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COORDINATE TRANSFORMATIONS AND PROGRAMMING FOR SMALL REVOLUTE COORDINATE ROBOTS

BY

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1. Introduction

In order for a robot end effector to make contact at any orientation with a workpiece in general 3-D space, the robot must be provided with six degrees of freedom. This is typically accomplished by the revolute-coordinate geometrical configuration shown in Fig.1, in which the manipulator arm consists of two links rotatable about parallel horizontal axes, and where rotation of the arm about a vertical axis through the robot base is also provided. These three degrees of freedom allow the end of the arm to reach any point within the 3D spherical working envelope around the robot (apart from a small 'no go' area arising from mechanical constraints which prevent full 360^o rotation of the joints). The other three degrees of freedom are usually provided by rotations of the robot end effector about three orthogonal axes, these motions being commonly referred to as roll, yaw and pitch.

In applications of such robots, the attainment of any new position in general xyz coordinate space is achieved by appropriate angular rotations of the six joints. This requires a transformation from spatial cartesian coordinates to joint angular coordinates. Such a transformation is achieved automatically in the 'teaching by doing' method of programming, where the end-effector is led by hand through the required sequence of points, and the corresponding joint coordinates are derived from angular position transducers on the joints and entered into computer memory. This method is very restrictive however, and particularly unsuitable for CADCAM applications where the required angular joint coordinates must be calculated by the computer.

In the case of industrial grade robot systems, the programming languages which are provided, such as Unimation's VAL, usually include algorithms which perform the required coordinate transformations. The robot programmer is only required to enter the spatial coordinates of the required positions and the necessary transformations are performed transparently to the programmer.

Unfortunately, such facilities are not usually provided with the growing range of small, low cost robots which are normally supplied in the form of a manipulator only, rather than as a complete robot system integral with a control computer. Such robots are primarily designed for the educational market, but they are also finding industrial application in some novel areas such as life-time testing of floppy discs and radiopharmaceuticals handling during radiation emission measurement.

The user of such small robots is normally restricted to using the facilities provided for programming in a 'teaching by doing' mode. This paper provides an alternative to this by developing the necessary coordinate transformation algorithms to allow direct programming of such robots by entry of the required cartesian coordinates of the robot end effector. A simplification of this algorithm is also presented for the important special case of the end-effector being required to move vertically downwards and pick up a component on a horizontal surface. The implementation of the algorithm is described for a Systems Control "Smart Arms 6E' robot, although a very similar approach is applicable to most other robots in this class.

2. General Kinematic Equations for 6 d.o.f. robot.

Fig.2. shows the general case of a six degree of freedom robot, where the required position and orientation of the end effector is specified by the spatial coordinates of both ends of the end-effector $(x_1 y_1 z_1)$ and $(x_2 y_2 z_2)$, and by the rotation of the end-effector about an axis through its line of action (Θ_6).

The required relative positions of $(x_1 y_1 z_1)$ and $(x_2 y_2 z_2)$ are achieved by rotation of the end-effector about horizontal and vertical axes (Θ_4 and Θ_5). Θ_6 describes only the twisting motion of the end-effector about the centre line along its length and cannot effect the relative position between $(x_1 y_1 z_1)$ and $(x_2 y_2 z_2)$.

The required rotations, Θ_4 and Θ_5, are calculated as follows:

$$\Theta_4 = \sin^{-1}\left(\frac{Z_2 - Z_1}{\ell 4}\right) \qquad \left[-\frac{\pi}{2} \leq \Theta_4 \leq \frac{\pi}{2}\right] \qquad (1)$$

$$\Theta_5 = \tan^{-1}\left(\frac{x_2 - x_1}{y_2 - y_1}\right) \qquad \left[x_2 \geq x_1,\ 0 \leq \Theta_5 \leq \pi\right]$$

$$= \tan^{-1}\left(\frac{x_2 - x_1}{y_2 - y_1}\right) \qquad \left[x_2 < x_1,\ 0 > \Theta_5 > -\pi\right] \qquad (2)$$

(with positive X,Y,Z axes as defined in Fig.2)

The necessary joint angles $(\Theta_1 \Theta_2 \Theta_3)$ required to achieve the given arm-end position $(x_1 y_1 z_1)$ can be calculated as follows:

$$\Theta_1 = \tan^{-1}(x_1/y_1) \qquad \left[x_1 \geq 0\ ,\ 0 \leq \Theta_1 \leq \pi\right]$$

$$= \tan^{-1}(x_1/y_1) \qquad \left[x_1 < 0\ ,\ 0 > \Theta_1 > -\pi\right] \qquad (3)$$

Referring to Fig.2.

$$z_1 = \ell_1 + \ell_2 \cos\Theta_2 + \ell_3 \cos(\Theta_2 + \Theta_3)$$

$$d = \sqrt{x_1^2 + y_1^2} = \ell_2 \sin \Theta_2 + \ell_3 \sin (\Theta_2 + \Theta_3) \quad (5)$$

From (4) $z_1 - \ell_1 = \ell_2 \sin (\pi/2 - \Theta_2) + \ell_3 \cos (\Theta_2 + \Theta_3)$ (6)

From (5) $d = \ell_2 \cos (\pi/2) - \Theta_2) + \Theta_3 \sin (\Theta_2 + \Theta_3)$ (7)

Squaring (6) and (7):

$$(z_1-\ell_1)^2 = \ell_2^2 \sin^2(\tfrac{\pi}{2} - \Theta_2) + \ell_3^2 \cos^2(\Theta_2+\Theta_3) + 2\ell_2\ell_3 \sin(\tfrac{\pi}{2}-\Theta_2) \cos(\Theta_2+\Theta_3) \quad (8)$$

$$d^2 = \ell_2^2 \cos^2(\tfrac{\pi}{2}-\Theta_2) + \ell_3^2 \sin^2(\Theta_2+\Theta_3) + 2\ell_2\ell_3 \cos(\tfrac{\pi}{2}-\Theta_2)\sin(\Theta_2+\Theta_3) \quad (9)$$

Adding (8) and (9):

$$(z_1-\ell_1)^2 + d^2 = \ell_2^2 + \ell_3^2 + 2\ell_2\ell_3 \cos\Theta_3 \quad (10)$$

$$\text{Thus } \Theta_3 = \cos^{-1} \left[\frac{(z_1-\ell_1)^2 + x_1^2 + y_1^2 - \ell_2^2 - \ell_3^2}{2\ell_2 \ell_3} \right] \quad (11)$$

From (4) $z_1 = \ell_1 + \cos\Theta_2 (\ell_2 + \ell_3\cos\Theta_3) - \ell_3 \sin\Theta_2 \sin\Theta_3$ (12)

Set $k_1 = \ell_2 + \ell_3 \cos\Theta_3$; $k_2 = \ell_3 \sin\Theta_3$

$k_3 = z_1 - \ell_1$

Then from (12)

$$k_3 = k_1 \cos\Theta_2 - k_2 \sin\Theta_2$$

$$= k_1 \cos\Theta_2 - k_2 \sqrt{1 - \cos^2\Theta_2}$$

or $k_2 \sqrt{1 - \cos^2 \Theta_2} = k_1 \cos\Theta_2 - k_3$ (13)

Squaring both sides of (13)

$$k_2^2(1-\cos^2\Theta_2) = k_1^2 \cos^2\Theta_2 + k_3^2 - 2k_1 k_3 \cos\Theta_2$$

or $(k_1^2 + k_2^2) \cos^2\Theta_2 - 2k_1k_3 \cos\Theta_2 + (k_3^2-k_2^2) = 0$

Solving this quadratic equation:

$$\cos\Theta_2 = \frac{k_1k_3 \pm k_2 \sqrt{k_1^2+k_2^2-k_3^2}}{k_1^2 + k_2^2}$$

Substituting back for k_1, k_2, k_3 and using C_3 and S_3 to represent $\cos\Theta_3$ and $\sin\Theta_3$:

$$\Theta_2 = \cos^{-1}\left[\frac{(\ell_2 + \ell_3 C_3)(z_1 - \ell_1) \mp \ell_3 S_3 \sqrt{\ell_2^2 + \ell_3^2 C_3^2 + 2\ell_2\ell_3 C_3 + \ell_3^2 S_3^2 - (z_1-\ell_1)^2}}{\ell_2^2 + 2\ell_2\ell_3 C_3 + \ell_3^2}\right] \qquad (14)$$

From (10): $C_3 = \dfrac{(z_1-\ell_1)^2 + (x_1^2 + y_1^2) - \ell_2^2 - \ell_3^2}{2\ell_2\ell_3}$

Thus, $2\ell_2\ell_3 C_3 + \ell_2^2 + \ell_3^2 = (z_1 - \ell_1)^2 + (x_1^2 + y_1^2)$

Using this relationship, (14) simplifies to

$$\Theta_2 = \cos^{-1}\left[\frac{(\ell_2+\ell_3 C_3)(z_1-\ell_1) \mp \ell_3 S_3 \sqrt{(z_1-\ell_1)^2 + (x_1^2+y_1^2) - (z_1-\ell_1)^2}}{(z_1-\ell_1)^2 + (x_1^2 + y_1^2)}\right]$$

$$= \cos^{-1}\left[\frac{(\ell_2+\ell_3 C_3)(z_1-\ell_1) \mp \ell_3 S_3 \sqrt{x_1^2 + y_1^2}}{(z_1-\ell_1^2) + (x_1^2 + y_1^2)}\right] \qquad (15)$$

In the above derivation of expressions for Θ_2 and Θ_3, the possibility of Θ_2 and Θ_3 lying in any of four possible quadrants has been ignored. The working envelope of the configuration shown in Fig.2. is part of a sphere. Negative and positive values of Θ_2 move the point P into opposite quadrants of the sphere. However, such movement between quadrants is also achieved by variations of Θ_1 (rotation about z axis) and hence the possibility of negative values of Θ_2 represents redundant motion. Hence the required values of Θ_2 in equation (15) are those within the range $|o \leq \Theta_2 \leq \pi|$.

Solving (11) yields two possible values (positive and negative) for Θ_3 and this is reflected in the two possible values of Θ_2 arising from (15). Again, however, negative values of Θ_3 represent redundant motion and the valid range for Θ_3 can be defined as $|0 \leq \Theta_3 \leq \pi|$. The solution of (15) corresponding to this is:

$$\Theta_2 = \cos^{-1} \frac{(\ell_2 + \ell_3 \cos\Theta_3)(z_1-\ell_1) + \ell_3 \sin\Theta_3 \sqrt{x_1^2 + y_1^2)}}{(z_1-\ell_1)^2 + (x_1^2 + y_1^2)} \qquad (16)$$

In the general case, therefore, the values of Θ to satisfy the defined spatial coordinates of the end effector are given by equation (1), (2), (3), (11) and (16).

2.1 Simplified equations with limited joint rotations

In common working implementations of the revolute coordinate geometry illustrated in Fig.1, joint rotations are often limited to the following ranges.

$$-\pi/2 \leq \Theta_1 \leq \pi/2 \qquad 0 \leq \Theta_2 + \Theta_3 \leq \pi$$
$$0 \leq \Theta_2 \leq \pi \qquad -\pi/2 \leq \Theta_4 \leq \pi/2$$
$$0 \leq \Theta_3 \leq \pi \qquad 0 \leq \Theta_5 \leq \pi$$

These limitations avoid ambiguities about which quadrant the required values of Θ lie in, and the equations for Θ reduce to the following:

$$\Theta_1 = \tan^{-1}(x_1/y_1) \tag{17}$$

$$\Theta_2 = \cos^{-1}\left[\frac{(\ell_2+\ell_3\cos\Theta_3)(z_1-\ell_1)+\ell_3\sin\Theta_3\sqrt{x_1^2+y_1^2}}{(z_1-\ell_1)^2+(x_1^2+y_1^2)}\right] \tag{18}$$

$$\Theta_3 = \cos^{-1}\left[\frac{(x_1-\ell_1)^2+x_1^2+y_1^2-\ell_2^2-\ell_3^2}{2\ell_2\ell_3}\right] \tag{19}$$

$$\Theta_4 = \sin^{-1}\left(\frac{z_2-z_1}{\ell_4}\right) \tag{20}$$

$$\Theta_5 = \tan^{-1}\left(\frac{x_2-x_1}{y_2-y_1}\right) \tag{21}$$

2.2 Special case of end effector in vertical orientation

A special case of equations (17) - (21) applies when the end-effector approaches vertically a workpiece located in a horizontal plane through the robot base. This is a very common working arrangement in handling applications.

Then $x_1 = x_2$

$y_1 = y_2$

$z_1 = z_2 + \ell_4$

$\Theta_4 = -\pi/2$

and Θ_5 is redundant.

The required equations for calculating the set of Θ are now equations (17) -(19), with Θ_4 set to the value $(-\pi/2)$.

3. Smart Arms 6E Robot Implementation

The Smart Arms 6E robot is a small robot designed for the educational market of the form shown in Fig.1. The arm has three degrees of freedom measured by the angles Θ_1, Θ_2, Θ_3, as shown in Fig.2. The end-effector is a simple gripper with two degrees of freedom measured by the angles Θ_4 and Θ_6 in Fig.2. The lengths of each link, and the range of rotation of each joint are as follows.

$\ell_1 = 76\text{mm}$ $\quad$ $\ell_3 = 142\text{mm}$

$\ell_2 = 144\text{mm}$ $\quad$ $\ell_4 = 66\text{mm}$

$-\pi/3 \leq \Theta_1 \leq \pi/3$ $\quad$ $-\pi/3 \leq \Theta_2 \leq \pi/3$

$0 \leq \Theta_3 \leq 2\pi/3$ $\quad$ $-\pi/3 \leq \Theta_4 \leq \pi/3$

$-\pi/3 \leq \Theta_6 \leq \pi/3$

These rotation ranges for each joint comply with the conditions stated for the application of the simplified transformation discussed in sections 2.1 except for Θ_2. In the specification, the range for Θ_2 is $\pm\ \pi/3$. However, negative values of Θ_2 correspond to arm configurations where the gripper cannot make contact with the horizontal table on which the robot base typically stands and such configurations are not required in normal working arrangements. Thus it is usual to limit values of Θ_2 to the range:

$$0 \leq \Theta_2 \leq \pi/3.$$

With this limitation, equations (17) - (21) of section 2.1 become directly applicable.

The coordinate transformation equations were tested by defining the xyz coordinates of three 20mm side wooden cubes placed on the robot table. The transformation algorithm was used to calculate the joint positions required to place the gripper around each cube such that it could be picked up. In this operation, the special case of section 2.2 applies, i.e. the gripper must be vertical during the pick up operation ($\Theta_4 = \pi/2$). The roll angle of the gripper must be set at the mid-point of its range (i.e. $\Theta_6 = 0$).

The robot was then programmed to move to the required points on the table in turn, pick up the bricks, and pile them one on top of another. For this latter operation, the required spatial coordinates of each brick in the pile were calculated and the coordinate transformation equations used to derive the corresponding robot joint angles. Figure 3 shows the robot proceeding with this operation.

4. Conclusions

An algorithm for transformation between spatial xyz coordinates of points within a robot working envelope and the corresponding angular coordinates of the joints of a revolute geometry robot necessary to achieve the defined end-effector position has been presented. Some simplifications of this general algorithm have also been described which are applicable under certain specified conditions. Use of the coordinate transformation algorithm has been illustrated by applying a small robot to a brick building task.

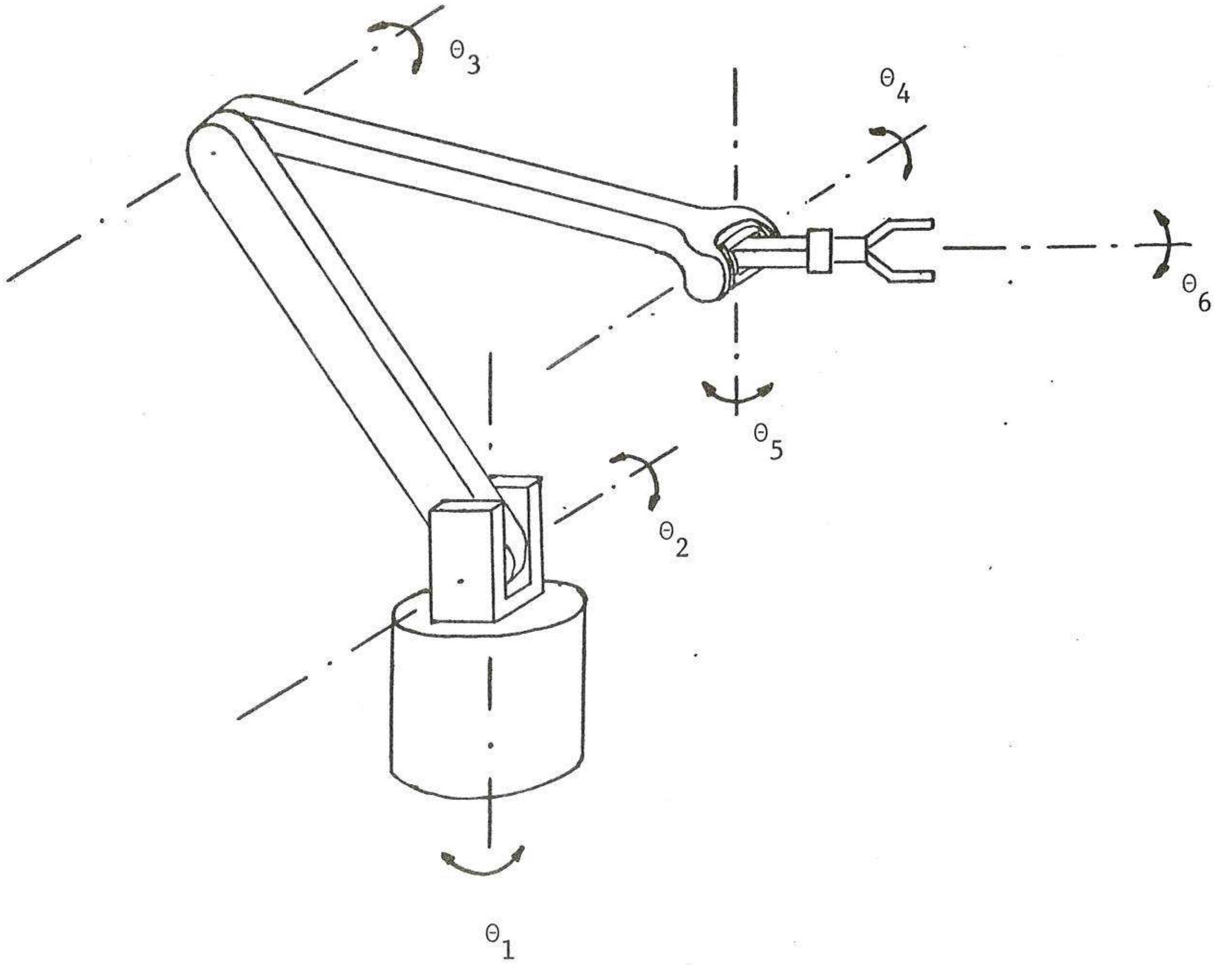
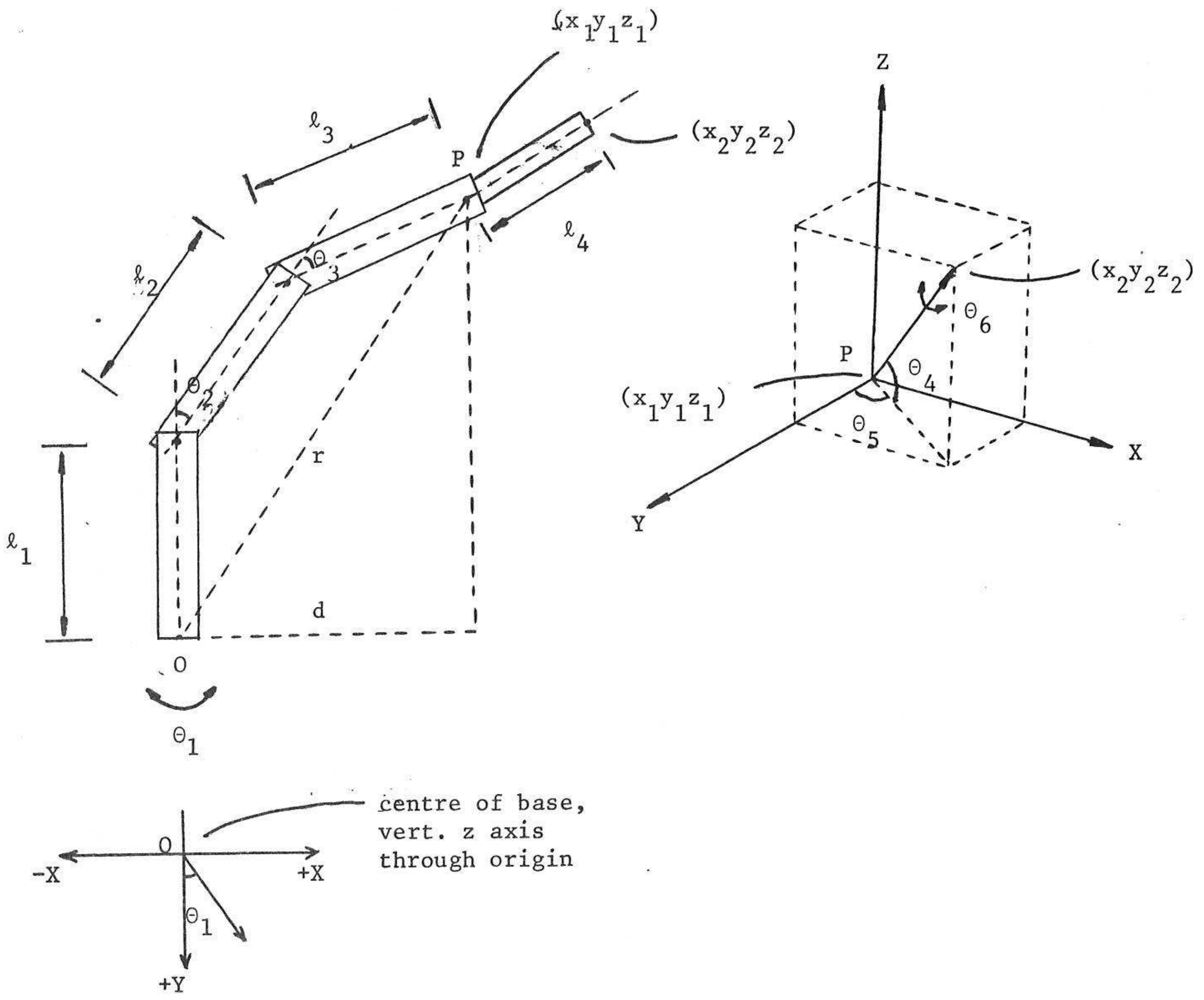


FIG.1.



$(x_1y_1z_1)$
$(x_2y_2z_2)$
ℓ_3
P
ℓ_2
Θ_3
ℓ_4
Θ_2
ℓ_1
r
d
O
Θ_1
Z
$(x_2y_2z_2)$
Θ_6
P
$(x_1y_1z_1)$
Θ_4
Θ_5
X
Y
centre of base,
vert. z axis
through origin
O
-X
+X
Θ_1
+Y

FIG.2.

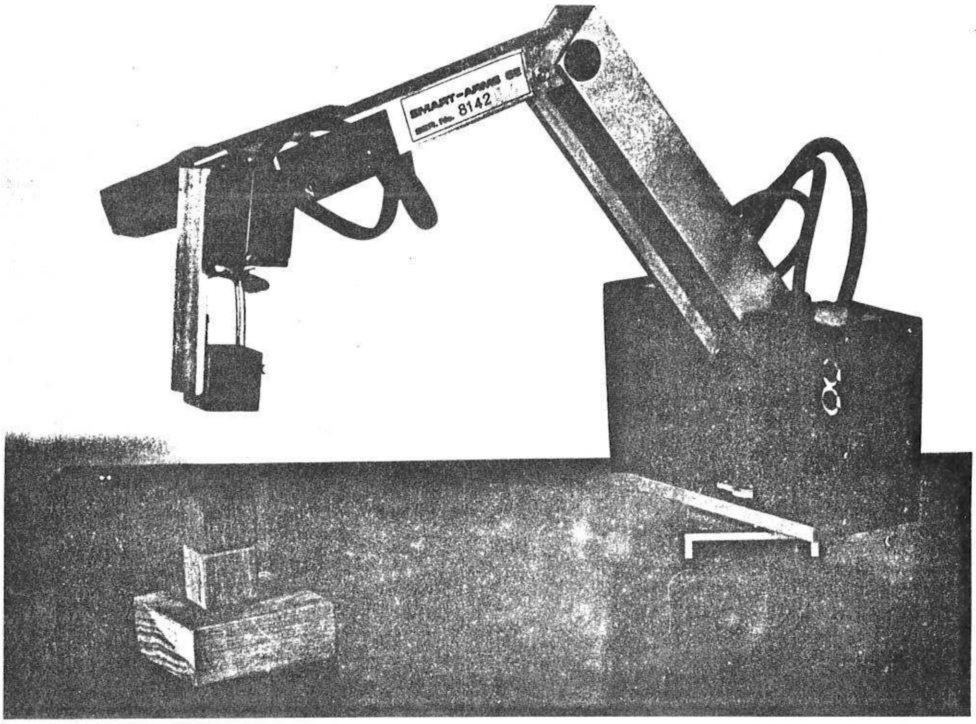


Figure 3. Brick-building test.