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Reply

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ABSTRACT

Hydrostatic tendency equations of pressure and geopotential have been used in various forms since the beginning of the twentieth century. In contrast to classical mass flux divergence formulations, all forms that involve vertical integrals of temperature tendencies contain an additional term related to geopotential tendency at the upper limit of the integral. In many previous studies, including a recent work by two of the authors on a case of unusual wintertime precipitation in tropical West Africa, it has been assumed that there exists a pressure level in the stratosphere (usually 100 or 50 hPa) where these tendencies become negligible. This assumption implies a direct relation between net column heating and surface pressure fall. Prompted by a critique of Spengler and Egger, the validity of the concept of a stratospheric level of insignificant dynamics is tested here for the previously studied case on the basis of operational analyses from the ECMWF. At least for low latitudes, significant tendencies with some spatial and temporal variations are found up to 10 hPa, which renders a general neglect of this term on a fixed pressure level problematic. These results call for a more detailed investigation of the dynamical causes of the analyzed tendencies, a reevaluation of some previous work, and a more careful design of future studies on this subject.

1. The pressure tendency equation

a. Introduction

In their comments on our recently published paper (Knippertz and Fink 2008, hereafter KF08), Spengler and Egger (2009, hereafter SE09) expressed concerns about the application of the surface pressure tendency equation based on Kong (2006)

$$\frac{1}{\rho_s} \frac{\partial p_s}{\partial t} = \frac{1}{\rho_t} \frac{\partial p_t}{\partial t} + g \int_{z_s}^{z_t} \left[\mathbf{v} \cdot \nabla_p \ln T - \frac{R}{g} \left(\frac{g}{c_p} + \frac{\partial T}{\partial z} \right) \frac{\omega}{p} - \frac{\dot{Q}}{c_p T} \right] dz \quad (1)$$

to a case of West African winter precipitation. In Eq. (1) p is pressure, ρ is air density, g is the acceleration of gravity, z is geopotential height, \mathbf{v} is the horizontal wind vector in Cartesian-pressure coordinates, T is temperature, R is the gas constant for dry air, ω is the vertical velocity in pressure coordinates, c_p is the specific heat capacity at constant pressure, and \dot{Q} represents the diabatic heating rate. Subscript s denotes the surface level and subscript t denotes a level in the upper atmosphere. The term in square brackets is an extension of local temperature tendency using the first law of thermodynamics. For the sake of simplicity, the effect of water vapor on density is neglected here and in section 1b, and the actual temperature T is used instead of the virtual temperature T_v as in KF08. The three main points of criticism of SE09 are 1) it is not possible to use the first law of thermodynamics in the derivation of a hydrostatic pressure tendency equation, because it contains non-hydrostatic pressure; 2) through the usage of ω , the right-hand side of Eq. (1) implicitly contains a local pressure tendency as well; and 3) the first term on the

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right-hand side of Eq. (1) cannot be assumed to become zero at the top of the atmosphere (TOA).

We will begin this reply with a short historical discourse to demonstrate that Eq. (1) has been employed in one or the other form since at least the 1940s (section 1b) and that points 1 and 2 above are not critical in its derivation. In the following section 1c the more problematic point 3 will be discussed in the light of the literature and calculations using analysis data from the European Centre for Medium-Range Weather Forecasts (ECMWF). Finally, we will shortly comment on the role of moisture in this problem in section 1d and give some concluding remarks in section 2.

b. Different forms of the pressure tendency equation

Pressure or geopotential tendency equations have been used since the beginning of the last century (e.g., Margules 1904). The classical approach to calculate the tendency of p_s is to integrate the vertical mass flux divergence in a Cartesian coordinate system from the surface to a level in the upper atmosphere, which can be expressed as

$$\frac{\partial p_s}{\partial t} - \frac{\partial p_t}{\partial t} = -g \int_{z_s}^{z_t} [\mathbf{v} \cdot \nabla_z \rho + \rho \nabla_z \mathbf{v}] dz + g \rho_s w_s - g \rho_t w_t, \quad (2)$$

where \mathbf{v} and w are the horizontal and vertical components of the wind vector, respectively. In the limit of t approaching the TOA the last terms on each side can be neglected. Panofsky (1944) used the ideal gas law to replace ρ by T in Eq. (2) and obtained

$$\begin{aligned} \frac{1}{p_s} \frac{\partial p_s}{\partial t} - \frac{1}{p_t} \frac{\partial p_t}{\partial t} \\ = \frac{g}{R} \int_{z_s}^{z_t} \left[\frac{1}{T^2} \mathbf{v} \cdot \nabla_z T + \frac{w}{T^2} (\Gamma - \alpha) - \frac{1}{T^2} \frac{dT}{dt} \right] dz, \quad (3) \end{aligned}$$

where Γ is the dry-adiabatic lapse rate (or the saturated-adiabatic lapse rate if the air is saturated), α is the actual lapse rate, and the final term represents diabatic temperature changes. Note that the second term on the left-hand side does not necessarily disappear in the limit against the TOA as further discussed in section 1c.

Four years later, Godson (1948) derived a similar equation in a Cartesian-pressure coordinate system that he therefore calls the isobaric tendency equation:

$$\begin{aligned} \frac{1}{\rho_s} \frac{\partial p_s}{\partial t} - \frac{1}{\rho_t} \frac{\partial p_t}{\partial t} \\ = g \int_{z_s}^{z_t} \left[\mathbf{v} \cdot \nabla_p \ln T - \frac{dp}{dt} \left(\frac{d \ln T}{dp} - \frac{\partial \ln T}{\partial p} \right) - \frac{d \ln T}{dt} \right] dz. \quad (4) \end{aligned}$$

This equation becomes identical to Eq. (1) after the introduction of $\omega = dp/dt$ and \dot{Q} , and a conversion of the stability term in the following way:

$$\frac{d \ln T}{dp} - \frac{\partial \ln T}{\partial p} = \frac{1}{T \rho g} \left(-\frac{dT}{dz} + \frac{\partial T}{\partial z} \right) = \frac{R}{g p} \left(\frac{g}{c_p} + \frac{\partial T}{\partial z} \right). \quad (5)$$

Thus, the method used in KF08 is based on classical concepts that have been used in similar forms in many studies since the 1940s. One example is the work of Hirschberg and Fritsch (1991) who evaluated tendencies of geopotential height at two pressure levels b and t , which is more adequate for modern (re) analysis products:

$$\frac{\partial z_b}{\partial t} - \frac{\partial z_t}{\partial t} = \frac{R}{g} \int_{p_b}^{p_t} \left[-\mathbf{v} \cdot \nabla_p T + S_d \omega + \frac{\dot{Q}}{c_p} \right] d \ln p. \quad (6)$$

The dry-static stability parameter S_d in Eq. (6) is related to the respective term in Eq. (1) in the following way (θ is potential temperature):

$$S_d := -\frac{T}{\theta} \frac{\partial \theta}{\partial p} = \frac{RT}{p c_p} - \frac{\partial T}{\partial p} = T \frac{R}{g p} \left(\frac{g}{c_p} + \frac{\partial T}{\partial z} \right). \quad (7)$$

As for Eq. (3), it is possible to replace θ by the equivalent potential temperature θ_e in cases of moist ascent. Steenburgh and Holton [1993; their Eq. (4)] showed that Eq. (6) is closely related to surface pressure tendency:

$$\frac{\partial p_s}{\partial t} = \rho_s g \frac{\partial z_t}{\partial t} + \rho_s R \int_{p_s}^{p_t} \left[-\mathbf{v} \cdot \nabla_p T + S_d \omega + \frac{\dot{Q}}{c_p} \right] d \ln p. \quad (8)$$

All forms of the pressure or geopotential tendency equation presented so far are founded on the hydrostatic assumption from the very first to the very last step of their derivation. All equations including the one used in KF08 and criticized by SE09 (point 1 in section 1a) are only valid in situations in which vertical accelerations are small enough to make the hydrostatic pressure a good approximation for the actual pressure. The first law of thermodynamics is certainly valid in such situations and can therefore be used for the derivation. In our view the hydrostatic assumption is well justified in KF08 given that data with 1° horizontal resolution generated by an assimilation system based on a hydrostatic numerical model is used and that there is no heavy precipitation in the area of interest. Regarding point 2 of SE09, it is important to stress that the motivation of earlier studies and of KF08 to employ one or the other

form of the pressure tendency equation was mainly to use it as a diagnostic tool (i.e., assigning the instantaneous pressure change to different processes in the atmospheric column above the surface). Certainly this requires the knowledge of the fields of the three-dimensional wind (including the vertical pressure velocity ω), temperature, moisture, and diabatic heating rates for the time the integral is evaluated.

c. The problem of the upper boundary

The summary in section 1b has shown that all forms of the pressure tendency equation, which use temperature in the integral [i.e., Eqs. (1), (3), (4), and (8)], contain a tendency term at the upper boundary of the following form:

$$\frac{1}{p_t} \frac{\partial p_t}{\partial t}, \tag{9a}$$

$$\frac{1}{\rho_t} \frac{\partial \rho_t}{\partial t}, \text{ and} \tag{9b}$$

$$\rho_s g \frac{\partial z_t}{\partial t}. \tag{9c}$$

We agree with point 3 by SE09 that it is in fact not trivial that these terms become zero at the TOA. Several authors have discussed this question in their application of the pressure tendency equation. In the 1930s Raethjen (1939) argued that term 9b should vanish near 30 km and neglected it in his analysis. Godson (1948) on the other hand used observed 12-h pressure changes at the 15 000-ft level to estimate term 9b, while Tsou et al. (1987), Boyle and Bosart (1986), and Hirschberg and Fritsch (1991) assumed height tendencies to be negligible above the highest level available in the analysis dataset used to evaluate the vertical integral (i.e., 100 hPa in the former two and 50 hPa in the latter study). In another study Hirschberg and Fritsch (1993) termed this level in the stratosphere the “level of insignificant dynamics” (LID) and confirmed its existence for scales less than ~ 5000 km using results from a quasigeostrophic analytic model. The assumption of a LID implies that all ageostrophic circulations that evacuate a column of air to cause falling surface pressure take place underneath the LID.

To test whether the assumption of a LID is in fact justified in the case investigated by KF08, 24-h tendencies of term 9c [from Eq. (4) of Steenburgh and Holton 1993] were calculated from ECMWF analysis data at 100, 50, 30, and 10 hPa for 0000 UTC 18 January–0000 UTC 19 January 2004 and are displayed in Fig. 1. The density ρ_s was calculated from p_s and surface T_v averaged over the four analysis times 0000, 0600, 1200, and 1800 UTC. The fields have units of hectopascals per day

and should be compared to typical tendencies of p_s as in Figs. 12c and 13c of KF08. The pronounced trough over northern Africa discussed in KF08 is clearly reflected in a lowering of the 100-hPa surface over eastern Algeria associated with tendencies of almost -15 hPa day^{-1} , while weakly positive values are found for the northwestern and southeastern parts of the domain (Fig. 1a). This clearly demonstrates that this level does not qualify as a LID in contrast to the assumptions made in KF08 and some previous studies. Even at 50 hPa, maximum tendencies of -5 hPa day^{-1} are still too large to be considered insignificant (Fig. 1b), although at least over tropical West Africa the signal is largely below -2 hPa day^{-1} . The tendencies at 30 hPa in contrast show little structure with values hardly exceeding $\pm 2 \text{ hPa day}^{-1}$ (Fig. 1c), making it appear a promising candidate for a LID. To our surprise, however, the tendencies increase again in magnitude at the 10-hPa level with a maximum of 10 hPa day^{-1} in the southwestern corner of the domain (Fig. 1d). This behavior is not a peculiarity of 19 January, as comparably large signals of both signs are found for the following day (not shown). The physical mechanisms (or maybe data analysis problems) that cause these large tendencies are unclear, and a detailed investigation of them is left for future studies. In any case, this example challenges the assumption of a LID and suggests that atmospheric dynamics as high as 10 hPa have to be considered in an examination of surface pressure tendencies.

The foregoing discussion motivated us to investigate the full balance involved in the pressure tendency equation. The following modified form of Eq. (8) was used for this exercise (note the usage of virtual temperature):

$$\frac{\partial p_s}{\partial t} = \frac{gp_s}{RT_{vs}} \frac{\partial z_t}{\partial t} - \frac{p_s}{T_{vs}} \int_{p_t}^{p_s} \frac{\partial T_v}{\partial t} d \ln p + \text{RES}. \tag{10}$$

The first term on the right-hand side is identical to what is shown in Fig. 1. The second term corresponds to the vertical integral in KF08, but using actual 24-h tendencies of T_v instead of advection and diabatic terms [cf. Eq. (1)]. The last term is the residuum that indicates any errors in the balance. When computing these terms for 19 and 20 January 2004 with the standard pressure levels used in KF08, rather noisy residuum terms of locally more than $\pm 2 \text{ hPa day}^{-1}$ are obtained (not shown). A potential source for this error is an inconsistency in the analysis data from one day to the next. We therefore repeated the calculations for a (physically more consistent) 24-h operational forecast, but found similar magnitudes (not shown). It appears that the main source of error lies in the computation of the vertical integral with a limited number of levels. Therefore, the 60 ECMWF model levels were

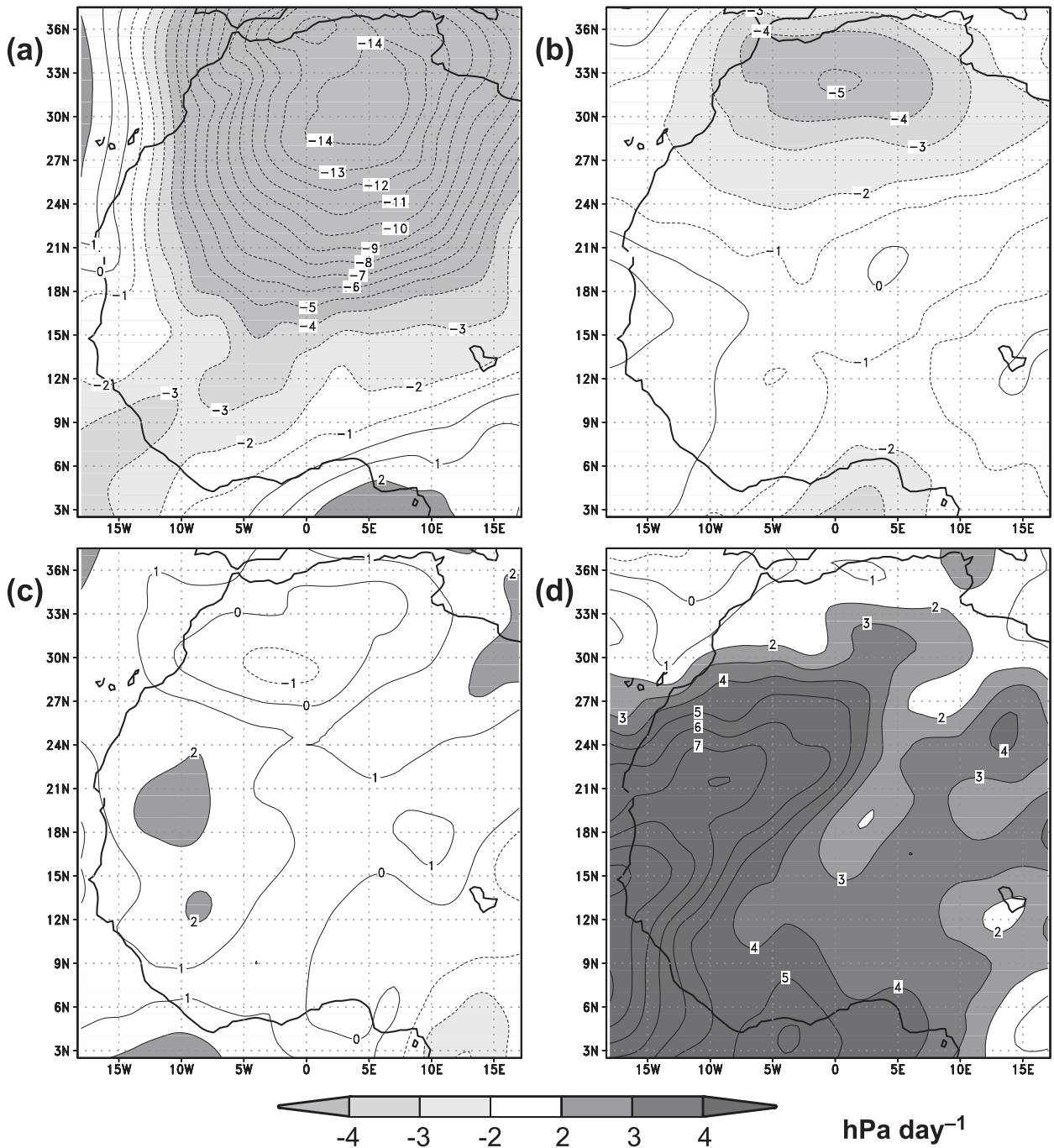


FIG. 1. Contributions to surface pressure tendencies related to height changes of the upper boundary (term 9c in the text) for the 24-h period 0000 UTC 18 Jan–0000 UTC 19 Jan 2004. The upper boundary is set to (a) 100, (b) 50, (c) 30, and (d) 10 hPa. Employed data are operational analyses from the ECMWF on pressure levels. The contour interval is 1 hPa day⁻¹.

used instead. Given that 30 hPa shows rather small geopotential tendencies (Fig. 1c), the upper boundary was set to model level 19, which corresponds to ~ 28.9 hPa.

For 19 January the first term clearly shows the widespread pressure fall of more than 6 hPa day⁻¹ over northern Africa and weakly increasing pressure close to

the Iberian Peninsula and near 21°N, 10°E (Fig. 2a). The overall pattern is highly correlated with column heating and cooling, respectively (Fig. 2b). In addition, the heating causes a slight lifting of the upper boundary over quite large parts of the domain (Fig. 2c). These results support the interpretations given in KF8 at least in a

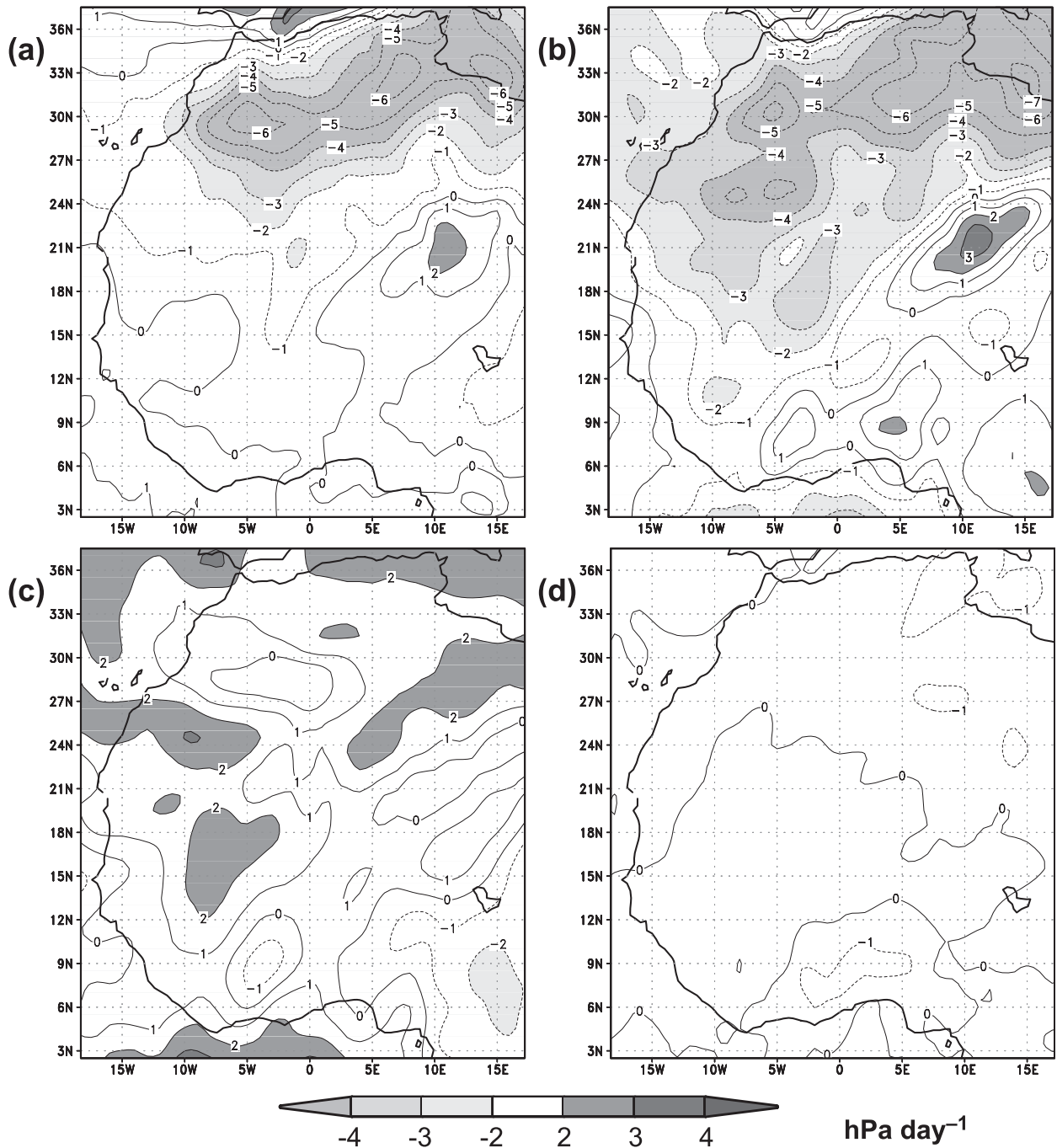


FIG. 2. Evaluation of the four terms in Eq. (10) for the period 0000 UTC 18 Jan–0000 UTC 19 Jan 2004: (a) surface pressure tendency, (b) contribution of vertically integrated virtual temperature tendency, (c) contribution of geopotential tendency at the upper boundary, and (d) residuum. Employed data are operational analyses from the ECMWF on model levels with level 19 (~28.9 hPa) as the upper boundary. The contour interval is 1 hPa day⁻¹.

qualitative sense (see their Fig. 12c). Using the model levels, the residuum term is significantly reduced to values mostly below ± 1 hPa day⁻¹ (Fig. 2d). The same analysis for the following day equally shows a close correspondence between surface pressure (Fig. 3a) and

column T_v tendencies (Fig. 3b) in the subtropics, but less clear in the tropics, where a considerable lifting of the upper boundary is found (Fig. 3c). Again there is a reasonable qualitative agreement with Fig. 13c in KF08. The residuum term is again very small (Fig. 3d).

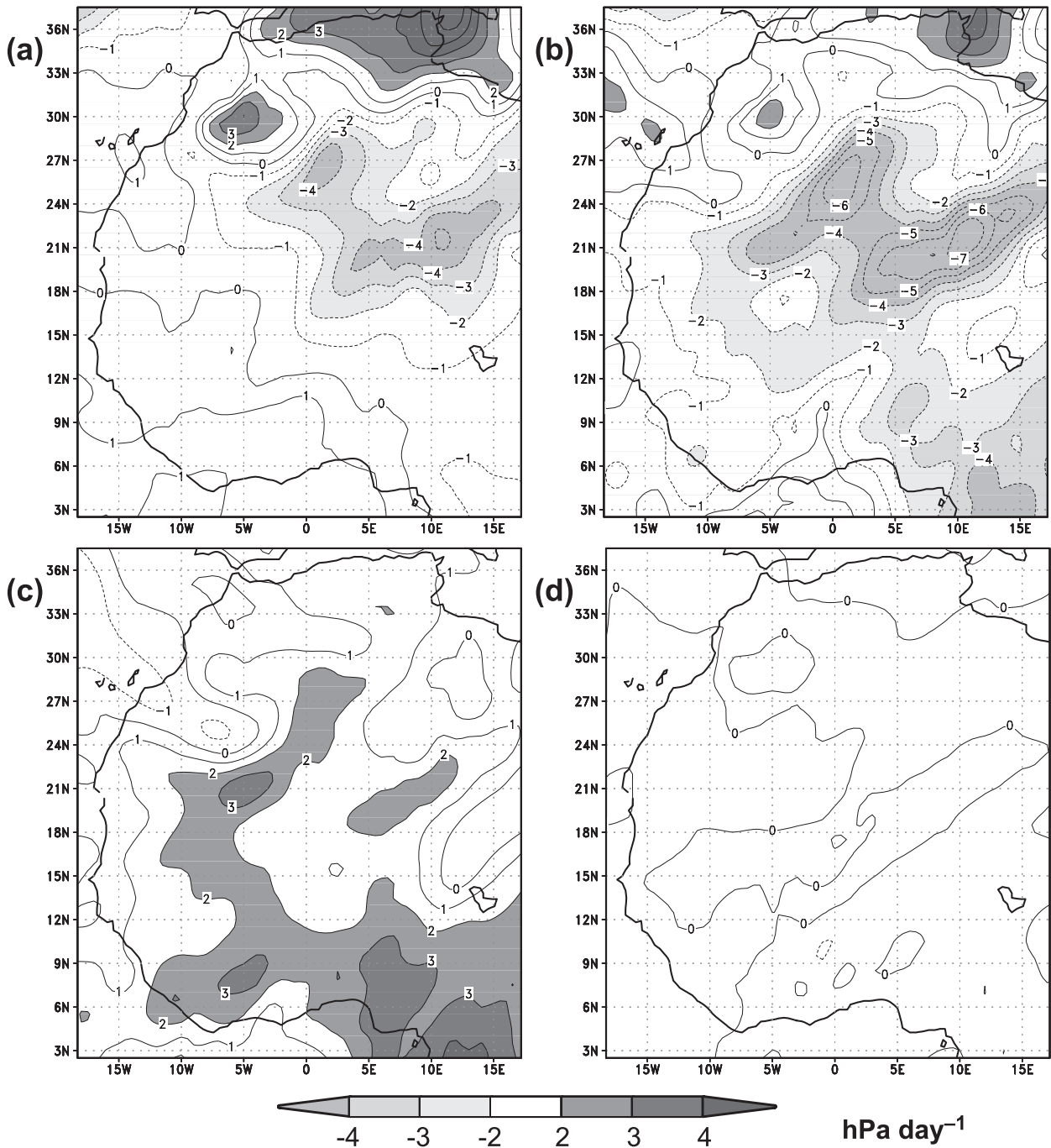


FIG. 3. As in Fig. 2, but for 19–20 Jan 2004.

d. Effects of moisture

As pointed out by SE09, an interesting extension of the problem is to include the effects of moisture [their Eq. (8)]. One possible way to do this in Eq. (1) is to use the so-called density temperature T_p , which accounts for the effects of water vapor, condensate, and ice on den-

sity in the ideal gas law (Emanuel 1994). This allows to simply replace T by T_p in the first two terms of the above equation, while the third term will contain diabatic heating and total tendencies of the mixing ratios of water vapor, condensate, and ice with the latter being closely related to processes such as evaporation and precipitation as pointed out by SE09. On scales of

several hundred kilometers, however, typical differences between T_v and T_p are on the order of 0.01 K and therefore it is justifiable to concentrate on the usage of T_v instead of T , as done here and in KF08.

2. Conclusions

The pressure tendency equation employed by KF08 [Eq. (1)] is closely related to a whole family of similar equations used since at least the 1940s and is based on physically sound concepts. The overall interpretation of KF08 that regions of falling surface pressure tend to be located underneath warming columns of air is confirmed in a broad sense. In particular, their conclusion that the presence of a cloud band over the Sahara caused a warming and thereby supported the pressure drop in its vicinity remains valid. However, to assess the magnitude and extent of the pressure fall, the assumption made by KF08 and other authors that geopotential tendencies can be neglected from about 100–50 hPa upward appears somewhat problematic, as significant values are found in the investigated case as high as 10 hPa. This is a truly unexpected result with profound implications for the interpretation of previous work and the design of future studies using pressure tendency concepts. We are grateful to SE09 for pointing us to this crucial aspect. A more technical outcome of this reply is that the usage of all available vertical levels is advisable to keep errors in the calculation of the vertical integral at an acceptable level. More work is necessary to better understand the causes of the surprisingly large upper-level geopotential tendencies and their impacts on the dynamics in the lower atmosphere.

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REFERENCES

- Boyle, J. S., and L. F. Bosart, 1986: Cyclone-anticyclone couplets over North America. Part II: Analysis of a major cyclone event over the eastern United States. *Mon. Wea. Rev.*, **114**, 2432–2465.
- Emanuel, K. A., 1994: *Atmospheric Convection*. Oxford University Press, 580 pp.
- Godson, W. L., 1948: A new tendency equation and its application to the analysis of surface pressure changes. *J. Meteor.*, **5**, 227–235.
- Hirschberg, P. A., and J. M. Fritsch, 1991: Tropopause undulations and the development of extratropical cyclones. Part II: Diagnostic analysis and conceptual model. *Mon. Wea. Rev.*, **119**, 518–550.
- , and —, 1993: On understanding height tendencies. *Mon. Wea. Rev.*, **121**, 2646–2661.
- Knippertz, P., and A. H. Fink, 2008: Dry-season precipitation in tropical West Africa and its relation to forcing from the extratropics. *Mon. Wea. Rev.*, **136**, 3579–3596.
- Kong, K.-Y., 2006: Understanding the genesis of Hurricane Vince through the surface pressure tendency equation. Preprints, *27th Conf. on Hurricanes and Tropical Meteorology*, Monterey, CA, Amer. Meteor. Soc., 9B.4. [Available online at <http://ams.confex.com/ams/pdfpapers/108938.pdf>.]
- Margules, M., 1904: Über die Beziehung zwischen Barometerschwankungen und Kontinuitätsgleichung (On the relation between barometer fluctuations and the continuity equation). *Festschrift Ludwig Boltzmann*, J. A. Barth, Ed., Druck von Metzger & Wittig, 585–589.
- Panofsky, H. A., 1944: The effect of vertical motion on local temperature and pressure tendencies. *Bull. Amer. Meteor. Soc.*, **25**, 271–275.
- Raethjen, P., 1939: Advective und konvektive, stationäre und gegenläufige Druckänderungen (Advective and convective stationary and opposing pressure changes). *Meteor. Z.*, **56**, 133–142.
- Spengler, T., and J. Egger, 2009: Comments on “Dry-season precipitation in tropical West Africa and its relation to forcing from the extratropics.” *Mon. Wea. Rev.*, **137**, 3149–3150.
- Steenburgh, W. J., and J. R. Holton, 1993: On the interpretation of geopotential height tendency equations. *Mon. Wea. Rev.*, **121**, 2642–2645.
- Tsou, C.-H., P. J. Smith, and P. M. Pauley, 1987: A comparison of adiabatic and diabatic forcing in an intense extratropical cyclone system. *Mon. Wea. Rev.*, **115**, 763–786.