DESIGN OF SMITH CONTROL SCHEMES FOR TIME-DELAY SYSTEMS BASED ON PLANT STEP DATA

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Abstract

Systematic design techniques for Smith control schemes are presented that enable off-line designs based on approximate plant models to be achieved that take explicit account of plant/model mismatch as observed in the open-loop plant and model unit step responses.

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1. Introduction

Consider an ℓ-input/m-output linear, convolution plant expressed in the separable form TG where the m-input/m-output element T represents (notionally) output delays or similar dynamic effects and the ℓ-input/m-output element G represents (notionally) strictly proper delay-free dynamics. The destabilizing effect of the delay T can be offset with a considerable improvement in performance by the use of the Smith Predictor control scheme illustrated in Fig.1(a) (Marshall 1979, Owens and Raya 1982,) where K represents a proper m-input/ℓ-output delay free, convolution, control element and G_A and T_A represent convolution models of G and T respectively. It is well-known that this scheme can be represented in the standard feedback form of Fig.1(b) where the forward path controller

$$K^* \triangleq (I_\ell + K(I-T_A)G_A)^{-1}K \qquad \ldots(1)$$

and that the mismatch $TG-T_AG_A$ between the plant and model can be a serious source of stability and performance problems on implemenetation unless it is adequately analysed at the design stage.

This paper presents a systematic analysis of the effect of mismatch on stability and closed-loop performance deterioration in the situation when mismatch is characterized by errors in modelling of the plant open-loop unit step response. This situation represents both the case when the plant TG is structurally and parametrically uncertain and the case when the plant TG is of such complexity that the use of an approximate model $T_A G_A$ is required to simplify computation and reduce storage requirements. The concepts and techniques used in the analysis are similar in structure to those of the recent work of Owens and Chotai (1983) on approximation and process control. Several useful results of that paper will be used here and, for clarity of exposition, the same notation will be used where possible.

As in Åstrom (1980) and Owens and Chotai (1983) a basic assumption made in the analysis is that, for each pair of indices (i,j), the response $Y_{ij}(t)$ from zero initial conditions of the ith plant output to a unit step in the jth input has been reliably estimated from plant trials or simulations of an available complex model. The step data is then assumed to be represented by the 'step-response matrix'

$$Y(t) \triangleq \begin{pmatrix} Y_{11}(t) & \cdots & Y_{1\ell}(t) \\ \vdots & & \\ Y_{m1}(t) & \cdots & Y_{m\ell}(t) \end{pmatrix} \qquad \ldots(2)$$

Let $Y_A(t)$ be the step response matrix of the model $T_A G_A$ and define the error

$$E(t) \triangleq Y(t) - Y_A(t) \qquad \ldots(3)$$

The problems considered below are how the error E can be used to guarantee the stability of the implemented scheme of Fig.1(a) if the predictor is designed off-line based on the assumption that the plant TG is equal to the model $T_A G_A$ and how the same information can be used to bound the deterioration in predicted transient performance due to the mismatch.

2. Mismatch and Frequency Domain Stability Criteria

The following basic result relates the stability of the Smith scheme of Fig.1(a) to the off-line scheme of Fig.1(c) obtained by replacing T by T_A and G by G_A. The elements G, G_A, T, T_A, K are assumed to be represented in transfer function matrix form in this development.

Theorem 1: The Smith scheme of Fig.1(a) is input/output stable in the L_2 sense if

(a) the Smith scheme of Fig.1(c) is stable,

(b) both G and G_A are stable, and

(c) the 'spectral bound' $\lambda_o \triangleq \sup_{s \in D} r\left(\|(I_\ell + K(s)G_A(s))^{-1}K(s)\|_P \Delta(s)\right)$

$$< 1 \qquad \ldots(4)$$

where $\Delta(s)$ is any available real-valued $m \times \ell$ matrix function of the complex variable satisfying

$$\Delta(s) \geq \|T(s)G(s) - T_A(s)G_A(s)\|_P \qquad \ldots(5)$$

for all s on D.

(Remarks: (i) r(M) denotes the spectral radius of M,

(ii) D is the usual Nyquist contour in the right-half complex plane,

(iii) $\|M\|_P$ denotes the matrix obtained by replacing M_{ij} by $|M_{ij}|$, and

(iv) $A \leq B$ denotes the inequalities $A_{ij} \leq B_{ij}$ for all i,j.)

Proof: Following Owens and Raya (1982) write the i/o equations in the Laplace transformed form

$$u = (I_\ell + KG_A)^{-1}K(r - (TG - T_A G_A)u)$$

$$= u_A - (I_\ell + KG_A)^{-1}K(TG - T_A G_A)u \qquad \ldots(6)$$

where u_A is the input response transform when TG is replaced by $T_A G_A$. Note that the assumptions ensure that $(I + KG_A)^{-1}K(TG-T_A G_A)$ is bounded and analytic in the closed right-half complex plane and that $u_A(s)$ has the same property whenever $r \in L_2^m(0,\infty)$. Let s be arbitrary and Re s $\geq$ 0 and write (6) in the form

$$(I_\ell + (I_\ell + KG_A)^{-1}K(TG - T_A G_A))u = u_A \qquad \ldots(7)$$

It is clear that u(s) is well-defined iff the coefficient matrix in the left-hand-side is nonsingular ie u(s) is bounded and analytic in the right-half-plane if

$$\inf_{Res>0} \left|\det(I_\ell + (I_\ell + KG_A)^{-1}K(TG - T_A G_A))\right| > 0 \qquad \ldots(8)$$

The principle of the maximum enables us to replace the infimum over Res>0 by an infimum over D whence (8) is satisfied if

$$\sup_{s \in D} r((I_\ell + KG_A)^{-1}K(TG - T_A G_A)) < 1 \qquad \ldots(9)$$

The arguments in Owens and Chotai (1983) can now be used to show that condition (4) implies (9) and the proof is complete.

The result expresses in the form of (4) a relationship between the model G_A, its controller K and the mismatch $TG-T_A G_A$ that ensures stability. Noting that (4) is trivially satisfied in the absence of mismatch (ie $TG = T_A G_A$), it can be interpreted as providing lower estimates of the largest permissible mismatch that retains stability. The evaluation of λ_o is a straightforward computational task that can be simplified in a number of obvious ways and situations:

(a) If $\lambda_o' \triangleq \sup_{s \in D} \gamma_o(s) < 1$ where $\gamma_o(s)$ is a conveniently computable upper bound for $r(\|(I_\ell + KG_A)^{-1}K\|_P \Delta)$ then (4) is automatically satisfied. Any vector-induced matrix norm (Owens and Chotai, 1983) is a candidate for γ_o including, for example, the maximum singular value.

(b) If $m = \ell$ (which includes the scalar case of $m = \ell = 1$) and K and G_A are taken to be diagonal due to a belief that interaction is small enough to be neglected and a desire to simplify the off-line design then the following graphical result can be obtained, and has the flavour of the well-known inverse Nyquist array (Rosenbrock, 1974).

Theorem 2: If $m = \ell$ and G_A and K are diagonal of the form

$$G_A(s) = \text{diag}\,\{g_j(s)\}_{1 \le j \le m} \quad , \quad K(s) = \text{diag}\{k_j(s)\}_{1 \le j \le m} \qquad \ldots(10)$$

then the conclusions of theorem 1 remain valid with condition (c) replaced by

(i) the inequalities

$$\limsup_{\substack{\text{Re}\, s \ge 0 \\ |s| \to \infty}} |k_j(s)| \sum_{k=1}^{m} \Delta_{jk}(s) < 1 \qquad \ldots(11)$$

$$1 \le j \le m$$

and (ii) the requirement that the (-1,0) point does not lie in or touch the 'confidence bands' generated by plotting the

<u>inverse</u> Nyquist loci of $g_j(s)k_j(s)$ with superimposed 'confidence circles' at each point of radius

$$r_j(i\omega) \triangleq |g_j^{-1}(i\omega)| \sum_{k=1}^{m} \Delta_{jk}(i\omega) \qquad \ldots(12)$$

<u>Proof:</u> Replace (4) by the <u>sufficient</u> condition

$$\sup_{s \in D} \max_{1 \leq j \leq m} \left| \frac{k_j(s)}{1+g_j(s)k_j(s)} \right| \sum_{k=1}^{m} \Delta_{jk}(s) < 1 \qquad \ldots(13)$$

as in theorem 3 of Owens and Chotai (1983). Condition (11) follows from the strict properness of G_A and properness of K and consideration of the semi-circular part of D, whilst (ii) follows by considering the imaginary axis and writing (13) in the form

$$|1 + (g_j(i\omega)k_j(i\omega))^{-1}| > r_j(i\omega), \quad \omega \geq 0, 1 \leq j \leq m \qquad \ldots(14)$$

The result has an identical graphical interpretation to theorem 3 of Owens and Chotai (1983). The choice of $\Delta(s)$ is open to the designer and the above results do not prejudge his choice. For the purposes of this paper however we will follow Åstrom (1980) and Owens and Chotai (1983) by concentrating on error bounds Δ that can be easily deduced from graphical inspection of the transient data E(t). The relevant quantity here is the 'matrix total variation' of E,

$$N_\infty^P(E) \triangleq \begin{pmatrix} N_\infty(E_{11}) & \cdots & N(E_{1\ell}) \\ \vdots & & \\ N_\infty(E_{m1}) & \cdots & N_\infty(E_{m\ell}) \end{pmatrix} \qquad \ldots(15)$$

where (Owens and Chotai, 1983) $N_\infty(E_{ij})$ is the norm of E_{ij} regarded as a function of bounded variation on $[0,\infty)$. The relevant result is an elementary extension of lemma 2 in that paper:

Lemma 1:

$$||T(s)G(s) - T_A(s)G_A(s)||_P \leq N_\infty^P(E) \quad , \quad \mathrm{Re}\,s \geq 0 \qquad \ldots(16)$$

$N_\infty^P(E)$ is hence a constant candidate for $\Delta(s)$ with the advantages that it can be evaluted graphically from step data and that the detailed structure and parameters of the plant TG need not be known (Owens and Chotai, 1983). To illustrate the application of the ideas consider the problem analysed in Owens and Chotai (1983) in a Smith predictor context. More precisely consider the scalar system with transfer function

$$T(s)G(s) = \frac{4e^{-s\tau}}{(s^2+2s+4)(s+1)} \quad , \quad \tau \geq 0 \qquad \ldots(17)$$

A commonly fitted simple model would have the delay-lag structure

$$T_A(s)G_A(s) = \frac{e^{-s\tau_A}}{1+s} \quad , \quad \tau_A = \tau+0.6 \qquad \ldots(18)$$

where we identify $T_A(s)$ with $e^{-s\tau_A}$. The step responses of plant and model are shown in Fig.2 for $\tau = 0$. The use of the approximate model (18) to represent the plant (17) in unity feedback control has been discussed in detail in the case of $\tau = 0$ by Owens and Chotai (1983). We concentrate here on the general use of $\tau \geq 0$ and the use of a Smith control scheme to illustrate the preceding theory, to demonstrate that the permissible errors in the predictor scheme can be larger than the errors allowed under normal feedback conditions and to indicate the improvement in input/output performance.

Considering initially the case of proportional control $K(s) = k_1$, it is trivially verified that condition (a) of theorem 1 is satisfied if

$$k_1 > -1 \qquad \ldots(19)$$

Condition (b) is clearly satisfied so it remains to check condition (c). This can be done analytically in this case by noting that, independent of $\tau \geq 0$,

$$\hat{\lambda}_o = \sup_{s \in D} \left| \frac{k_1(s+1)}{k_1+1+s} \right| = |k_1| \qquad \ldots(20)$$

(if attention is restricted to positive gains $k_1 > 0$) and that $N_\infty^P(E) = 0.45$ <u>independent of the value of $\tau \geq 0$</u>. The maximum permitted gain predicted by theorem 1 is hence described by the relation $\hat{\lambda}_o N_\infty^P(E) < 1$ ie

$$k_1 < \frac{1}{0.45} = 2.22 \qquad \ldots(21)$$

<u>independent of the value of $\tau \geq 0$</u>. This should be compared with the <u>lower</u> maximum gains of $k_1 < 1.32$ allowed (Owens and Chotai, 1983) in the standard feedback configuration in the case of $\tau = 0$ and $k_1 < 0.78$ when $\tau = 3$.

Turning our attention now to the case of proportional plus integral control $K(s) = k_1 + s^{-1}k_2$ we again check condition (c) of theorem 1 in the form of the conditions of theorem 2. Requirement (i) with $\Delta(s) = N_\infty^P(E)$ again reduces to (21). Choosing therefore $k_1 = 1.5$ within this range to obtain closed-loop time-constants of 0.4 from the approximating predictor of Fig.1(c) and integral gain

$k_2 = 1.0$ to obtain reset times of the order of 1.0, the corresponding inverse Nyquist plot is given in Fig.3 and indicates stability as the (-1,0) point does not lie in or on the confidence band at any frequency. This prediction is verified by the closed-loop unit step responses given in Fig.4 for the cases of $\tau = 0$ and $\tau = 3$ and illustrates that the approximate model is an adequate representation of the plant for predictive control purposes ensuring stability and providing reasonable estimates of closed-loop transient behaviour. The problem of prediction of the error $y-y_A$ using E(t) is considered in later sections.

Finally, it is known (Marshall and Salehi, 1982) that over-estimation of the plant delay can benefit performance of Smith schemes. The above example also indicates that overestimation of the plant delay can have benefits in stabilizability. To demonstrate this let τ and τ_A now vary independently and let $\delta\tau = \tau_A - \tau$. Suppose initially that the plant delay τ is zero and note that $\delta\tau = \tau_A$ and that any positive delay then represents an overestimation. Consider the delay-lag model

$$T_A(s)G_A(s) = \frac{e^{-s\tau_A}}{1+s} \quad , \qquad \ldots(22)$$

with $T_A(s)$ identified with $e^{-s\tau_A}$. The choice of $\delta\tau$ has no effect on conditions (a) and (b) of theorem 1 but it does affect the validity of condition (c). More precisely, taking $\Delta = N_\infty^P(E)$, it is clear that λ_0 is proportional to $N_\infty^P(E)$ and that, for a fixed gain K(s), λ_0 will be minimized by choosing $\delta\tau$ to minimize $N_\infty^P(E)$. The plot of $N_\infty^P(E)$ against $\delta\tau$ is given in Fig.5(a) and indicates that the stability predictions are least conservative by the choice of

$\delta\tau = \delta\tau^* = 0.7 > 0$. That is overestimation of the plant delay leads to improved stability characteristics and consequent robustness! This result still holds for all $\tau \geq 0$ as $N_\infty^P(E)$ is unchanged by this transformation and hence is minimized by $\delta\tau = \delta\tau^* > 0$ as above ie overestimation of the delay improves stability characteristics independent of the length of the plant delay. The corresponding plot with $\tau = 3$ is given in Fig.5(b) to underline the validity of this statement.

3. Mismatch and Time Domain Stability Criteria

It has been demonstrated in Owens and Chotai (1983) that the use of time-domain mismatch data $E(t)$ can be used as the basis of a time-domain simulation-based stability criterion. The potential advantages of such an approach are described in that paper. The following result is a direct parallel of theorem 4 in that paper and is interpreted in an identical manner:

Theorem 3: Suppose that

(a) The Smith scheme of Fig.1(c) is stable,

(b) both G and G_A are stable

and that $E^{(j)}(t)$ is the jth column of $E(t)$. Suppose also that the matrix

$$W_A(t) \triangleq [W_A^{(1)}(t), \ldots, W_A^{(\ell)}(t)] \qquad \ldots(23)$$

has been computed where $W_A^{(j)}(t)$ is the response of the 'delay-free' system $(I_\ell + KG_A)^{-1}K$ from zero initial conditions to the input vector $E^{(j)}(t)$. Then the Smith scheme is input/output stable in the L_∞ sense if the 'contraction constant'

$$\lambda_1 \triangleq r(N_\infty^P(W_A)) < 1 \qquad \ldots(24)$$

Proof: Regarding equation (6) as an equation in $L_\infty^\ell(0,\infty)$, stability is equivalent to the existence of a solution $u \in L_\infty^\ell(0,\infty)$. This is guaranteed (Owens and Chotai, 1983) by the contraction mapping theorem if $r(\|(I+KG_A)^{-1}K(TG-T_AG_A)\|_P) < 1$ where $\|L\|_P$ denotes the matrix of L_∞ induced-operator norms of elements of L. The result follows in a similar way to theorem 4 of Owens and Chotai (1983) by identifying $\|L\|_P$ with the matrix total variation of its step response matrix.

To illustrate the application of the result consider the example of section 2 with $K(s) = 1.5 + s^{-1}1.0$ and hence $(1+K(s)G_A(s))^{-1}$ $K(s) = \frac{1.5s^2+2.5s+1}{s^2+2.5s+1}$. Elementary simulation leads to the form for $W_A(t)$ in the case of $\tau = 0$ given in Fig.6 and hence $\lambda_1 = N_\infty^P(W_A) = 0.595 < 1$. Note that λ_1 is independent of τ as increasing τ simply delays E and hence W_A and has no effect on the total variation. This verifies the stability predictions of section 2. It also indicates that the time-domain approach is less conservative than the frequency domain approach and that the Smith scheme permits higher gains than the standard feedback scheme. These observations can be substantiated in a quantitative way by considering the case of proportional control $K(s) = k_1$ and plotting the contraction constants for the feedback control and Smith scheme as a function of k_1 as illustrated in Fig.7. The feedback scheme permits a maximum gain of $k_1^* = 1.55$ in the case of $\tau = 0$ and $k_1^* = 1.02$ when $\tau = 3$ whilst the Smith scheme permits a maximum gain of $k_1^* = 3.35$ independent of the value of $\tau \geq 0$.

Another advantage of the use of time-domain methods of analysis is that it leads to estimates of performance degradation due to mismatch (Owens and Chotai, 1983). This is discussed in the next section.

4. Mismatch and Performance

We now consider the problem of estimation of $u(t)$ and $y(t)$ given the data $u_A(t)$ and $y_A(t)$ deduced from off-line design and the plant mismatch data $E(t)$.

4.1. Input Degradation due to Mismatch

The following result is a direct parallel to theorem 5 of Owens and Chotai (1983) and is based on the extended contraction principle outlined in that paper. For simplicity of exposition, the result is stated for step inputs only and with the initial guess $u^{(o)}(t) \triangleq u_A(t)$ in the successive approximation scheme.

Theorem 4: Suppose that the conditions of theorem 3 hold and that

(a) $u_A(t)$ is the input response of the Smith scheme of Fig.1(c) to a step input demand $r(t) = \beta$, $t \geq 0$,

(b) $\xi(t)$ is the ℓx1 vector computed by the convolution

$$\xi(t) = -\left(\int_o^t W_A(t-t')H(t')dt'\right)\beta \qquad \ldots(25)$$

where $H(t)$ is the impulse response matrix of $(I_\ell + KG_A)^{-1}K$, and

(c) $$\varepsilon(t) = \begin{pmatrix} \varepsilon_1(t) \\ \vdots \\ \varepsilon_\ell(t) \end{pmatrix} \triangleq (I-P_t)^{-1}P_t \sup_{0 \leq t' \leq t} \|\xi(t')\|_P \qquad \ldots(26)$$

where $P_t = N_t^P(W_A)$ is the matrix total variation of W_A on the interval $[0,t]$ (Owens and Chotai, 1983).

Then, the real input response u(t) of the Smith scheme of Fig.1(a) to the step demand r(t) from zero initial conditions satisfies the bound

$$|u_j(t) - u_j^{(1)}(t)| \leq \varepsilon_j(t) \quad , \quad 1\leq j\leq \ell \ , \ t\geq 0 \qquad \ldots(27)$$

where $u^{(1)}(t) = u_A(t)+\xi(t)$ is the first order correction to the approximate response $u_A(t)$.

Proof: The proof is identical to the proof of theorem 5 of Owens and Chotai (1983) with $G-G_A$ replaced by $TG-T_AG_A$ and $u^{(o)} = (I+KG_A)^{-1}Kr = u_A$ and is omitted for brevity.

The graphical interpretation of (27) is simply that $u_j(t)$ lies in the region between the curves $u_j^{(1)}(t)\pm\varepsilon_j(t)$. Both $u^{(1)}$ and ε are easily evaluated (see Owens and Chotai, 1983) by simulation of low order feedback systems generated by K and G_A. The details of the procedure are illustrated in section 4.3 by an example.

4.2. Output Degradation due to Mismatch

The following result is proved in a similar manner to Corollary 3 of Owens and Chotai (1983):

Theorem 5: With the conditions of theorem 4, we have

$$\|y(t) - y^{(1)}(t)\|_P \leq N_t^P(Y)\varepsilon(t) + N_t^P(E) \sup_{0\leq t'\leq t} \|u^{(1)}(t')\|_P \qquad \ldots(28)$$

where $y^{(1)}$ is the response of T_AG_A from zero initial conditions to $u^{(1)}(t)$.

A better bound is however available in the case of scalar systems ($m = \ell = 1$) when the contraction principle can be used to prove the following result:

<u>Theorem 6</u>: Suppose that $m = \ell = 1$, and that the conditions of theorem 3 hold. Then the controller K will stabilize the Smith scheme of Fig.1(a) and the response $y(t)$ from zero initial conditions to a <u>unit</u> step demand $r(t)$ satisfies the bound

$$|y(t) - y^{(1)}(t)| \leq \varepsilon(t) \quad , \qquad t \geq 0 \qquad \ldots(29)$$

where

$$\varepsilon(t) \triangleq \frac{N_t(W_A)}{1-N_t(W_A)} \max_{0 \leq t' \leq t} |\eta(t')| \quad , \quad t \geq 0 \ , \qquad \ldots(30)$$

$$y^{(1)}(t) \triangleq y_A(t) + \eta(t) \quad , \quad t \geq 0 \qquad \ldots(31)$$

and $\eta(t)$ is the response from zero initial conditions of the system $I-(I+G_AK)^{-1}G_AKT_A$ to the input $W_A(t)$.

<u>Proof</u>: The proof has a similar structure to that of theorem 6 in Owens and Chotai (1983). Consider the output equation obtained from Fig.1(b)

$$y = TGK^* r - TGK^* y \qquad \ldots(32)$$

Adding $T_AG_AK^*y$ to both sides of the equation leads, after a little manipulation using (1) and the commutation of scalar convolution operators, to

$$\begin{aligned} y &= (I+T_AG_AK^*)^{-1}(TGK^*r - (TG-T_AG_A)K^*y) \\ &= (I+KG_A)^{-1}KTGr - (I+KG_A)^{-1}K(TG-T_AG_A)y \qquad \ldots(33) \end{aligned}$$

or

$$y = y_A + (I+KG_A)^{-1}K(TG-T_AG_A)(r-y) \qquad \ldots(34)$$

where $y_A = (I+G_AK)^{-1}G_AKT_A$ is the response of the Smith scheme of Fig.1(c). This equation has the form $y = W_t(y)$ where W_t maps $L_\infty(0,t)$ into itself for all $t \geq 0$. Following the argument in the proof of theorem 6 in Owens and Chotai (1983), stability is guaranteed if, taking $t = +\infty$, we have the contraction condition

$$\lambda_\infty = \|(I+KG_A)^{-1}K(TG-T_AG_A)\|_P < 1 \qquad \ldots(35)$$

which is valid by the conditions of theorem 3 and the identification of $\|(I+KG_A)^{-1}K(TG-T_AG_A)\|_P$ with the total variation $N_\infty(W_A)$ of its step response $W_A(t)$ on $[0,\infty)$.

Condition (35) also guarantees that W_t is a contraction for all $t \geq 0$ with contraction constant $\lambda_t \triangleq N_t(W_A)$ and that y can be obtained successfully by the successive approximation scheme $y^{(k+1)} = W_t(y^{(k)})$ with $y^{(o)}$ arbitrary. Letting $y^{(o)} = y_A$, we obtain, using commutation and the definition of W_A,

$$\begin{aligned} y^{(1)} &= y_A + (I+KG_A)^{-1}K(TG-T_AG_A)(I-(I+G_AK)^{-1}G_AKT_A)r \\ &= y_A + (I - (I+G_AK)^{-1}G_AKT_A)W_A \\ &= y_A + \eta \end{aligned} \qquad \ldots(36)$$

and the result follows from the standard contraction mapping error estimate

$$\|y - y^{(1)}\|_\infty \leq \frac{\lambda_t}{1-\lambda_t} \|y^{(1)} - y^{(o)}\|_\infty \qquad \ldots(37)$$

in $L_\infty(0,t)$.

4.3. Illustrative Example

Considering the example plant (17) with model (18) and controller $K(s) = 1.5 + s^{-1}1.0$ used in previous discussions and consider the system input and output responses u and y to a unit step demand r(t) at t = 0 from zero initial conditions. The responses u_A and y_A of the ideal Smith scheme of Fig.1(c) are shown in Fig.8 for the case of $\tau = 0$. The corresponding responses for $\tau>0$ are obtained by shifting the output response only to the right by τ time units.

The deterioration in input characteristics predicted by $u_A(t)$ are obtained by evaluation of $\xi(t)$ using (25) or, equivalently in the scalar case, evaluating $\xi(t)$ as the response from zero initial conditions of $(I+KG_A)^{-1}K$ to the input W_A. The resultant ξ for the case of $\tau = 0$ is given in Fig.9(a) (the ξ for $\tau = 3$ is obtained as a delayed version). This is used in (26) to obtain the error bound (27). These are represented graphically in Fig.9(b),(c) in the cases of $\tau = 0$ and $\tau = 3$ respectively.

The deterioration in output characteristics is obtained by the use of theorem 6. Given the data E(t) the response W_A can be computed and used to obtain η by simulation of $(I+G_AK)^{-1}G_AK$ from zero initial conditions to obtain the response $\psi(t)$ to the input $W_A(t)$. The signal η is then obtained from $\eta(t) = W_A(t)-\psi(t-\tau-0.6)$ and is given in Fig.10 for the cases of $\tau = 0$ and $\tau = 3$. These can be used directly to evaluate $y^{(1)}$ and ε. The resultant error bounds are illustrated in Fig.11 for the cases of $\tau = 0$ and $\tau = 3$. Note that the performance predicted by the ideal scheme was a reasonable indicator of the performance to be expected of the implemented scheme.

5. Conclusions

The paper has demonstrated that the approximation ideas introduced by Owens and Chotai (1983) for the design of multivariable feedback systems based on approximate plant models carry over to the design of Smith control schemes in the presence of mismatch. The approaches are very similar in structure, being based upon systematic use of plant step data obtained from plant tests or model simulations and producing both graphical frequency domain and simulation-based time-domain methods of off-line computer-aided design. Both stability and performance degradation can be analysed and it is not necessary, in principle, to obtain a detailed plant model to apply the results provided that plant step data can be obtained or synthesized.

References

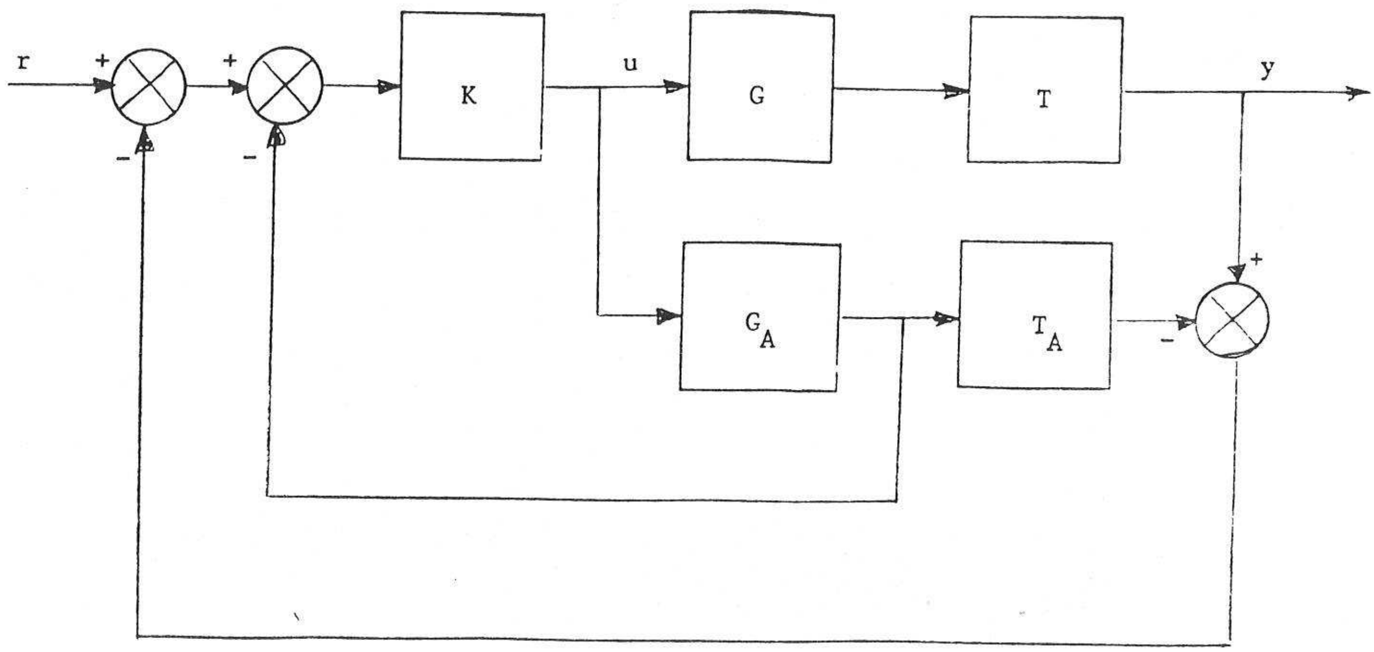
ÅSTROM, K. J., 1980, Automatica, 16, 313.

MARSHALL, J. E., 1979, Control of time-delay systems, (Peter Peregrinus).

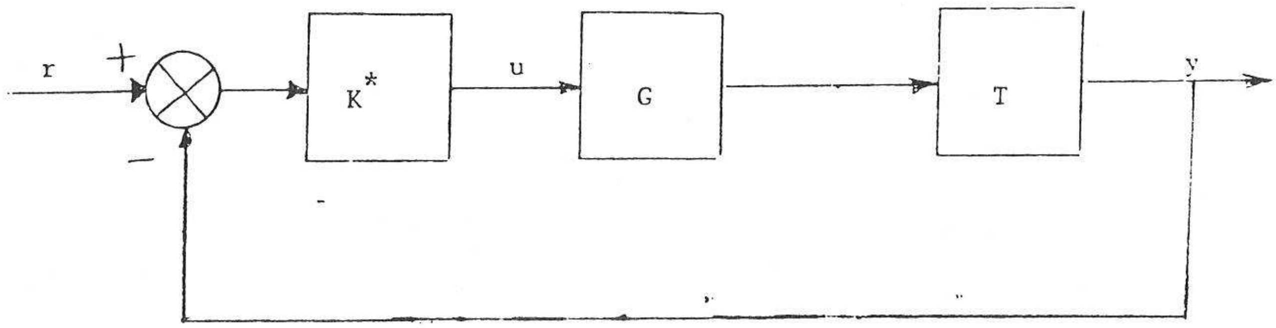
MARSHALL, J. E., and SALEHI, S. V., 1982, IEE Proc., Pt.D, Control Theory and Appl., 129, (5), 177.

OWENS, D. H., and CHOTAI, A., 1983, IEE Proc., Pt.D, Control Theory and Appl., 130, (2), 45.

OWENS, D. H., and RAYA, A., 1982, IEE Proc., Pt.D, Control Theory and Appl., 129, (6), 298.



(a)



(b)

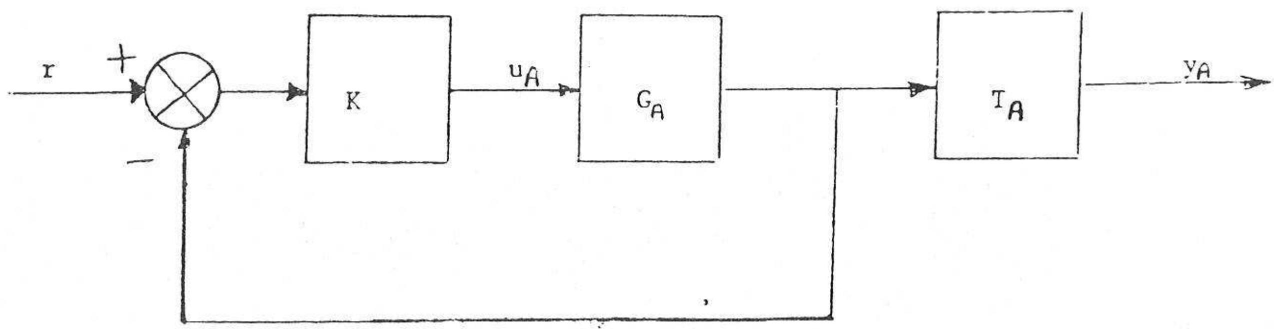


Fig. 1 (c)

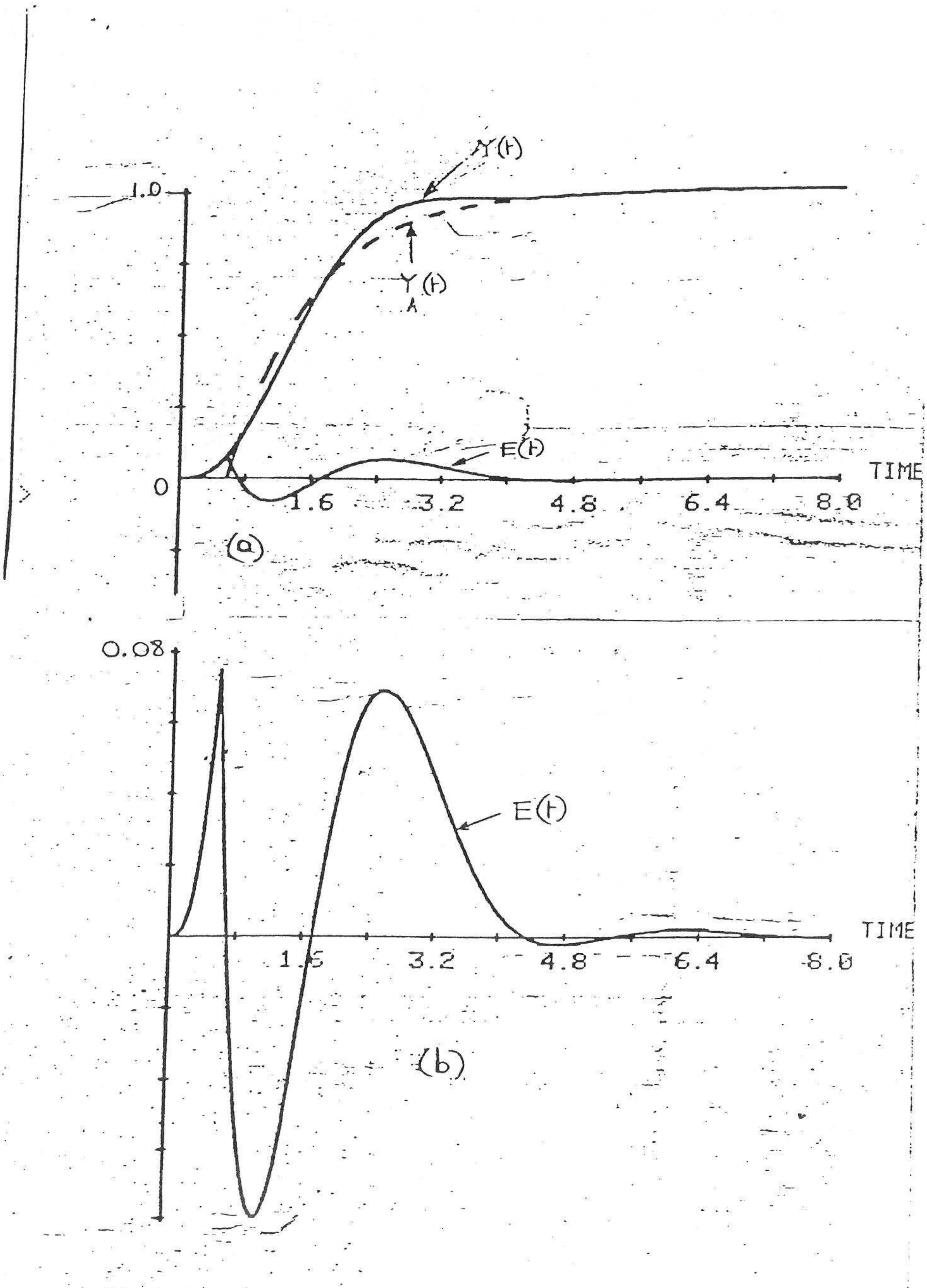


Fig. 2

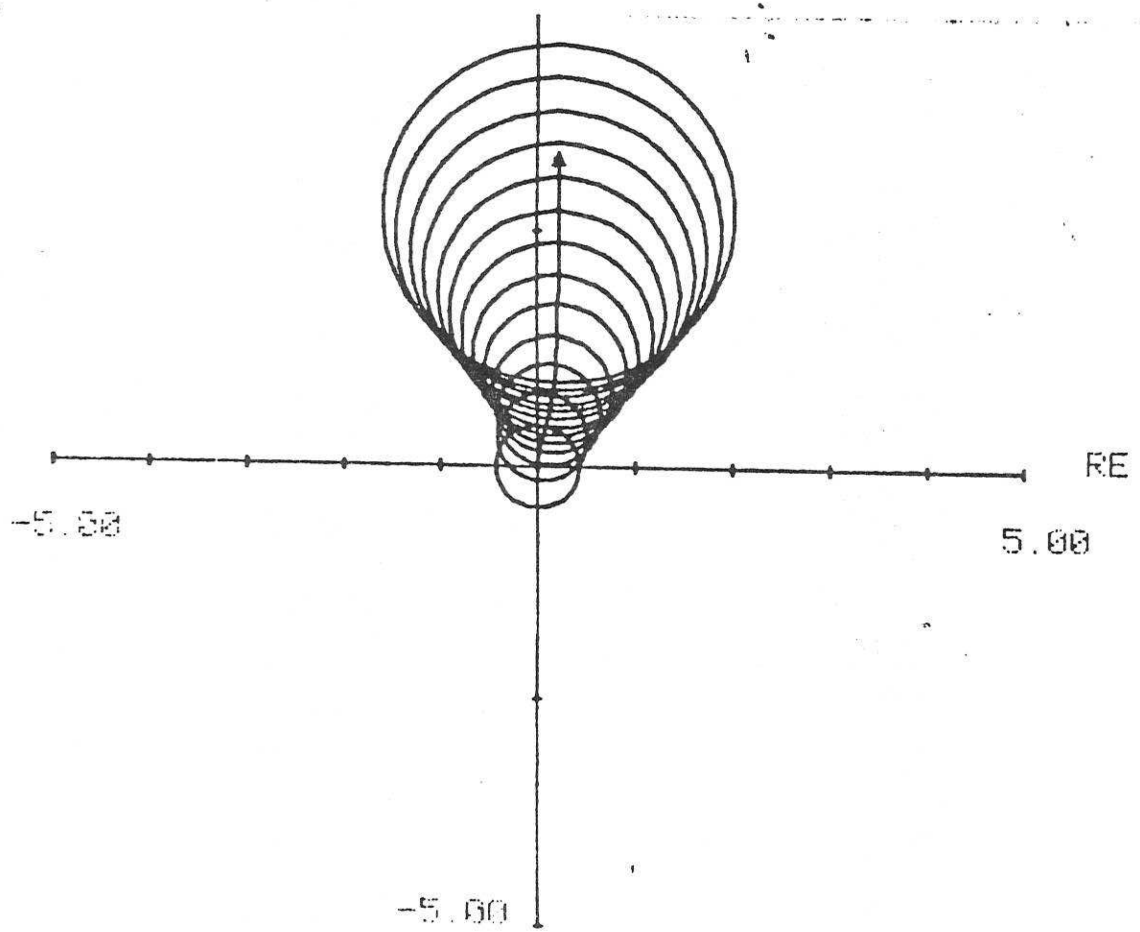
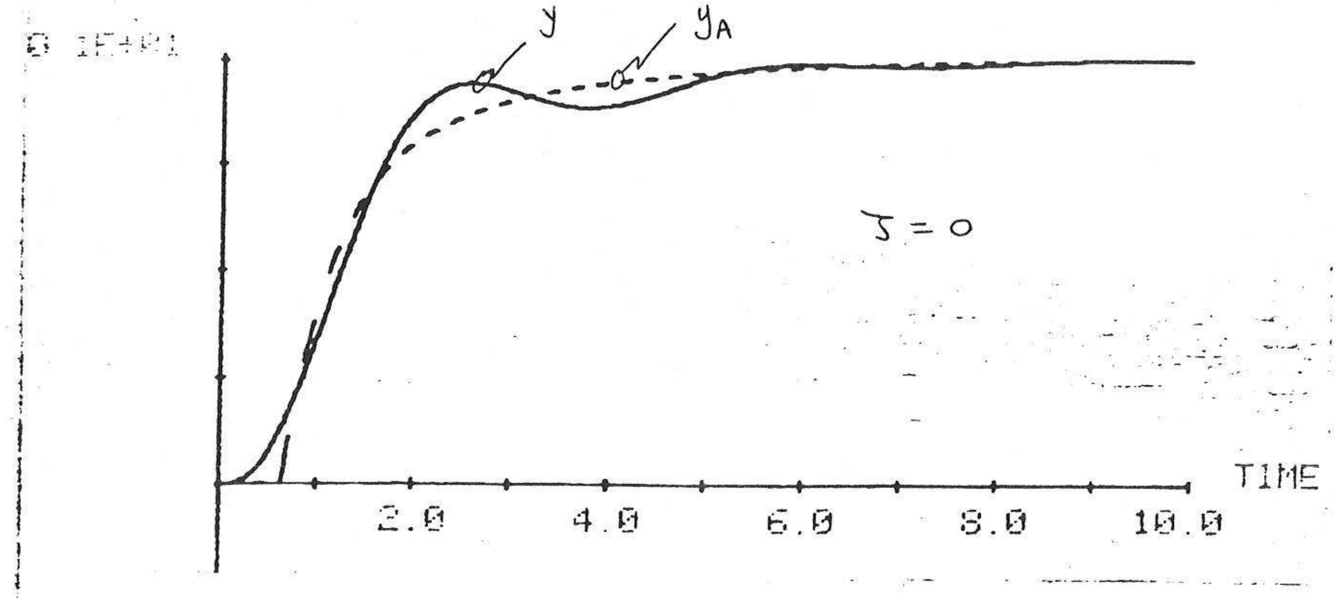
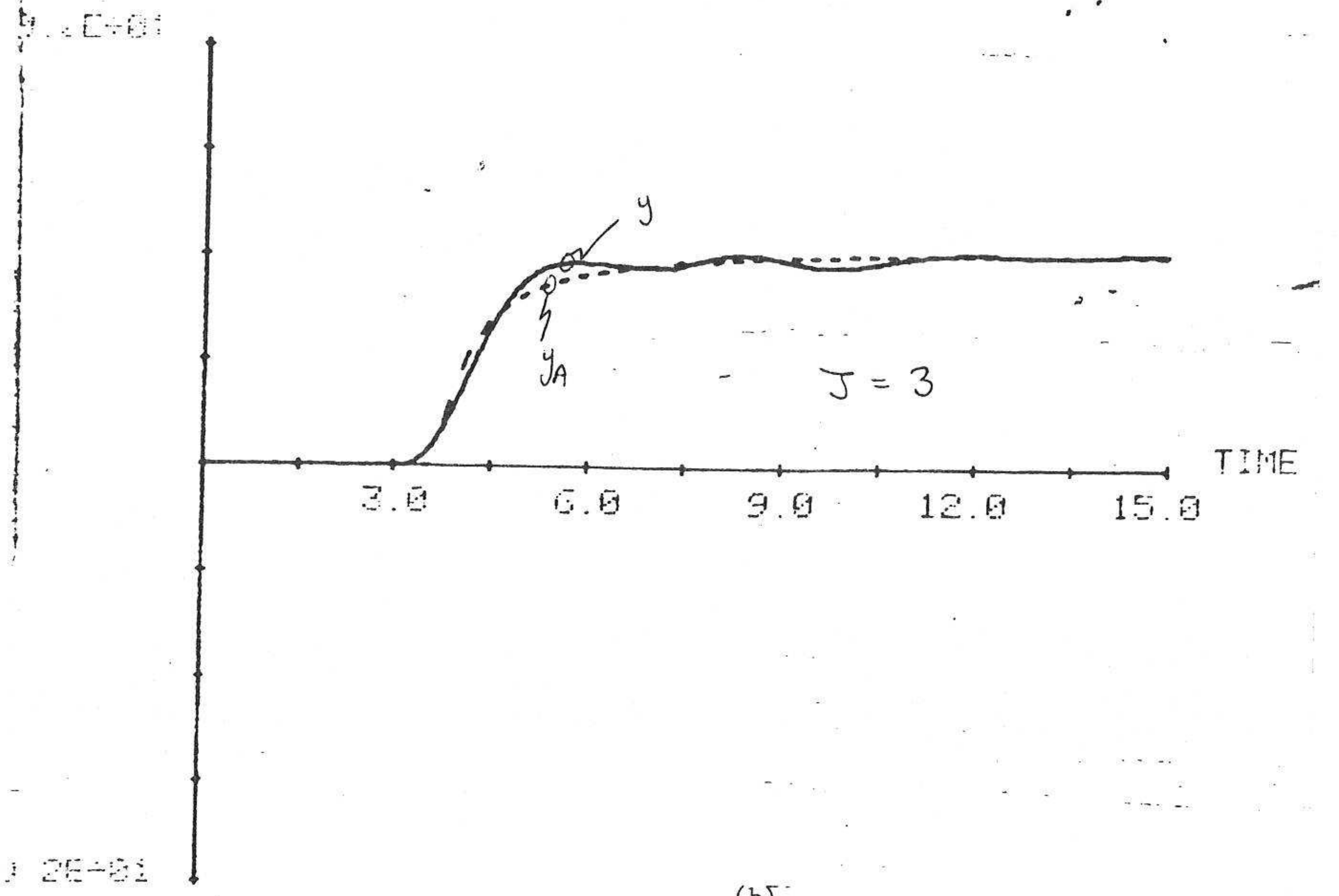


Fig. 3

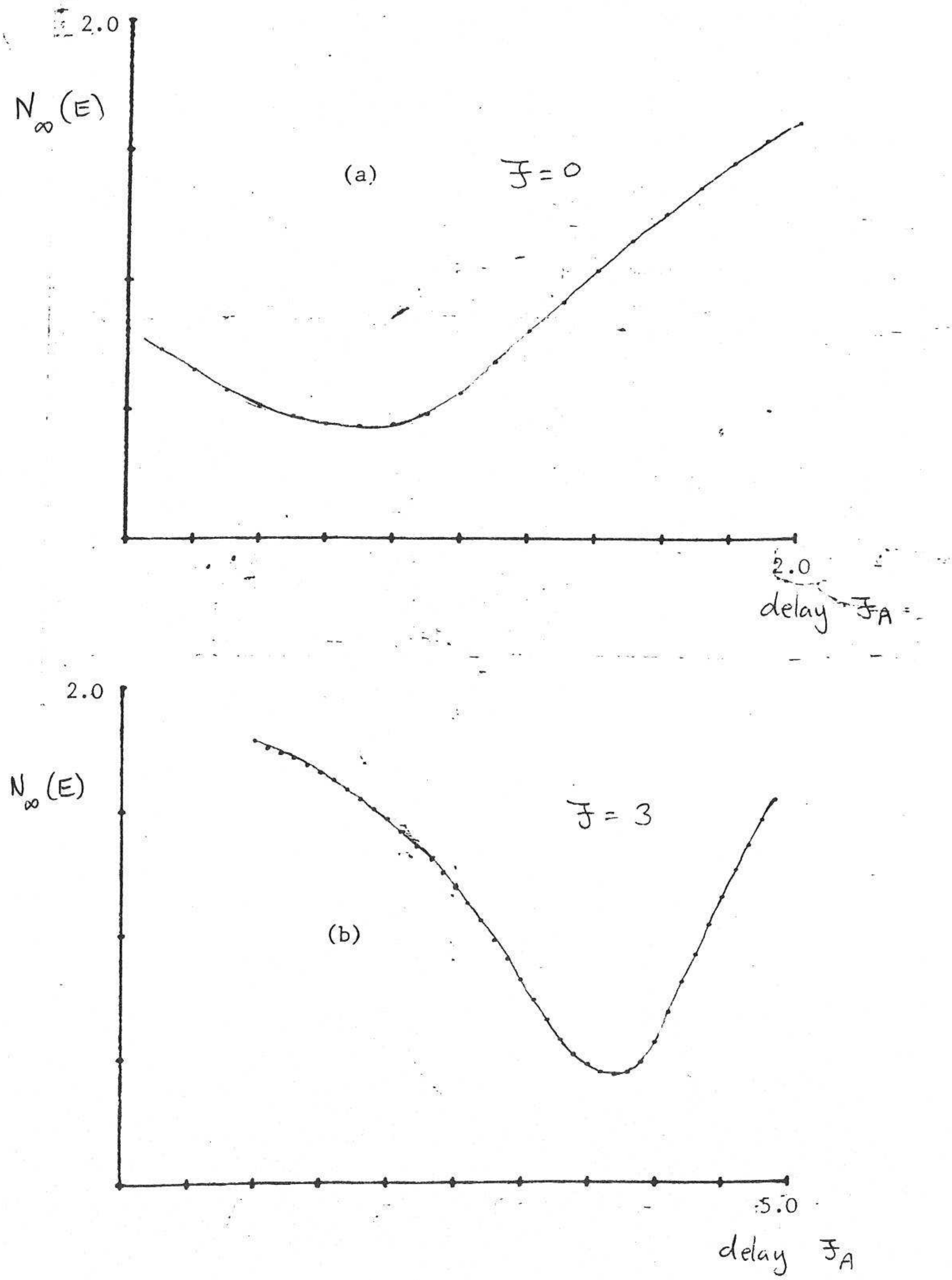


(a)



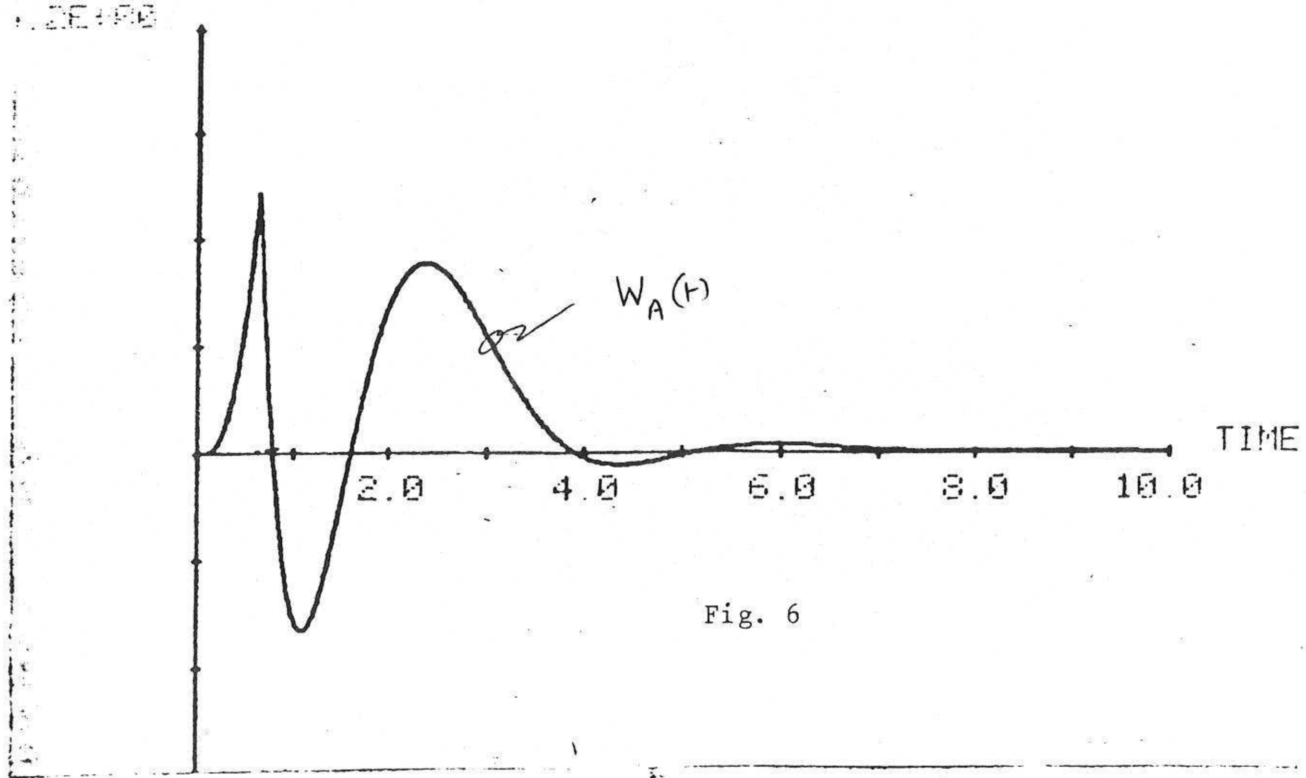
(b)

Fig. 4



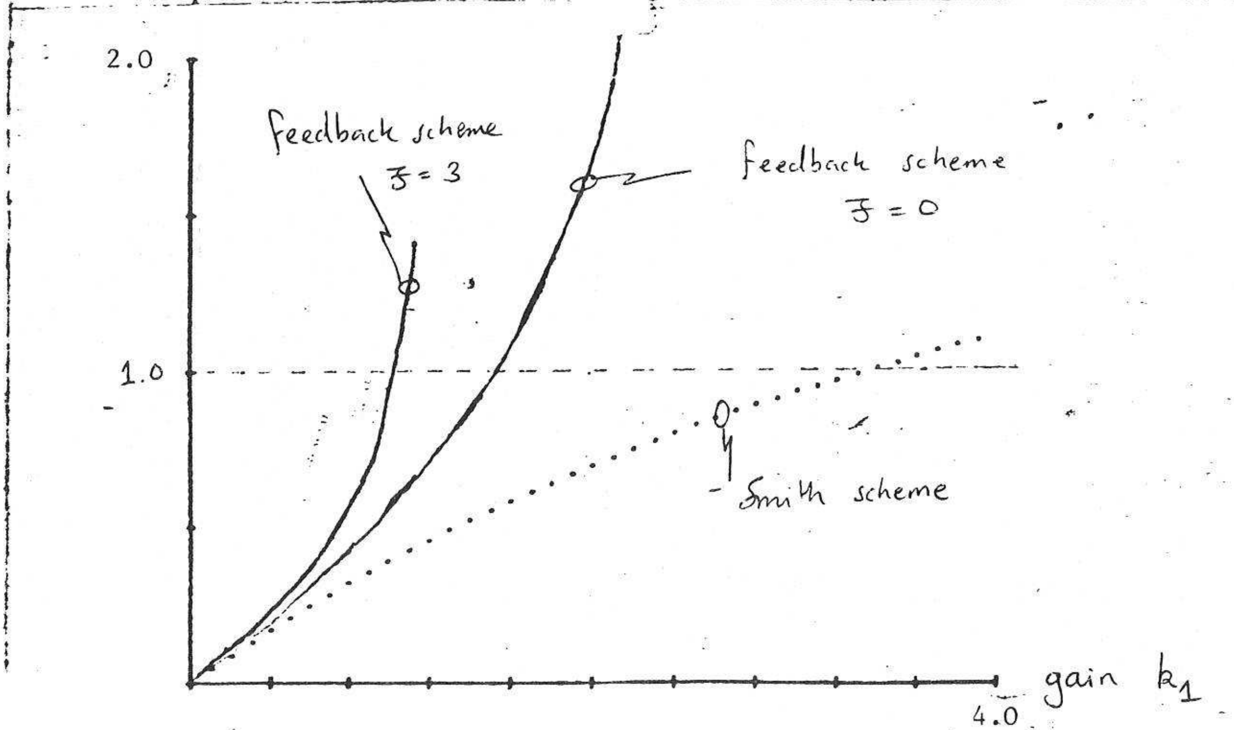
2.0
$N_\infty(E)$
(a)
$\mathcal{F} = 0$
2.0
delay $\mathcal{F}_A$
2.0
$N_\infty(E)$
$\mathcal{F} = 3$
(b)
5.0
delay $\mathcal{F}_A$

Fig. 5



.2E+00
$W_A(t)$
TIME
2.0
4.0
6.0
8.0
10.0

Fig. 6



2.0
1.0
feedback scheme
ξ = 3
feedback scheme
ξ = 0
Smith scheme
gain k_1
4.0

Fig. 7

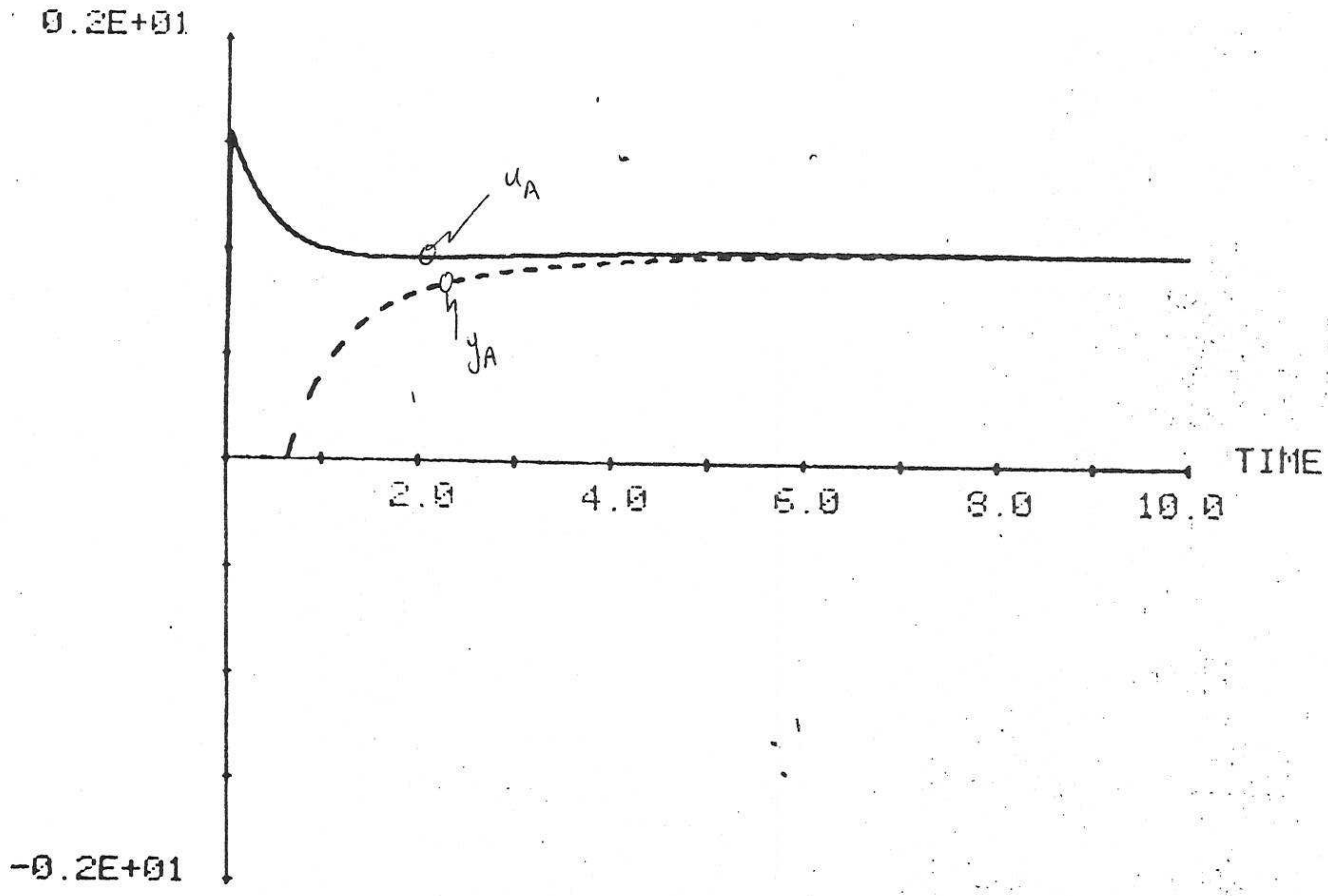
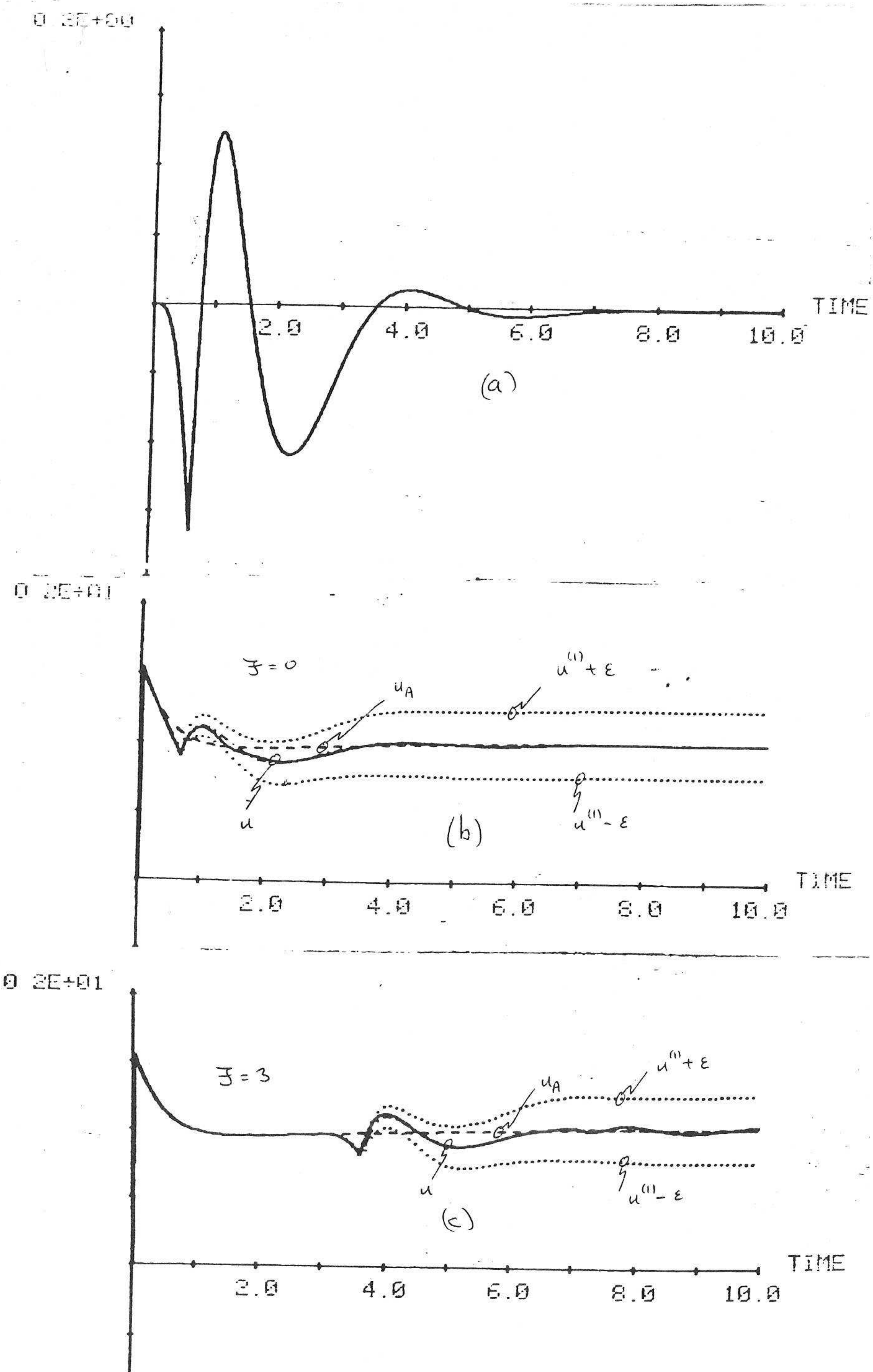


Fig. 8



0 2E+00
TIME
2.0
4.0
6.0
8.0
10.0
(a)
0 2E+01
ξ = 0
u^(1) + ε
u_A
u
u^(1) − ε
(b)
TIME
2.0
4.0
6.0
8.0
10.0
0 2E+01
ξ = 3
u_A
u^(1) + ε
u
u^(1) − ε
(c)
TIME
2.0
4.0
6.0
8.0
10.0

Fig. 9

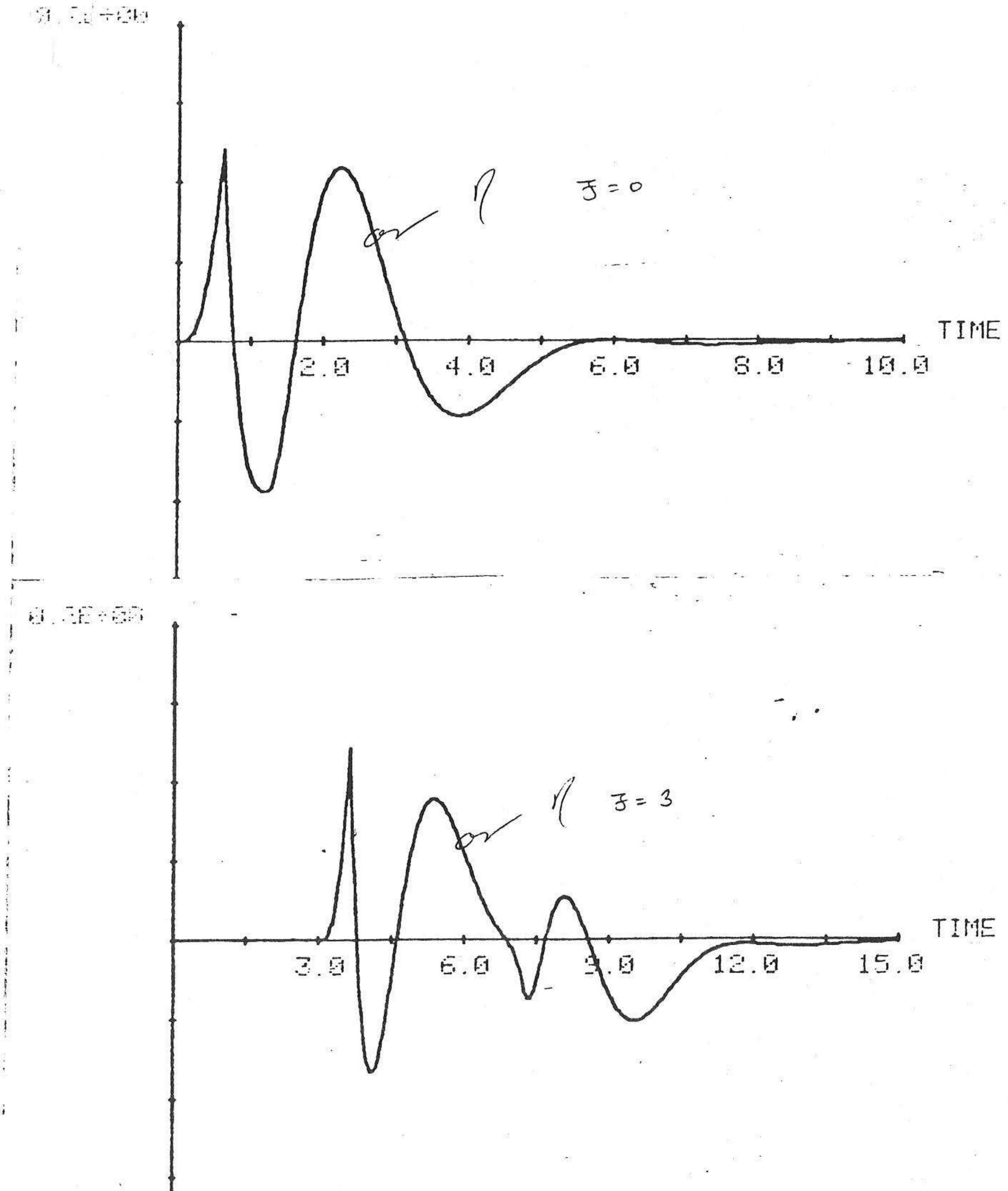


Fig. 10

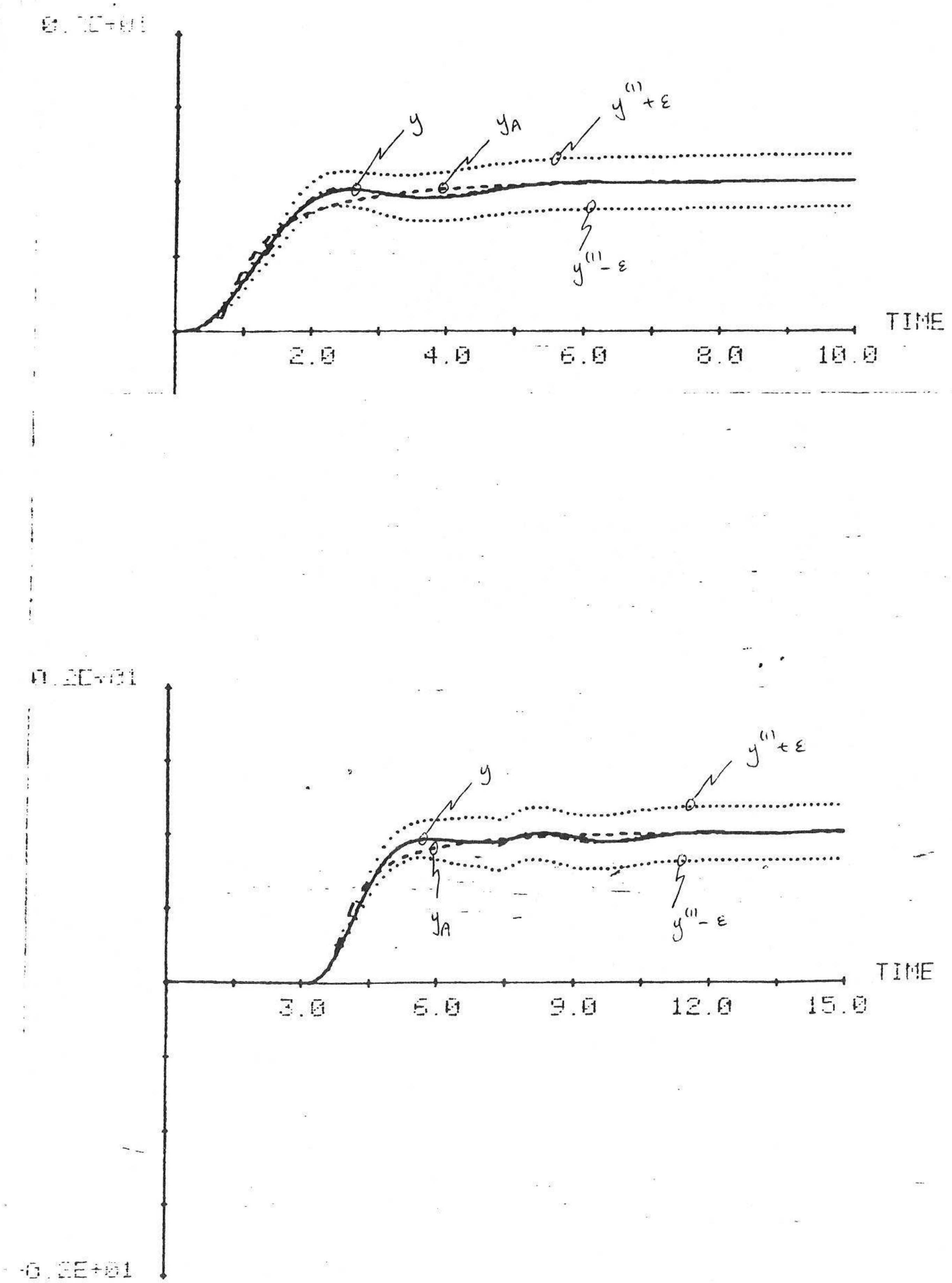


Fig. 11