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STABILITY AND PERFORMANCE DETERIORATION

DUE TO MODELLING ERRORS

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STABILITY AND PERFORMANCE DETERIORATION DUE TO MODELLING ERRORS

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1. Introduction

Although plant models are invariably nonlinear in structure, it is common practice to represent these models by linearized versions for the purposes of control systems design in the vicinity of a known operating condition. In such circumstances it is possible to use well-established linear design techniques described, for example, in Raven (1978), Rosenbrock (1974), Owens (1978) and MacFarlane (1980) to ensure desired stability and performance characteristics from the linear model. The success of the design procedure relies heavily on the accuracy of the linearized model and on the assumptions of linear actuator and transducer dynamics. If one or more of these requirements is violated, then the confidence that can be invested in the design is severely limited. Unfortunately, many linear models are highly inaccurate, being only representative of plant dynamics due to difficulties in plant modelling or deliberate use of simplified plant models. It is also true that many actuators and transducers have severe nonlinear characteristics even within the range of signal levels where the plant is reasonably linear. Consider, for example, the existence of measurement deadzones, quantization or transducers with static nonlinear characteristics. In such cases, it is clearly important to be able to assess, in a quantitative manner, the effect of modelling errors and nonlinearities on the stability and performance predictions obtained from linear techniques.

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or

(b) G is stable and known but is regarded as being too complex for the design exercize to be undertaken but simulations are undertaken to record Y(t), $t \geq 0$.

In both situations, design must proceed based on the use of the data Y(t), $t \geq 0$. Suppose therefore that visual or computer-aided analysis of Y(t) suggests that a stable approximate model with transfer function $G_A(s)$ is representative of plant dynamics. Note that it is not assumed that G_A is an accurate representation of G! The complexity of G_A is left to the designer who is free to choose either a simple form of G_A to ease analysis (recognizing that modelling errors may be large) or a more complex model G_A with smaller modelling errors and a consequent increase in analytical complexity. Whatever his choice, the model G_A can be used as the basis of control system design to choose a controller K to produce the required stability and performance characteristics from the approximate feedback scheme illustrated in Fig.1(b). The aim of this exercize is, of course, to implement the controller on the real plant but the presence of modelling errors throws some doubt over the success of this operation. More precisely, the implementation of K on the plant G will only be successful if

(a) the stability of the approximate feedback scheme of Fig.1(b) guarantees the stability of the implemented scheme of Fig.1(a) in the presence of the modelling error, and

(b) the performance deterioration $y(t)-y_A(t)$ due to the modelling errors is acceptable.

For the design exercize to guarantee these properties, information on the modelling error must be explicitly used. If $Y_A(t)$, $t \geq 0$, is the unit step response of the model G_A (obtained by direct simulation), then the 'step response error'

$$E(t) = Y(t) - Y_A(t) \quad , \quad t \geq 0 \qquad \ldots(1)$$

contains, in principle, all the information on the modelling error and hence should be usable in stability and performance studies. There is no unique way of using E(t). For example, Owens and Chotai (1983) use E(t) as the basis of frequency response stability studies to guarantee property (a) above. Here, however, we concentrate on time-domain simulation based methods similar to those described in that paper.

Although E(t) is used directly, it is also necessary to construct other forms of 'error measure' deduced from this data. Consider a continuous signal f(t), $t \geq 0$, with local minima and maxima at times $t_o < t_1 < t_2 < \ldots$ with $t_o = 0$, then the total variation of f on $[0,t]$ (Owens and Chotai, 1983) is defined to be the function

$$N_t(f) \triangleq |f(t_o+)| + \sum_{j=1}^{k} |f(t_j) - f(t_{j-1})|$$

$$+ |f(t) - f(t_k)| \quad , \quad t \geq 0 \qquad \ldots(2)$$

where k is the largest index such that $t_k \leq t$. $N_t(f)$ can easily be deduced from graphical analysis of f(t) as illustrated in Fig.2. The function can also be defined for most stable, well-defined signals in the case of $t = +\infty$. Formally, we need only write

$$N_\infty(f) \triangleq \sup_{t \geq 0} N_t(f) \qquad \ldots(3)$$

but, in practice (Owens and Chotai, 1983), $N_\infty(f)$ can be regarded as being $N_T(f)$ where T is long compared with the time constants of the signal f.

2.1. Stability Assessment using Error Data

With the above definitions, the following result provides a technique for assessing the stability of the implemented control scheme.

Theorem 1 (Owens and Chotai, 1983)

If the controller K stabilizes the model G_A in the configuration of Fig.1(b) and both G and G_A are stable, then K stabilizes the plant G in the configuration of Fig.1(a) if

(i) GKF is controllable and observable, and

(ii) $N_\infty(W_A) < 1$ ...(4)

where (Fig.3) $W_A(t)$, $t \geq 0$, is the response of the system $(1+KFG_A)^{-1}KF$ from zero initial conditions to the error signal E(t), $t \geq 0$.

The application of the result is straightforward. Condition (ii) is checked by undertaking a simple simulation to obtain the response W_A of the known (normally low order system) $(1+KFG_A)^{-1}KF$ to the known data E(t), $t \geq 0$, and verifying that its total variation satisfies equation (4). If (4) is satisfied then condition (i) must be checked. If however (4) is violated then stability cannot be guaranteed as the modelling error is too large. All is not lost however as (4) can be satisfied by either

(1) increasing the accuracy of the model G_A to reduce the modelling error E and hence $N_\infty(W_A)$,

or

(2) reducing control gains to reduce the magnitude of W_A.

Choice of the first option essentially admits that the modelling error must be reduced to achieve the required performance, whilst choice of the second option is an acceptance that the loop must be detuned to cope with the modelling error. Both options are open to the design engineer.

The checking of condition (i) of the theorem is straightforward if the model G is known as it is equivalent to requiring no pole-zero cancellations in the transfer function GKF. If G is not known this is not possible and the designer must check the condition by indirect means or rely on the fact that it is almost always valid. The choice of indirect means depends primarily on the structure of K! For example, if K is a standard proportional plus integral control, it is easily verified that it is only necessary that the plant d.c. gain is non-zero or, equivalently, that the steady state value $Y(\infty)$ of the step data is nonzero.

To illustrate the application of the ideas consider a process with transfer function

$$G(s) = \frac{1}{(s+1)^3} \qquad \ldots(4)$$

with step response Y shown in Fig.4(a). Following the common practice of using delay-lag process models, visual inspection of the response suggests the approximate model

$$G_A(s) = \frac{e^{-s0.7}}{1 + s2.4} \qquad \ldots(5)$$

leading to the step response Y_A illustrated in Fig.4(a) and the error $E = Y-Y_A$ given in Fig.4(b). Standard techniques applied to G_A, assuming unit feedback $F = 1$, suggest the use of the PI controller

$$K(s) = k_1 + \frac{k_2}{s} \quad , \quad k_1 = 1.0 \quad , \quad k_2 = 0.5 \qquad \ldots(6)$$

leading to the predicted, closed-loop step response y_A shown in Fig.5. To check that the controller will also stabilize G, W_A is computed as shown in Fig.6 and its total variation deduced to be $N_\infty(W_A) = 0.5 < 1$. Stability is hence guaranteed provided that the controllability and observability condition is satisfied. Given G and K it is easily seen that this is so. If however G was not known, note that this has no real practical impact as GK is controllable and observable for almost all k_1 and k_2 (more precisely, for all $k_1 \neq k_2$).

2.2. Performance Assessment using Error Data

The analysis of section 2.1 can be extended (Owens and Chotai, 1983) to bound the performance deterioration due to the modelling error E(t).

Thoerem 2 (Owens and Chotai, 1983)

Suppose that the conditions of theorem 1 are satisfied and define a 'corrected response prediction' by

$$y^{(1)}(t) \triangleq y_A^c(t) + \eta(t) \quad , \qquad t \geq 0 \qquad \ldots(7)$$

where

(a) $y_A^c(t)$ is the response of the approximating feedback system from zero initial conditions to a unit step input, and

(b) $\eta(t)$ is the response of the system $(1+KFG_A)^{-1}K(1+KFG_A)^{-1}$ from zero initial conditions to the error data E(t).

Then the unit step response y(t) of the implemented scheme of Fig.1(a) satisfies the inequality, $t \geq 0$,

$$|y(t) - y^{(1)}(t)| \leq \varepsilon(t) \triangleq \frac{N_t(W_A)}{1-N_t(W_A)} \max_{0 \leq t' \leq t} |\eta(t')| \qquad \ldots(8)$$

(Remarks (i) This result follows from theorem 6 of Owens and Chotai (1983) choosing $H_1 = (1+G_A KF)^{-1} G_A K$ and hence $y^{(o)} = y_A^c$.

(ii) If F has a stable inverse, η can be computed more simply as the response of the configuration $(1+G_A KF)^{-1}F^{-1}$ to the input $W_A(t)$, $t \geq 0$.

(iii) $\varepsilon(t)$ is monotonically increasing and hence the left-hand side of (8) can be replaced by $\max_{0 \leq t' \leq t} |y(t')-y^{(1)}(t')|$).

In graphical terms the result states that the worst performance deterioration due to the known modelling error is such that the unit step response y(t) will lie between the two responses $y^{(1)}(t) \pm \varepsilon(t)$, $t \geq 0$. This is illustrated below.

The application of the result can be illustrated by considering the example of section 2.1. Elementary simulations lead to the 'correction' $\eta(t)$ given in Fig.7(a) and to the bound $\varepsilon(t)$ of Fig.7(b). The resultant bounds $y^{(1)} \pm \varepsilon$ and the actual response y are given in Fig.8 verifying the validity of the prediction (8).

3. Modelling Errors and Nonlinear Design

Having completed the incorporation of the modelling errors into the linear design it is now possible to assess the effect of measurement nonlinearities on stability and performance. It is assumed throughout the remainder of the chapter that F is a scalar gain approximating a nonlinear measurement characteristic N and that the implemented scheme has the structure shown in Fig.9. The nonlinearity is assumed to be static and memoryless and can be decomposed into the form (Owens and Chotai, 1981)

$$N(y) \quad = \quad Fy + n_1(y) + n_2(y) \qquad \qquad ...(9)$$

where

(i) $n_1(y)$ is a nonlinearity of finite incremental gain ν

ie for all y_1, y_2,

$$|n_1(y_1) - n_1(y_2)| \leq \nu |y_1 - y_2| \qquad \qquad ...(10)$$

(Remark: although this idea includes some nondifferentiable nonlinearities, for differentiable nonlinearities ν is just the maximum modulus of the derivative of n_1).

(ii) $n_2(y)$ is a bounded output nonlinearity of the form, for all y,

$$|n_2(y)| \quad \leq \quad q/2 \qquad \qquad ...(11)$$

where $q \geq 0$ is a real scalar.

(iii) $n_1(0) = 0$ (and hence $|n_1(y)| \leq \nu|y|$) ...(12)

(Remark: the nonlinearity N is linear if, and only if, we can choose F such that $\nu = q = 0$).

This separation of the nonlinearity is slightly nonstandard as most authors take $n_2 \equiv 0$. The inclusion of n_2 however allows the analysis of nonlinearities with bounded discontinuities or phenomena such as deadzones. The following results indicate that stability is independent of n_2 and hence separating it out tends to reduce the conservation of the predictions by decreasing the 'effective' gain ν and, in some circumstances, permits the application of the theory to discontinuous nonlinearities that do not have finite incremental gain.

To illustrate how the decomposition could be applied consider the nonlinearity

$$N(y) = \begin{cases} 0 & , \quad |y| \le 1 \\ y - \operatorname{sgn} y & , \quad 1 \le |y| \le 2 \\ 3y - 5\operatorname{sgn} y & , \quad 2 \le |y| \end{cases} \qquad \ldots(13)$$

illustrated in Fig.10(a). Choosing F = 2, then it is easily verified that N has the decomposition (9) with (Fig.10(b))

$$n_1(y) = \begin{cases} -y & , \quad |y| \le 2 \\ y - 4\operatorname{sgn} y & , \quad |y| \ge 2 \end{cases} \qquad \ldots(14)$$

with incremental gain $\nu = 1$, and $n_2(y)$ has the form (Fig.10(c))

$$n_2(y) = \begin{cases} -y & , \quad |y| \le 1 \\ -\operatorname{sgn} y & , \quad |y| \ge 1 \end{cases} \qquad \ldots(15)$$

with $q = 2$.

3.1. Stability Assessment

With the above definitions, the following result provides a technique for assessing the stability of the nonlinear feedback scheme (the proof is outlined in appendix 7.1). The notation of theorems 1 and 2 is assumed throughout.

Theorem 3 (Owens and Chotai, 1981)

Suppose that the conditions of theorem 1 are satisfied and define the monotonically increasing function

$$\lambda(t) = N_t(y^{(1)}) + \frac{N_t(W_A)N_t(\eta)}{1 - N_t(W_A)} \qquad \ldots(16)$$

Then the feedback system of Fig.9 is input/output stable (in the L_∞ sense) in the presence of the controller K if the contraction condition

$$\lambda(\infty)\nu < 1 \qquad \ldots(17)$$

is satisfied.

The interpretation of the result is similar to that of theorem 1. Note that the stability result depends only on the incremental gain ν of n_1 and is independent of q and hence n_2. The validity of (17) depends clearly on the magnitude of ν but it also depends explicitly on the magnitude of modelling error E as can be seen by examination of (16). In general terms an increase in the modelling error requires a decrease in the gain ν to satisfy (17), indicating that the presence of modelling errors can make the design analysis more sensitive to measurement nonlinearities at implementation.

To illustrate the application of the ideas, consider the example of section 2.1 with the specified controller but where the implemented scheme is subject to the nonlinearity

$$N(y) = \frac{(0.9 + 9.9y^2)}{(1 + 9y^2)}\, y \qquad \text{...(18)}$$

Choosing unity feedback (F = 1) as in section 2 to represent a linear form of the nonlinearity and $n_2(y) \equiv 0$ defines $n_1(y)$ uniquely by

$$n_1(y) = 0.1\, y\, \frac{(9y^2-1)}{(9y^2+1)} \qquad \text{...(19)}$$

which has incremental gain $\nu = 0.1$. The data $y^{(1)}$, W_A and η obtained in the linear design can now be used to deduce that $\lambda(\infty) = 1.98$. It is easily verified that condition (17) is hence satisfied and that the nonlinearity will not affect the stability predictions based on the linear approximating model G_A and the linear approximation F = 1 to N.

3.2. Performance Assessment

It can be anticipated that the nonlinear characteristic will also affect performance predictions. More precisely, we expect that the results of theorem 2 will remain essentially valid but with an increase in performance uncertainty represented by an increase in $\varepsilon(t)$! This intuition is formalized in the following statement (Owens and Chotai, 1981) proved in Appendix 7.2. The result is stated for unit step responses but steps of any magnitude can be obtained by prescaling of variables.

Theorem 4

Suppose that the conditions of theorems 1, 2 and 3 are satisfied. Then the response $y_{n\ell}(t)$, $t\geq 0$, from zero initial conditions of the nonlinear feedback system of Fig.9 to a unit step input satisfies the inequality, $t\geq 0$,

$$|y_{n\ell}(t) - y^{(1)}(t)| \leq \varepsilon_{n\ell}(t) \triangleq \varepsilon(t) + \frac{\lambda(t)}{1-\lambda(t)\nu}(\lambda(t)\nu + q/2) \qquad \ldots(20)$$

The interpretation of this result is identical to that of theorem 2 with ε replaced by $\varepsilon_{n\ell}$. Note that $\varepsilon_{n\ell}(t) \geq \varepsilon(t)$, $t\geq 0$, and hence that the nonlinearity increases the uncertainty in $y(t)$, $t\geq 0$.

A similar result can also be proved as follows (see appendix 7.3) and is expected, on intuitive grounds, to give better results.

Theorem 5

The conclusions of theorem 4 remain valid with $\varepsilon_{n\ell}(t)$ replaced by

$$\varepsilon_{n\ell}(t) \triangleq \frac{\varepsilon(t)}{1-\lambda(t)\nu} + \frac{\lambda(t)\nu}{1-\lambda(t)\nu}\max_{0\leq t'\leq t}|y^{(1)}(t')| + \frac{\lambda(t)}{1-\lambda(t)\nu}\frac{q}{2} \qquad \ldots(20a)$$

The results are illustrated by considering the example of section 3.1 using (20a). The performance bounds $y^{(1)}\pm\varepsilon_{n\ell}$ are given in Fig.11 together with the unit step response of Fig.9. Note that $y_{n\ell}$ lies between the bounds as expected.

4. Conclusions

In the presence of modelling errors in a linear process G and measurement nonlinearities N, the natural approach to control systems design is to base the design on an approximate model G_A and a linear version F of the nonlinearity. This procedure leads to severe uncertainty in predicting the stability and performance of the implemented scheme unless some information on modelling error and nonlinear characteristics is included in the linear design. This chapter has illustrated recent developments to address this problem that rely on the availability of step response data for the plant and an upper bound for the incremental gain of the nonlinearity. This data can be processed using simulation techniques to predict the stability of the implemented scheme and the construction of a 'response envelope' containing the scheme response. The stability prediction has obvious value and the response envelope provides a measure of the maximum and minimum possible values of the effect of the modelling errors and nonlinearity on predicted linear performance.

Finally, the chapter has described the ideas for single-input/single-output continuous control schemes. The ideas carry over with little change to the sampled-data case and also to cope with multivariable systems. The reader is referred to the references for details of these generalizations.

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7. Appendix

7.1. Proof of Theorem 3

Let $r \in L_\infty(0,\infty)$. The closed-loop equation of Fig.9 is written as

$$y_{n\ell} = GK(r - (Fy_{n\ell} + n_1(y_{n\ell}) + n_2(y_{n\ell}))) \qquad \ldots(21)$$

in $L_\infty^{ext}(0,\infty)$. If L_c is the linear operator in $L_\infty(0,\infty)$ defined as the map $r \to y$ of Fig.1(a), then, noting that $y = L_c r$, (21) can be rewritten as

$$\begin{aligned} y_{n\ell} &= L_c(r - n_1(y_{n\ell}) - n_2(y_{n\ell})) \qquad \ldots(22) \\ &= y - L_c(n_1(y_{n\ell}) + n_2(y_{n\ell})) \triangleq \psi_{n\ell}(y_{n\ell}) \end{aligned}$$

Regarding $r - n_2(y_{n\ell}) \in L_\infty(0,\infty)$ as fixed, the contraction mapping theorem (Holtzmann, 1970) guarantees the existence of a unique solution of (21) in $L_\infty(0,\infty)$ (and hence stability) if we can choose a real number μ (termed the contraction constant) satisfying

$$\|L_c\|_\infty \nu \leq \mu < 1 \qquad \ldots(23)$$

where $\|\cdot\|_\infty$ denotes both the norm of any point in $L_\infty(0,\infty)$ and the induced norm of an operator in $L_\infty(0,\infty)$ as required by the context.

The input/output equation $y = L_c r$ can be written (Owens and Chotai, 1983) as

$$y = GKr - GKFy \qquad \ldots(24)$$

or, after a little manipulation

$$\begin{aligned} y &= (1 + G_A KF)^{-1}(KGr - KF(G-G_A)y) \\ &\triangleq \psi(y) \qquad \ldots(25) \end{aligned}$$

Let P_t be the natural projection of $L_\infty(0,\infty)$ onto $L_\infty(0,t)$ with $P_\infty = I$ (the identity map). The work of Owens and Chotai (1983) then indicates that $P_t \psi P_t$ is a contraction on $L_\infty(0,t)$ for all $t\geq 0$ with contraction constant

$$||P_t(1+KFG_A)^{-1}KF(G-G_A)||_\infty = N_t(W_A) \leq N_\infty(W_A) < 1 \qquad \ldots(26)$$

Let $v \in L_\infty(0,\infty)$ be arbitrary and use the triangle inequality to obtain

$$||P_t y||_\infty \leq ||P_t v||_\infty + ||P_t(y-v)||_\infty \qquad \ldots(27)$$

Let $y_o = y_A = (1+G_AKF)^{-1}G_AKr$ and consider the successive approximation scheme $y_{k+1} = \psi(y_k)$, $k\geq 0$, to obtain, in particular

$$y_1 = (1+G_AKF)^{-1}G_AKr + (1+KFG_A)^{-1}K(1+KFG_A)^{-1}(G-G_A)r$$

$$= y_o + (1+KFG_A)^{-1}K(1+KFG_A)^{-1}(G-G_A)r \qquad \ldots(28)$$

Let $v = y_1$ and deduce that

$$||P_t v||_\infty \leq ||P_t((1+G_AKF)^{-1}G_AK+(1+KFG_A)^{-1}K(1+KFG_A)^{-1}(G-G_A))||_\infty \; ||P_t r||_\infty$$

$$= N_t(y^{(1)}) \; ||P_t r||_\infty \qquad \ldots(29)$$

by using lemma 4 in Owens and Chotai (1983) to identify the operator norm with the total variation of its step response and noting that its step response is just $y_A^c+\eta = y^{(1)}$. Next, use standard contraction formula to deduce that

$$||P_t(y-y_1)||_\infty \leq \frac{N_t(W_A)}{1-N_t(W_A)} \; ||P_t(y_1-y_o)||_\infty \qquad \ldots(30)$$

and use (28) to yield

$$\|P_t(y-v)\|_\infty \le \frac{N_t(W_A)}{1-N_t(W_A)} \|P_t(1+KFG_A)^{-1}K(1+KFG_A)^{-1}(G-G_A)\|_\infty \|P_t r\|_\infty$$

$$= \frac{N_t(W_A)N_t(\eta)}{1-N_t(W_A)} \|P_t r\|_\infty \qquad \ldots(31)$$

as the operator norm can (again) be identified with the total variation of its step response which is just η! Substituting (29) and (31) into (27) yields $\|P_t y\|_\infty \le \lambda(t) \|P_t r\|_\infty$ and hence $\|P_t L_c\|_\infty \le \lambda(t)$. The result follows by taking $t = +\infty$ and $\mu = \lambda(\infty)\nu$.

Proof of Theorem 4

The analysis of section 7.1 indicates that $P_t\psi_{n\ell}P_t$ is a contraction on $L_\infty(0,t)$ with contraction constant $\lambda(t)\nu$. Choose r as a unit step and apply successive approximation with initial guess $y_{n\ell}^{(o)} = x$ leads to the first iterate

$$y_{n\ell}^{(1)} = y - L_c(n_1(x) + n_2(y_{n\ell})) \qquad \ldots(32)$$

whence, using $\|P_t L_c\|_\infty \le \lambda(t)$,

$$\|P_t(y_{n\ell}^{(1)} - y)\|_\infty \le \|P_t L_c\|_\infty (\|P_t n_1(x)\|_\infty + \|P_t n_2(y_{n\ell})\|_\infty)$$

$$\le \lambda(t)(\nu \|P_t x\|_\infty + \frac{q}{2}) \qquad \ldots(33)$$

and

$$\|P_t(y_{n\ell} - y_{n\ell}^{(1)})\|_\infty \le \frac{\lambda(t)\nu}{1-\lambda(t)\nu} \|P_t(y_{n\ell}^{(1)} - x)\|_\infty \qquad \ldots(34)$$

Clearly

$$|y_{n\ell}(t) - y^{(1)}(t)| \leq |y(t) - y^{(1)}(t)| + |y_{n\ell}(t) - y(t)|$$

$$\leq \varepsilon(t) + |y_{n\ell}(t) - y_{n\ell}^{(1)}(t)| + |y_{n\ell}^{(1)}(t) - y(t)|$$

$$\leq \varepsilon(t) + \frac{\lambda(t)\nu}{1-\lambda(t)\nu} \|P_t(y_{n\ell}^{(1)}-x)\|_\infty + \|P_t(y_{n\ell}^{(1)}-y)\|_\infty$$

$$\leq \varepsilon(t) + \frac{\lambda(t)\nu}{1-\lambda(t)\nu} \|P_t(y-x)\|_\infty + \frac{1}{1-\lambda(t)\nu} \|P_t(y_{n\ell}^{(1)}-y)\|_\infty \quad \ldots(35)$$

Choosing $x = 0$, noting that $\|P_t y\|_\infty \leq \|P_t L_c\|_\infty \|r\|_\infty \leq \lambda(t)$ and using (33) leads to (20) directly. The result is hence proved.

7.3. Proof of Theorem 5

Applying the argument of section 7.2, choose $x = y^{(1)}$ in (35) and use (8) to verify that $\|P_t(y-y^{(1)})\|_\infty \leq \varepsilon(t)$ and hence that

$$|y_{n\ell}(t) - y^{(1)}(t)| \leq \frac{\varepsilon(t)}{1-\lambda(t)\nu} + \frac{1}{1-\lambda(t)\nu} \|P_t(y_{n\ell}^{(1)}-y)\|_\infty$$

$$\leq \frac{\varepsilon(t)}{1-\lambda(t)\nu} + \frac{\lambda(t)\nu}{1-\lambda(t)\nu} \|P_t y^{(1)}\|_\infty + \frac{\lambda(t)}{1-\lambda(t)\nu} \frac{q}{2} \quad \ldots(36)$$

which is just (20a) as required.

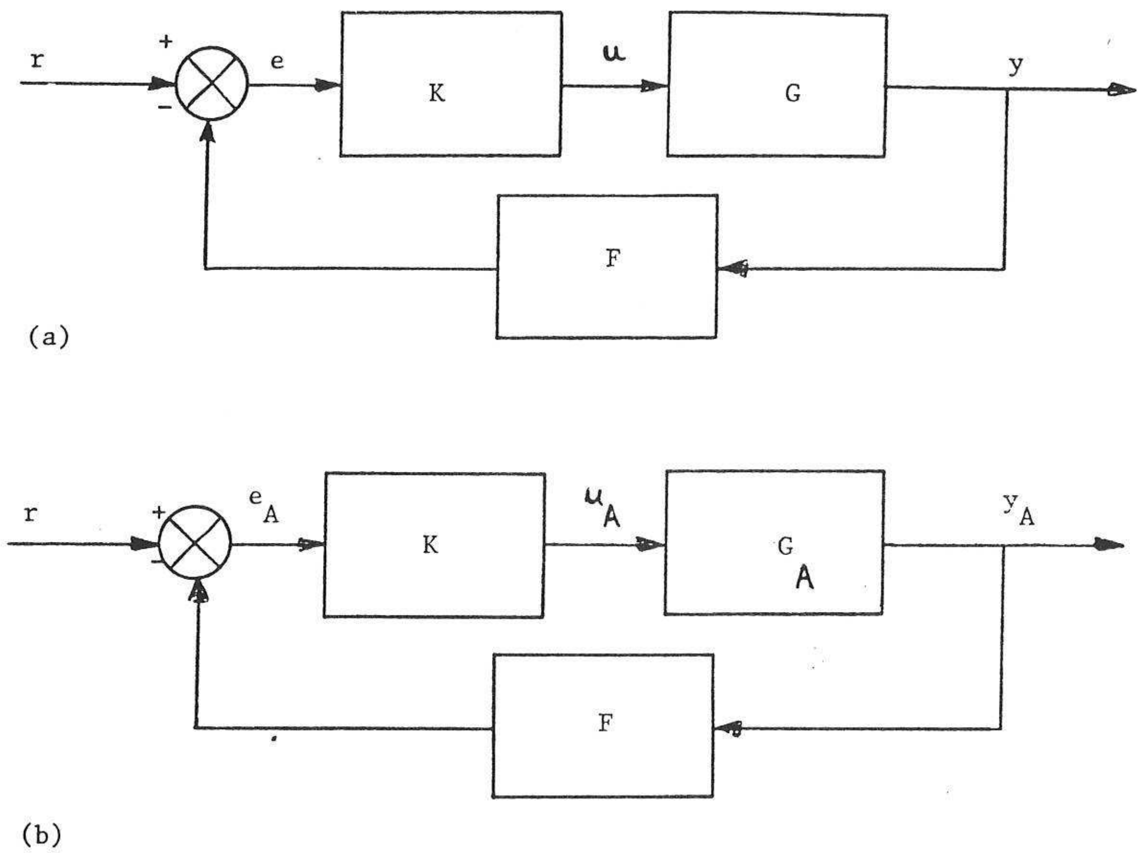


Fig. 1 (a) Real and (b) Approxmating Feedback Systems

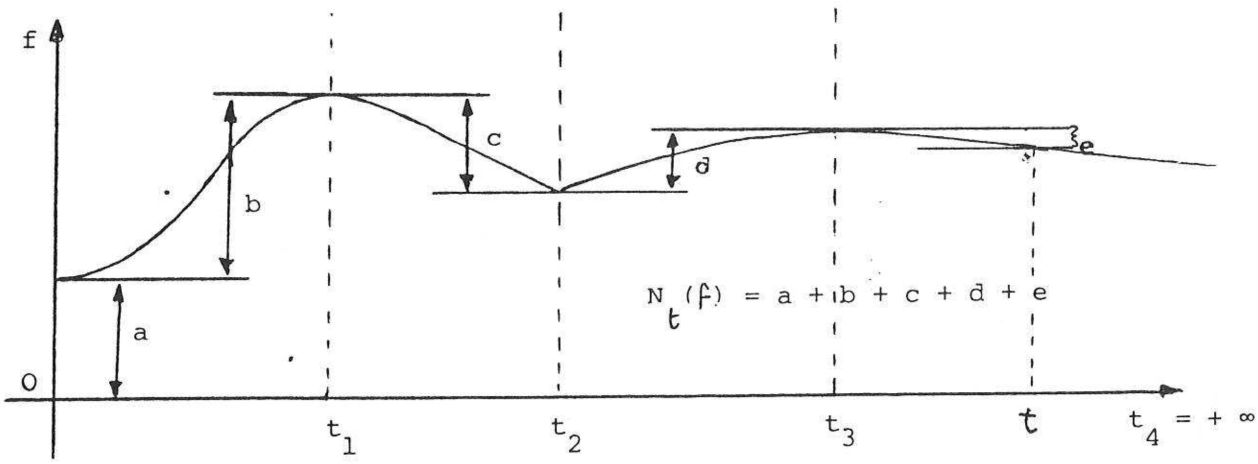


Fig. 2

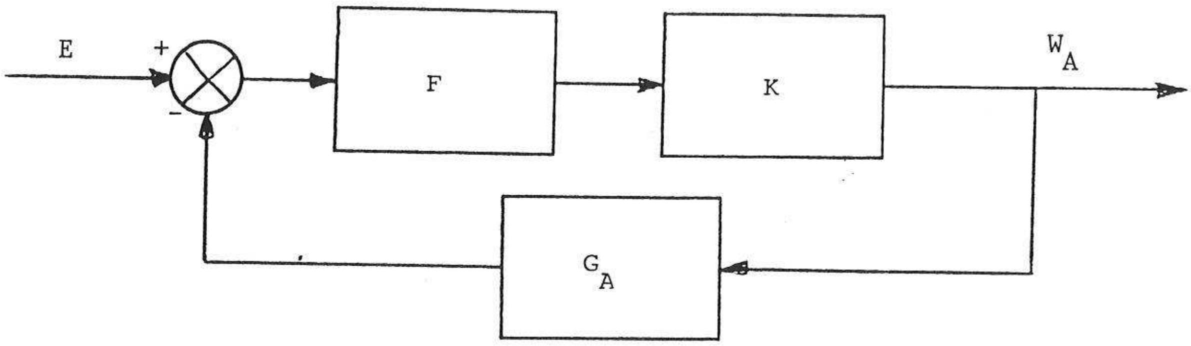


Fig. 3

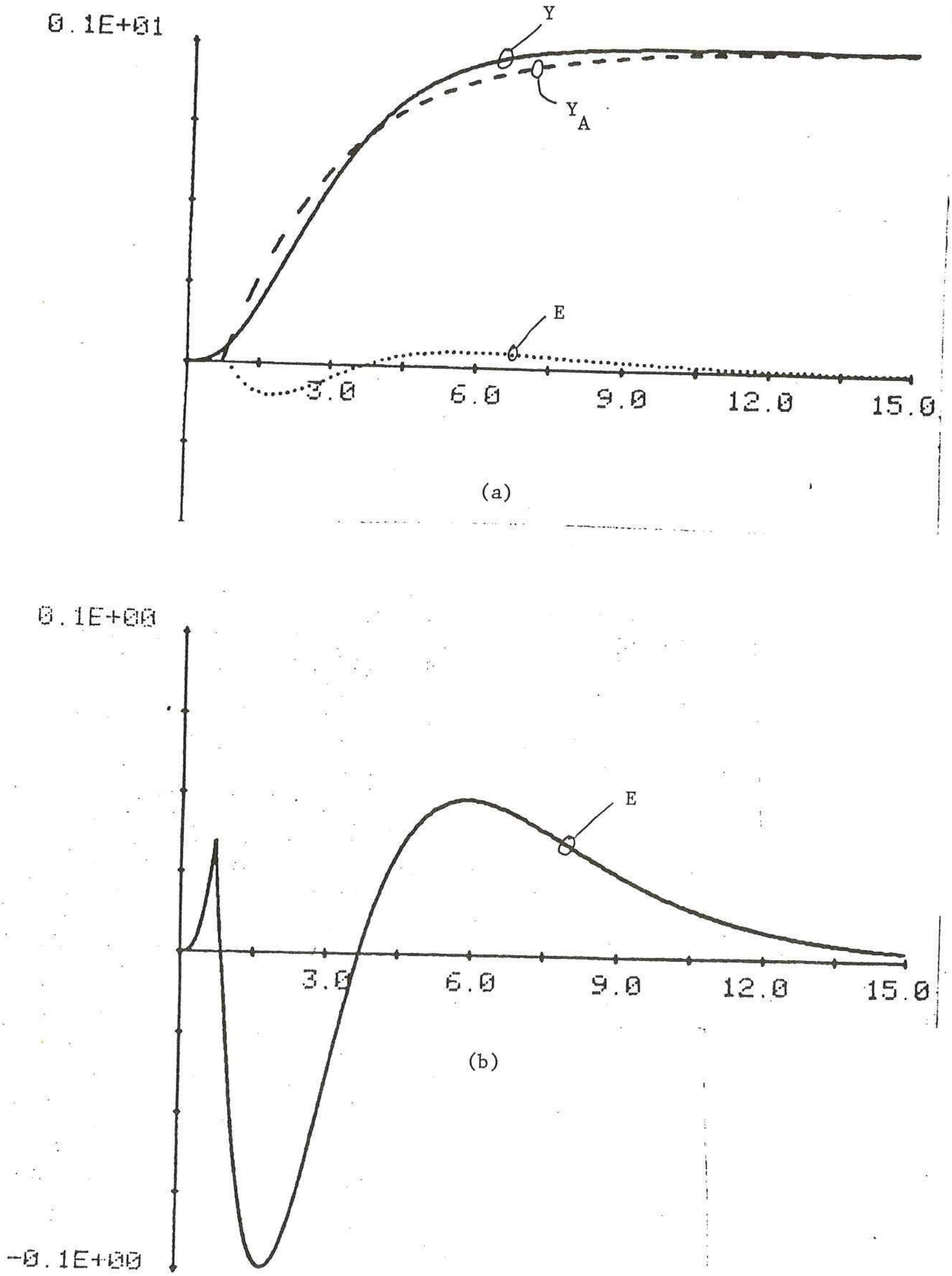
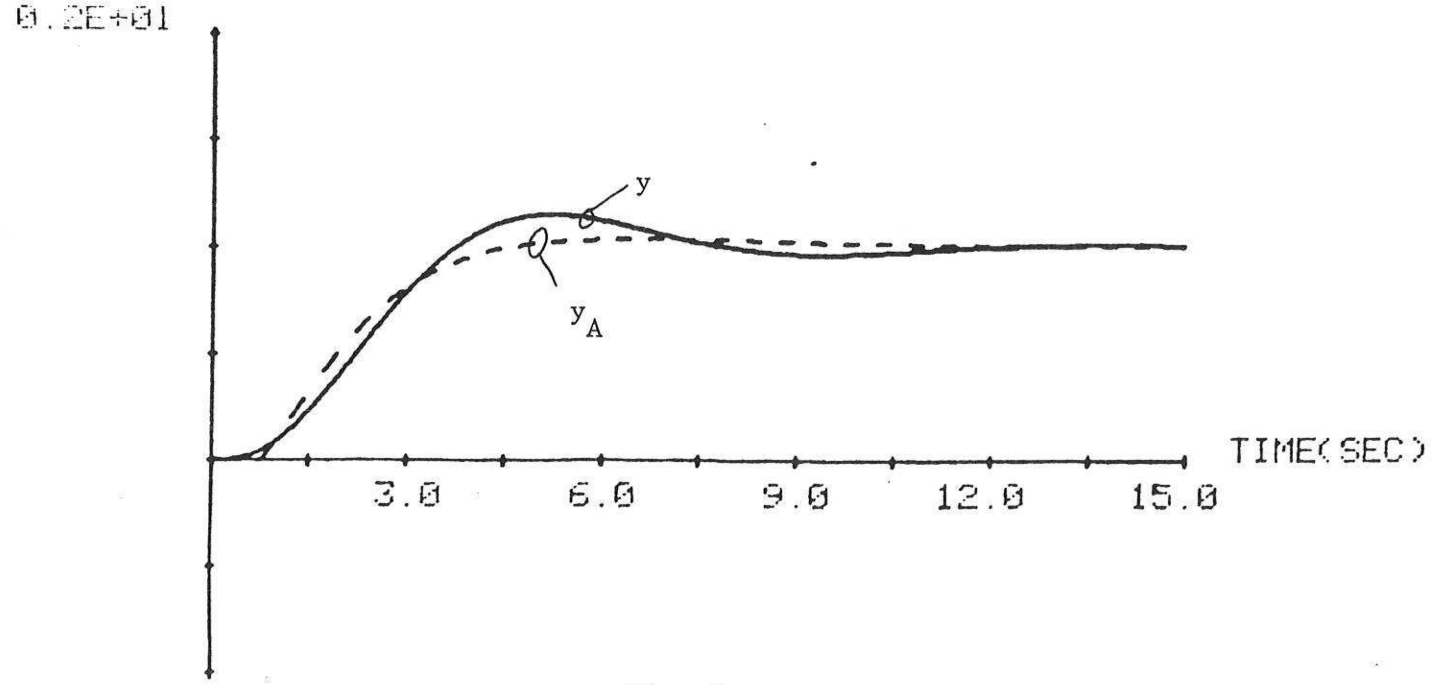


Fig. 4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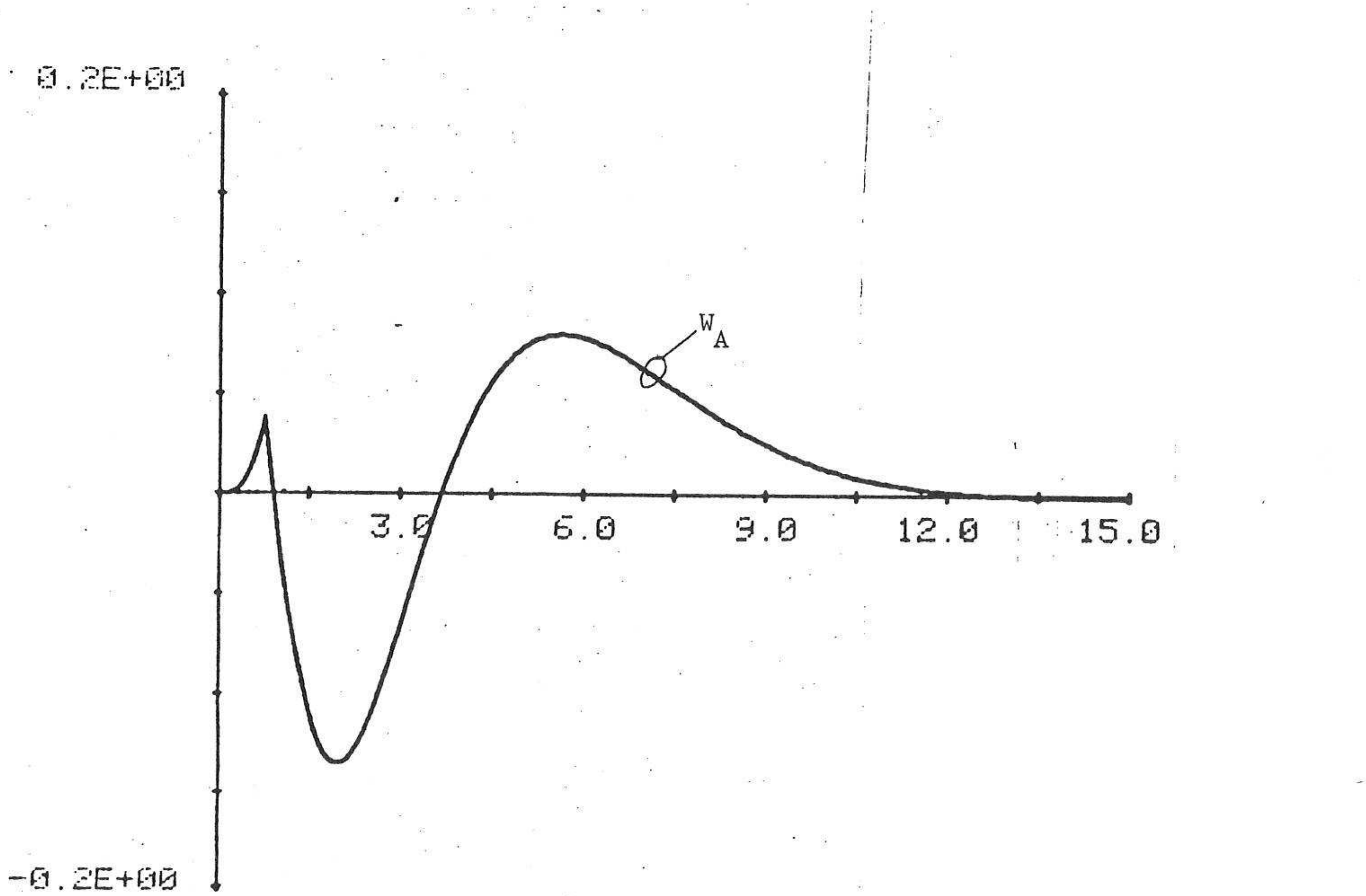
y
y_A
TIME(SEC)
3.0
6.0
9.0
12.0
15.0

Fig. 5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W_A
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6.0
9.0
12.0
15.0
-0.2E+00

Fig. 6

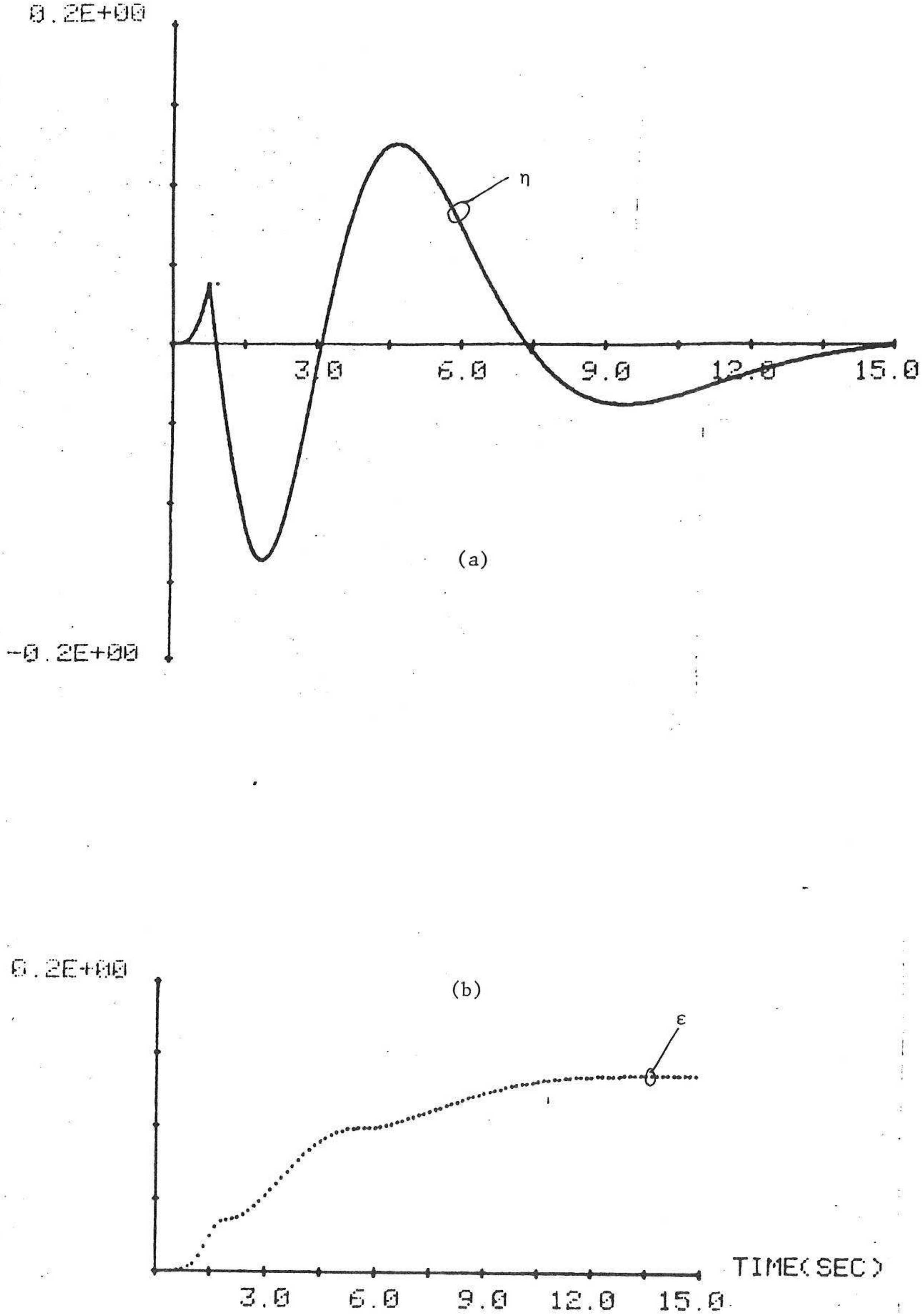
0.2E+00
η
3.0
6.0
9.0
12.0
15.0
(a)
-0.2E+00
0.2E+00
(b)
ε
TIME(SEC)
3.0
6.0
9.0
12.0
15.0

Fig. 7

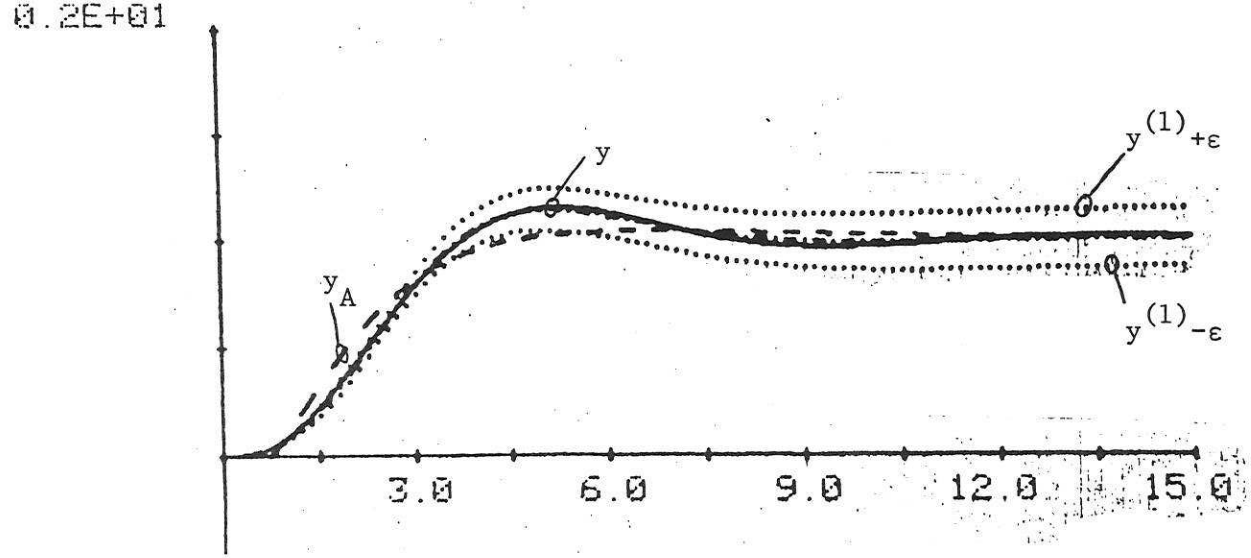

Fig. 8

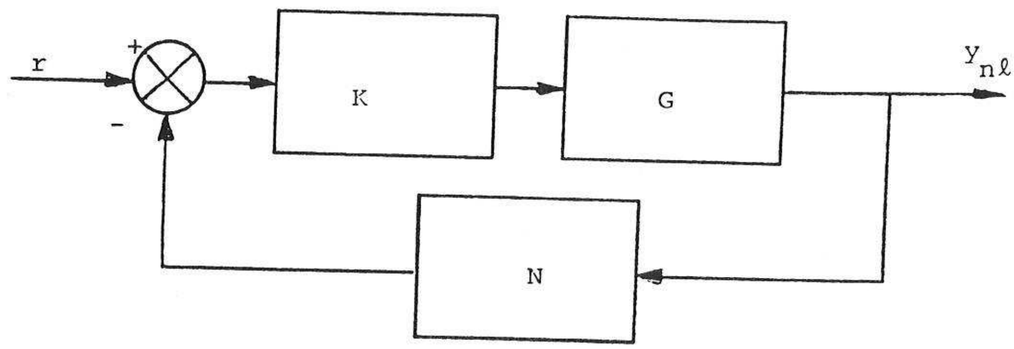

Fig. 9

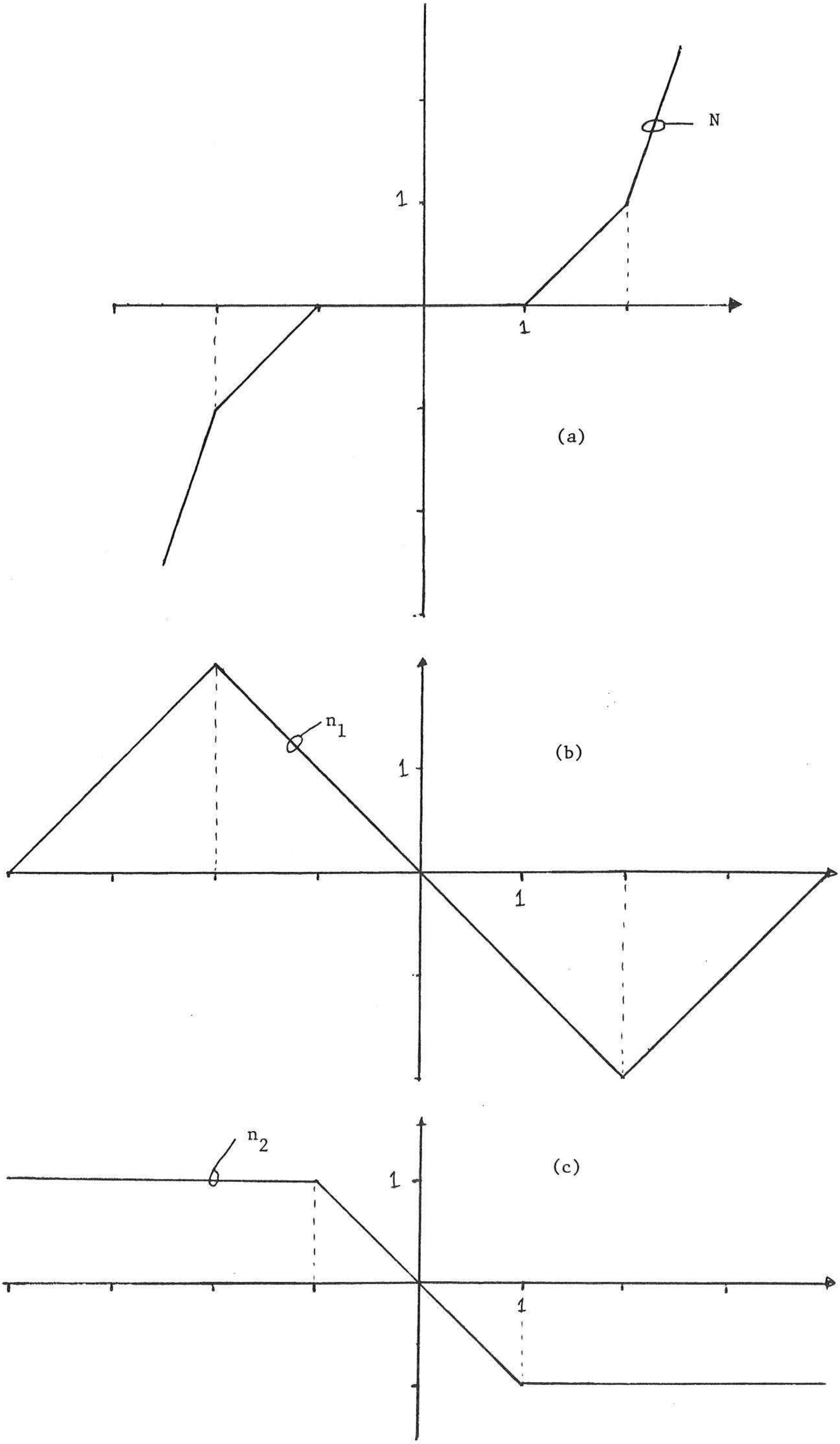

Fig. 10

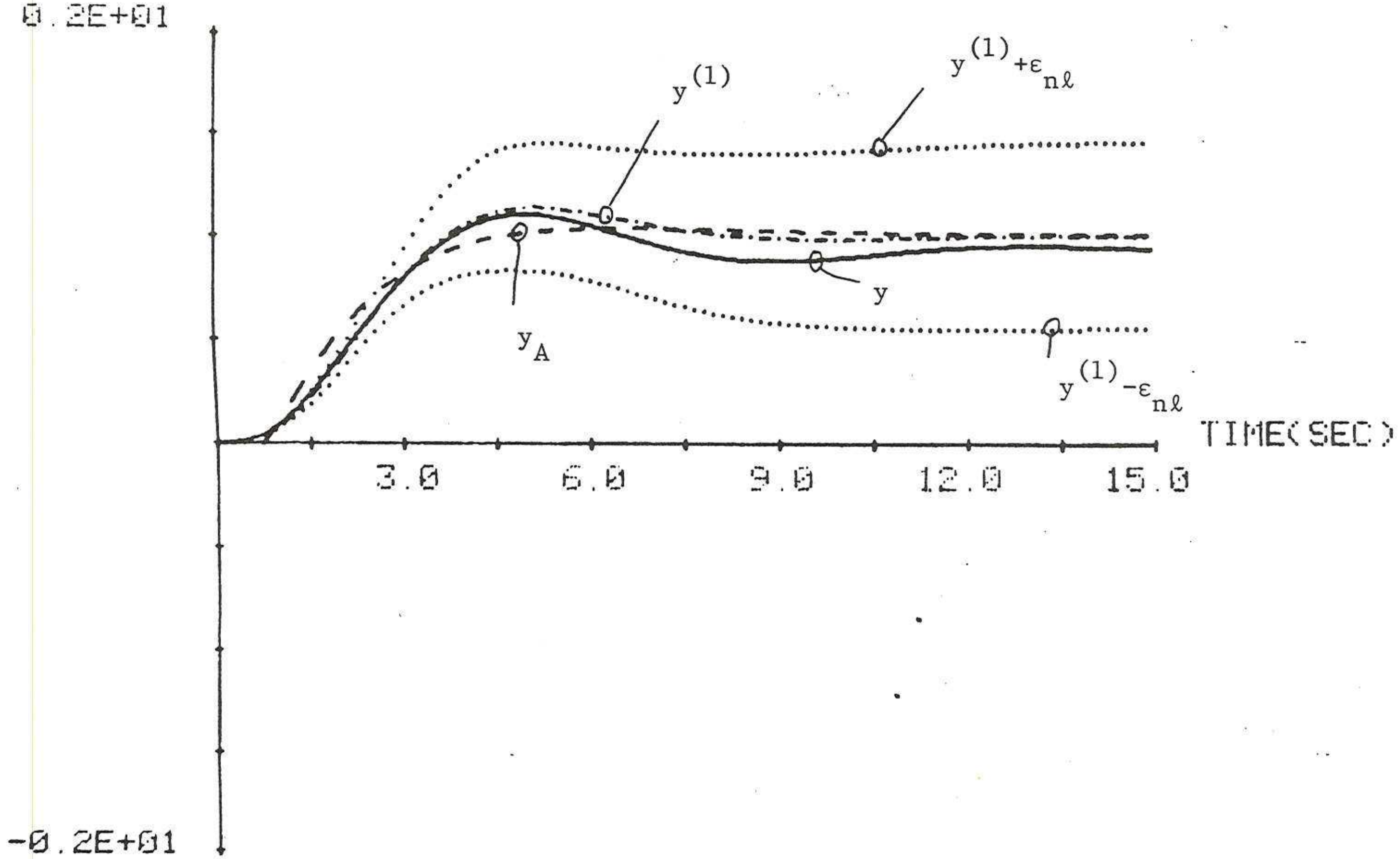

Fig. 11