-1)-0.05788y^2(t-1)$$
$$+0.04298y(t-1)y(t-2)-0.02345y(t-1)u(t-1)$$
$$-0.01323u^2(t-1)+\xi(t) \qquad (16)$$

The model in eqn (16) includes linear dynamic terms and four second degree nonlinear dynamic terms as suggested in the linear analysis. From all the eighty four candidate terms only those in eqn (16) are selected by the ERR_i test with a prespecified significance level.

The prediction accuracy of eqn (16) is much better than that of the linear model eqn (14) but the model validity tests illustrated in Fig.6 show that $\Phi_{\xi\xi u}(\tau)$ and $\Phi_{u^{2'}\xi}(\tau)$ are now acceptable while $\Phi_{\xi\xi}(\tau)$, $\Phi_{u\xi}(\tau)$ and $\Phi_{u^{2'}\xi^2}(\tau)$ are not. This indicates [Billings and Voon 1986b] that the deficiency may be due to unmodelled linear terms. A comparison of

Fig.6 with Fig.3 reveals that $\Phi_{\xi\xi}(\tau)$ and $\Phi_{u\xi}(\tau)$ are very similar in both diagrams and suggests that a fourth order linear noise model may alleviate the problem as in the linear analysis. Re-estimating produced

$$
\begin{aligned}
y(t)=&2.113+1.118y(t-1)-0.2119y(t-2)+0.4975u(t-1)\\
&-0.03032y^2(t-1)+0.01497y(t-1)y(t-2)\\
&-0.02657y(t-1)u(t-1)-0.01194u^2(t-1)\\
&-0.1348\xi(t-1)+0.01108\xi(t-2)+0.2553\xi(t-3)\\
&+0.04077\xi(t-4)+\xi(t) \qquad (17)
\end{aligned}
$$

The model validity tests for this model showed that $\Phi_{\xi\xi}(\tau)$ was slightly outside the 95% confidance bands at lag 5 and $\Phi_{u^{2'}\xi^2}(\tau)$ was slightly outside at several lags. This may suggest that $\xi(t-5)$ should be included in the model. A comparison with Billings and Fadzil's [1985] results reveals that both fits included the constant term, y(t-1), u(t-1), $y^2(t-1)$ and $u^2(t-1)$ terms. The other terms do not match because the above fit includes two more second degree nonlinear terms y(t-1)y(t-2), y(t-1)u(t-1) and a linear term y(t-2) in place of two cubic nonlinear terms $y^2(t-1)u(t-1)$, $u^3(t-1)$ and a u(t-1) term in Billings and Fadzil [1985]. A lower threshold [C_d=0.04, C_{de}=0.01] was selected to bring in some cubic terms but no improvement in $\Phi_{u^{2'}\xi^2}(\tau)$ was achieved. As a result, inclusion of higher order nonlinear dynamic terms did not appear to be justified. Thus re-estimating with a fifth order linear noise model gave

$$
\begin{aligned}
y(t)=&2.017+1.172y(t-1)-0.2465y(t-2)+0.4879u(t-1)\\
&-0.3557y^2(t-1)+0.01937y(t-1)y(t-2)\\
&-0.02566y(t-1)u(t-1)-0.01278u^2(t-1)\\
&-0.1223\xi(t-1)-0.001474\xi(t-2)+0.2486\xi(t-3)\\
&+0.04464\xi(t-4)-0.1674\xi(t-5)+\xi(t) \qquad (18)
\end{aligned}
$$

The model validity tests applied to this model confirmed that this fit was adequate (Fig.7). The advantage of building up a nonlinear model becomes apparent from an inspection of the prediction accurancy (Fig.8) which is clearly an improvement on the linear model case (Fig.5). Eqn (18) was therefore accepted as the 'best' nonlinear model. Note that the validity tests for this fit are virtually the same as those of Billings and Fadzil's [1985]. The advantage of the present algorithm is that it is much simpler and computationally more efficient.

7. Conclusions

The NARMAX difference equation model can provide a concise representation for nonlinear systems. Although numerous simulations have been completed using the orthgonal estimator coupled with the ERR_i test, this paper investigated the practical application of the algorithm to a heat exchanger. The ERR_i test can provide a very powerful tool for the determination of the structure of nonlinear systems using a NARMAX model representation and the final model can be checked by computing correlation model validity tests and examining predicted outputs. The ERR_i test as reported above is dependent upon the order that terms are included in the model and new algorithms which overcome this deficiency and extend the results to the multivariable case are in press.

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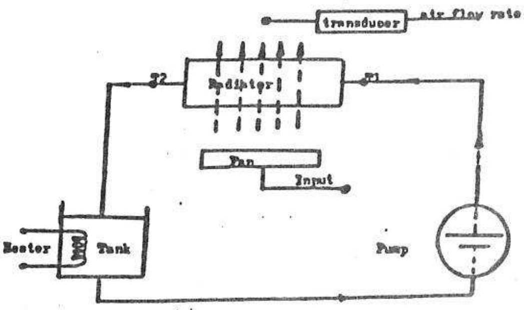

Fig. 1 The Heater Exchanger

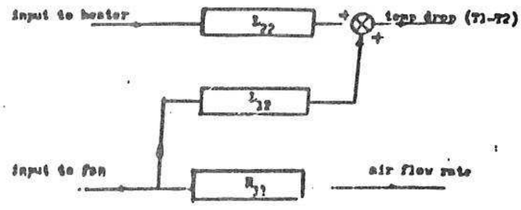

Fig. 2 Block Diagram

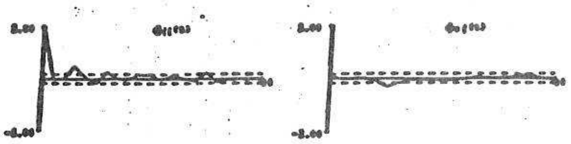

Fig. 3 Linear Model Validity Tests

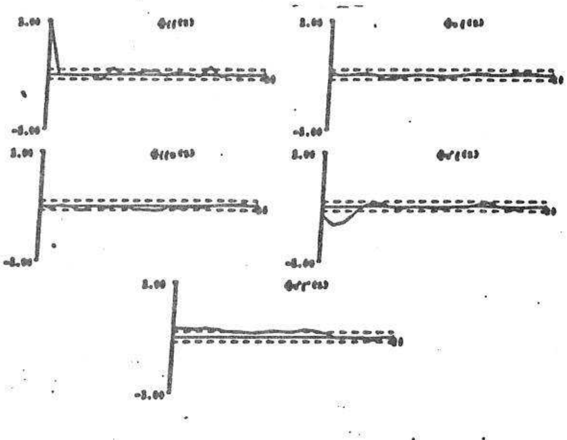

Fig. 4 Best Linear Fit

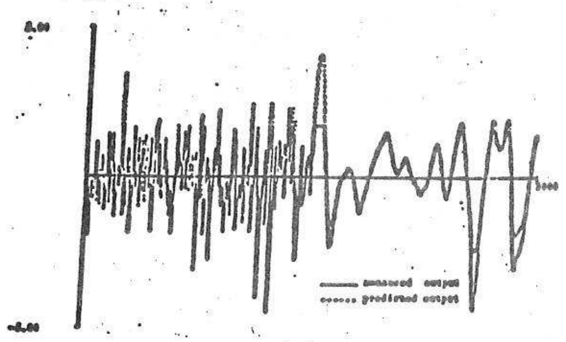

Fig. 5 Best Linear Model

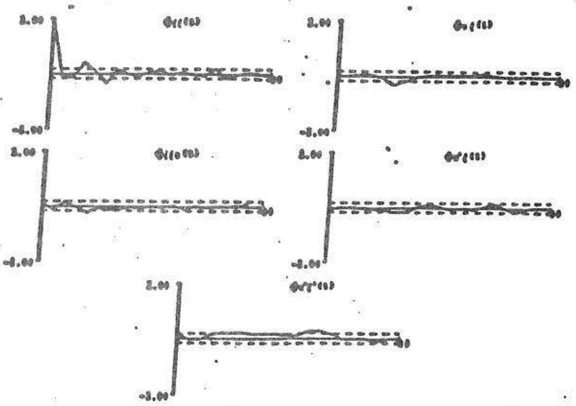

Fig. 6 Nonlinear Model

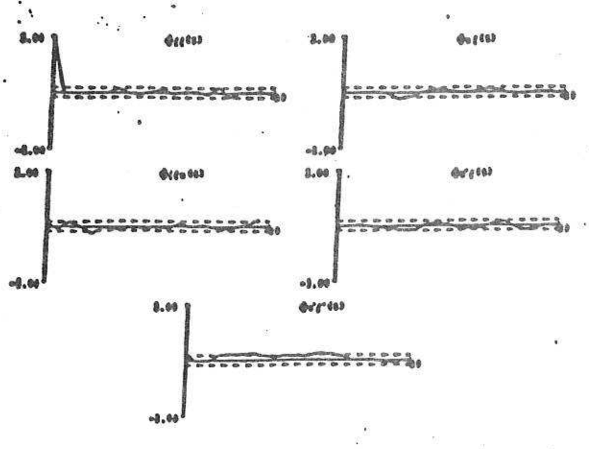

Fig. 7 Best Nonlinear Model

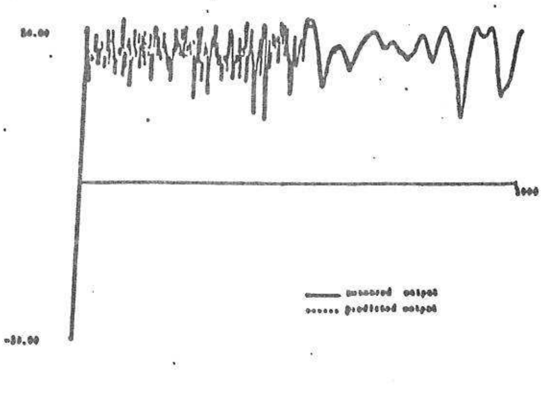

Fig. 8 Best Nonlinear Model