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PRECOMPENSATION, APPROXIMATION AND AN INVERSE NYQUIST ARRAY DESIGN TECHNIQUE USING PLANT STEP DATA ONLY

by

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ABSTRACT

Given a multivariable plant with uncertain or unknown dynamics, the use of step response data in control design is described based upon a simple diagonal model of a precompensated plant. The technique is identical in structure to the inverse Nyquist array design technique with Gershgorin circles replaced by 'confidence circles' deduced from simple graphical operations on plant step data. At no stage is a plant model required! The choice of constant precompensator and simplified model by iterative means is described as a sequence of dual optimization problems.

Introduction

The inverse Nyquist array (INA) design technique[1-3] is well-established as a systematic design technique using separate loop controllers on a precompensated plant. It can be highly successful in practice provided that the plant model is a good representation of plant dynamics in the signal range of interest and that a precompensator of suitably simple form can be found. In this paper, we consider the development of an INA design technique suitable for situations where the plant model is uncertain or unknown but the designer has access to plant step data. These assumptions will also cover the case when a model is available but also regarded as unnecessarily complex for design work. The plant step responses can then be obtained by simulation studies. The technique has the same structure as the INA with Gershgorin bands replaced by so-called 'Confidence bands' deduced from elementary and numerically robust graphical operations on the plant step data and has the advantages that

(a) at no stage is it necessary to have an accurate plant model available,

(b) matrix inversions are eliminated from the computational procedure, and

(c) the simplified model used for design calculations can have any desired complexity.

The technique will however, tend to produce more conservative designs than the INA due to the fact that the detailed dynamic structure of the plant is not used. This may be unacceptable in a given application if high performance controls are required, but could be acceptable in process control applications where emphasis is placed on stability, tracking of set-point changes and disturbance rejection rather than fast, high performance, low-interaction closed-loop dynamics. This problem must be weighed against the advantages of (a),(b) and (c) above in each design situation

The technique is based on the recent results of Owens and Chotai[(4)]. These are summarized in Section 2 and extended to include the possibility of control precompensation and the use of integrated modelling error in the design process. The choice of precompensator is discussed in Section 3 and the techniques illustrated by an example in Section 4. Section 5 outlines a technique for incorporating derivative information in the design when available and Section 6 indicates the theoretical possibility of using dynamic rather than constant precompensation and simultaneous pre-and post compensation.

2. Stability and Graphical Design Criteria

In a recent paper[(4)], fundamental theoretical results were introduced to enable the design of feedback controls for an m-input/m-output plant $G(s)$ based on off-line control design using an approximate model $G_A(s)$. The control system is designed to produce the required stability and performance characteristics for the model G_A, followed by a graphical procedure suitable for CAD based on the open-loop modelling error that ensures that the controller will also stabilize the real plant G. The results described in that paper require modification for the purposes of this paper.

It is assumed that the designer has access to a reliable estimate of the real plant step response matrix

$$Y(t) \triangleq \begin{bmatrix} Y_{11}(t) & \cdots & Y_{1m}(t) \\ \vdots & & \vdots \\ Y_{m1}(t) & \cdots & Y_{mm}(t) \end{bmatrix} \quad , t \geq o \qquad (1)$$

where $Y_{ij}(t)$ is the response from zero initial conditions of the i^{th} output y_i to a unit step in the j^{th} input $u_j(t)$ with $u_\ell(t) \equiv 0$, $\ell \neq j$. Let K_p be an mxm (as yet unspecified) forward path constant precompensator, $K_c(s)$ = diag $\{k_j(s)\}$ an mxm diagonal forward path compensator of loop controllers and $F(s)$ = diag $\{f_j(s)\}$ an mxm diagonal representation of output measurement transducer dynamics. The resultant feedback scmeme is illustrated in Fig. 1(a) where GK_p is the 'plant seen by the compensator K_c'.

The step response matrix of GK_p is easily seen to be given by

$$Y_p(t) \triangleq Y(t)K_p \qquad , t \geq o \qquad (2)$$

Let $G_A(s)$ = diag$\{ g_j(s)\}_{1\leq j\leq m}$ be a diagonal approximate model of the compensated plant GK_p deduced by inspection of $Y_p(t)$ followed by the construction of models of the diagonal terms of the desired compexity and neglecting the off-diagonal interaction effects. Suppose that G_A has step response matrix

$$Y_A(t) = \text{diag } \{h_j(t)\}_{1\leq j\leq m} \qquad (3)$$

deduced from simulation or analytical studies and represent the modelling error by the matrix function of time

$$E^p(t) \triangleq Y_p(t) - Y_A(t) \tag{4}$$

Theorem 3 in Owens and Chotai[4] can now be expressed in the form below by replacing G by GK_p, K by K_c and E by E^p. The proof is therefore omitted.

Theorem 1: Suppose that the control elements $k_j(s)$, $1 \leq j \leq m$, are designed to produce stability and desirable loop dynamics from the approximating non-interacting feedback scheme illustrated in Fig. 1(b) and that

(i) both the plant and model are stable, and

(ii) the composite system GK_pK_cF is both controllable and observable,

then the resultant scheme will stabilize the real plant G in the configuration of Fig. 1 (a) if

(iii) the inequality

$$\limsup_{\substack{Re s > o \\ |s| \to \infty}} \left| \frac{k_j(s) f_j(s)}{1 + k_j(s) f_j(s) g_j(s)} \right| < \frac{1}{\sum_{\ell=1}^{m} N_\infty(E^p_{j\ell})} \triangleq \frac{1}{d_j(\infty)} \tag{5}$$

is satisfied for $1 \leq j \leq m$, and

(iv) the 'confidence bands' generated by plotting the inverse Nyquist locus of $g_j(s)k_j(s)f_j(s)$ for $s = i\omega$, $\omega \geq o$ with superimposed 'confidence circles' at each point of radius

$$r^p_j(s) \triangleq |g_j^{-1}(s)| d_j(\infty) \tag{6}$$

does not contain or touch the (-1,0) point of the complex plane.

(Remarks: (1) the graphical interpretation of (iv) has been given in Owens and Chotai[4],

(2) $N_\infty(E^p_{j\ell})$ is the 'total variation' of $E^p_{j\ell}(t)$ and can be obtained by graphical analysis of its time variation as discussed in Owens and Chotai[4],

(3) the radii of the confidence circles increase as the modelling error E^p increases in the time-domain and are small if G_A is a good representaion of the step response dynamics of GK_p,

(4) requirement (ii) holds generically but can be reduced to stabilizability and detectability of GK_pK_cF if stable uncontrollable and/or unobservable modes are acceptable in the closed-loop system

and (5) Note that the inverse plant $\hat{G}$ is not required as it is in the INA technique.

The theorem has a design interpretation identical in structure to that of the INA:

Step 1: Obtain the plant step response matrix $Y(t), t \geq o$, from simulation or plant tests.

Step 2: Choose a constant precompensator K_p and compute $Y_p(t)$, $t \geq o$.

Step 3: Construct an approximate diagonal model G_A by inspection of the dynamic characteristics of the diagonal terms of $Y_p(t)$ and calculate the total variations $N_\infty(E^p_{ij})$ of the elements of the error E^p.

Step 4: Design compensation networks $k_j(s)$, $1 \leq j \leq m$, to produce the stability and performance required from the approximating feedback scheme of Fig. 1(b) using condition (iii) to put a preliminary bound on the control gains allowed.

Step 5: Check condition (iv) graphically to assess stability. The condition plays the same role as that of diagonal row dominance in the INA with the confidence circles replacing Gershgorin circles.

The choice of precompensator K_p and its effect on the design are discussed in the next section. The only other major design problem that could arise is that step 5 is unsuccessful. This problem could be offset by changing K_p as discussed in the next section or by 'reducing the control gains $k_j(s)$'. More precisely, if $f_j(s)$ is stable with no zeros at the origin of the complex plane then conditions (ii) and (iv) can be satisfied by reducing loop gains except in the case where integral control action is involved when (iv) also requires that $r_j^p(o) < 1$, $1 \leq j \leq m$, to ensure that the (-1,0) point does not lie in the low frequency confidence circles. Equivalently, the permitted modelling error is bounded by

$$d_j(\infty) = \sum_{\ell=1}^{m} N_\infty(E_{j\ell}^p) < |g_j(o)| \quad , \quad 1 \leq j \leq m \tag{7}$$

This situation can be improved by reducing the radii of the confidence circles at low frequencies in a number of ways. Consider for example the matrix total variation[(4)]

$$N_\infty^P(E^p) \triangleq \begin{pmatrix} N_\infty(E_{11}^p) & \cdots & N_\infty(E_{1m}^p) \\ \vdots & & \vdots \\ N_\infty(E_{m1}^p) & \cdots & N_\infty(E_{mm}^p) \end{pmatrix} \tag{8}$$

then Gershgorins theorem (1-3) implies that the spectral radius

$$r(\infty) \triangleq r(N_\infty^P(E^p)) \leq \max_{1\leq j\leq m} \sum_{\ell=1}^{m} N_\infty(E_{j\ell}^p) = \max_j d_j(\infty) \tag{9}$$

Clearly, if (7) is violated for some index j, it may be true that $r(\infty) < |g_j(o)|$ for all $1 \leq j \leq m$, and hence that the following theorem can be applied in the presence of integral control action:

Theorem 2: The conclusions of theorem 1 still holds if $d_j(\infty)$ is replaced by $r(\infty)$, $1 \leq j \leq m$.

Proof: Using the procedures and notation of ref(4), the closed-loop system of Fig. 1(a) is stable if

$$\sup_{s\in D} r(||(I_m+K_c(s)F(s)G_A(s))^{-1}K_c(s)F(s)||_P \, N_\infty^P(E^P)) < 1 \tag{10}$$

This inequality is satisfied if

$$\sup_{s \in D} \max_{1 \leq j \leq m} \left| \frac{k_j(s) f_j(s)}{1+k_j(s) f_j(s) g_j(s)} \right| \; r(N_\infty^P(E^P)) < 1 \qquad (11)$$

by equation (20) in ref(4). This requirement reduces to conditions (iii) and (iv) of theorem 1 above with $r(\infty)$ replacing $d_j(\infty)$, $1 \leq j \leq m$, by considering separately the semi-circular and imaginary axis components of the Nyquist D-contour respectively.

The potential benefits of using this form of the result can be illustrated by supposing that

$$N_\infty^P(E) = \begin{bmatrix} 0.0 & 0.1 \\ 10.0 & 0.0 \end{bmatrix} \qquad (12)$$

where $d_1(\infty) = 0.1$ and $d_2(\infty) = 10.0$. In contrast $r(\infty) = 1$ which reduces the width of the confidence band in the second loop by a factor of 10.0. There are, of course, many situations where it can be envisaged that $r(\infty) \geq |g_j(o)|$ for some index j and hence that theorem 2 cannot be applied with integral action in the j^{th} loop. In this situation, the width of the confidence band can be reduced at low frequencies by using a better upper bound on the modelling error as follows:

<u>Lemma 1</u>: Defining the <u>integrated</u> modelling error

$$z^P(t) \triangleq \int_o^t (E^P(t') - E^P(\infty))dt' \qquad (13)$$

then, for $\mathrm{Re}\, s \geq o$, $1 \leq i,j \leq m$,

$$|(G(s)K_p)_{ij} - (G_A(s))_{ij}| \le d_{ij}(s) \triangleq \min\{|E^p_{ij}(\infty)| + |s| \ N_\infty(z^p_{ij}), N_\infty(E^p_{ij})\} \qquad (14)$$

Proof: Lemma 2 in Owens and Chotai[4] indicates that $|(G(s)K_p)_{ij} - (G_A(s))_{ij}| \le N_\infty(E^p_{ij})$. The result follows by noting that

$$\frac{1}{s}((G(s)K_p)_{ij} - (G_A(s))_{ij} - E^p_{ij}(\infty)) = \int_o^\infty e^{-st}(E^p_{ij}(t) - E^p_{ij}(\infty))dt \qquad (15)$$

and hence that, for Res $\ge$ o,

$$|(G(s)K_p)_{ij} - (G_A(s))_{ij} - E^p_{ij}(\infty)| \le |s| \int_o^\infty |E^p_{ij}(t) - E^p_{ij}(\infty)|dt$$

$$= |s| \ N_\infty(z^p_{ij}) \qquad (16)$$

by Proposition 2 in ref (4). Equation (16) implies equation (14) after a little rearrangement.

The lemma has the following two implications:

Theorem 3: The conclusion of theorem 1 remains valid if $d_j(\infty)$ is replaced in (6) by the frequency dependent function,

$$d_j(s) \triangleq \sum_{\ell=1}^{m} d_{j\ell}(s) \qquad (17)$$

Proof: The result follows from Theorem 1 in Owens and Chotai (4) and lemma 1 above with $\Delta_{ij}(s) = d_{ij}(s)$ and

$$\gamma(s) = \max_{1\le i\le m} \sum_{j=1}^{m} \left| \frac{k_i(s)f_i(s)}{1+k_i(s)f_i(s)g_i(s)} \right| d_{ij}(s) \qquad (18)$$

by interpreting the stability criterion sup $\gamma(s) < 1$ in graphical terms.

QED

Theorem 4: The conclusions of theorem 1 remain valid if we define

$$r(s) = r(D(s)) \quad , \quad \text{Re } s \geq o \tag{19}$$

where $D(s)$ is the mxm matrix with $(i,j)^{th}$ element $d_{ij}(s)$ and $d_j(\infty)$ is replaced in (5) and (6) by $r(\infty)$ and $r(s)$ respectively.

Proof: The result follows in a similar manner to theorem 3 by choosing $\Delta_{ij}(s) = d_{ij}(s)$ and

$$\gamma(s) = \max_{1 \leq i \leq m} \left| \frac{k_i(s) f_i(s)}{1+k_i(s) f_i(s) g_i(s)} \right| \quad r(s) \tag{20}$$

In practical terms, the main improvement made possible by theorem 3 and 4 is that the width of the confidence band is smaller at low frequencies as is seen by noting that $d_{ij}(o) = |E_{ij}(\infty)| \leq N_\infty(E_{ij})$, $1 \leq i,j \leq m$, and hence that

$$d_j(o) = \sum_{\ell=1}^{m} |E_{j\ell}(\infty)| \leq d_j(\infty) \quad , \quad 1 \leq j \leq m \tag{21}$$

and

$$r(o) = r(||E(\infty)||_p) \leq r(N_\infty^P(E)) = r(\infty) \tag{22}$$

In particular, if we choose, for example, $K_p = G^{-1}(o)$ $(= Y^{-1}(\infty))$, and $g_j(o) = 1$, $1 \leq j \leq m$, then $E(\infty) = 0$ and hence $d_j(o) = r(o) = 0$, $1 \leq j \leq m$, indicating that the confidence bands reduce continuously to zero width at low frequencies. The price that is paid for this improvement is the need to calculate the total variation of the integrated error z^p by graphical inspection of its time variation. This should not be a problem as integration is a robust operation with the added advantages of partially smoothing the data and reducing the effect of noise.

Next, we note the following result characterizing the integrity of the design:

Theorem 5: If the conditions of any one of theorems 1 - 4 are satisfied, then the resultant feedback scheme is stable in the presence of simultaneous failures in the sensors $f_j(s)$, and/or actuators $k_j(s)$, $j = j_1, j_2, \ldots, j_p$ provided that these networks are stable.

(Remark: In practice, this reduces to the requirement that there are no integrators in loops $j_1, j_2, \ldots, j_p$).

Proof: Considering, for example, theorem 1, then conditions (iii) and (iv) are equivalent to

$$\sup_{s \in D} \left| \frac{k_j(s) f_j(s)}{1+g_j(s) k_j(s) f_j(s)} \right| d_j(\infty) < 1 \quad , \quad 1 \leq j \leq m \tag{23}$$

The failed conditions described correspond to the situation where $k_j(s)\ f_j(s) \equiv 0$, $j = j_1, \ldots, j_p$ when (23) is immediately satisfied but condition (ii) is violated unless the defined networks are stable.

Finally, we note the following result extending theorems 1 and 3 to a form analogous to the diagonal column dominance in the INA:

Theorem 6: The conclusions of theorem 1, 3 & 5 hold with $d_j(\infty)$ and $d_j(s)$ replaced by, respectively,

$$d_j'(\infty) = \sum_{\ell=1}^{m} N_\infty(E_{\ell j}^p) \tag{24}$$

and

$$d_j'(s) = \sum_{\ell=1}^{m} d_{\ell j}(s) \tag{25}$$

Proof: The result in theorem 1 of ref (4) can be written as

$$\sup_{s \in D} r(\Delta(s) \| (I_m + K_c(s) F(s) G_A(s))^{-1} K_c(s) F(s) \|_P) < 1 \tag{26}$$

The above result follows by using the dual vector-induced matrix norm,

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$$||M|| = \max_{1 \le j \le m} \sum_{\ell=1}^{m} |M_{\ell j}| \qquad (27)$$

and $\Delta(s) = N_{\infty}^{P}(E)$ and $\Delta(s) = D(s)$ respectively.

---

3. Choice of Precompensation

Although theorems 2 - 4 produce some refinements to reduce the width of the confidence band, it would appear that the choice of K_p will play a more important role. In general terms, it is necessary to choose the constant precompensator K_p to make the error $E^p(t)$ as 'diagonal as possible' for $t \geq o$. This problem is clearly also connected with the choice of G_A (and hence $Y_A(t)$) and could proceed in the following iterative manner:

Step 1: Choose an initial precompensator $K_p^{(o)}$. Set $j = o$.

Step 2: Evaluate $Y_p(t)$ and use this data to construct an approximate model $G_A^{(j)}(s)$ of diagonal form.

Step 3: Evaluate the error $E^p(t), t \geq o$, and, if theorems 3 or 4 are to be used $Z^p(t)$, $t \geq o$, and compute the radii of the confidence bands for $s = i\omega$, $\omega \geq o$.

Step 4: If these are acceptable for design work let $K_p = K_p^{(j)}$ and $G_A = G_A^{(j)}$. Otherwise choose a new precompensator $K_p^{(j+1)}$, replace j by j+1 and return to step 2.

For any given trial precompensator $K_p^{(j)}$, the model $G_A^{(j)}$ can be constructed by any convenient technique to produce a model of the desired complexity. The main problem anticipated in practice therefore is the systematic improvement in the trial precompensator in step 4.

Alternatively, the designer could proceed as follows:

Step 1: Choose a precompensator K_p such that the off-diagonal terms of YK_p are small in the sense of the total variation.

Step 2: Choose G_A to model the diagonal terms to the required accuracy.

Again the main problem is the systematic choice of K_p.

3.1 Intuitive Choices of K_p

As noted in Owens and Chotai[4,5] a plant G with relatively small interaction in the open-loop will be characterized by a step response matrix Y with dominant diagonal characteristics. In such a situation, it is natural to choose the unit precompensator

$$K_p = I_m \tag{24}$$

with the added advantage that the controller $K = K_p K_c$ will consist of m separate loop controllers. In this situation, theorem 1 reduces to previously reported work[4,5] where examples indicate that successful designs can be achieved. More generally, however, it is expected that K_p will not be diagonal. An alternative in this situation is to diagonalize $Y(t)$ at a time of interest (say, the time of peak interaction effects) by choosing

$$K_p = Y^{-1}(t_o) \tag{25}$$

provided the inverse exists. Taking, for example, the case of $t_o = \infty$ yields

$$K_p = Y^{-1}(\infty) = G^{-1}(o) \tag{26}$$

which will remove all steady state interaction in the plant and have the added bonus that $E(\infty) = 0$ if $g_j(o) = 1$, $1 \leq j \leq m$. The use of theorems 3 or 4 will then produce narrow confidence bands in the vicinity of $s = o$.

3.2 The Choice of K_p as an Optimization Problem

In the INA design technique, the choice of precompensator can be formulated as a quadratic optimization problem such as pseudo-diagonalisation[1-3] or more directly as a mathematical programming problem[6]. This is also possible in the context of the results presented in this paper as outlined below.

Consider the situation where the model G_A has been specified and the designer wishes to reduce the width of the confidence bands by updating his choice of K_p. In terms of theorem 1, this could be expressed in the form of the optimization problem

$$\min_{K_p} \max_{1 \leq j \leq m} d_j(\infty) a_j \qquad (27)$$

where $a_j > o$, $1 \leq j \leq m$, are 'weights' to reflect the importance of the loop in closed-loop dynamics. For theorem 2, this problem could be replaced by

$$\min_{K_p} r(\infty) \qquad (28)$$

and, for theorem 6, by the problem

$$\min_{K_p} \max_{1 \leq j \leq m} d'_j(\infty) a_j \qquad (29)$$

Focussing attention on (29) and writing K_p in columns

$$K_p = [K_1, K_2, \ldots, K_m] \qquad (30)$$

it is clear that $d'_j(\infty)$ depends only on K_j and hence (29) can be separated into the equivalent form

$$\min_{K_j} d'_j(\infty) \quad , \quad 1 \leq j \leq m \qquad (31)$$

which substantially reduces the dimension of the problems considered. (Note: Problems (27) and (29) represent minimum norm problems in the product space $(BV[0,T])^{m^2}$ of m^2 dimensional vectors with elements of bounded variation in the data interval $[0,T]$. The minimization is performed over finite-dimensional linear variety defined by (4) and hence[7] it has a solution due to a standard result (theorem 2, p. 121) in Luenberger[8] and the observation that $BV[0,T]$ is the dual space of the space $C(0,T)$ of continuous functions on $[0,T]$).

Efficient numerical procedures for the solution of these problems are under consideration. For the purposes of this paper, we concentrate on the development of a quadratic approximation scheme paralleling that of pseudo-diagonalization in the INA.

Let T be the length of the data sequence available and let

$$0 = t_o < t_1 < \ldots\ldots < t_{N-1} < t_N = T \tag{32}$$

be a partition of $[0,T]$. Let G_A be specified (see Appendix 9 for the case of G_A unspecified) and consider the approximation

$$d_j'(\infty) \equiv \sum_{\ell=1}^{m} N_\infty(E_{\ell j}^p)$$

$$\approx \sum_{\ell=1}^{m} \sum_{k=1}^{N} |E_{\ell j}^p(t_k) - E_{\ell j}^p(t_{k-1})| \tag{33}$$

then it is natural to replace (31) by the discrete approximations

$$\min_{K_j} \sum_{\ell=1}^{m} \sum_{k=1}^{N} |E_{\ell j}^p(t_k) - E_{\ell j}^p(t_{k-1})| \quad , 1 \le j \le m \tag{34}$$

In general, these problems cannot be solved analytically, but analytical procedure can be used if they are relaxed to the <u>quadratic</u> form

$$\min_{K_j} \sum_{\ell=1}^{m} \sum_{k=1}^{N} (E_{\ell j}^p(t_k) - E_{\ell j}^p(t_{k-1}))^2 \ , 1 \le j \le m \tag{35}$$

It is convenient to introduce the notation $\{e_j\}_{1 \le j \le m}$ to denote the natural basis in R^m (i.e. $e_1 = (1\ 0\ .\ .\ .\ 0)^T$, $e_2(0\ 1\ 0\ .\ .\ .0)^T$ etc.) and define the operations

$$\Delta_k M = M(t_k) - M(t_{k-1}) \quad , \quad 1 \le k \le N \tag{36}$$

for any matrix function M(t) on $[0,T]$. The objective functional in (35) now takes the form

$$\sum_{\ell} \sum_{k} (\Delta_k E_{\ell j}^p)^2 = \sum_{\ell} \sum_{k} (e_\ell^T(\Delta_k Y)K_j - e_\ell^T(\Delta_k Y_A)e_j)^2$$

$$= K_j^T Q K_j - 2 b_j^T K_j + c_j \tag{37}$$

where

$$Q = \sum_{\ell} \sum_{k} (\Delta_k Y)^T e_{\ell} e_{\ell}^T (\Delta_k Y)$$

$$= \sum_{k} (\Delta_k Y)^T (\Delta_k Y) \qquad (38)$$

as $I_m = \sum_{\ell} e_{\ell} e_{\ell}^T$,

$$b_j^T = e_j^T B^T \qquad (39)$$

with

$$B \triangleq \sum_{\ell} \sum_{k} (\Delta_k Y)^T e_{\ell} e_{\ell}^T (\Delta_k Y_A)$$

$$= \sum_{k} (\Delta_k Y)^T (\Delta_k Y_A) \qquad (40)$$

and

$$c_j = \sum_{\ell} \sum_{k} e_j^T (\Delta_k Y_A)^T e_{\ell} e_{\ell}^T (\Delta_k Y_A) e_j$$

$$= e_j^T C e_j \qquad (41)$$

with

$$C \triangleq \sum_{k} (\Delta_k Y_A)^T (\Delta_k Y_A) \qquad (42)$$

A minimizing column K_j^* then satisfies the equation

$$QK_j^* = B e_j \quad , \quad 1 \leq j \leq m \qquad (43)$$

and hence the optimal precompensator K_p^* is given by

$$K_p^* = Q^{-1} B \qquad (44)$$

provided that the inverse exists. This will normally be the case as clearly $Q = Q^T \geq 0$. In fact $Q > 0$ in general, as $x^T Q x = 0$ is equivalent to the condition

$$\begin{pmatrix} \Delta_1 Y \\ \Delta_2 Y \\ \cdot \\ \cdot \\ \cdot \\ \Delta_N Y \end{pmatrix} x = o \tag{45}$$

which implies that x = o if, and only if,

$$\text{rank} \begin{pmatrix} \Delta_1 Y \\ \Delta_2 Y \\ \cdot \\ \cdot \\ \cdot \\ \Delta_N Y \end{pmatrix} = m \tag{46}$$

This condition is generic in the sense that it holds for almost all plants and almost all partitions. This can be illustrated by writing (46) in the form

$$\text{rank} \begin{pmatrix} I_m & O & \cdots & & O \\ -I_m & I_m & & & \vdots \\ O & & & & \vdots \\ \vdots & & & & O \\ O & \cdots\ O & & -I_m & I_m \end{pmatrix} \begin{pmatrix} Y(t_1) \\ \cdot \\ \cdot \\ \cdot \\ Y(t_N) \end{pmatrix} = m \tag{47}$$

This reduces to the requirement that

$$\text{rank} \begin{pmatrix} Y(t_1) \\ \cdot \\ \cdot \\ \cdot \\ Y(t_N) \end{pmatrix} = m \tag{48}$$

which holds true for almost all partitions and almost all systems. For example, if $|G(o)| \neq 0$ and T is larger than the dominant time constants of the system then $G(o) - Y(t_N) = Y(\infty) - Y(T)$ is small indicating that $|Y(t_N)| \neq 0$.

4. Illustrative Example

Consider the boiler-furnace in Rosenbrock[1] with 4x4 transfer function matrix

$$G(s) = \begin{bmatrix} \frac{1.0}{1+4s} & \frac{0.7}{1+5s} & \frac{0.3}{1+5s} & \frac{0.2}{1+5s} \\ \frac{0.6}{1+5s} & \frac{1.0}{1+4s} & \frac{0.4}{1+5s} & \frac{0.35}{1+5s} \\ \frac{0.35}{1+5s} & \frac{0.4}{1+5s} & \frac{1.0}{1+4s} & \frac{0.6}{1+5s} \\ \frac{0.2}{1+5s} & \frac{0.3}{1+5s} & \frac{0.7}{1+5s} & \frac{1.0}{1+4s} \end{bmatrix}$$

$$\equiv \frac{1}{1+4s} I_4 + \frac{1}{1+5s} (G(o) - I_4) \tag{49}$$

Considering initially the simple possibility of loop controllers only, choose $K_p = I_4$ and the diagonal compensator $K_c(s) = k\, I_4$ with identical proportional gain $k > o$ in all loops to reflect the identical diagonal plant dynamics. Choosing the model $G_A(s) = I_4/(1+4s)$ then G_A is stabilized for all $k > o$ and noting that the interaction effects in all loops of $G K_p$ are monotonic leads to the total variation

$$N_\infty^P(E^P) = G(o) - I_m \tag{50}$$

and hence the 'column errors' (see theorem 6)

$$d_1'(\infty) = d_4'(\infty) = 1.15 \;,\; d_2'(\infty) = d_3'(\infty) = 1.4 \tag{51}$$

A preliminary estimate of the permissible gain range can now be obtained from (5) i.e.

$$k < \min_j \left(\frac{1}{d_j'(\infty)}\right) = 0.71 \tag{52}$$

Simulation studies indicate closed-loop stability in this range but with sluggish response characteristics and large steady state errors. Consideration is therefore given to the use of a precompensator. (Note: the above result should be compared with that in Rosenbrock[1] where it is shown using the INA that the controller stabilizes the system for all $k > o$. The techniques on this paper are therefore more conservative than the INA. This is to be expected however, as the total variation represents and needs only minimal plant information. The conservatism can be removed considerably by the use of the precompensator described below in this case. Conservatism is still, of course, present but this problem does not prevent the achievement of a practical design and the techniques have the advantage of eliminating the need to evaluate the inverse $\hat{G}$ of the plant and requiring little information on plant detailed dynamics. The design is also robust in the sense defined in Owens and Chotai[4].)

Following Rosenbrock[1] consider the use of the steady state precompensator

$$K_p = G^{-1}(o) = Y^{-1}(\infty) = \begin{pmatrix} 1.75 & -1.21 & -0.16 & 0.17 \\ -0.98 & 1.87 & -0.23 & -0.32 \\ -0.32 & -0.23 & 1.87 & -0.98 \\ 0.17 & -0.16 & -1.21 & 1.75 \end{pmatrix} \quad (53)$$

leading to the compensated plant

$$G(s)\ K_p = \frac{1}{1+4s} G^{-1}(o) + \frac{1}{1+5s} \left(I_4 - G^{-1}(o)\right) \quad (54)$$

with step response matrix

$$Y^p(t) = (1 - e^{-t/4})G^{-1}(o) + (1 - e^{-t/5})(I_4 - G^{-1}(o)) \quad (55)$$

Choosing $G_A(s)$ to model the diagonal terms to high accuracy leads to

$$g_1(s) = g_4(s) = \frac{1 + 5.75s}{(1+4s)(1+5s)}, \quad g_2(s) = g_3(s) = \frac{1 + 5.87s}{(1+4s)(1+5s)} \quad (56)$$

with total variation of the error equal to

$$N_\infty^P(E^P) = \begin{bmatrix} 0 & 1.21 & 0.16 & 0.17 \\ 0.98 & 0 & 0.23 & 0.32 \\ 0.32 & 0.23 & 0 & 0.98 \\ 0.17 & 0.16 & 1.21 & 0 \end{bmatrix} N_\infty(e^{-t}/5 - e^{-t}/4) \quad (57)$$

where $N_\infty(e^{-t}/5 - e^{-t}/4) = 0.164$. The corresponding column sums are

$$d_1'(\infty) = d_4'(\infty) = 0.24 \quad , \quad d_2'(\infty) = d_3'(\infty) = 0.26 \quad (58)$$

Noting the dynamic similarity of all the $g_i(s)$ suggests the use of the compensator $K_c(s) = \text{diag}\,\{k_i(s)\}_{1\leq i\leq 4}$ with diagonal term $k_i(s) = k_1^{(i)} + s^{-1}k_2^{(i)}$, $i = 1,4$. The choice of $k_i(s)$ is now based on the stabilization and performance of the approximating feedback scheme. At this stage, it is useful to use condition (5) (with row sums replaced by column sums) to obtain an initial estimate of the gains allowed i.e.

$$|k_1^{(i)}| < \frac{1}{d_i'(\infty)} = \begin{cases} 4.1 & , \quad i=1 \quad , i=4 \\ 3.8 & , \quad i=2 \quad , i=3 \end{cases} \quad (59)$$

The approximate model G_A indicates that these gains allow an increase in response speed of up to six times open-loop response speeds which is normally more than necessary for process control applications.

Choosing the networks

$$k_i(s) = 3 + \frac{1}{2s} \quad , \quad i = 1,4 \quad (60)$$

to produce closed-loop time-constants of approximately 1.0 and reset times of approximately 6.0 for the approximating feedback systems, then the condition (59) is satisfied and the stability of the real plant G with the resulting control scheme is checked by checking condition (iv) of theorem 1 with (theorem 6) row sums replaced by column sums.

The resulting inverse Nyquist plots with confidence bands for the first two loops are given in Fig. 2. The plots for loops 3 and 4 are obtained by symmetry by interchanging. Note that the (-1,0) point does not lie in or on the confidence band and hence the controller successfully stabilizes the plant. The resulting closed-loop performance is illustrated in Fig. 3 indicating excellent loop performance with low interaction effects.

Although the design is successful without the low frequency modifications described in theorems 3 and 6, confidence in the design can be increased by calculating the integrated modelling error $z^p(t)$ with elements (in analytic form)

$$(z^p(t))_{ij} = \begin{cases} 0 , & i = j \\ (G^{-1}(o))_{ij}(1 - 5e^{-t/5} + 4\, e^{-t/4}) , & i \neq j \end{cases} \tag{61}$$

and $E(\infty) = 0$. The corresponding total variation is given by

$$N^p_\infty(z^p) = \begin{bmatrix} 0 & 1.21 & 0.16 & 0.17 \\ 0.98 & 0 & 0.23 & 0.32 \\ 0.32 & 0.23 & 0 & 0.98 \\ 0.17 & 0.16 & 1.21 & 0 \end{bmatrix} \tag{62}$$

Using column sums defined by (25) then yields the inverse Nyquist plots with confidence bands indicated in Fig. 4. Note the decrease in the width of the confidence bands at low frequencies by comparison with the plots in Fig. 2.

5. Notes on the Use of Derivative Information

Consider the following result:

Lemma 2: If the derivative $\dot{E}^p$ is continuous on $[0, +\infty)$ and

$$V^p(t) \triangleq \frac{dE^p(t)}{dt} \tag{63}$$

then, for Re $s \geq 0$, $1 \leq i,j \leq m$, we have

$$|(G(s)K_p)_{ij} - (G_A(s))_{ij}| \leq \tilde{d}_{ij}(s) \triangleq \min\{d_{ij}(s), |s|^{-1}N_\infty(V^p_{ij})\} \tag{64}$$

Proof: The result follows from lemma 1 and the inequality

$$|s((G(s)K_p)_{ij} - (G_A(s))_{ij})| = |\dot{E}^p_{ij}(0+) + \int_0^\infty e^{-st}\ddot{E}^p_{ij}(t)\,dt|$$

$$\leq |\dot{E}^p_{ij}(0+)| + \int_0^\infty |\ddot{E}^p_{ij}(t)|dt = N_\infty(V^p_{ij}) \tag{65}$$

from lemma 2 in ref (4) and the identity

$$\dot{E}^p_{ij}(t) = \dot{E}^p_{ij}(0+) + \int_0^t \ddot{E}^p_{ij}(t')dt' \tag{66}$$

Similar consideration to the proofs of theorems 3 and 6 indicate that

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Theorem 7: The conclusion of theorems 3 - 6 hold true with the replacements, respectively,

$$d_j(s) \rightarrow \tilde{d}_j(s) \triangleq \sum_{\ell=1}^{m} \tilde{d}_{j\ell}(s) \tag{67}$$

$$r(s) \rightarrow \tilde{r}(s) \triangleq r(\tilde{D}(s)) \tag{68}$$

where $\tilde{D}(s)$ is the matrix with $(i,j)^{th}$ element $\tilde{d}_{ij}(s)$ and

$$d'_j(\infty) \to 0 \tag{69}$$

$$d'_j(s) \to \tilde{d}'_j(s) = \sum_{\ell=1}^{m} \tilde{d}_{\ell j}(s) \tag{70}$$

<u>Proof</u>: the result is proved in a similar way to theorem 3,4 and 6 with $\Delta(s) = \tilde{D}(s)$ as an upper bound on the modelling error $GK_p - G_A$. The details are omitted except to note that (64) indicates that

$$\lim_{|s|\to\infty} \tilde{d}_{ij}(s) = 0 \quad , \quad 1 \leq i,j \leq m \tag{71}$$

and hence all error estimates tend to zero at high frequency.

The application of the result is identical to those described in Section 3 with the bonus that (see (71)) the radii of the confidence circles at <u>high</u> frequencies are reduced considerably and that condition (5) of theorem 1 is trivially satisfied. The main problem in practice is that derivative measurements tend to be unreliable due to noise on plant data. It is expected therefore that the result will only be of practical use when a <u>detailed</u> model of the plant is available to obtain Y(t).

In the example of section 4, the plant model is known and hence V^p can be computedd to give

$$V^p_{ij}(t) \equiv \begin{cases} 0 & , \quad i = j \\ (G^{-1}(0))_{ij} \left(\frac{1}{4} e^{-t/4} - \frac{1}{5} e^{-t/5}\right) & \end{cases} \tag{72}$$

with total variation easily evaluted to be

$$N^p_\infty(V^p) = \begin{pmatrix} 0 & 1.21 & 0.16 & 0.17 \\ 0.98 & 0 & 0.23 & 0.32 \\ 0.32 & 0.23 & 0 & 0.98 \\ 0.17 & 0.16 & 1.21 & 0 \end{pmatrix} \times 0.114 \tag{73}$$

The confidence bands for the example of section 4 are shown in Fig. 5. Note the substantial reduction in the radii of the confidence circles at high frequencies as proved by the identity

$$\lim_{|s|\to\infty} |g_i^{-1}(s)|\tilde{d}_i'(s) = \begin{cases} 0.58 & , \quad i=1,\ i=4 \\ 0.62 & , \quad i=2,\ i=3 \end{cases} \tag{74}$$

as compared with

$$\lim_{|s|\to\infty} |g_i^{-1}(s)|d_i'(s) = +\infty \tag{75}$$

Finally, it has been noted that the use of bounds on the modelling error such as (64) are easily obtained but tend to be conservative when compared with the bounds obtained from a detailed plant model. This is illustrated in Fig. 6 for a typical element of $G - G_A$ where it is seen that the gain characteristics are overestimated by up to 150%.

6. A Note on Dynamic Precompensation and Postcompensation

As in the INA, the use of constant precompensation can, in severe cases, be insufficient to reduce interactions in Y_p to the desired level. One way out of this problem is to use both constant pre-and postcompensation as illustrated in Fig. 7. The 'plant seen by the compensator' is then $K_p^y\ G\ K_p^u$ which has step response matrix

$$Y_p(t) = K_p^y\ Y(t)\ K_p^u \tag{76}$$

If the model G_A is chosen to reflect the diagonal dynamics of $Y_p(t)$, then it is easily verified that the techniques of section 2 carry through with no change. For implementation purposes, the post-compensator K_p^y can be 'passed around the loop' to form the forward path controller

$$K(s) = K_p^u\ K_c(s)\ F\ K_p^y\ F^{-1} \tag{77}$$

provided that F(s) is non-dynamic and nonsingular.

The potential power of this idea can be illustrated by the example of section 4 by choosing $K_p^u = T$ and $K_p^y = T^{-1}$ where T is an eigenvector matrix of the matrix $G(o) - I_4$. Equation (49) immediately reveals that $K_p^y \; G \; K_p^u$ and hence Y_p is exactly diagonal (G is, in fact, a dyadic transfer function matrix, as defined by Owens[2]).

Dynamic precompensation can also be incorporated in this theoretical framework. Let $K_p(s)$ represent a dynamic precompensator, then the previous theory still holds with step response matrix of GK_p represented by the convolution

$$Y_p(t) = \int_o^t Y(t-t')\, H_p(t')dt' \tag{78}$$

where $H_p(t')$ is the impulse response matrix of the chosen $K_p(s)$. The use of this relation in the choice of K_p is under consideration but in practice it will probably reduce to the application of standard INA methodology to obtain K_p by reducing interaction effects in a non-diagonal but still approximate model G_I of G. The 'intermediate model' G_I could be obtained by fitting models to all elements of Y but its sole purpose would be to attempt to reduce interaction in Y_p. Controller design can still proceed based on a diagonal model G_A of Y_p.

7. Conclusions

The paper has demonstrated that recent results due to the authors[4] can be used to construct a frequency-domain design technique for multivariable plant that requires and uses only easily obtained graphical information from plant step response characteristics. The technique is an exact parallel of the well-established INA technique for multivariable feedback design including the possibility of using constant precompensators to reduce interaction and a stability check based upon the inverse Nyquist loci of an approximate model of desired complexity with a 'confidence band' replacing the well-known Gershgorin band of the INA. It has been demonstrated that the resultant design has well-defined integrity characteristics but it has not been found possible to obtain a generalization of the Ostrowski band for assessment of closed-loop performance. Closed-loop performance can only be assessed by producing the required characteristics from the approximating diagonal model and, perhaps, assessing performance degradation using the time-domain results described in Owens and Chotai[4].

The confidence band is obtained by a number of possible techniques paralleling the concepts of row-dominance and column-dominance of the INA. The simplest technique simply requires the graphical evaluation of the total variation of the error is modelling the open-loop compensated plant step response by a diagonal model. This, however, can lead to the preclusion of integral control but a modified technique based on the total variation of the integral of the modelling error has been shown to remove this problem. Derivative information can also be used to improve the results if it can be obtained reliably.

The techniques have the advantages that detailed plant model structure and data is not required during the design, inversion of plant transfer function matrices is not necessary and the approximate model used can have any desired complexity. It can therefore be used directly on plant step data in a similar manner to Owens and Chotai[4] and Åström[9]. It does however, tend to lead to more conservative designs than the INA as only minimal plant information is required and used. This problem must be offset againstthe above mentioned advantages in any given application.

The techniques have conceptual relationships with the work of Åström[9] Davison[10] and the recent work of Lünze[11] and Nwokah[12] in that they enable successful multivariable design to be achieved in the presence of model uncertainty. The strongest links are however, with the INA and other techniques in the UK frequency-domain school.

Finally, the choice of precompensator is open to the designer to reduce the interaction in the open-loop compensated plant step response. The paper has indicated that it can be chosen on intuitive bases such as diagonalization in the steady state but its choice can also be posed as an optimization problem. This will form the basis of further studies but the paper has indicated that an analytic quadratic approximation technique paralleling the well-known methods of pseudo-diagonalization is possible and could merit further study.

7. Acknowledgements

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9. Appendix

If, during the choice of K_p, G_A is as yet unspecified then $E^p_{jj}(t)$ is unknown, $1 \leq j \leq m$, and the designer can only achieve a reduction in the off-diagonal terms of the error. The errors in the diagonal terms can be made arbitrarily small by choosing G_A by subsequent inspection of the diagonal terms of Y_p. The quadratic problem corresponding to (35) in this case can be written as

$$\min_{K_j} \sum_{\substack{\ell=1 \\ \ell \neq j}}^{m} \sum_{k=1}^{N} p_{\ell j}(t_k) - E^p_{\ell j}(t_{k-1}))^2$$

$$= \min_{K_j} \sum_{\substack{\ell=1 \\ \ell \neq j}}^{m} \sum_{k=1}^{N} (e_\ell^T \Delta_k \; Y \; K_j)^2$$

$$= \min_{K_j} \; K_j^T \, Q_j \, K_j \tag{79}$$

where

$$Q_j = \sum_{\substack{\ell=1 \\ \ell \neq j}}^{m} \sum_{k=1}^{N} (\Delta_k Y)^T \, e_\ell \, e_\ell^T (\Delta_k Y) \tag{80}$$

To avoid the trivial solution $K_j = 0$, an additional constraint

$$K_j^T \, R \, K_j = 1 \tag{81}$$

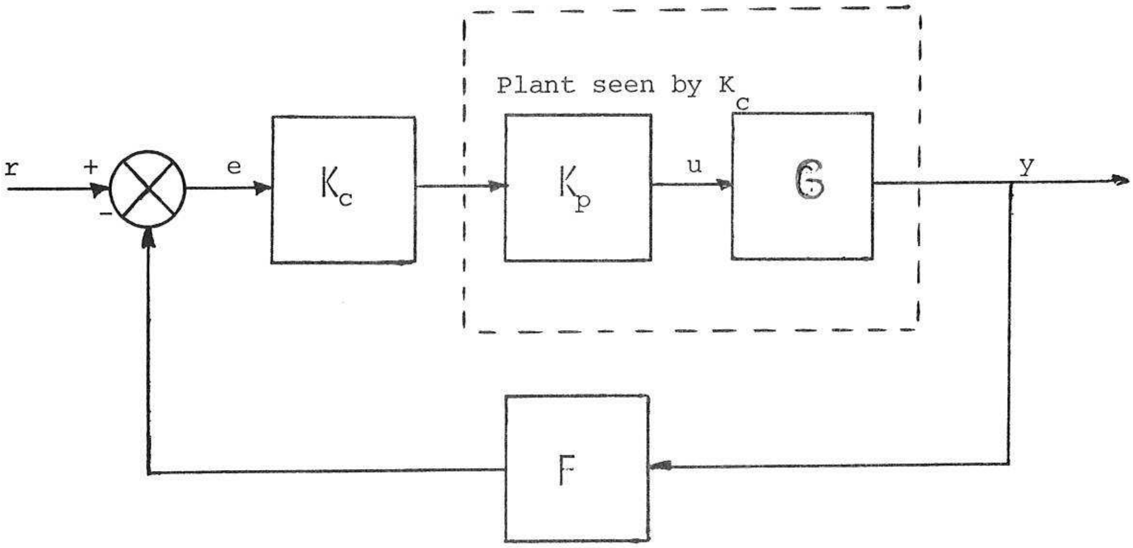
can be introduced where $R = R^T > 0$. Elementary analyses of (79) and (81) indicates that

$$Q \, K_j^* = \lambda \, R \, K_j^* \tag{82}$$

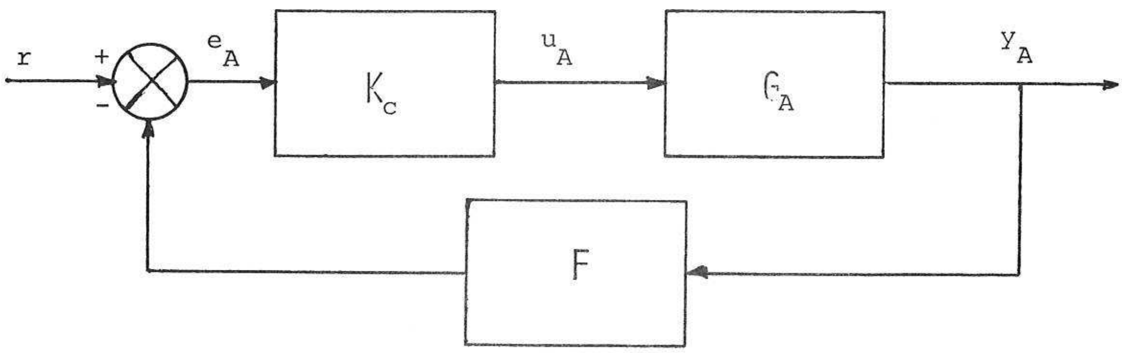
for some scalar (Lagrange multiplier) λ and

$$\min_{K_j} \; K_j^T \, Q \, K_j = \lambda \tag{83}$$

That is, K_j^* is an eigenvector of $R^{-1}Q$ corresponding to an eigenvalue of smallest magnitude.



(a)



(b)

Fig. 1 (a) Feedback scheme with precompensation

(b) Approximating non-interacting scheme

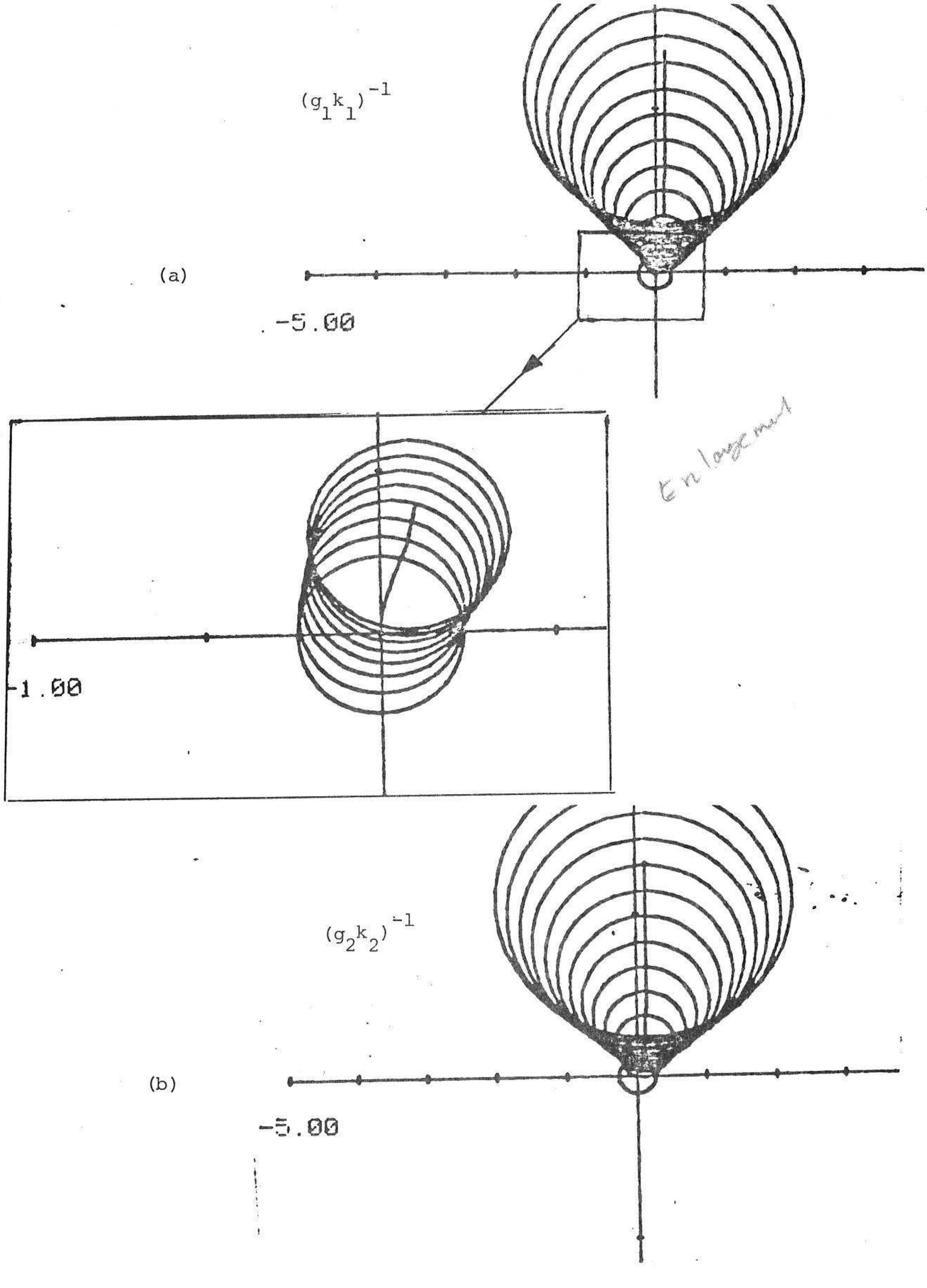


Fig. 2 Inverse Nyquist plots with confidence bands

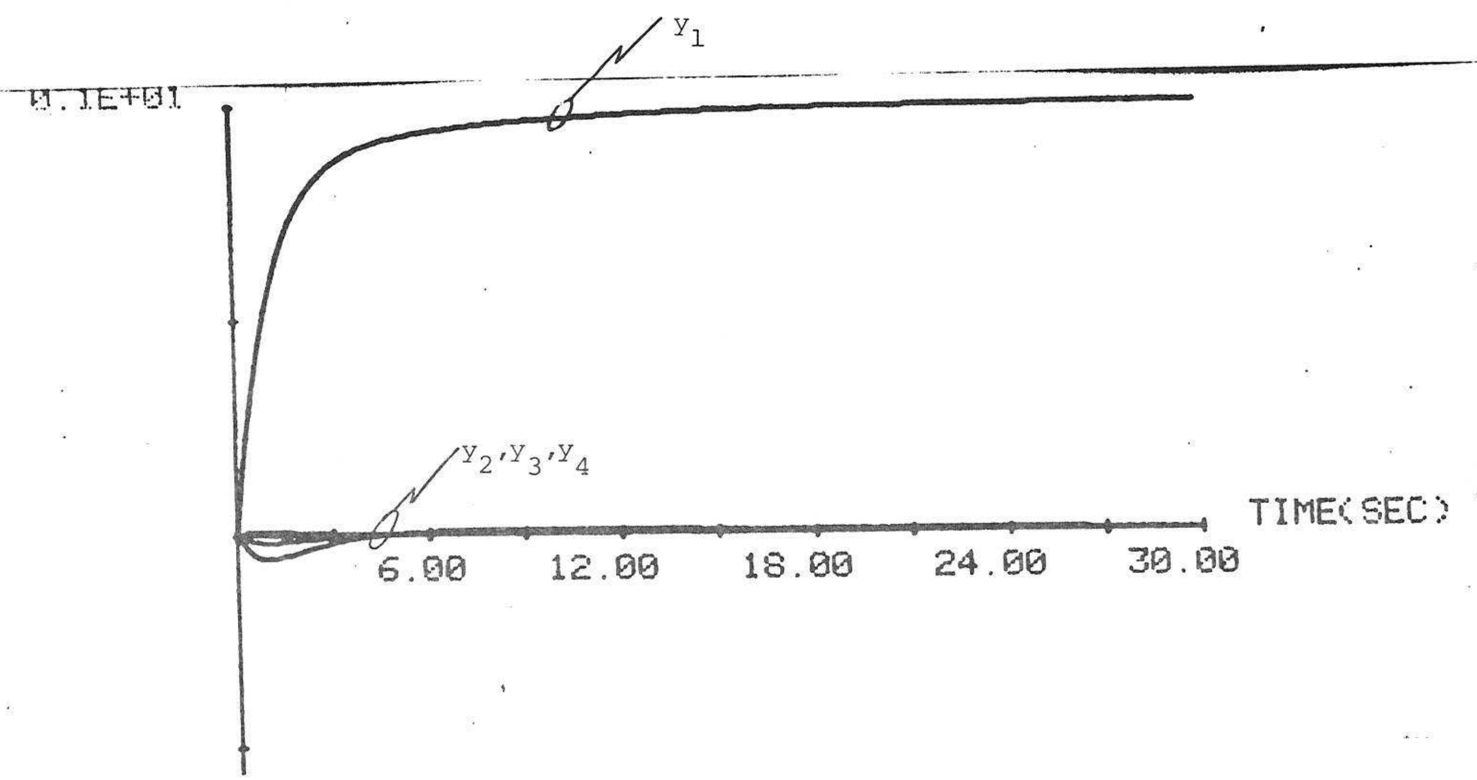
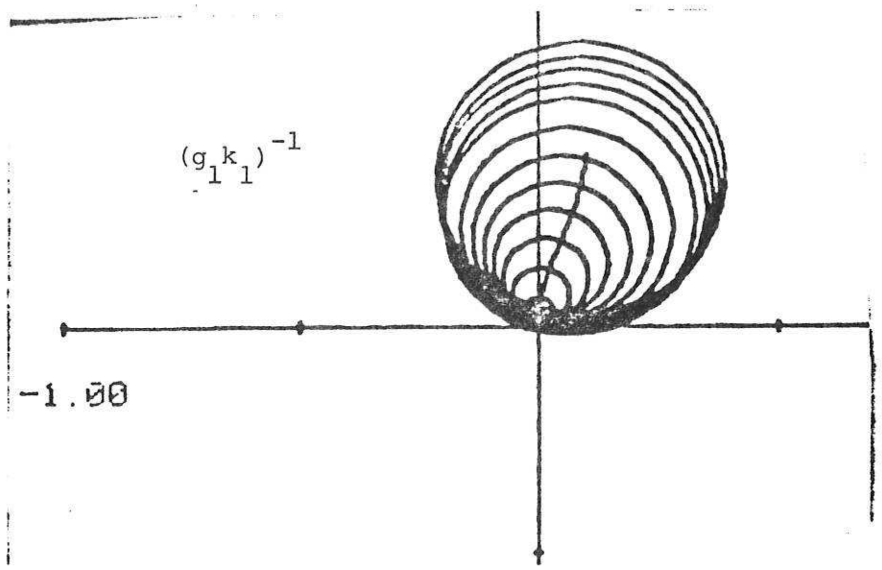


Fig. 3 Closed-loop responses to a unit step demand in y_1

(a)



(b)

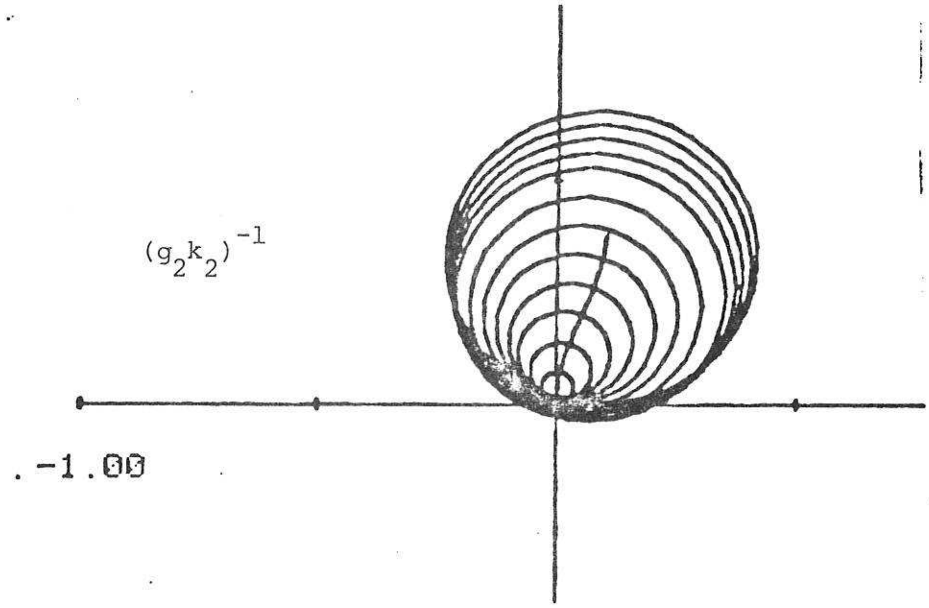


Fig. 4 Low frequency confidence bands with integrated error data

(a)

(b)

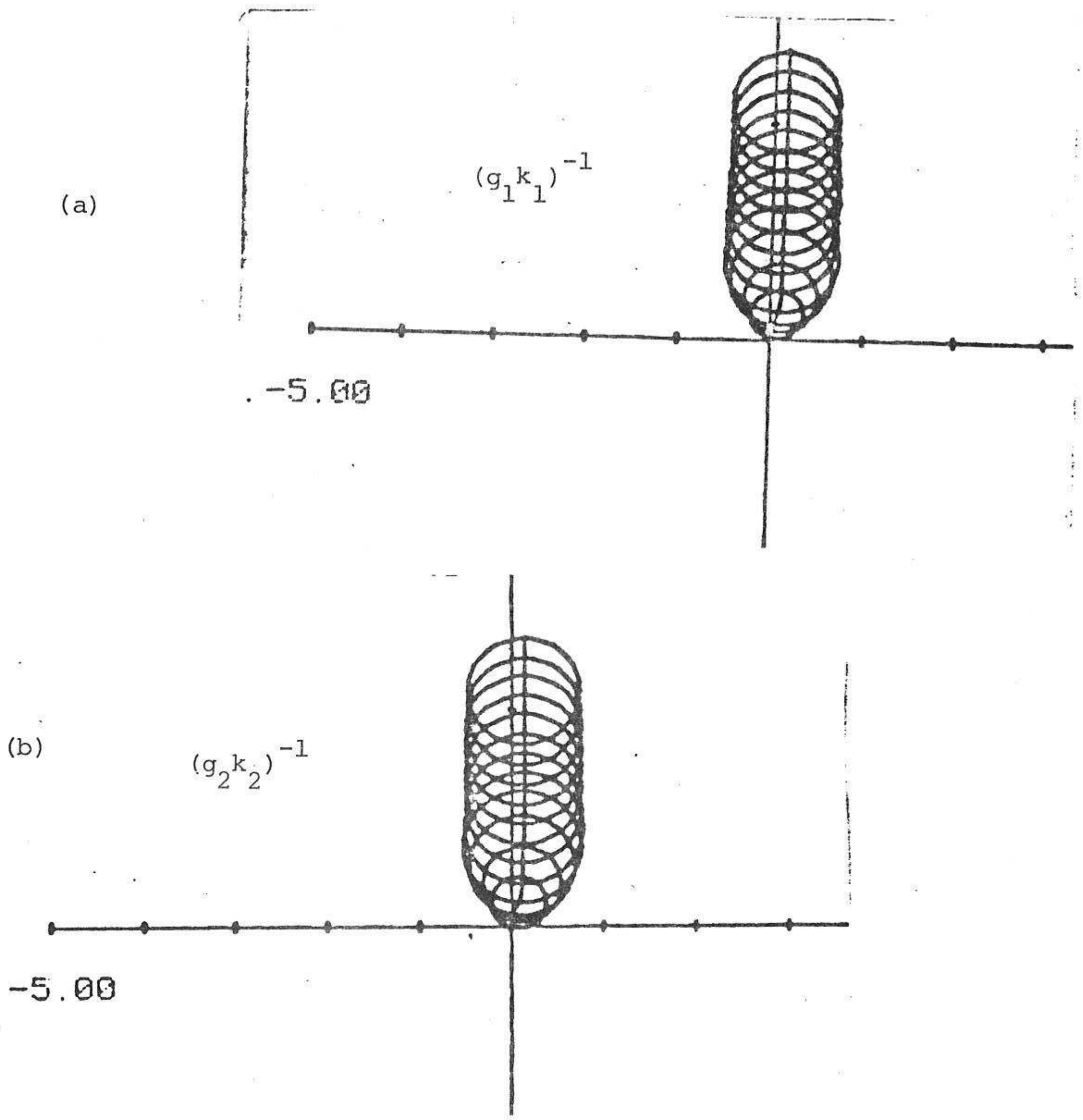


Fig. 5 Confidence bands using derivative data

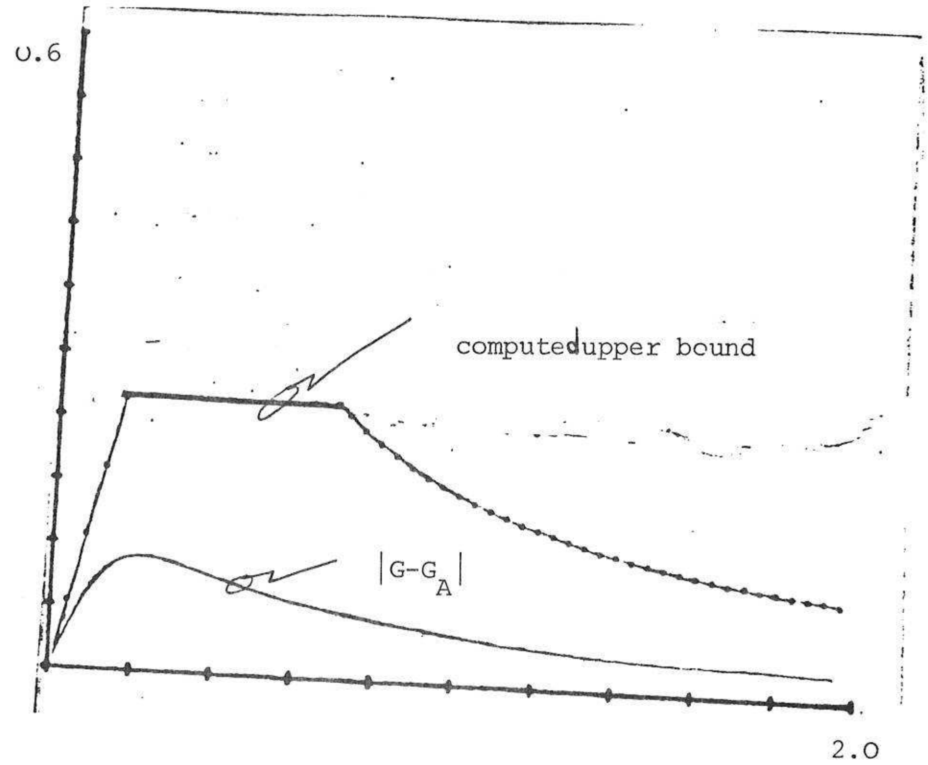


Fig. 6 The gain error and its computed upper bound

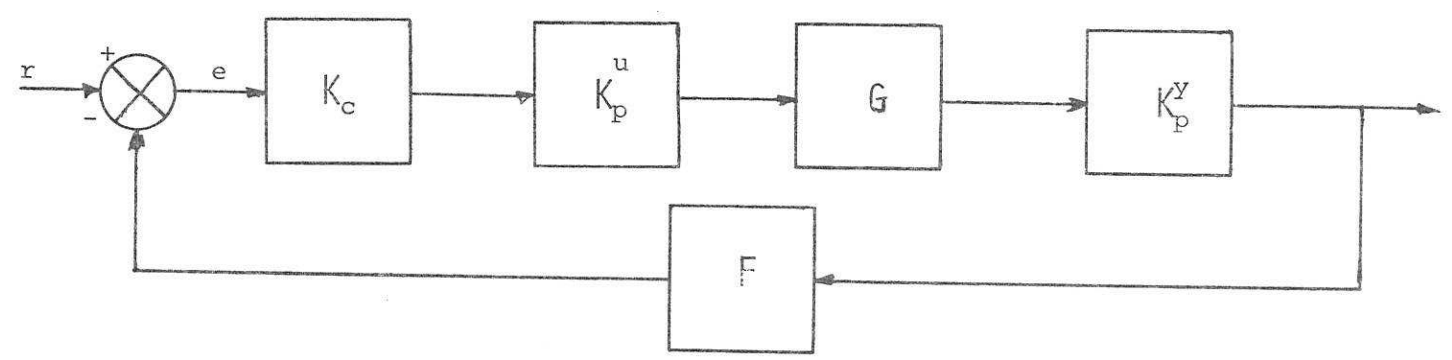


Fig. 7 Feedback with pre- and post-compensation